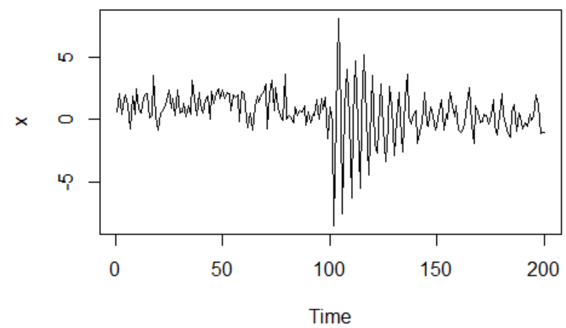
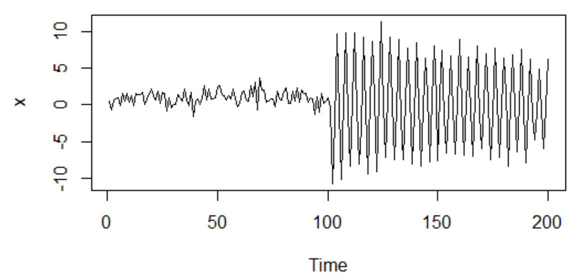
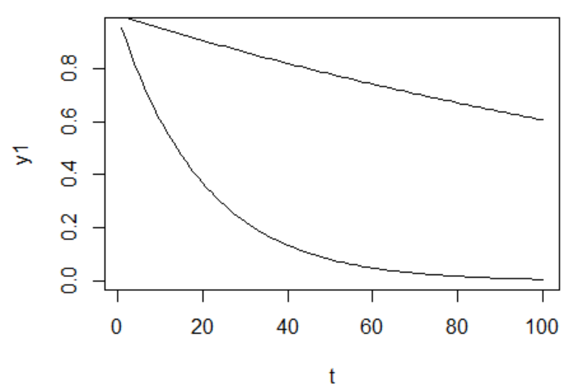
1.2 (a)



(b)



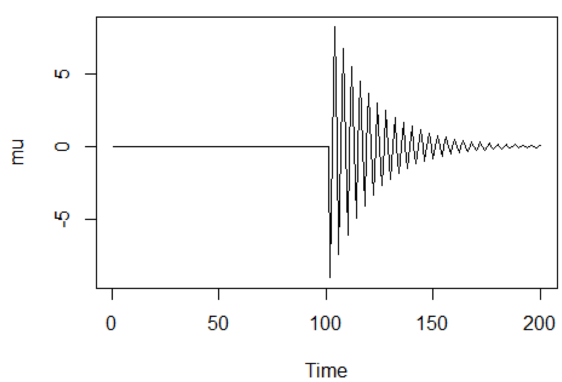
(c)
The series (a) and (b) both changed violently after $t = 100$, and become smooth over time.

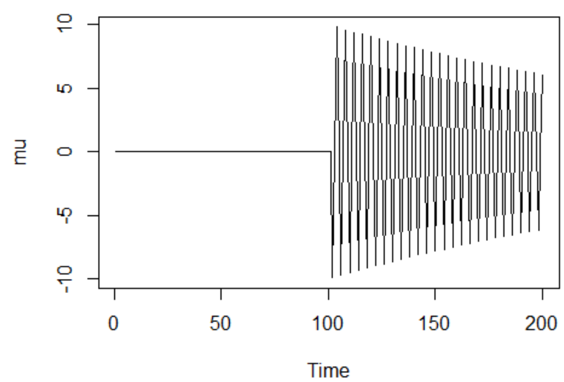


From the plot of $e^{-t/20}$ and $e^{-t/200}$ above we can find that in the period of $t = 100s$, $e^{-t/20}$ changed a lot, but $e^{-t/200}$ did not change much. That is in sympathy with we found from the plots in (a) and (b).

