Consequences of Probability Axioms

Review

Definitions

Basic definitions

Elementary outcomes

Sample space

Event

Probability

Set theory

$$\Omega = \{1, \cdots, 10\}, \quad A = \{1, 2, 3\} \quad B = \{2, 4, 5\}$$

$$A \cup B \qquad A \cap B \qquad A \backslash B$$

$$\emptyset \qquad disjoint$$

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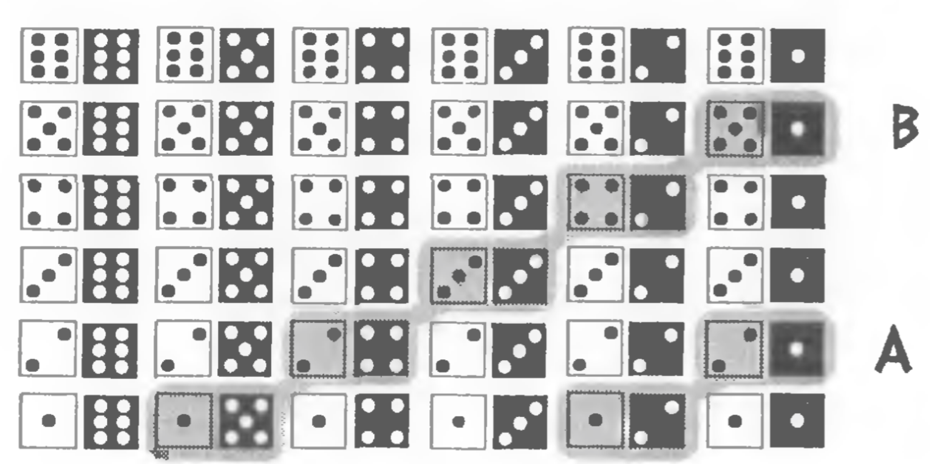
Review

Definitions

Basic operations

Let A, B be **disjoint** events. By Kolmogorov's 3rd axiom:



$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

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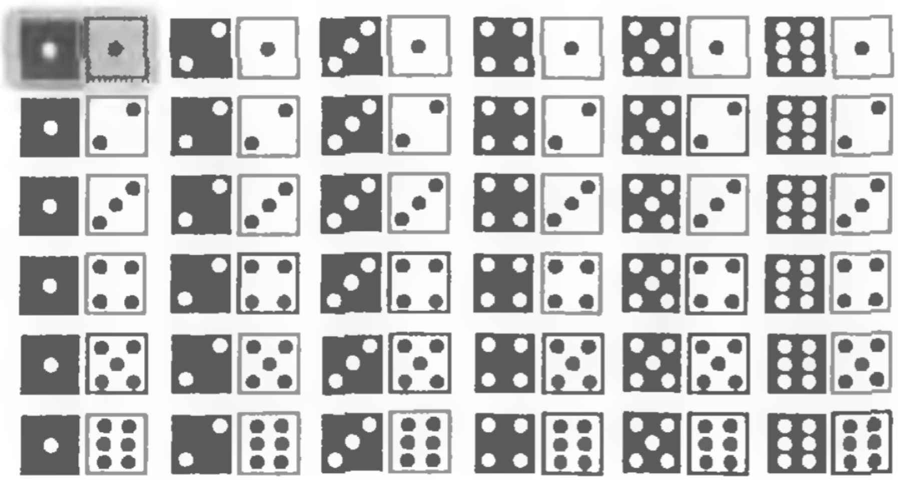
Review

Definitions

Basic operations

Let A be the event "double ones".



$$P(not\ A) = P(A^c) = P(\Omega \backslash A) = P(\Omega) - P(A) = 1 - P(A)$$

Review

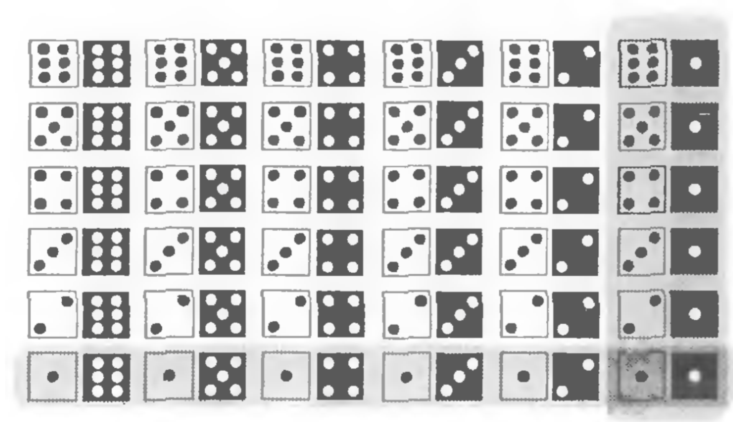
Definitions

Basic operations

Let E be the event "at least one die rolls one".

