Lab Report: Trapezoid and Simpon's Rule

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Introduction

I'll begin by calculating the exact value of the integral using the built-in function of integration in Matlab. Afterwards, I will code the equations of trapezoid and Simpson's rule and input the provided a = 1 and b = 2 to evaluate the integral as well as the error estimate of these methods. Using the results I can then compare them to the actual error and exact integral result. I repeat the procedure for the composite trapezoid and Simpson's rules with the addition of using the equally spaced points $x_i = 1 + ih \text{ where } i = 0, ..., n \text{ and } h = 1/n \text{ for } n = 10, 20, 50, 100$ and plotting the value of the numerical quadrature as a function of n.

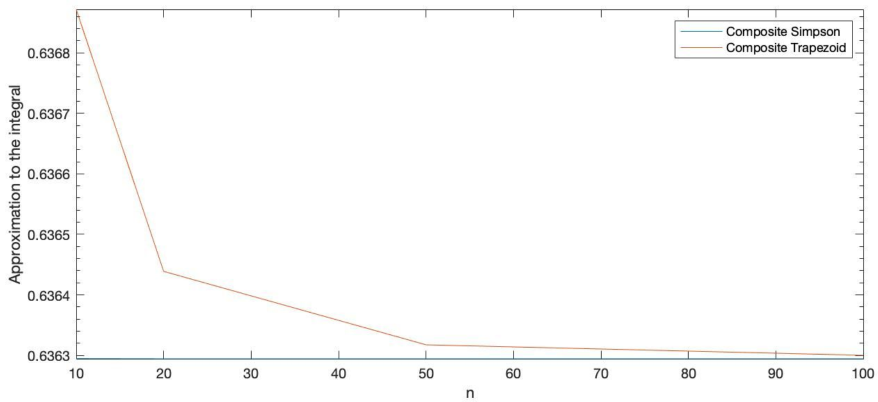
Algorithm Method

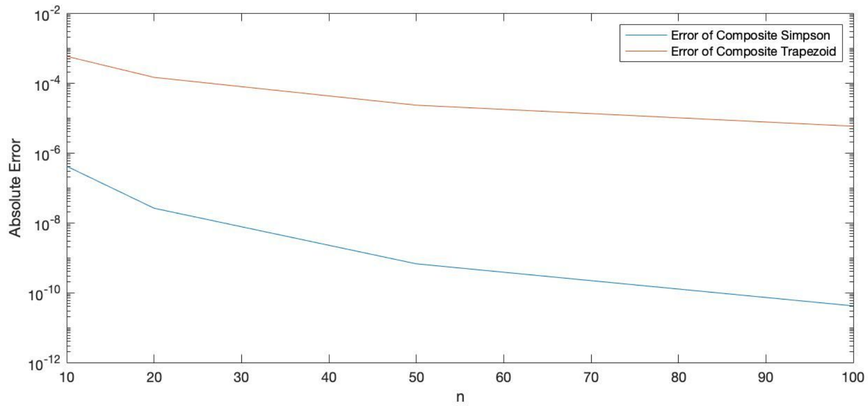
Part A: To calculate the exact value of the integral, I simply set up the function $f(x) = xlog(x)$ and use the built-in function *int* to get the result of 0.6363. As for the computation of the trapezoid rule, I code in the formula ((f(a) + f(b))*h_t/2) and get the result of trap = 0.6931. The exact error would then be exact_t $= |0.6363 - 0.6931| = 0.0569$ and the error estimate is error_t = 0.0833. Comparing the two errors, I notice that the exact error is a little bit smaller than the error estimate but still around the same so I can conclude that the exact error is justified. Similarly, I code in the formula (f(a) + 4*f(x_1) + f(b))*h_s/3 for the Simpson's rule, with x_1 being the midpoint between a and b, and get the result simp = 0.6365. The exact error would then be exact_s $= |0.6363 - 0.6365| = 2.1981e-04$ and the error estimate is error_s = 6.9444e-04. Comparing the two errors, I notice that the exact error is also smaller than the error estimate but still around the same so I can conclude that this exact error is justified as well.

Part B: The coding for the composite simpson's rule and trapezoid are longer and require for loops but nonetheless they are very similar to the formulas before. I follow the algorithm of the Simpson's rule by first setting h = (b-a)/n and the row $I_{n,0} = f(a) + f(b),\ I_{n,1} = I_{n,2} = 0$. Then beginning with a for loop for i that goes from 1 to n-1, every iteration would update the evaluation points $x = a + ih$ as well as summing up $I_{n,2} = I_{n,2} + f(x)$ for even i and

$I_{n,1} = I_{n,1} + f(x)$ for odd i. At the end of the loop for each n, the code would then add all the column of the row together using the formula $I_n = h/3(I_{n,0} + 2I_{n,2} + 4I_{n,1})$ to get the result of comp_simp = [0.6363, 0.6363, 0.6363, 0.6363]. Using this information, I can plot comp_simp against n to get the approximation curve of the integral as shown from the graph down below. By looking at the values alone, it is obvious that every n produce the same value of 0.6363. While each estimation is not exactly the same, since computing the exact error would produce error_comp_simp = 1.0e-06 *[0.4130, 0.0260, 0.0007, 0.0000] and the error plot shows curvature, the curve results in a straight line as the difference is too small to be plot (even when I used semilogy to plot). This can help conclude that the composite Simpson's rule computes an accurate integration even when n is small. As for the composite trapezoid rule, I repeat the same algorithm as the composite Simpson's rule with the main differences of I is now J and the final equation is replace to $J_n = h/2(J_{n,0} + 2J_{n,1})$. The result of the approximation is comp_trap = [0.6369, 0.6364, 0.6363, 0.6363] and the exact error is error_comp_trap = 1.0e-03 *[0.5775, 0.1444, 0.0231, 0.0058]. Compare to the result of composite Simpson's, the composite trapezoid method shows curvature in the plot of numerical quadrature and the errors are also bigger than that of the Simpson's rule as seen in the graph. The error estimate of composite Simpson's rule is error_est_simp = 1.0e-05 *[0.1111, 0.0069, 0.0002, 0.0000] and the error estimate of composite trapezoid rule is error_est_trap = 1.0e-03 *[0.8333, 0.2083, 0.0333, 0.0083]. Comparing these to the actual errors, I conclude that the actual error of Simpson is larger than the estimate and the actual error of trapezoid is smaller than the error estimate.

Graph





Conclusion

In comparison, the Simpson's rule and its composite method is more accurate in computing definite integrals numerically. This is justified by comparing the error exact_t = 0.0569 to exact_s = $2.1981e-04$ and by observing the absolute error graph above which shows the curve for the error of composite trapezoid is higher than the curve for the error of composite Simpson. Additionally, the composite Simpson's rule produce an accurate approximation faster than the composite trapezoid since the approximate integral graph shows that when n = 10, the approximation gives the true solution of 0.6363 while the trapezoid approximate the value to be 0.6369.