Math 105B
Computer Assignment 6
Due Friday 2/15, 5pm

The aim of this lab is to test the accuracy of the trapezoid and Simpson's rules, as well as their composite formulas, for computing definite integrals numerically.

Consider the integral $\int_1^2 x \ln x \, dx$.

(1). Calculate the exact value of the integral.

(2). Use the trapezoid rule and Simpson's rule over the whole interval to evaluate the integral. What is the error? Compare the result to the error estimate.

(3). Develop algorithms to perform numerical quadrature for a general indefinite integral $\int_a^b f(x)dx$ using the composite trapezoid and Simpson's rules.

(4). Apply the algorithms to the integral listed above. Use equally spaced points: $x_i = 1 + ih$, where $i = 0,..,n$ and $h = 1/n$. Use $n = 10,\ 20, 50, 100$. Produce *one* plot showing the value of the numerical quadrature as a function of n. There will be two curves on this plot: one for the composite Simpson's rule and one for the composite trapezoid rule.

(5). Compare the error to the exact error estimates given below (and in the book). Produce *one* plot showing the error of the numerical quadrature as a function of n. There will be two curves on this plot: one for the composite Simpson's rule and one for the composite trapezoid rule.

Theorem (Simpson's Rule). Let $f \in C^4[a,b]$, n be even, $h = (b-a)/n$ and $x_i = 1 + ih$, with $i = 1,..,n$. Then, there exists $c \in (a,b)$ such that the composite Simpson's rule satisfies

$$\int_a^b f\,dx = \frac{h}{3}\left(f(a) + 2\sum_{j=1}^{n/2-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b) \right) + \frac{b-a}{180} h^4 f^{(4)}(c)$$

SIMPSON'S RULE:

INPUT a, b and n, a positive even integer
OUTPUT I_n, the approximation to the integral

Step 1. Set $h = (b-a)/n$

Step 2. Set $I_{n,0} = f(a) + f(b)$

$I_{n,1} = 0$

$I_{n,2} = 0$

Step 3. For $i = 1,..,n-1$

Set $x = a + ih$
If i is even, then $I_{n,2} = I_{n,2} + f(x)$
Otherwise $I_{n,1} = I_{n,1} + f(x)$

Step 4. Set $I_n = \frac{h}{3}\left(I_{n,0} + 2I_{n,2} + 4I_{n,1}\right)$

Theorem (Trapezoid Rule). Let $f \in C^2[a,b]$, n be an integer, $h = (b-a)/n$ and $x_i = 1 + ih$, with $i = 1,..,n$. Then, there exists $c \in (a,b)$ such that the composite Trapezoid rule satisfies

$$\int_a^b f\,dx = \frac{h}{2}\left(f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right) + \frac{b-a}{12} h^2 f^{(2)}(c)$$