Note this can be broken as two events:

A: "white die is one" and B: "black die is one"



$$P(E) = P(A \text{ } or \text{ } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

These are all consequences of the three axioms!

Conditional probability

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Conditional probability

Formula

What is it?

It allows you to **introduce information** into your calculations.

How does it do it?

By reducing the sample space, then renormalizing the probabilities.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

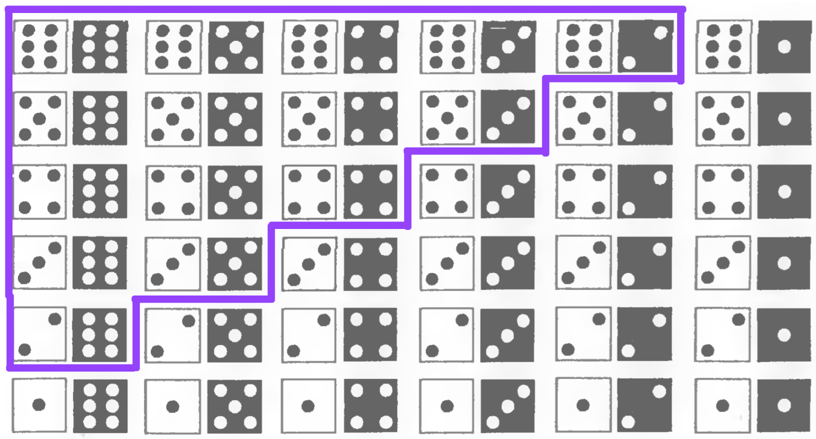
Conditional probability

Formula

Example 1

Let A be the event that the sum of two dice rolls is 8 or greater.

What is P(A)?



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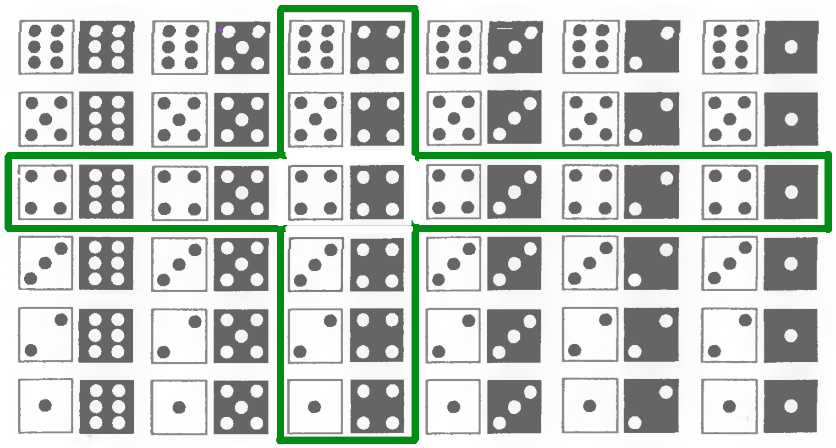
Conditional probability

Formula

Example 1

Now suppose you receive information that one of the die rolled a four.

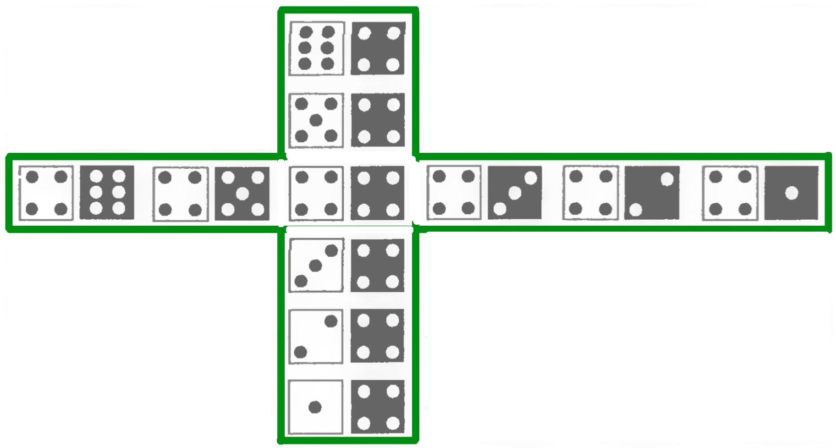


Conditional probability

Formula

Example 1

We can safely discard all other possibilities.



