3.10

```
#Problem 3.10(a)
fitCmort = ar.ols(cmort, order.max = 2, demean = F, intercept = T)
#Problem 3.10(b)
predictCmort = predict(fitCmort, n.ahead = 4)
intervals = matrix(c(predictCmort$pred + predictCmort$se * qnorm(0.025),
predictCmort$pred + predictCmort$se * qnorm(0.975)),4,2)
print(intervals)2212212121212
```

The fitted coefficients are $\phi_1 = 0.4286, \phi_2 = 0.4418, \sigma_w^2 = 11.45$. The intervals are (76.45777,98.74196), (74.64117,98.88581), (73.35431,101.31997) and (72.33079,102.09621).

3.15

By induction, the m-step forecast is

$$x_{t+m}^t = \phi^m x_t$$

Thus,

$$\begin{aligned}
\mathbb{E}\left[(x_{t+m} - x_{t+m}^t)^2\right] &= \mathbb{E}\left[\left(\phi^m x_t + \sum_{i=0}^{m-1} \phi^i w_{t+m-i} - \phi^m x_t\right)^2\right] \\
&= \mathrm{var}\left(\sum_{i=0}^{m-1} \phi^i w_{t+m-i}\right) \\
&= \frac{1-\phi^{2t}}{1-\phi^2}\sigma_w^2
\end{aligned}$$

3.16

According to Example 3.7, the model is an ARMA(1,1) model, it can be written as

$$x_t = w_t + 1.4\sum_{j=1}^{\infty} .9^{j-1} w_{t-j}$$

$$x_t = 1.4\sum_{j=1}^{\infty} (-.5)^{j-1} x_{t-j} + w_t$$

thus, by equation (3.92),

$$
\begin{aligned}
\tilde{x}^n_{n+m} &= 0.9\tilde{x}^n_{n+m-1} + 0.5\tilde{w}^n_{n+m-1} \\
&= 0.9\tilde{x}^n_{n+m-1} = 0.9^{m-1}\tilde{x}^n_{n+1} \\
&= 0.9^m x_n + 0.9^{m-1} 0.5\tilde{w}^n_n \\
\tilde{w}^n_n &= x_t - 0.9x_{t-1} - 0.5\tilde{w}^n_{n-1} \\
&= 1.4\sum_{j=0}^{t-1}(-0.5)^j x_{t-j} - 0.4x_t \\
-\sum_{j=1}^{m-1}\pi_j\tilde{x}^n_{n+m-j} &= 1.4\sum_{j=1}^{m-1}(-0.5)^{j-1}\tilde{x}^n_{n+m-j} \\
&= 1.4\sum_{j=1}^{m-1}(-0.5)^{j-1}(0.9^{m-j}x_n + 0.9^{m-j-1}0.5\tilde{w}^n_n) \\
\tilde{x}^n_{n+m} + \sum_{j=1}^{m-1}\pi_j\tilde{x}^n_{n+m-j} &= \tilde{w}^n_n + 0.4x_t \\
&= -\sum_{j=m}^{n+m-1}\pi_j x_{n+m-j}
\end{aligned}
$$

This is equal to equation (3.91).

3.40

Since in AR(p) model,

$$
x_{n+1} = \sum_{j=1}^{p}\phi_j x_{t-j} + w_t
$$

For any $g(x)$ in $\overline{\text{sp}}\{x_k\}$,

$$
\begin{aligned}
\mathbb{E}\left[(x_{n+1} - g(x))^2\right] &= \mathbb{E}\left[\left(x_{n+1} - \sum_{j=1}^{p}\phi_j x_{t-j} + \sum_{j=1}^{p}\phi_j x_{t-j} - g(x)\right)^2\right] \\
&= \mathbb{E}\left[\left(x_{n+1} - \sum_{j=1}^{p}\phi_j x_{t-j}\right)^2\right] + 2\mathbb{E}\left[\left(x_{n+1} - \sum_{j=1}^{p}\phi_j x_{t-j}\right)\left(\sum_{j=1}^{p}\phi_j x_{t-j} - g(x)\right)\right] \\
&+ \mathbb{E}\left[\left(\sum_{j=1}^{p}\phi_j x_{t-j} - g(x)\right)^2\right]
\end{aligned}
$$

since $g(x)$ and $\sum_{j=1}^{p} \phi_j x_{t-j}$ are in $\overline{\text{sp}}\{x_k\}$,

$$\mathbb{E}\left[\left(x_{n+1} - \sum_{j=1}^{p} \phi_j x_{t-j}\right)\left(\sum_{j=1}^{p} \phi_j x_{t-j} - g(x)\right)\right] = \mathbb{E}\left[w_t\left(\sum_{j=1}^{p} \phi_j x_{t-j} - g(x)\right)\right] = 0$$

$$\mathbb{E}\left[\left(\sum_{j=1}^{p} \phi_j x_{t-j} - g(x)\right)^2\right] \geq 0$$

thus,

$$\mathbb{E}\left[(x_{n+1} - g(x))^2\right] \geq \mathbb{E}\left[\left(x_{n+1} - \sum_{j=1}^{p} \phi_j x_{t-j}\right)^2\right]$$

Thus, $\sum_{j=1}^{p} \phi_j x_{t-j}$ is the BLP.