Math 105B
Computer Assignment 3
Due Friday 1/25, 10pm

The aim of this lab is to write a function to compute the coefficients for a Hermite polynomial interpolating function using divided differences and then to use the coefficients to compute the Hermite interpolating polynomial.

The algorithm for the divided difference formulas for the Hermite polynomial coefficients is:

INPUT $X = (x_0,...,x_n)$, $Y = (f(x_0),...,f(x_n))$, $YP = (f'(x_0),...,f'(x_n))$
OUTPUT $Q_{i,j}$, the appropriate divided differences, where the Hermite interpolating polynomial is $H_{2n+1}(x) = Q_{0,0} + \sum_{i=1}^{2n+1} Q_{i,i} \prod_{j=0}^{i-1} (z - z_j)$ and $z_{2i} = x_i$, $z_{2i+1} = x_i$ for $i = 0,...,n$

STEP 1. For i=0,1,…n

$z_{2i} = x_i$

$z_{2i+1} = x_i$

$Q_{2i,0} = f(x_i)$

$Q_{2i+1,0} = f(x_i)$

$Q_{2i+1,1} = f'(x_i)$

If $i \neq 0$ then

$$Q_{2i,1} = \frac{Q_{2i,0} - Q_{2i-1,0}}{z_{2i} - z_{2i-1}}$$

End If

End

STEP 2. For $i = 2,3,...,2n+1$

For $j = 2,3,...,i$

$$Q_{i,j} = \frac{Q_{i,j-1} - Q_{i-1,j-1}}{z_i - z_{i-j}}$$

End

End

STEP 3. OUTPUT $Q_{i,j}$

STOP

(1). Write a matlab routine called main.m that calls a function that you write to compute the generalized divided differences given above.
(2). Use a Hermite polynomial to interpolate the function $f(x) = e^{0.1x^2}$ given below in tabular form, at the point x=1.25. Use $H_5(1.25)$ with nodes x_0, x_1, x_2 and $H_3(1.25)$, with nodes x_0, x_1.

x	$f(x)$	$f'(x)$
$x_0 = 1$	1.105170918	0.2210341836
$x_1 = 2$	1.491824698	0.5967298792
$x_2 = 3$	2.459603111	1.475761867

(3). What is your estimate of the errors? Justify your answer. Find error bounds for these approximations.