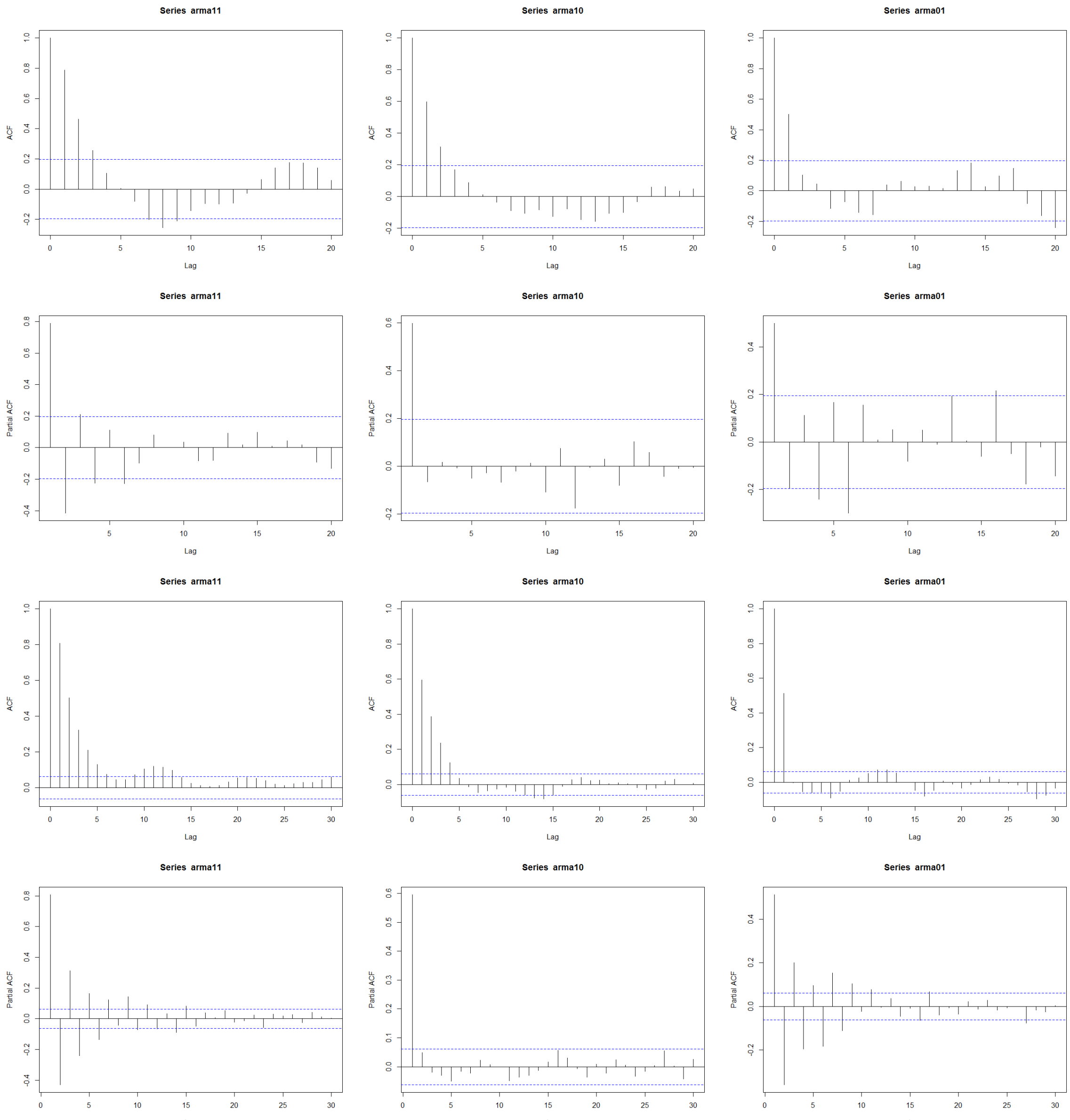
3.9 The plots is as follows. The plot is not so good because of white noise. If we sample more data, like $n = 1000$, which is shown below, it fits Table 3.1 better.




```
#Problem 3.9

n = 1000
arma11 = arima.sim(n = n, list(ar = 0.6, ma = 0.9))
arma10 = arima.sim(n = n, list(ar = 0.6))
arma01 = arima.sim(n = n, list(ma = 0.9))

acf(arma11); acf(arma10); acf(arma01)

pacf(arma11); pacf(arma10); pacf(arma01)
```

