Math 105B Lab Report 5

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Objective

The goal of this project is to use Matlab to test the accuracy of forward difference, backward difference, and central difference approximation formulas for computing derivatives of functions numerically. Using the equally-spaced grid points, $x_i = -5 + ih$, where h = 10/n and i = 0,..,n, I'll be computing and plotting numerous results to compare the difference in using different n values and see how the approximation gets better or worse.

Introduction

I will first be setting up the domain of x_i and function I want to compute the differentiation of. Then I'll implement the forward, backward, and central differences in my code instead of creating a function for them as they are simply one line of code each. After the computation goes through, I will be able to use the data to plot a graph that's an estimation of $f'(x)$ *or* $f''(x)$ and calculate the error from the true solution by using the same x_i on the actual differentiation. At the end of the code, I'll compare and contrast the results with the actual differentiation by looking at the maximum error and theoretical error estimate.

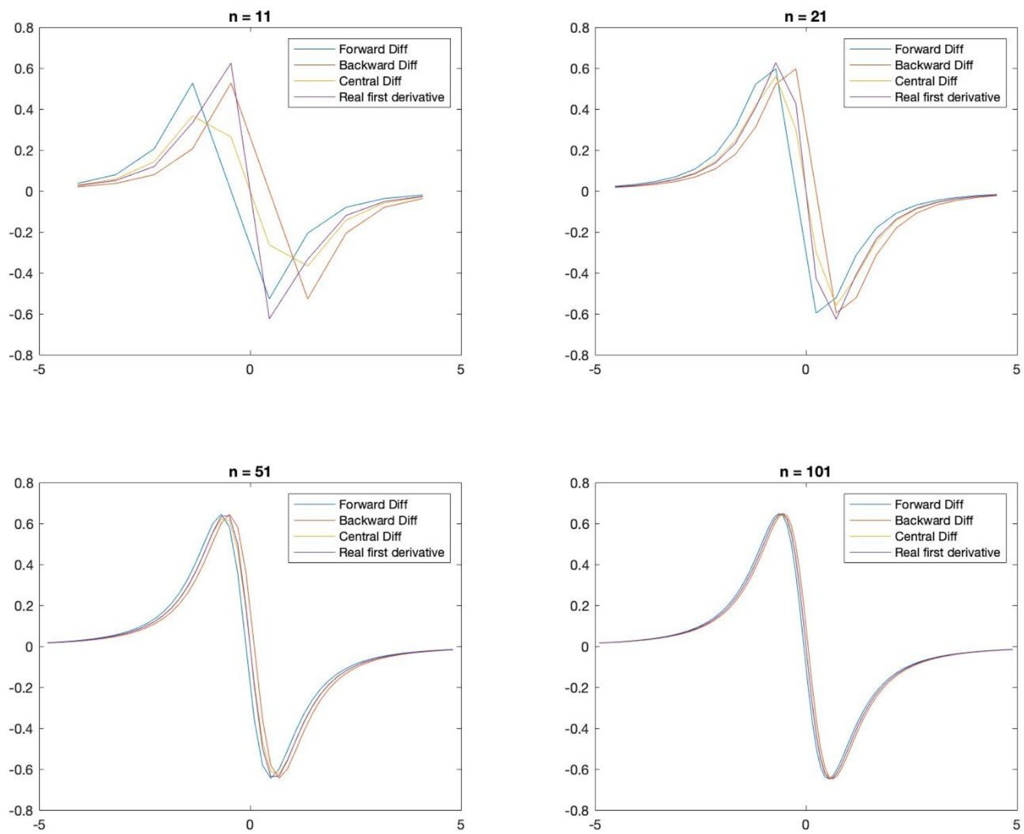
Algorithm Method

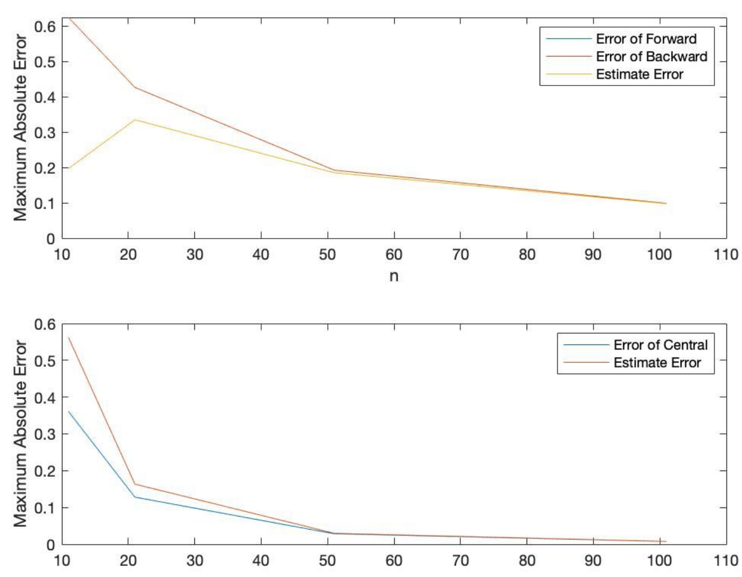
Part A: In order to test out all the values of n = [11, 21, 51, 101] without rewriting the code repeatedly, I start with a for loop that goes from 1 to 4. Within the loop, I first initiate four empty lists: F, B, C, E to store my computation of forward, backward, central, and actual values (this is needed since if I don't, the code gives an error when running it for the second time). Here, I also include an empty list to store values of the theoretical error estimate. Now I got the set up ready, I can include another for loops for i that goes from 1 to n-1. As for the formulas, I just code the differences that estimate the first and second derivative, in which they include the computation of f(x) with the respected x and h. These values then get stored to the list of forward, backward, and central differences.

