Math 105B
Computer Assignment 1
Due Friday 1/11, 10pm

Consider the Lagrange interpolating polynomial. Given input coordinates, $X = (x_1,...,x_n)$ and corresponding function values $Y = (y_1,...,y_n) = (f(x_1),...,f(x_n))$, the Lagrange interpolant allows us to interpolate the function values at any point x using the formula: $P(x) = \sum_{j=1}^{n} f(x_j) L_{n-1,j}(x)$, where $L_{n-1,k}(x) = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{x - x_j}{x_k - x_j}$. Note that the index above runs from 1 to n instead of 0 to n that is used in the textbook. This is because Matlab does not allow the index 0.

The aim of this lab is to write a function for interpolation using Lagrange Polynomials. We wish to write a function that will take (X,Y) and a point x as inputs and return $P(x)$ as output.

Consider the following Matlab code:

```
n=3;
X = linspace(1,2.9,n)';   % equally-spaced x – coordinates. We will vary n.
Y = 1./X;   %y – coordinates– > f(x) = 1/x
% we want to interpolate for values of f(x) at different x values given below.
x = linspace(1,3,100)'   %interpolation points.
for i = 1:100
y(i) = f_lagrange(X,Y,x(i))
end
% here the function f_lagrange takes two vector inputs X, Y
% the point at which we want the interpolated value is the scalar input x
plot(x,y)   %plot the interpolated values
hold on
plot(x,1./x,'-r')   %plot the actual function for comparison in red
```

(1) Write a Matlab code for the function f_lagrange.
(2) Find the difference f(3)-P(3) using n=3, as above
(3) Plot the difference |f(3)-P(3)| versus n, using n=3, 4, 5 and 6.
(4) Produce a single plot showing the exact function f(x)=1/x and the Lagrange interpolants using n=3, 4 and 6.