Differences of Types*

Subtitle[†]

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See Introduction.

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1 INTRODUCTION

This document is a place to record my thoughts on the Coq proof repair project headed by Talia Ringer, Nate Yazdani, and Dan Grossman. Everything in it is preliminary.

2 DIFFERENCES OF TYPES

Dependently-typed functional programs and mathematical objects which can be manipulated algebraically. One algebraic operation that can be performed is to take the difference of two types. Given types A:T and B:T of the same sort T this difference B-A can be expressed as a function from A to B:

$$f: A \to B$$

This is perhaps not very interesting, and of course not in general unique. We can also look at the difference between two terms of different types. Given a: A and b: B, express b-a as

$$f_{ab}: A \to B \ni f(a) = b$$

This function is again not unique in general, and in fact always has the trivial solution $f_{ab}(\cdot) = b$.

The existence of a should provide some help in calculating b. We would like to capture somehow the notion of using a "maximally", and then defining b-a to be the function that thus takes the minimal amount of effort to go from a to b.

Given that a and b are themselves algebraic structures, it should be possible to quantify the size of a term as either the size or depth of its AST. Denote |b| as the size of b. Our definition of b-a could then be a function $f_{ab}:A\to B$ such that $|f_{ab}(a)|$ is minimal, where we set |a| to be 1.

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Note that we do not say what happens when the argument to f_{ab} is not a. We could arbitrarily set the output to b always in this case. More interesting would be to handle any term of type A, using only information about the types A and B. In this case we could define B-A to be any function $f:A\to B$ that minimizes |f(a)| for all $a\in A$.