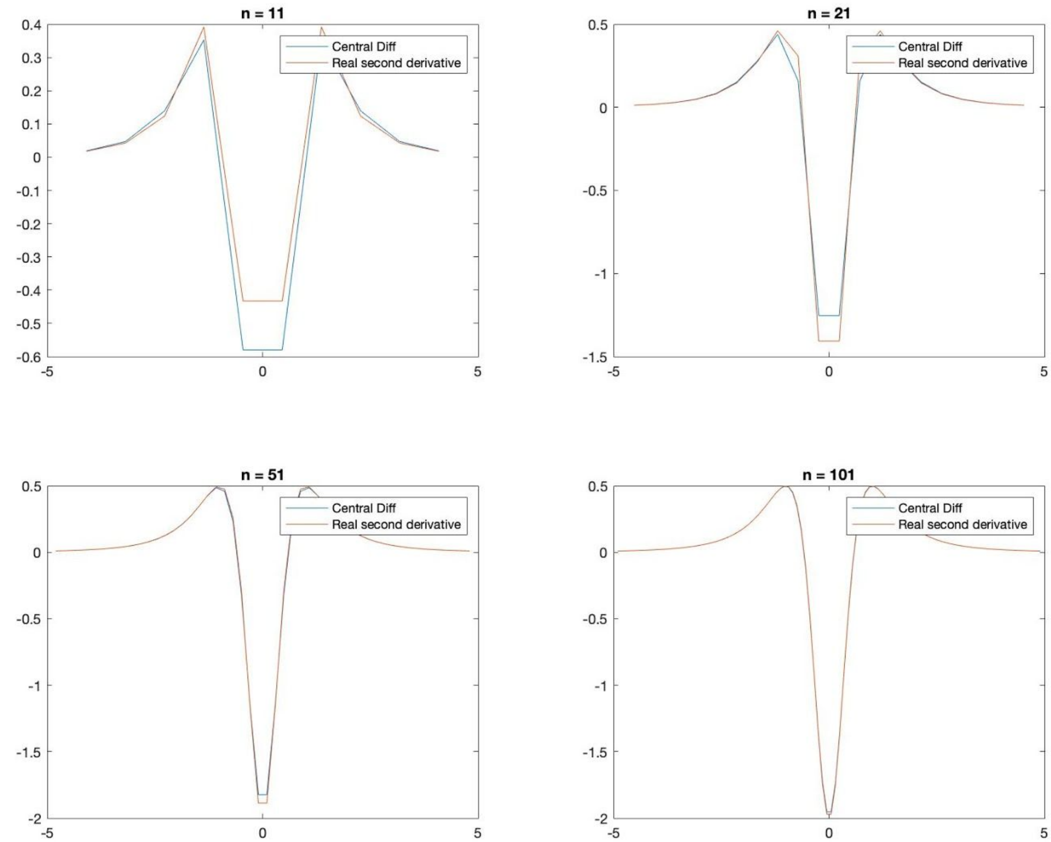
Part B: Using the result of the differences, I can plot them against x inside the loop to make one graph for each n values. Every graph have 4 plots on it that includes the actual differentiation and the estimated differentiation using the three formulas. Since the actual error between the numerical solution and the exact solution would be too much to print, I take the maximum absolute differences/error for each n and plot them against n (as seen under graph below). The maximum actual differences between forward difference/backward difference and exact solution when n = 11, 21, 51, 101 is max_1 = max_2 = [0.6244, 0.4265, 0.1924, 0.0985] respectively (note that this is to approximate $f'(x)$). As for the maximum error for central difference, I got max_3 = [0.3609, 0.1281, 0.0285, 0.0075]. To calculate the error estimate, I use the function $f''(x)h/2$ and $f''(x)h^2/6$ for x between x_0 *and* $x_0 + h$. Taking only the maximum error estimate, I got the following result of max_est_error_1 = [0.1967, 0.3350, 0.1850, 0.0976] and max_est_error_2 = [0.5624, 0.1634, 0.0296, 0.0076]. By observing these numbers alone, I see that the maximum actual error when n = 21, 51, 101 is around the same size as the maximum estimate error. It is only when n is the smallest, n = 11, that the maximum estimate error is smaller than the actual error of forward/backward and bigger than the actual error of central difference. From observing the graph, I can reasoned that this is due to how off the estimated differentiation is from the real f(x) compared to when n = 21, 51, 101. Similarly, when approximating $f''(x)$ using only the central difference formula, I can see the graph for n = 11 is the least accurate when compared to the other n's graph. The maximum actual error of central difference for approximating $f''(x)$ is the following: max_cent = [0.1468, 0.1535, 0.0641, 0.0187] and the maximum error estimate is max_est_error = [0.0154, 0.1546, 0.0663, 0.0189]. I can conclude that the maximum actual error is around the same as the error estimate.