3.12

$\forall h > 0$, by Cauchy Inequality,

$$
\begin{aligned}
|\gamma(h)| &= |\mathbb{E}\left[(x_0 - \mu)(x_h - \mu)\right]| \\
&\leq \sqrt{\mathbb{E}\left[(x_0 - \mu)^2\right]\mathbb{E}\left[(x_h - \mu)^2\right]} \\
&= \gamma(0)
\end{aligned}
$$

The equation holds iff the distribution of x_0 and x_h are equal. Since the pick of x_0 is alternative, if $\gamma(h) = \gamma(0)$, we can prove that for any positive integer k, $x_0 = x_{kh}$, thus,

$$\gamma(kh) = \gamma(0) > 0$$

Let $k \to \infty$, it is contradict to $\gamma(h) \to 0$. Thus, $\forall h > 0, |\gamma(h)| < \gamma(0)$. It means that Γ is a strictly diagonally dominant matrix. So $|\Gamma| \neq 0$. By the proof of problem 1.25, the sample autocovariance is a non-negative definite funcition. So Γ is positive semidefinite. Since $|\Gamma| \neq 0$, Γ is positive definite.

3.14 (a)

$$
\begin{aligned}
\underset{g(x)}{\arg\min}\,\mathrm{MSE} &= \underset{g(x)}{\arg\min} \int \left[\int (y - g(x))^2 f(y|x)\,\mathrm{d}\,y\right] f(x)\,\mathrm{d}\,x \\
&= \underset{g(x)}{\arg\min} \int (y - g(x))^2 f(y|x)\,\mathrm{d}\,y \\
&= \underset{g(x)}{\arg\min} \left(\mathbb{E}\left[y^2|x\right] - 2\mathbb{E}\left[yg(x)|x\right] + \mathbb{E}\left[g^2(x)\right]\right) \\
&= \mathbb{E}\left[y|x\right]
\end{aligned}
$$

(b)

$$
\begin{aligned}
\mathrm{MSE} &= \mathbb{E}\left[(x^2 + z - \mathbb{E}\left[x^2 + z|x\right])^2\right] \\
&= \mathbb{E}\left[z^2\right] \\
&= 1
\end{aligned}
$$

(c)Apply $g(x) = a + bx$ to the function of MSE,

$$
\begin{aligned}
\mathrm{MSE} &= \int \left[\int (y - a - bx)^2 f(y|x)\,\mathrm{d}\,y\right] f(x)\,\mathrm{d}\,x \\
&= \mathbb{E}\left[\mathbb{E}\left[(y - bx)^2|x\right] - 2a\mathbb{E}\left[y - bx|x\right] + a^2\right] \\
&= \mathbb{E}\left[(y - bx)^2\right] - 2a\mathbb{E}\left[y - bx\right] + a^2 \\
\underset{a}{\arg\min}\,\mathrm{MSE} &= \mathbb{E}\left[y - bx\right] = 1
\end{aligned}
$$

thus,

$$
\begin{aligned}
\text{MSE} &= \int \left[\int (y - 1 - bx)^2 f(y|x) \, \mathrm{d}\, y \right] f(x) \, \mathrm{d}\, x \\
&= \mathbb{E} \left[\mathbb{E} \left[(y-1)^2 | x \right] - 2b\mathbb{E} \left[xy - x | x \right] + b^2 x^2 \right] \\
&= b^2 \mathbb{E} \left[x^2 \right] - 2b\mathbb{E} \left[xy - x \right] + \mathbb{E} \left[(y-1)^2 \right] \\
\arg\min_b &= \frac{\mathbb{E} \left[xy - x \right]}{\mathbb{E} \left[x^2 \right]} \\
&= \frac{\mathbb{E} \left[xy \right]}{\mathbb{E} \left[x^2 \right]} = 0 \\
\text{MSE} &= \int \left[\int (y-1)^2 f(y|x) \, \mathrm{d}\, y \right] f(x) \, \mathrm{d}\, x \\
&= \mathbb{E} \left[(x^2 + z - 1)^2 \right] \\
&= \mathbb{E} \left[x^4 + z^2 + 1 - 2x^2 \right] \\
&= 3 + 1 + 1 - 2 = 3
\end{aligned}
$$

Since x has zero mean, the slope is zero, and the intercept equals to mean of y. MSE measures the variance of y. In part (b), MSE measures the variance of z