CSCI 301, Winter 2017
Math Exercises #4

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1. The language $\{100, 10, 011\}$. Regular expression: $(100 \cup 10 \cup 011)$
2. The language $\{100, 10, 011\}$. DFA:

   See page 2.

3. The set of all strings that begin or end with a doubled digit, either 11 or 00. Regular expression: $(((1 \cup 0)^*(11 \cup 00)) \cup ((11 \cup 00)(1 \cup 0)^*))$
4. The set of all strings that begin or end with a doubled digit, either 11 or 00. DFA:

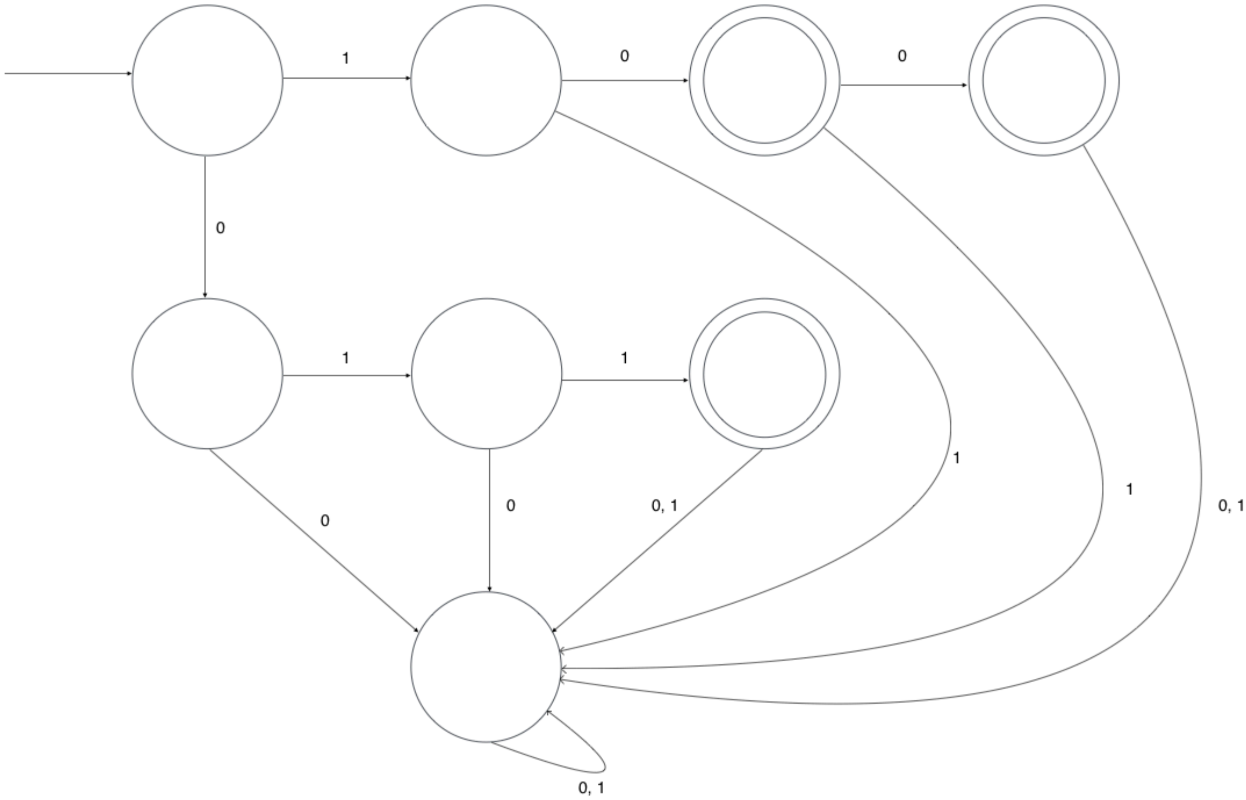
   See page 3.

5. The set of all strings that have exactly one doubled digit in them. In other words, either 11 or 00 occurs in the string, but not both, and it only occurs once. Regular expression: $(((01)^*0 \cup (10)^* \cup 0)11(0 \cup (01)^* \cup 0(10)^*) \cup ((10)^*1 \cup (10)^* \cup 1)00(1 \cup (10)^* \cup 1(01)^*))$
6. The set of all strings that have exactly one doubled digit in them. In other words, either 11 or 00 occurs in the string, but not both, and it only occurs once. DFA:

