Math 105B
Computer Assignment 4
Due Friday 2/1, 10pm

The aim of this lab is to write a function to compute the coefficients for a cubic spline interpolating function and then to use the coefficients to compute a cubic spline interpolating polynomial. We will focus on natural splines.

The algorithm for the coefficients of a cubic spline interpolating polynomial is (Algorithm 3.4 in the text). The algorithm for solving the tridiagonal system is Algorithm 3.7 in the text.

INPUT $X = (x_0,...,x_n)$, $Y = (f(x_0),...,f(x_n)) = (a_0,...,a_n)$,
OUTPUT a_j, b_j, c_j, d_j for $j = 0,..,n-1$ the coefficients, where the cubic spline is:
$S(x) = S_j(x) = a_j + b_j(x-x_j) + c_j(x-x_j)^2 + d_j(x-x_j)^3$, for $x_j \le x \le x_{j+1}$.

STEP 1. For i=0,1,…n-1

$$h_i = x_{i+1} - x_i$$

End For

STEP 2. For i=1,…n-1

$$\alpha_i = \frac{3}{h_i}(a_{i+1} - a_i) - \frac{3}{h_{i-1}}(a_i - a_{i-1})$$

End For

STEP 3. Set %STEPS 3, 4, 5 and part of 6 solve the tridiagonal linear system.

$l_0 = 1$

$\mu_0 = 0$

$z_0 = 0$

STEP 4. For i=1,…n-1

$l_i = 2(x_{i+1} - x_{i-1}) - h_{i-1}\mu_{i-1}$

$\mu_i = h_i / l_i$

$z_i = (\alpha_i - h_{i-1}z_{i-1}) / l_i$

End For

STEP 5. Set

$l_n = 1$

$z_n = 0$

$c_n = 0$

STEP 6. For j=n-1, n-2,…,0

$c_j = z_j - \mu_j c_{j+1}$

$b_j = \left(a_{j+1} - a_j\right) / h_j - h_j\left(c_{j+1} + 2c_j\right) / 3$

$d_j = \left(c_{j+1} - c_j\right) / \left(3h_j\right)$

End For

STEP 7. OUTPUT a_j, b_j, c_j, d_j for $j = 0,..,n-1$

STOP

1. Compute and graph the natural cubic spline interpolant for the data shown in the table below. To plot, use these points, together with the uniformly spaced points given in problem 3 below.

x	f(x)
-2.4061	-0.3984
-1.0830	-0.7611
-0.6440	-0.9688
-0.4068	-0.9791
-0.2448	-0.7899
-0.1158	-0.4397
0	0
0.1158	0.4397
0.2448	0.7899
0.4068	0.9791
0.6440	0.9688
1.0830	0.7611
2.4061	0.3984

2. Use your Lagrange interpolation algorithm from computer assignment 1 to compute the corresponding Lagrange polynomial interpolant, at the same points as listed in part 3. How does the result compare to that for the cubic spline? Explain your results.

3. The function given the table above is: $f(x) = \frac{x}{1/4 + x^2}$. Use the cubic spline interpolant you found above to approximate the function at the uniformly spaced grid

points z=linspace(-2,2,10). What is the maximum error for the cubic spline interpolant? How does the result compare to that predicted by the theoretical error estimate?