3.28

For IMA(1, 1) model

$$x_t = x_{t-1} + w_t - \lambda w_{t-1}$$

Assume $y_t = w_t - \lambda w_{t-1}$, y_t is invertible, and

$$\begin{aligned} x_t &= x_{t-1} + y_t \\ y_t &= \sum_{j=1}^{\infty} \lambda^j y_{t-j} + w_t \\ x_t - x_{t-1} &= \sum_{j=1}^{\infty} \lambda^j (x_{t-j} - x_{t-j-1}) + w_t \\ x_t &= \sum_{j=1}^{\infty} \lambda^{j-1}(1-\lambda) x_{t-j} + w_t \end{aligned}$$

3.29 (a).

$$(1 - \phi B) y_t = \delta + w_t$$

assume $z_t = y_t - \mu, \mu = \dfrac{\delta}{1-\phi}$,

$$(1 - \phi B) z_t = w_t$$

in AR(1) model, z_{n+j}^n is only related to z_n

$$\begin{aligned} z_{n+j}^n &= \phi^j z_n \\ y_{n+j}^n &= \phi^j y_n + \frac{\delta(1-\phi^j)}{1-\phi} \qquad \text{(i)} \\ &= \phi^j y_n + \delta[1 + \phi + \cdots + \phi^{j-1}] \end{aligned}$$

