Demo: Counterpoint Analysis and Synthesis

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Abstract

We present *Music Tools*, an Agda library for analyzing and synthesizing music. The library uses dependent types to simplify encoding of music rules, thus improving existing approaches based on simply typed languages. As an application of the library, we demonstrate an implementation of first-species counterpoint, where we use dependent types to constrain the motion of two parallel voices.

Keywords: counterpoint, Haskell, SMT

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1 Introduction

We demonstrate work in progress on a tool to assist in the analysis of synthesis of musical counterpoint. Since the mid-18th century, the composition of counterpoint has been guided by principles enunciated in Fux's *Gradus ad Parnassum* [Fux 1965], first published in 1725. Fux presents an increasingly sophisticated series of "species" (one note against one note, two notes against one note, etc.) along with rules governing intervals between notes and motion between intervals designed to ensure consonance and independence of voices. It is well documented that composers including Haydn, Mozart and Beethoven both studied and taught from this text, and its fundamentals continue to taught to music students today (for example [Aldwell and Cadwallader 2018; Kennan 1999]).

In previous work, Cong and Leo [Cong and Leo 2019] encode the rules of first-species (note against note) counterpoint as type constructors in the dependenty-typed language Agda. This enforces correct-by-construction counterpoint and allows use of the Agda typechecker to return errors with no additional effort required. On the down side, these errors can be difficult to interpret for those less familiar with type errors. Furthermore, encoding rules into constructors proved awkward for handling more complex species and more global constraints. One could separate the construction of the pure music from the constraints which can be added as a local or global predicate, but one may also wish to deliberately violate some of the rules of strict counterpoint, as composers often do in practice, and simply be informed of where the violations occur without being prohibited from incorporating them.

Another approach then is to write a special-purpose "type checker" which can be run on previously-created music and which can generate clear and precise error messages which are musically meaningful. The checker is ideally easily customizable in terms of what constraints one would like to impose. It turns out these constraints can be expressed in the quantifier-free logic of linear arithmetic and uninterpreted functions (QF-UFLIA [Barrett et al. 2010]), which allows one to not only analyze existing counterpoint, but also synthesize counterpoint satisfying the constraints using an SMT solver.

This tool has been implemented in Haskell and is available at [Leo 2022]. Haskell was chosen as a high-quality and feature-rich SMT library called SBV [Erkök 2022] is available.

Cong [Cong 2022].

To analyze and synthesize tonal music, researchers have attempted to encode these rules into programming languages. Functional programming languages seem ideally suited for this task, and in particular, those with a static type system can further guarantee that *well-typed music does not sound wrong*. In the past decade, Haskell has been a popular choice for encoding the rules of harmony [De Haas et al. 2011, 2013; Koops et al. 2013; Magalhães and de Haas 2011; Magalhães and Koops 2014] as well as counterpoint [Szamozvancev and Gale 2017]. An interesting observation is that, many of the existing encodings rely on some form of *dependent types*, i.e., types that depend on terms. While Haskell is not a dependently typed language, it has various language extensions (such as GADTs [Cheney and Hinze 2002] and singleton types [Eisenberg and Weirich 2013]) for simulating dependent types, and as shown by previous studies, the extended type system is expressive enough for a wide class of music applications. On the other hand, the need for simulating dependent types can also result in code duplication, making the implementation less elegant [Monnier and Haguenauer 2010]. This motivates us to explore music programming in a language with intrinsic support for dependent types.

We present Music Tools[1], a library of small tools that can be combined functionally to help analyze and synthesize music. To allow simple and natural encoding of rules, we build our library in Agda [Norell 2007], a functional language with full dependent types. As an application of the library, we demonstrate an implementation of species counterpoint,

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[1] https://github.com/halfaya/MusicTools

based on the rules given by Fux [1965]. Thanks to Agda's rich type system, we can express the rules in a straightforward manner, and thus ensure by construction that well-typed counterpoint satisfies all the required rules.

2 The Music Tools Library

The goal of Music Tools is to provide a core collection of types and functions that can be used to easily build applications to analyze and synthesize music. It is intended to evolve into a dependently-typed replacement for Euterpea [Hudak and Quick 2018], and borrows many ideas from that library. Currently, Music Tools is restricted to the chromatic scale for simplicity, and like Euterpea is designed to work well with MIDI.

The fundamental type for melody is `Pitch`, which is just a natural number with 0 representing the lowest expressible pitch (it is intended to correspond to MIDI note 0, but the interpretation can change depending upon the application). One can also express pitch as a pair of an octave and relative pitch within the octave, which is more convenient for some applications, and convert between this representation and absolute pitch. One can then prove that converting back and forth is the identity function.

