(T-10) + исследовать на состоятельность

d) $H_0: \xi \sim R(0,1)$ $\qquad \alpha = 0{,}05$

$G_{кр}: x_{min} < c$

$P(\vec{x}_n \in G_{кр} | H_0) = \alpha$

$P(x_{min} < c \,|\, H_0) = \alpha$

$\alpha = P(x_{min} < c) = 1 - (1 - F_0(c))^n \Rightarrow (1 - F_0(c))^n = 1 - \alpha$

$\Rightarrow c = 1 - \sqrt[n]{1-\alpha}$

$\Rightarrow G_{кр}: x_{min} < 1 - \sqrt[n]{1-\alpha}$

$\alpha_1 = \alpha$ (ошибка $I^{го}$ рода)

$W = P(\vec{x}_n \in G_{кр} | H_1) = P(x_{min} < c \,|\, H_1) = 1 - \left(1 - F_1(1 - \sqrt[n]{1-\alpha})\right)^n$

$$H_1: \xi \sim p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0 & x \notin (0,1) \end{cases}$$

$$F_1(x) = \int_{-\infty}^{x} p_1(t)\,dt = \int_0^x e^{-t}dt \cdot \frac{e}{e-1} = \frac{e}{e-1}(1 - e^{-x})$$

$$W = 1 - (1 - F_1(1 - \sqrt[n]{1-\alpha}))^n = 1 - \left(1 - \frac{e}{e-1}\left(1 - e^{\sqrt[n]{1-\alpha} - 1}\right)\right)^n =$$

$$= \cancel{\ldots}\; 1 - \left(1 - \frac{1}{e-1}\left(e - e^{e^{\ln(1-\alpha)^{\frac{1}{n}}}}\right)\right)^n =$$

$$= 1 - \left(1 - \frac{1}{e-1}\left(e - e^{e^{\frac{1}{n}\ln(1-\alpha)}}\right)\right)^n =$$

$$= 1 - \left(1 - \frac{1}{e-1}\left(e - e^{\left(1 + \frac{\ln(1-\alpha)}{n} + O\left(\frac{1}{n}\right)\right)}\right)\right)^n =$$

$$= 1 - \left[1 - \frac{e}{e-1}\left(1 - \left(1 + \frac{\ln(1-\alpha)}{n} + o\left(\frac{1}{n}\right)\right)\right)\right]^n =$$

$$= 1 - \left[1 - \frac{e}{e-1}\left(-\frac{\ln(1-\alpha)}{n} + o\left(\frac{1}{n}\right)\right)\right]^n =$$

$$= 1 - \left[1 + \frac{e}{e-1} \cdot \frac{\ln(1-\alpha)}{n} + o\left(\frac{1}{n}\right)\right]^n \longrightarrow 1 - e^{\frac{e}{e-1}\cdot\ln(1-\alpha)} = 0{,}08$$

$\not\to 1 \Rightarrow$ ~~оценка~~ критерий не является состоятельным

$\alpha_2 = 1 - W = 0{,}92$ (ошибка $2^{го}$ рода)

T-13

$\xi \sim N(a, \sigma_x^2)$ $\quad \eta \sim N(b, \sigma_y^2)$ – независимые

$x = \{-1,11;\ -6,10;\ 2,42\}$ $\quad \bar{x} = -1,6 \quad n = 3$

$y = \{-2,29;\ -2,91\}$ $\quad \bar{y} = -2,6 \quad m = 2$

$H_0: a = b \qquad H_1: a \neq b,\ a > b,\ a < b$

H_0: ~~$\Delta = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}} \sim N(0,1)$~~

~~$\Delta = \frac{-1,6+2,6}{\sqrt{\frac{2}{3}+\frac{1}{2}}}$~~

$$\frac{\bar{x} - a}{\sigma_x}\sqrt{n} \sim N(0,1) \qquad \frac{\bar{y} - b}{\sigma_y}\sqrt{m} \sim N(0,1)$$

$$\bar{x} - \bar{y} - (a - b) \sim N\left(0, \frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}\right)$$

$$\Delta = \frac{(\bar{x} - \bar{y})\sqrt{mn}}{\sqrt{2m + n}} \sim N(0,1)$$

$$\tilde{\Delta} = 0,93$$

$a \neq b$: ~~p-value $= P(|\Delta| \geq |\tilde{\Delta}|) = 2P(\Delta \geq |\tilde{\Delta}|)$~~

$$\text{p-value} = P(|\Delta| \geq |\tilde{\Delta}|\,|H_0) = 2P(\Delta \geq |\tilde{\Delta}|\,|H_0) = 2\int_{0,93}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}\,dx = 0,352 > \alpha = 0,05$$