1.5 (a)

$$\mu_{1x}(t) = s_t = \begin{cases} 0, & t = 1, \cdots, 100; \\ 10e^{-(t-100)/20}\cos(2\pi t/4), & t = 101, \cdots, 200. \end{cases}$$
$$\mu_{2x}(t) = s_t = \begin{cases} 0, & t = 1, \cdots, 100; \\ 10e^{-(t-100)/200}\cos(2\pi t/4), & t = 101, \cdots, 200. \end{cases}$$





(b)

$$\begin{aligned} \gamma_x(s,t) &= \mathbb{E}(X_s - \mu_s)(X_t - \mu_t) \\ &= \mathbb{E}w_s w_t \\ &= 0 \end{aligned}$$

1.8 (a)

Assume that $x_{t-1} = \delta t - 1 + \sum_{k=1}^{t-1} w_k$, then

$$\begin{aligned} x_t &= \delta + x_{t-1} + w_t \\ &= \delta t + \sum_{k=1}^{t} w_k \end{aligned}$$

Since

$$x_1 = \delta + 0 + w_1$$

By mathematical induction, the model can be written as $x_t = \delta t + \sum_{k=1}^{t} w_k$ (b)

$$\begin{aligned} \mu_x t &= \delta t \\ \gamma_x(s,t) &= \mathbb{E}\sum_{k\le t} w_k \sum_{j\le s} w_j \\ &= \min(s,t)\sigma^2 \end{aligned}$$

(c)
From the calculation in (b), $\gamma(s,t)$ not only depends on $|s-t|$, so x_t is not stationary.
(d)

$$\begin{aligned}\rho_x(t-1,t) &= \frac{\gamma(t-1,t)}{\sqrt{\gamma(t-1,t-1)\gamma(t,t)}} \\ &= \frac{(t-1)\sigma^2}{\sqrt{(t-1)t\sigma^4}} \\ &= \sqrt{\frac{t-1}{t}}\end{aligned}$$

It implicates that as $t \to \infty$, x_t is nearly proportional to x_{t-1}.
(e)

Let

$$y_t = x_t - x_{t-1}$$

Then

$$\begin{aligned}y_t &= \delta + w_t - w_{t-1} \\ w_t - w_{t-1} &\sim (0, 2\sigma_w^2)\end{aligned}$$

So

$$\begin{aligned}\mu_y t &= \delta \\ \gamma_y(s,t) &= \begin{cases} 0, & |s-t| > 1; \\ -2\sigma_w^2, & |s-t| = 1. \end{cases}\end{aligned}$$

1.14

(a)

$$\begin{aligned}\mathbb{E}(y_t) &= \mathbb{E}e^{x_t} \\ &= e^{\mu_x + \frac{1}{2}\gamma(0)}\end{aligned}$$

(b)

$$\begin{aligned}\gamma_y(s,t) &= \mathbb{E}(e^{x_t} - \mu_y)(e^{x_s} - \mu_y) \\ &= \mathbb{E}(e^{x_t + x_s} - 2\mu_y(e^{x_s} + e^{x_t}) + \mu_y^2) \\ &= e^{\mu_x + \gamma(0)} - 3e^{2\mu_x + \gamma(0)}\end{aligned}$$

1.18

Since $\sum |\phi_j|$ is converge, when j is big enough,

$$|\phi_j| < 1,$$

so,

$$|\phi_j||phi_{j+h}| < \max(|\phi_j|, \|\phi_{j+h}|)^2 < 1,$$

thus,

$$\gamma(h) = \sigma_w^2 \sum |\phi_j||phi_{j+h}|$$

is converge.

1.25

We directly prove (b), for any vector $\mathbf{y}$

$$\begin{aligned}
& mathbfy^{\mathsf{T}} \mathbb{E}\left[(X - \mathbb{E}X)(X - \mathbb{E}X)^{\mathsf{T}}\right] mathbfy \\
& = \mathbb{E}\left[mathbfy^{\mathsf{T}}(X - \mathbb{E}X)\right]^2 \\
& \geq 0
\end{aligned}$$

Thus, the sample autocovariance is a non-negative definite function.