

CSCI 301, Winter 2017

Math Exercises # 5

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Construct a context-free grammar for each of the languages in questions 1 to 7.

1. $\{0^{2n}1^n : n \geq 0\}$

$$S \rightarrow 00S1 \mid \epsilon$$

2. $\{w : w \text{ contains at least three 1s}\}$

$$\begin{aligned} S &\rightarrow Z1Z1Z1Z \mid \epsilon \\ Z &\rightarrow 0Z \mid 1Z \mid \epsilon \end{aligned}$$

3. $\{w : \text{the length of } w \text{ is odd and its middle symbol is } 0\}$

$$S \rightarrow 0S0 \mid 1S1 \mid 1S0 \mid 0S1 \mid 1S0 \mid 0 \mid$$

4. $\{w : w \text{ is a palindrome}\}$

$$\begin{aligned} S &\rightarrow 0Z0 \mid 1Z1 \mid 0 \mid 1 \mid \epsilon \\ Z &\rightarrow 0Z \mid 1Z \mid \epsilon \end{aligned}$$

5. $\{w : w \text{ starts and ends with the same symbol}\}$

$$\begin{aligned} S &\rightarrow 0Z0 \mid 1Z1 \mid \epsilon \\ Z &\rightarrow 0Z \mid 1Z \mid \epsilon \end{aligned}$$

6. $\{w : w \text{ starts and ends with different symbols}\}$

$$\begin{aligned} S &\rightarrow 1Z0 \mid 0Z1 \mid \epsilon \\ Z &\rightarrow 0Z \mid 1Z \mid \epsilon \end{aligned}$$

7. $\{a^m b^n : 0 \leq m \leq n \leq 2m\}$

$$S \rightarrow aSb \mid aSbb \mid \epsilon$$

8. Let G be the grammar:

$$\begin{aligned} S &\rightarrow aB \mid bA \\ A &\rightarrow a \mid aS \mid bAA \\ B &\rightarrow b \mid bS \mid aBB \end{aligned}$$

For the string $aaabbabbba$, find a

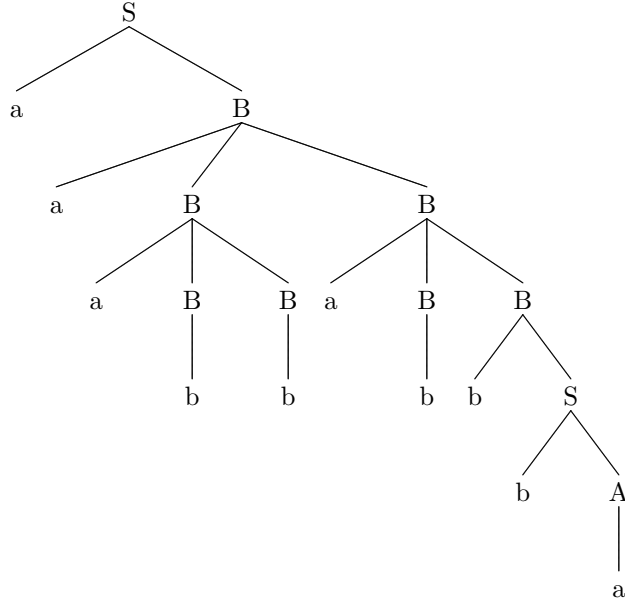
(a) leftmost derivation,

$$S \rightarrow a\underline{B} \rightarrow aa\underline{BB} \rightarrow aaa\underline{BBB} \rightarrow aab\underline{BB} \rightarrow aaabb\underline{B} \rightarrow aaabba\underline{BB} \rightarrow aaabbab\underline{B} \rightarrow aaabbabb\underline{S} \rightarrow aaabbabbb\underline{A} \rightarrow aaabbabbba$$

(b) rightmost derivation,

$$S \rightarrow a\underline{B} \rightarrow aa\underline{BB} \rightarrow aaBa\underline{BB} \rightarrow aaBaBb\underline{S} \rightarrow aaBaBbb\underline{A} \rightarrow aaBaBbbba \rightarrow aa\underline{B}abbbba \rightarrow aaa\underline{B}abbbba \rightarrow aaa\underline{B}abbbba \rightarrow aaabbabbba$$

(c) parse tree.



9. Convert the following grammar to Chomsky normal form:

$$\begin{aligned} S &\rightarrow bA \mid aB \\ A &\rightarrow bAA \mid aS \mid a \\ B &\rightarrow aBB \mid bS \mid b \end{aligned}$$

Follow the steps documented in my notes and the text, and show the resulting grammar after each step.

Step 1 Eliminate the start variable from the right-hand side of rules.

$$\begin{aligned} S_0 &\rightarrow bA \mid aB \\ S &\rightarrow bA \mid aB \\ A &\rightarrow bAA \mid aS \mid a \\ B &\rightarrow aBB \mid bS \mid b \end{aligned}$$

Step 2 Eliminate ϵ -rules. *No ϵ -rules to eliminate.*

Step 3 Eliminate unit-rules. *No unit-rules to eliminate.*

Step 4 Eliminate all rules having more than two symbols on the right-hand side.

$$\begin{aligned}
S_0 &\rightarrow bA \mid aB \\
S &\rightarrow bA \mid aB \\
A &\rightarrow bA_1 \mid aS \mid a \\
B &\rightarrow aB_1 \mid bS \mid b \\
A_1 &\rightarrow AA \\
B_1 &\rightarrow BB
\end{aligned}$$

Step 5 Eliminate all rules of the form $A \rightarrow u_1u_2$ where u_1 and u_2 are not both variables.

$$\begin{aligned}
S_0 &\rightarrow B_2A \mid A_2B \\
S &\rightarrow B_2A \mid A_2B \\
A &\rightarrow B_2A_1 \mid A_2S \mid a \\
B &\rightarrow A_2B_1 \mid B_2S \mid b \\
A_1 &\rightarrow AA \\
B_1 &\rightarrow BB \\
A_2 &\rightarrow a \\
B_2 &\rightarrow b
\end{aligned}$$