Нет основания отвергнуть H_0

$a > b$: $\text{p-value} = P(\Delta > \tilde{\Delta}\,|H_0) = \int_{0,93}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx = 0,176 > \alpha$

Нет основания отвергнуть H_0

$a < b$: $\text{p-value} = P(\Delta < \tilde{\Delta}\,|H_0) = \int_{-\infty}^{-0,93} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx = 0,176 > \alpha$

Нет основания отвергнуть H_0.

T-11

$\xi \sim p(x) = \left(\frac{8}{24} + \theta\right)\{4\} + \left(\frac{4}{24} - \theta\right)\{3\} + \frac{1}{4}\{2\} + \frac{1}{4}\{1\}$

$H_1: \ p(x) = \frac{1}{4}\{4\} + \frac{1}{4}\{3\} + \frac{1}{4}\{2\} + \frac{1}{4}\{1\} \Rightarrow$

$\Rightarrow \ \theta = -\frac{1}{12}$

$H_0: \ \theta_1 = 0 \qquad H_1: \ \theta_2 = -\frac{1}{12}$

$\vec{X}_n = (x_1, x_2) \qquad l(\vec{X}_n) = \frac{L_1}{L_0} = \frac{p(x_1, \theta_2)\, p(x_2, \theta_2)}{p(x_1, \theta_1)\, p(x_2, \theta_2)}$

$G_{кр}: \ l \geq C$

l:

$x_2 \backslash x_1$	1	2	3	4
1	1	1	$\frac{3}{2}$	$\frac{3}{4}$
2	1	1	$\frac{3}{2}$	$\frac{3}{4}$
3	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{9}{8}$
4	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{8}$	$\frac{9}{16}$

H_0:

$x_2 \backslash x_1$	1	2	3	4
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
3	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{18}$
4	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{9}$

H_1:

$x_2 \backslash x_1$	1	2	3	4
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

$\alpha_1 = P(\vec{X}_n \in G_{кр} \mid H_0) \leq 0{,}2 = \alpha$

$\alpha_1 = \frac{1}{36} + 4 \cdot \frac{1}{24} = \frac{7}{36} = 0{,}194 < \alpha$

Если взять минимально хотя бы $\frac{1}{18}$ $\Rightarrow$ сумма будет $> \alpha$

$W = P(\vec{X}_n \in G_{кр} \mid H_1) = 5 \cdot \frac{1}{16} = \frac{5}{16} \Rightarrow \alpha_2 = 1 - W = \frac{11}{16}$

T-12

$H_0: \sigma^2 = 0{,}1 \qquad H_1: \sigma^2 > 0{,}1$

$n = 25 \qquad S'^2 = 0{,}2$

$P(\Delta > C \mid H_0) = \alpha \Rightarrow$

$$\Rightarrow \int_C^{+\infty} q_{\chi^2(24)}(x)\,dx = \alpha \Rightarrow C = 36{,}4$$