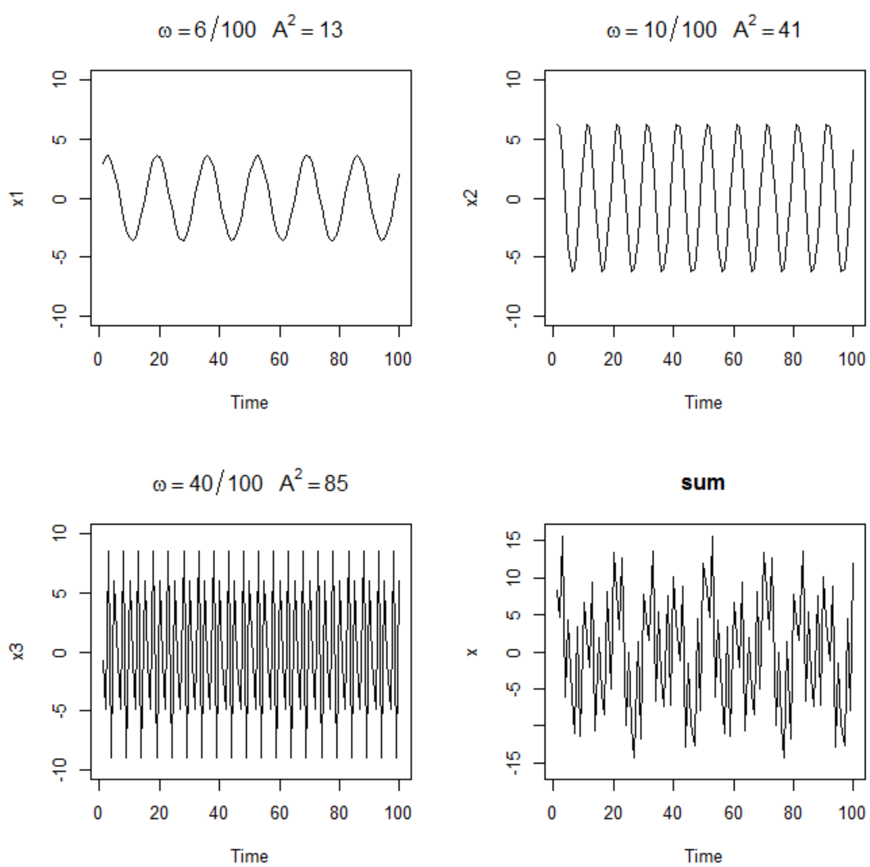
(b).From (i),

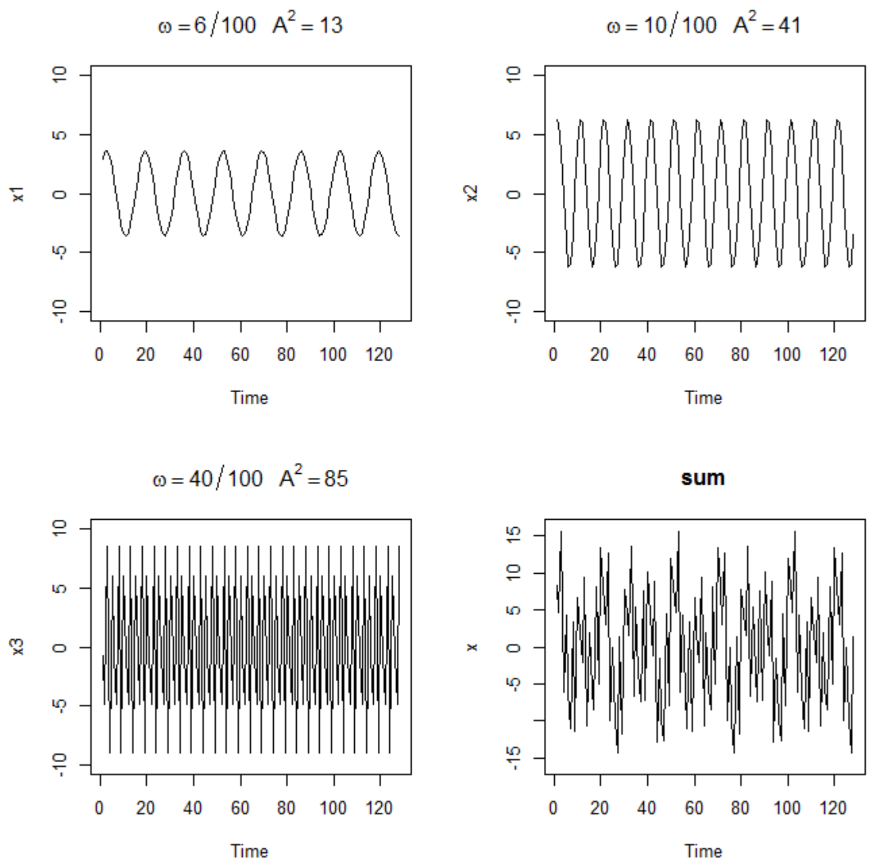
$$\begin{aligned} x_{n+m}^n - x_{n+m-1}^n = y_{n+m}^n &= \phi^j y_n + \frac{\delta(1-\phi^m)}{1-\phi} \\ &= \phi^m (x_n - x_{n-1}) + \frac{\delta(1-\phi^m)}{1-\phi} \\ x_{n+m}^n - x_n &= \sum_{k=1}^{m} \phi^k (x_n - x_{n-1}) + \sum_{k=1}^{m} \frac{\delta(1-\phi^k)}{1-\phi} \\ x_{n+m}^n &= x_n + (x_n - x_{n-1}) \frac{\phi(1-\phi^m)}{1-\phi} + \delta \left[m - \frac{\phi(1-\phi^m)}{1-\phi} \right] \end{aligned}$$

(c)

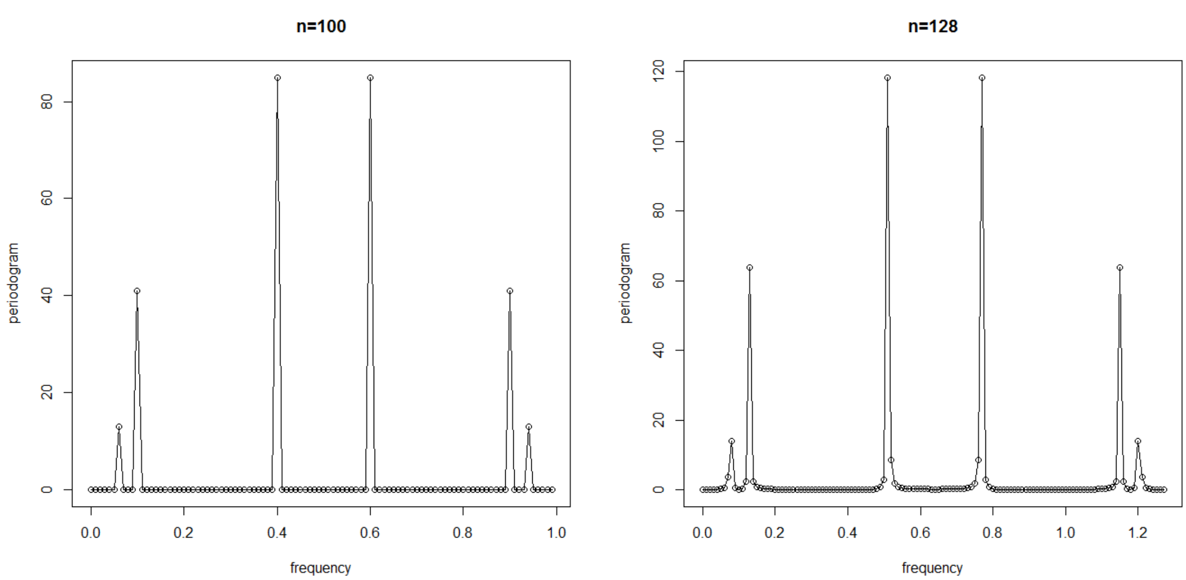
$$
\begin{aligned}
\theta(z) &= 1 - \phi z \\
\phi(z) &= z \\
\psi^*(z) &= \phi(z)/\theta(z)(1-z) \\
&= \frac{z}{(1-z)(1-\phi z)} \\
&= z + (1+\phi)z + \cdots + \sum_{k=1}^{n} \phi^{k-1} z^n \\
P_{n+m}^n &= \sigma_w^2 \sum_{j=0}^{m-1} \psi_j^{*2} \\
&= \sigma_w^2 \sum_{j=0}^{m-1} (\sum_{k=1}^{j} \phi^{k-1})^2 \\
&= \sigma_w^2 \sum_{j=0}^{m-1} \frac{(1-\phi^{j+1})^2}{(1-\phi)^2}
\end{aligned}
$$

4.1 (a).