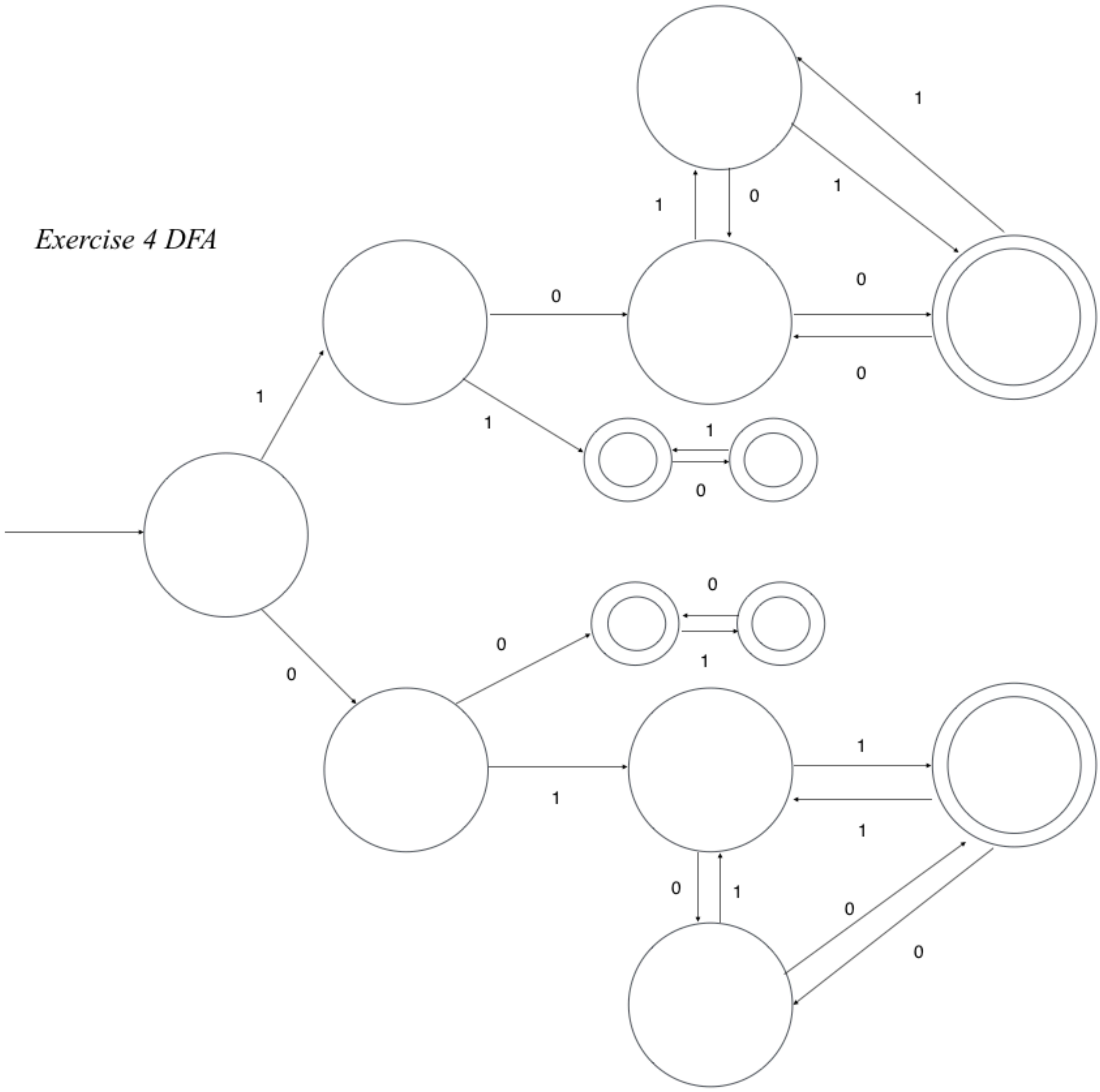
   See page 4.

7. Use the pumping lemma to show that the language that consists of all *palindromes* over $\Sigma = \{0, 1\}$ is not regular. A palindrome is a string that reads the same backwards and

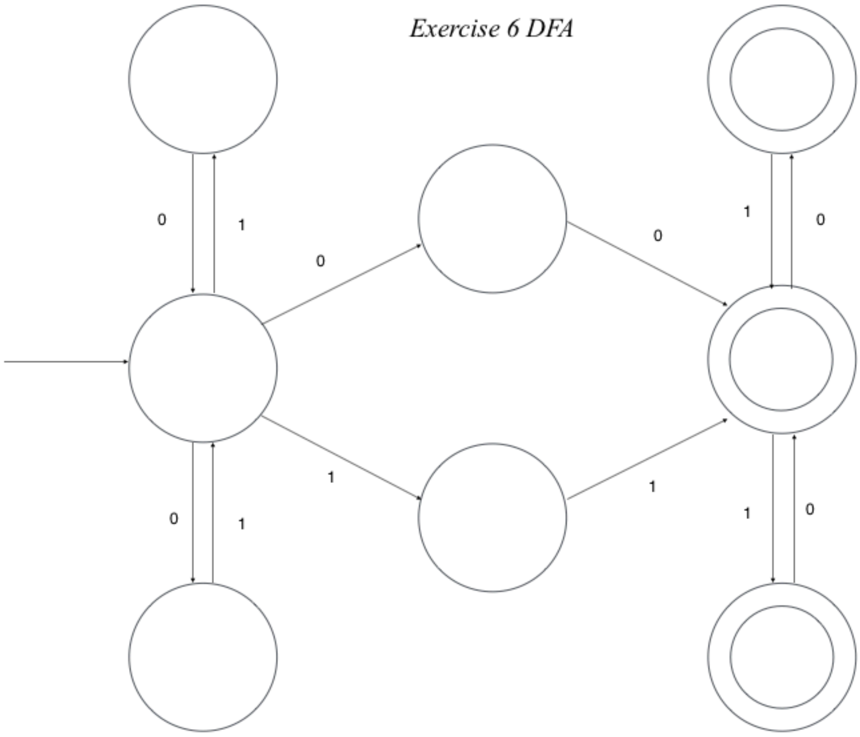
Exercise 2 DFA



Exercise 4 DFA



Exercise 6 DFA



forwards, for example 11011, 01010, 0, and 0110.