$G_{кр}: \ \Delta > 36{,}4$

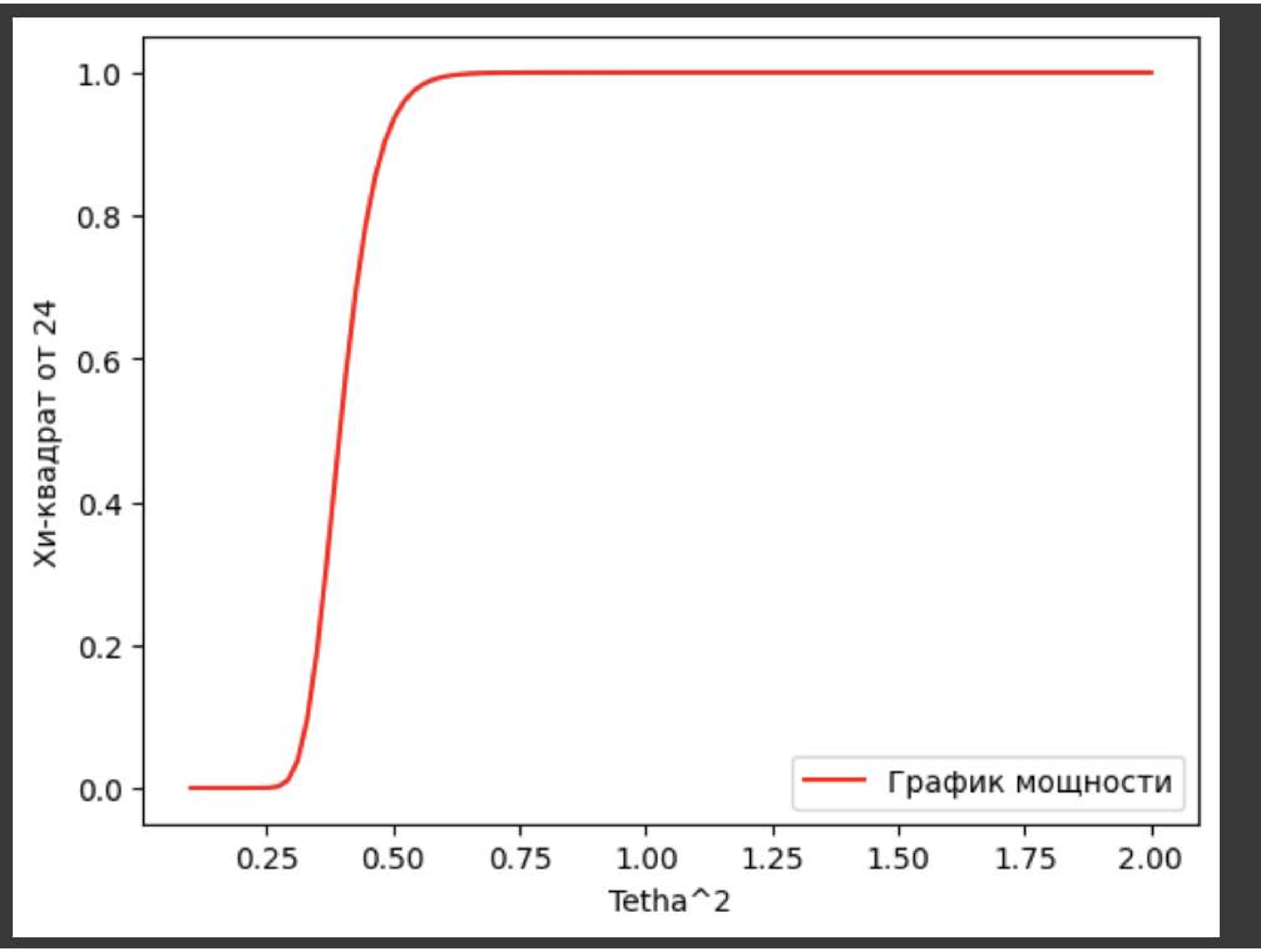
$W = P(\vec{X_n} \in G_{кр} \mid H_1) = P(\Delta > C \mid H_1) =$

$$= \left\{ \Delta = \frac{S'^2(n-1)}{\theta^2} \sim \chi^2(n-1) \text{ по т. Фишера} \right\} =$$

$$= P\left(\frac{S'^2(n-1)}{\sigma^2} > C \mid H_1\right) = P\left(\frac{S'^2(n-1)}{\theta^2} > \frac{C\cdot\sigma^2}{\theta^2} \mid H_1\right) =$$

$$= \int_{\frac{C\sigma^2}{\theta^2}}^{\infty} q_{\chi^2(24)}(x)\,dx$$

Т.е. график мощности: $q_{\chi^2(24)}(x)dx \quad \forall x = \frac{C\sigma^2}{\theta^2}: \ \theta^2 > 0{,}1$



1.0
0.8
0.6
0.4
0.2
0.0
Хи-квадрат от 24
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Tetha^2
График мощности