

Math 105B
Computer Assignment 5
Due Friday 2/8, 10pm

The aim of this lab is to test the accuracy of several finite difference formulas for computing derivatives of functions numerically.

Consider the function $f(x) = (1 + x^2)^{-1}$ defined on $[-5, 5]$. Choose equally-spaced grid points $x_i = -5 + ih$, where $i = 0, \dots, n$ and $h = 10/n$ with $n = 11, 21, 51, 101$.

- (1) Numerically approximate $f'(x_i)$ for $i = 1, \dots, n-1$ using forward difference, backward difference, central difference approximations. Plot the results on a graph.
- (2) What is the error between the numerical solution from part (1) and the exact solution? Take the maximum of the absolute value of the errors. Plot the results versus n . How does the error scale with n ? Compare the result to the error estimate. Is the actual error about the same size as the error estimate? Explain your answer.
- (3) Numerically approximate $f''(x_i)$ using the central difference formula at the interior grid points $i = 1, \dots, n-1$. Plot the result on a graph.
- (4) What is the error between the numerical solution from part (3) and the exact solution? Take the maximum of the absolute value of the errors. Plot the results versus n . How does the error scale with n ? Compare the result to the error estimate. Is the actual error about the same size as the error estimate? Explain your answer.
- (5) Calculate the derivative $f'(0)$ using the forward difference formula:
$$D_N f(0) = \frac{f(10^{-N}) - f(0)}{10^{-N}}$$
 for $N = 1, 2, 5, 10, 20, 40$ and describe what happens. Justify your observations.