$$L = \{w \in \{0,1\}^* \mid w = w^R\}$$

Assume that L is a regular language. Let $s = 0^p10^p$, $s \in L$ and is in the set of all palidromes. If L is a regular language then by the pumping lemma there is a pumping length, $p \geq 1$, such that there exists xy^iz such that $s = xyz$, $|xy| \leq p$, $|y| \geq 1$, $|s| = 2p+1$ and $i \in \mathbb{N} \cup \{0\}$, $xy^iz \in L$.
From observing these conditions we can see that s is some derivation of the following form $x = 0^k$, $y = 0^q$ and $z = 0^{p-q-k}10^p$ such that $k + q \leq p$ and $q \geq 1$.
Let's examine the string $xy^iz \in L$ at $i = 2$. Then

$$xy^2z = xyyz = 0^k0^q0^q0^{p-q-k}10^p = 0^q0^p10^p = 0^{p+q}10^p.$$

Since we are observing string that are in the language of all palindromes xy^2z should be equal to its reverse xy^2z^R. Observe that the reverse string of xy^2z is equal to 0^p10^{p+q} and

$$0^{p+q}10^p \neq 0^p10^{p+q}.$$

Therefore, $xy^2z \notin L$ under all possible derivations of string xyz and under all aforementiond conditions. Thus, L is a non-regular language.

8. Use the pumping lemma to show that the following language is not regular. For examples, 00110000, 01110000 and 00 are in the language, but 110 is not.

$$L = \{0^i1^j0^{i+j} \mid i, j \in \{0, 1, 2, \ldots\}\}$$

Assume that L is a regular language. Then there exists an integer $p \geq 1$, called the pumping length as given by the pumping lemma. Consider the string $s = 0^p1^p0^{2p}$. Then $s \in L$ and $|s| = 4p \geq p$. By the pumping lemma, s can be written as $s = xyz$, where $y \neq \epsilon$, $|xy| \leq p$, and $xy^iz \in L$ for all $i \geq 0$.
Since $|xy| \leq p$, the string y contains only 0's. Since $y \neq \epsilon$, y contains at least one 0, however, in the definition of the language L we can see that i, the number of 0's, can be equal to 0. Therefore, there is no string defined within the parameters of the pumping lemma such that *all* strings in language L can be accounted for. Because L cannot be described in its entirety by the pumping lemma, L is a non-regular langauge.

9. Give an example of a regular language R and a nonregular language N such that $R \cup N$ is regular. Describe all three languages in English and either prove they are regular/nonregular, or show that they are instances of languages with known regularity.

Consider the language $A_1 = \{a^nb^n|n \geq 0\}$ and $A_2 = \{a^nb^m|n.m \geq 0\}$. Language A_1 represents all strings with the same number of a's and b's and is non-regular, while language A_2 represents strings with the same number of a's and b's *and* strings with an unequal number of a's and b's making $A_1 \subseteq A_2$, as every element of A_1 is also in A_2. As proof of A_2's regularity, consider that no matter how many times we pump some number of a's within the however many a's we will still be adhering to the language's rule of having some number of a's and b's making A_2 regular. However, if we were to pump A_1 we would end up getting an uneven amount of a's and b's which does not adhere to the rule of A_1, having the same number of a's and b's. Since A_2 is larger than A_1 we can think of the union between the two languages as just A_2, and since A_2 is regular the language created by the union of A_1 and A_2 is regular.

10. Give an example of a regular language R and a nonregular language N such that $R \cup N$ is non-regular. Describe all three languages in English and either prove they are regular/nonregular, or show that they are instances of languages with known regularity.

Consider the regular language $A = \{11011, 01010, 0, 0110\}$ and the language $L = \{w \in \{0,1\}^* \mid w = w^R\}$ proven to be non-regular above. From the previous question's examples we know that the strings $11011, 010100$ and 0110 are all defined within the language L and they can all easily be defined by a DFA or a regular expression. Since these strings are all palidromes and therefore exist within L, the set of all palindromes, we can say that $A \subseteq L$. Since A is regular and a subset of L we can say that the intersection of A and L is regular however, some of the strings present only in L will be non-regular as we have proven L to be an non-regular language in the previous exercise. Thus, this third language, the union of a regular and non-regular language, which includes all elements of A and L, which we can think of as being just L, a non-regular language, is non-regular.