For rhythm the fundamental type is `Duration`, also a natural number, which represents some unspecified unit of time (when the music is played the unit is then specified). A combination of a pitch and a duration gives a `Note`, and notes in turn are made into `Music` via sequential and parallel composition, as in Euterpea.

To play music on synthesizers, one can convert it to MIDI by calling Haskell MIDI libraries via Agda's Haskell FFI. Details can be found in the GitHub repository.

3 Application: First-Species Counterpoint

We now explain how to implement the rule system of first-species counterpoint[2]. In first-species counterpoint, one starts with a base melody (the *cantus firmus*), and constructs a counterpoint melody note-by-note in the same rhythm. The two voices are represented as a list of pitch-interval pairs, where intervals must not be dissonant (2nds, 7ths, or 4ths).

```
data IntervalQuality : Set where
  min3  : IntervalQuality
  maj3  : IntervalQuality
  per5  : IntervalQuality
  min6  : IntervalQuality
  maj6  : IntervalQuality
  per8  : IntervalQuality
  min10 : IntervalQuality
  maj10 : IntervalQuality
```

```
PitchInterval : Set
PitchInterval = Pitch × IntervalQuality
```

In addition, it is prohibited to move from any interval to a perfect interval (5th or octave) via parallel or similar motion. Therefore, we define a predicate that checks whether a motion is allowed or not.

```
motionOk : (i1 : Interval)
           (i2 : Interval) → Set
motionOk i1 i2 with motion i1 i2
         | isPerfectInterval i2
motionOk i1 i2 | contrary | _     = ⊤
motionOk i1 i2 | oblique  | _     = ⊤
motionOk i1 i2 | parallel | false = ⊤
motionOk i1 i2 | parallel | true  = ⊥
motionOk i1 i2 | similar  | false = ⊤
motionOk i1 i2 | similar  | true  = ⊥
```

The last requirement is that the music must end with a cadence, which is a final motion from the 2nd or 7th degree to the tonic (1st degree). We impose this requirement by declaring two cadence constructors as the base cases of counterpoint (note that the final interval of the cadence is always `(p , per8)` and hence is not explicitly specified). Thus, we arrive at the following datatype for well-typed counterpoint[3].

```
data FirstSpecies : PitchInterval →
                    Set where
  cadence2 : (p : Pitch) →
    FirstSpecies (transpose (+ 2) p , maj6)
  cadence7 : (p : Pitch) →
    FirstSpecies (transpose -[1+ 0 ] p , min10)
  _::_ : (pi : PitchInterval) →
         {pj : PitchInterval} →
         {_ : motionOk pi pj} →
         FirstSpecies pj →
         FirstSpecies pi
```

Observe that `motionOk` is an implicit argument of the `_::_` constructor, here denoted by the curly braces. The argument can be resolved automatically by the type checker, hence there is no need to manually supply this proof.

Now we can write valid first-species counterpoint as in the example below.

```
example : FirstSpecies (g 4 , per8)
example =
  (g 4 , per8) :: (c 5 , maj10) ::
  (c 5 , per8) :: (c 5 , maj10) ::
  (e 5 , min10) :: (g 5 , per8) ::
  (cadence2 (c 6))
```

[2]The code is available at https://github.com/halfaya/MusicTools/blob/master/agda/Counterpoint.agda.

[3]For readability, we have omitted explicit conversions from `PitchInterval` (which ensures the interval is not dissonant) to the general `Interval`.

4 Future Work

We intend to expand the Music Tools library to include richer functionality for analysis and synthesis. We are particularly interested in expressing the work done in Haskell on functional harmony in the library, to see how much dependent types can simplify the representation.

Music is a rich yet circumscribed domain in which issues of equivalence [Tabareau et al. 2018] and ornamentation [Dagand 2017] naturally arise. For example, a musical score can be interpreted either horizontally (counterpoint) or vertically (harmony), and it is important to be able to seamlessly convert between these representations. Similarly, one may wish to treat pitch or rhythm separately as well as combine them, which is a natural application of ornamentation. We plan to examine the extent to which this research can be put to use in a practical domain.

For counterpoint specifically, we intend to represent higher species counterpoint by extending the rules, and to explore automatic generation of species counterpoint. It would be interesting to compare our correct-by-construction counterpoint with that created by machine learning [Huang et al. 2017], which does not have correctness guarantees.

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