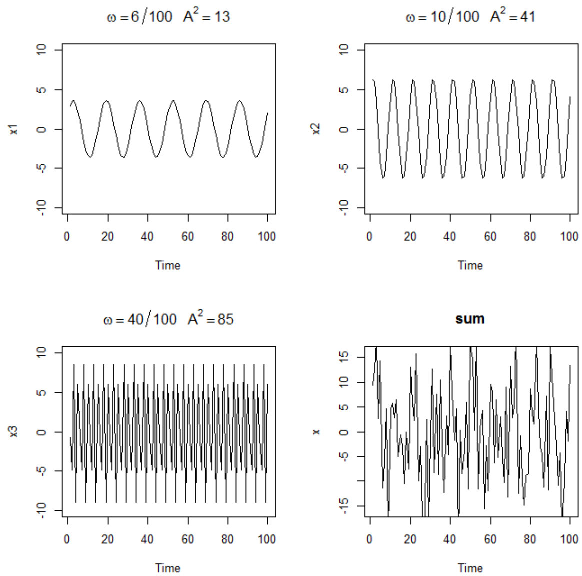


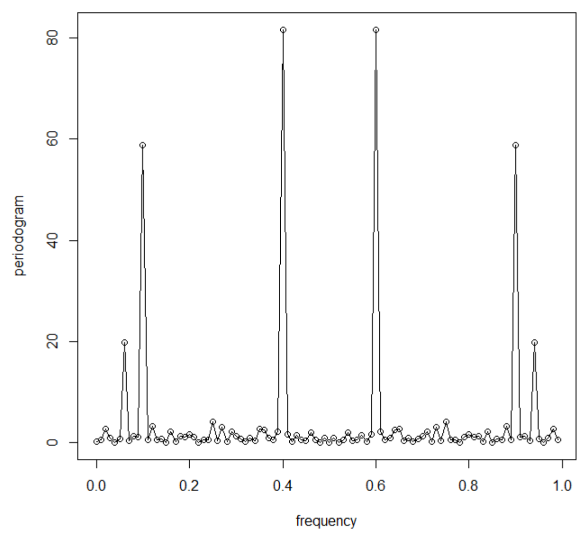


The major difference between these series is that there is two completed peroid in the plot of sum.
(b).



The periodogram becomes larger with the change of n.
(c).





When the noise is added, the peroid of plot of sum is not so clear. In the periodogram plot, periodogram is not always zero in the points outside ω.

```
#Example 4.1
n = 100
x1 = 2*cos(2*pi*1:n*6/100) + 3*sin(2*pi*1:n*6/100)
x2 = 4*cos(2*pi*1:n*10/100) + 5*sin(2*pi*1:n*10/100)
x3 = 6*cos(2*pi*1:n*40/100) + 7*sin(2*pi*1:n*40/100)
x = x1 + x2 + x3 + rnorm(100, sd=5)
par(mfrow=c(2,2))
plot.ts(x1, ylim=c(-10,10), main=expression(omega==6/100~~~A^2==13))
plot.ts(x2, ylim=c(-10,10), main=expression(omega==10/100~~~A^2==41))
plot.ts(x3, ylim=c(-10,10), main=expression(omega==40/100~~~A^2==85))
plot.ts(x, ylim=c(-16,16), main="sum")

#Example 4.2
P = abs(2*fft(x)/100)^2
Fr = 0:(n-1)/100
plot(Fr, P, type='o', xlab="frequency", ylab="periodogram")
```

4.3

$$
\begin{aligned}
\mu_x(t) &= \mathbb{E}\left[\sum_{k=1}^{q}[U_{k1}\cos(2\pi\omega_k t) + U_{k2}\sin(2\pi\omega_k t)]\right] \\
&= 0 \\
\gamma(h) &= \mathbb{E}\left[x_t x_{t+h}\right] \\
&= \mathbb{E}\left[\sum_{k=1}^{q}[U_{k1}\cos(2\pi\omega_k t) + U_{k2}\sin(2\pi\omega_k t)]\sum_{k=1}^{q}[U_{k1}\cos(2\pi\omega_k (t+h)) + U_{k2}\sin(2\pi\omega_k (t+h))]\right] \\
&= \sum_{k=1}^{q}\sum_{l=1}^{q}\mathbb{E}\left[[U_{k1}\cos(2\pi\omega_k t) + U_{k2}\sin(2\pi\omega_k t)][U_{l1}\cos(2\pi\omega_k (t+h)) + U_{l2}\sin(2\pi\omega_k (t+h))]\right] \\
&= \sum_{k=1}^{q}\mathbb{E}\left[[U_{k1}\cos(2\pi\omega_k t) + U_{k2}\sin(2\pi\omega_k t)][U_{k1}\cos(2\pi\omega_k t) + U_{k2}\sin(2\pi\omega_k t)]\right] \\
&= \sum_{k=1}^{q}\sigma_k^2\cos(2\pi\omega_k h)
\end{aligned}
$$