Year 12 January Further Mathematics Test

Section A: Pure (Total: 94 marks)

Question 1

a) Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{3}$.

i. $\sqrt{108}+\sqrt{3}$. **(1)**

ii. $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{2}+1}$. **(3)**

b) Solve the equation

$$(5-x)^{\frac{3}{2}}=8.$$ **(3)**

Detailed working must be shown in this question.

Question 2

The curve C has equation

$$y=\sqrt{2x}, \quad x \geq 0$$

Find an equation of the normal to C at the point where $x=2$, giving the answer in the form $y=mx+c$, where m and c are constants. **(6)**

Question 3

Solve the following trigonometric equation in the range given.

$$\frac{3+\sin^2\theta}{\cos\theta-2}=3\cos\theta, \quad 0° \leq \theta < 360°.$$ **(8)**

Question 4

A circle C has equation

$$x^2 + y^2 + ax + by + 43 = 0,$$

where a and b are constants.

a) Given that the points $(-4,7)$ and $(-2,5)$ lie on C, determine the coordinates of the centre of C and the size of its radius. **(6)**

A straight line passes through the point $P(4,5)$ and is a tangent to C at the point Q.

b) Show that the length of PQ is $4\sqrt{3}$. **(3)**

Question 5

$$f(x) = (3-2x)^2(1+2x)^6.$$

Find the binomial expansion of $f(x)$ in ascending powers of x, up and including the term in x^3. **(6)**

Question 6

Find the range of values of the non zero constant k, given that the quadratic equation

$$3kx^2 - 2kx - 4x + 3 = 0$$

has two different real roots. **(7)**

Question 7

The curve C has equation

$$y = 5x - 4x^2 - x^3.$$

a) Express y as a product of linear factors. **(2)**

b) Sketch the graph of C.

The sketch must include the coordinates of all the points where the curve meets the coordinate axes. **(3)**

c) Hence sketch the curve with equation

$$y = 5(x-2) - 4(x-2)^2 - (x-2)^3,$$

clearly showing the coordinates of all the points where the curve meets the coordinate axes. **(3)**

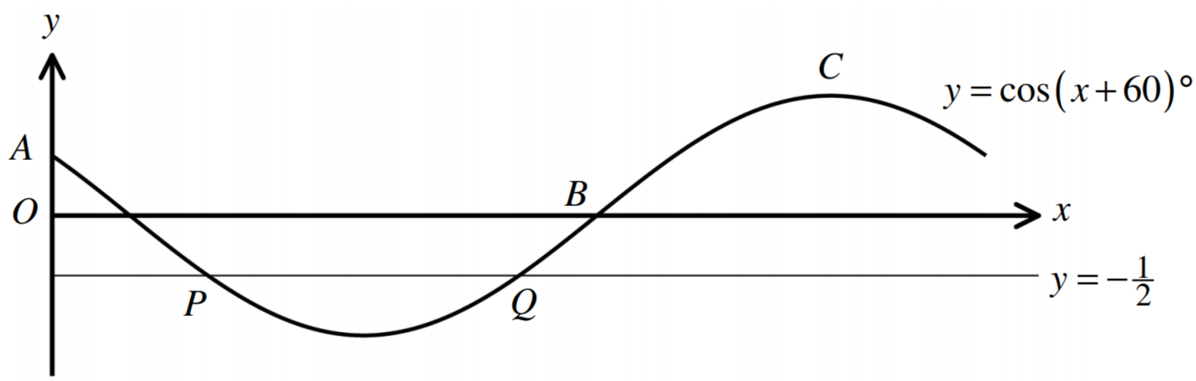
Question 8

Solve each of the following equations.

a) $\frac{1}{2} \times 4^{3x+1} = 600^{600}$. **(6)**

b) $\log_3(2y+5) = 1 - \log_3 y$. **(6)**

Question 9



The figure above shows the graph of

$$y = \cos(x+60)°,\ 0 \le x \le 360.$$

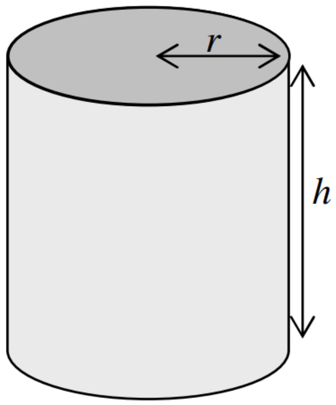
The graph meets the y axis at the point A and the point B is one of the two x intercepts of the curve. The point C is the maximum point of the curve.

a) State the coordinates of A, B and C. **(3)**

The straight line with equation $y = -\frac{1}{2}$ meets the graph of $y = \cos(x+60)°$ at the points P and Q.

b) Determine the coordinates of P and Q. **(5)**

Question 10



The figure above shows a **closed** cylindrical can of radius r cm and height h cm.

a) Given that the surface area of the can is 192π cm^2, show that the volume of the can, V cm^3, is given by

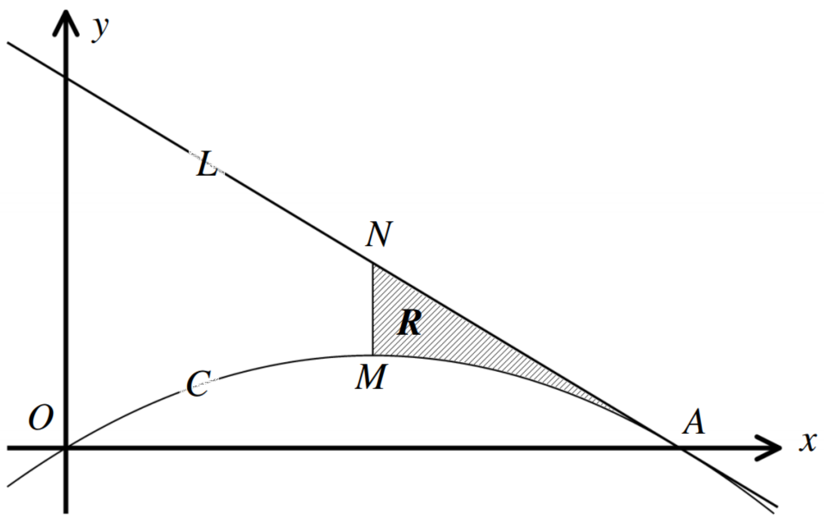
$$V = 96\pi r - \pi r^3 .$$ **(4)**

b) Find the value of r for which V is stationary. **(5)**

c) Justify that the value of r found in part **(b)** gives the maximum value for V. **(2)**

d) Calculate the maximum value of V. **(1)**

Question 11



The figure above shows the graph of the curve C with equation

$$y = 6x - x^2, \; x \in \mathbb{R}.$$

The curve meets the x axis at the origin O and at the point A. The straight line L is the tangent to C at A.

a) Find an equation of L. **(4)**

The point M is the maximum point of C. The point N lies on L so that MN is parallel to the y axis. The finite region R, shown shaded in the figure above, is bounded by C, L and the straight line segment MN.

b) Determine the area of R. **(7)**