Lab Report: Adaptive Two-point Gaussian

Introduction

I'll begin by calculating the exact value of the integral using the built-in function of integration in Matlab. Afterward, I will code out the basic two-point Gauss procedure *Gauss(f,a,b)* to approximate the integral of f(x). This function takes the input f, function, and it's interval [a,b] and return the approximation of the integral. Using this function I can then implement it in my recursive function *Adaptive(f,S,a,b,level,tol,N)* that takes the additional input of S for sum that starts from zero, level for the number of subintervals, tol for tolerance, and N for maximum depth. I will then print out the contribution of each subinterval and the depth at which the approximate value over the subinterval is accepted. I will be testing the *Adaptive* function by using a maximum depth of 100 subintervals and tolerance of 10^{-7}.

Algorithm Method

Part A: To calculate the exact value of the integral, I simply set up the function $f(x) = \frac{sin 10/x}{x}$ with a = 1 and b = 3 and use the built-in function *int* to get exact = -0.3627. To code the function Gauss, I begin by setting t = 0.5773502692 which would be used as a change of variable to transformed the integral over interval [a,b] to the integral over [-1,1]. Then I set up x1 and x2 with the formula x1 = ((b-a)*t+a+b)/2 and x2 = ((b-a)*(-t)+a+b)/2. I finish the code by adding the evaluation of f(x1) and f(x2) together to get the approximation of the integral: -0.3507.

Part B: As to write the function for *Adaptive*, I follow the procedure provided by the professor. The recursive function will check to see if the maximum depth is exceeded, and if so it prints an error message and stops. If the maximum depth is not exceeded, then the function continues by dividing the interval [a,b] in half, and calling the procedure Gauss on the left sub-interval, the right subinterval, and the whole interval, then checking to see if the tolerance test is accepted. If yes, then it adds the approximate value over the whole interval to the variable sum. Otherwise, it calls the recursive procedure Adaptive on the left and right subintervals and increases the value of the depth variable. The tolerance test checks to see if the difference in the absolute value between the approximate value over the whole interval and the sum of the approximate values over the left and right subintervals is less than a variable tolerance. Running the function would produce an estimate of -0.1880.

Conclusion

In comparison, the adaptive two-point Gaussian integration produce a better approximation of the integral than using the basic two-point Gauss procedure. The adaptive Gaussian divided the interval [1,3] into 33 subintervals with 33 contributions to add up to the sum of -0.188.