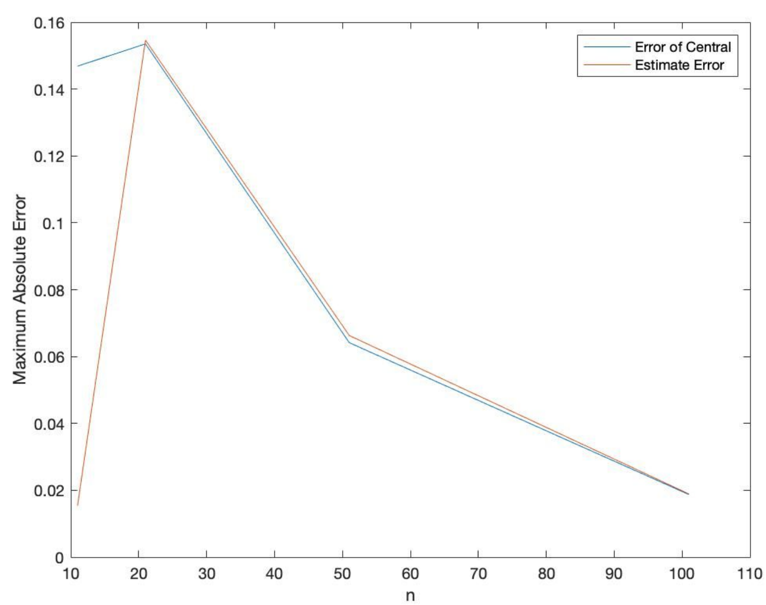
Part C: For the last few lines of the code, I calculated the derivative $f'(0)$ using the forward difference formula with N = 1, 2, 5, 10, 20, 40 and got the result of D = [-0.0990, -0.0100. -0.0000, -0.0000, -0.0000, -0.0000]. From this result, I see that the estimation of $f'(0)$ does not equal to zero when N = 1 and N = 2. I can interpret this as that N needs to be big enough for the estimation of forward difference to give us an accurate estimation of differentiation. The forward difference formula requires $f(10^{-N})$ to be very close to $f(0)$ in order for them to cancel out and equal to zero, the actual solution $f'(0)$.

Graph


n = 11
n = 21
n = 51
n = 101
Forward Diff
Backward Diff
Central Diff
Real first derivative


Maximum Absolute Error
n
Error of Forward
Error of Backward
Estimate Error
Error of Central


n = 11
Central Diff
Real second derivative
n = 21
Central Diff
Real second derivative
n = 51
Central Diff
Real second derivative
n = 101
Central Diff
Real second derivative


Error of Central
Estimate Error
Maximum Absolute Error
n

Conclusion

The accuracy of finite difference formulas depends on how many xi or, in terms of this example, how big n is. In other words, the spread of the grid points is crucial in providing a close approximation to the first and second derivatives of functions numerically. In terms of which difference formulas produce a more accurate result, this lab shows that forward and backward method computes almost the same result of approximation and error. Meanwhile, the central difference gives even a more accurate calculation as the actual error is smaller than the actual error of forward/backward.