

Problem 1

Assume X_2 is k -dimensional,

$$\begin{aligned} f(X_1|X_2 = x_2) &= \frac{f((X_1, X_2)^\top)}{f(X_2^\top)} \\ &= \frac{\sqrt{(2\pi)^k |\Sigma_{22}|} \exp\left(-\frac{1}{2}((X_1, X_2)^\top - \mu)^\top \Sigma^{-1}((X_1, X_2)^\top - \mu)\right)}{\sqrt{(2\pi)^d |\Sigma|} \exp\left(-\frac{1}{2}(X_2^\top - \mu_2)^\top \Sigma_{22}^{-1}(X_2^\top - \mu_2)\right)} \end{aligned}$$

denote $X'_1 = X_1 - \mu_1, X'_2 = X_2 - \mu_2$,

$$= M \exp\left(-\frac{1}{2} \left[(X'_1, X'_2) \Sigma^{-1} (X'_1, X'_2)^\top - X'_2 \Sigma_{22}^{-1} X'_2{}^\top \right] \right) \quad (M \text{ is a constant.})$$

denote $\Sigma^{-1} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, where A is a $(d-k) \times (d-k)$ matrix.

$$= M \exp\left(-\frac{1}{2} \left[X'_1 A X'_1{}^\top + X'_1 B X'_2{}^\top + X'_2 C X'_1{}^\top + X'_2 D X'_2{}^\top \right] \right)$$

since Σ is symmetric, $B = C^\top$, assume $\mu' = B X'_2$,

$$\begin{aligned} &= M \exp\left(-\frac{1}{2} \left[(X'_1 + \mu') A (X'_1 + \mu')^\top - X'_2 E X'_2{}^\top \right] \right) \quad (E \text{ is a } k \times k \text{ matrix.}) \\ &= M \exp\left(-\frac{1}{2} (X_1 - \mu_1 + \mu') A (X_1 - \mu_1 + \mu')^\top \right) \end{aligned}$$

This is a Multiple Normal Distribution with mean $\mu_1 - \mu'$ and covariance A^{-1} .

Problem 2

```
library(MASS)
x = seq(-5,5,0.02)
ker2 <- function(x,y){
  exp(-(abs(x-y)^2/2))
}
ker1 <- function(x,y){
  exp(-(abs(x-y)/2))
}
covMat <- function(x, kerFun){
  toReturn = matrix(rep(x, length(x)), length(x), length(x))
  apply(toReturn, 1, kerFun, y=x)
}
tshw9Plot <- function(isKer2 = T){
  if (isKer2) {
    kerFun = ker2
    main = 'Kernel with exponential 2'
  } else {
    kerFun = ker1
    main = 'Kernel with exponential 1'
  }
}
```

```

}
y = mvrnorm(5, mu = rep(0, length(x)), Sigma = covMat(x, kerFun))
for (i in 1:5){
  plot(x, y[i,], xlim=c(-5, 5), ylim = c(-5,5), lty = i, ylab='',
main=main, type = 'l')
  par(new = T)
}
}
tshw9Plot(T)
par(new = F)
tshw9Plot(F)

```

The kernel function with exponential 2 is much more smoother.