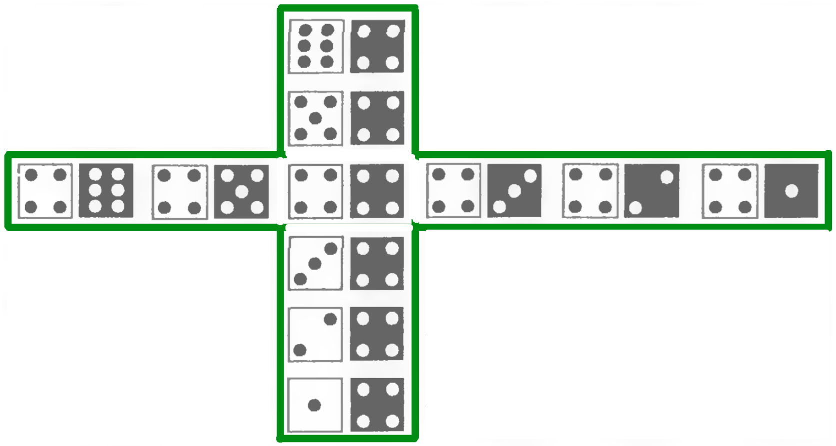
What happened to the sample space?

Conditional probability

Formula

Example 1

We can safely discard all other possibilities.



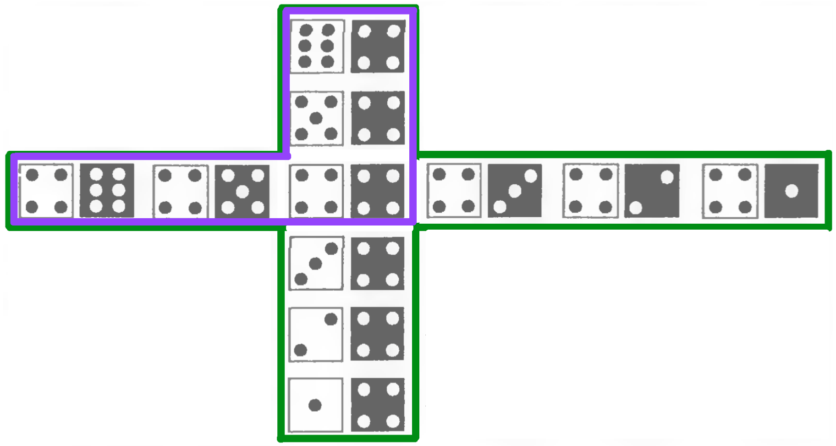
What happened to the sample space?

Conditional probability

Formula

Example 1

Now these are the only elementary outcomes that matter.



What set is this?

How many favorable elementary outcomes are there?

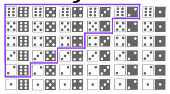
Out of how many possible outcomes?

Conditional probability

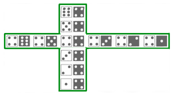
Formula

Example 1

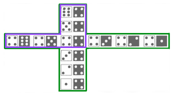
We started by looking at A:



But then we learned only a subset B could have happened:



The relevant outcomes became those in $P(A \cap B)$



Since there are 5 elements in $A \cap B$ and 12 in B

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{5}{12}$$

Conditional probability

Formula

Example 1

Example 2

Non-naive probability example

		Intelligence low	Intelligence high	
Grade	A	0.07	0.18	0.25
	B	0.28	0.09	0.37
	C	0.35	0.03	0.38
		0.7	0.3	1

Think about the probabilities of:

$$P(High \cup A)$$

$$P(High), P(C)$$

$$P(High|C), P(C|High)$$

More examples later [ ]

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Conditional probability

Formula

Example 1

Example 2

Takeaway

"Conditioning is the soul of statistics"*

Conditional probability shows up all the time *in the wild*.

Unconditional probabilities are boring:

P(College), P(Return on Investment), P(Click on Ad)

Much more informative:

P(College | Rich, Female, Hispanic)

P(RoI | Technology, Startup, Cambridge)

P(Click on Ad | Browse history, Ad Placement)

* Joe Blitzstein (link)