Probability

Basics

Experiment

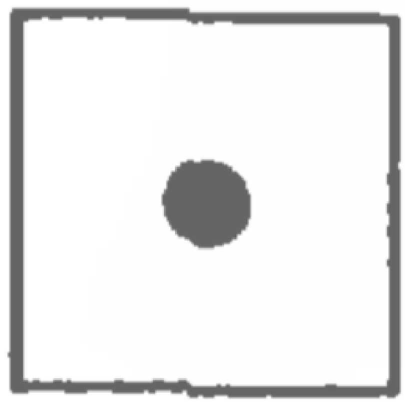
- Observation of random occurence
- Toy examples:
  - Rolling a die, tossing a coin
- Serious examples:
  - Dropping a ball and a feather in vacuum
  - Measuring light distorion
- How about these?
  - Measuring economic effect of earthquakes
  - Vietnam draft lottery effect on education and wages

Basics

Experiment

Elementary outcomes

- Each unique possible outcome
- Toy example:



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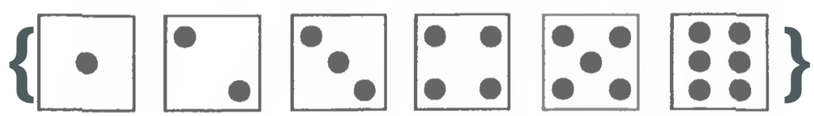
Basics

Experiment

Elementary outcomes

Sample space and event

- The **sample space** is the set of elementary outcomes, usually denoted by Ω



- An **event is a subset of the sample space.**

* **A set is a collection of distinct objects**

Basics

Experiment

Elementary outcomes

Sample space and event

Remember your set operations?

Let A = {1,2,3}, B = {2,4,5} be two subsets of Ω = {1, ..., 10}

$$A \cup B$$

$$A \cap B$$

$$A - B = A \backslash B$$

$$A^c$$

$$|A|$$

$$\emptyset = \{\,\}$$

Basics

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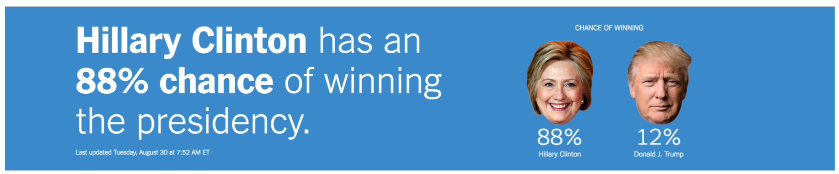
Probability

Defining probability

That's hard! Our first impulse is to talk about **frequencies**:

If we toss a coin an infinite number of times, heads will come up 50% of the time.

However*:



We need a working definition.

*See more on page 35 of the textbook.

Probability

Naive definition

Classical or Naive probability

$$P_{Classical}(A) = \frac{|A|}{|\Omega|} = \frac{\text{\# Favorable outcomes}}{\text{\# Total outcomes}}$$

When is this definition applicable?

- Symmetry
- Design

When is it not applicable?

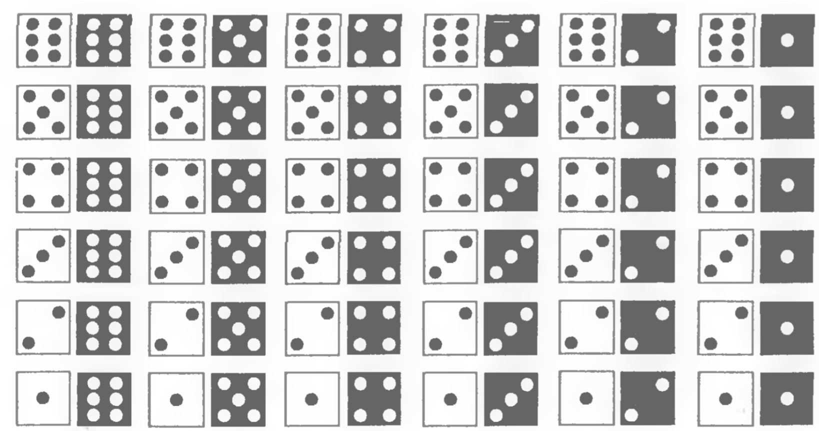
Probability

Naive definition

Examples

Two dice are rolled

Take the sum of the pips. Is an 11 more likely than a 12?



Probability

Naive definition

Examples

Counting

Multiplication rule

In English: There are 6 ways of pairing 2 entrees and 3 deserts

- When an experiment E can be split into two sub-experiments A and B, the total number of outcomes in E equals the numbers of outcomes in A and B multiplied.

"Proof": Tree branching.

Applications [ ]:

- Sampling with replacement
- Permutations
- Sampling without replacement

Probability

Naive definition

Examples

Counting

Non-naive definition

Kolmogorov's axioms:

Let Ω be the sample space, and let A be an event.

1. Positivity

$$P(A) \geq 0$$

2. Unitarity

$$P(\Omega) = 1$$

3. Additivity* If A and B are disjoint

$$P(A \cup B) = P(A) + P(B)$$

All other properties can be derived from these [ ]

* Actually, Kolmogorov's axioms hold for an infinite collection of sets.

Probability

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Examples

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Proof example

Proving properties

Let's prove that

$$P(A^c) = 1 - P(A)$$

Thee probabiliy of A not happening equals 1 minus the probabiliy of it happening

$$P(\Omega) = 1 \qquad \text{From axiom 2}$$

$$P((\Omega \backslash A) \cup A) = 1 \qquad \text{Def of union}$$

$$P(A^c \cup A) = 1 \qquad \text{Def of complement}$$

$$P(A^c) + P(A) = 1 \qquad \text{Axiom 3}$$

$$P(A^c) = 1 - P(A) \qquad \text{Rearranging}$$

Done! Have fun doing your homeworks.