# ST2131 Cheatsheet

## 1. Random Variables

### Cumulative Distribution Function (c.d.f.)

$$F_X(x) = P(X \in (-\infty, x]) = P(X \leq x)$$

### Probability Mass Function (p.m.f.)

$$f_X(x) = P(X = x)$$

### Probability Density Function (p.d.f)

$$f_X(x) = \frac{dF_X(x)}{dx}$$

- $\int_{-\infty}^{\infty} f_X(x)\ dx = 1$
- $P(a < x < b) = \int_a^b f_X(x)$

### Expectation

$$E[X] = \begin{cases} \Sigma_i\ x_i \cdot f_X(x_i) & \text{if discrete} \\ \int_{-\infty}^{\infty} x \cdot f_X(x)\ dx & \text{if continuous} \end{cases}$$

$$E[g(X)] = \begin{cases} \Sigma_i\ g(x_i)_X(x_i) & \text{if discrete} \\ \int_{-\infty}^{\infty} g(x) \cdot f_X(x)\ dx & \text{if continuous} \end{cases}$$

- Comparison: if $P(X \geq a) = 1$, then $E[X] \geq a$
- Linearity: $E[aX + b] = aE[X] + b$
- $E[XY] = E[X] \cdot E[Y]$ if X and Y are independent

### Variance

$$Var(X) = E[(X - [X])^2] = E[X^2] - E[X]^2$$
$$\sigma_X = \sqrt{Var(X)}$$

- $Var(aX + b) = a^2 \cdot Var(X)$
- $Var(X + Y) = Var(X) + Var(Y)$ if X and Y are independent

## 2. Discrete Prob Distributions

### Bernoulli Distribution

$$X \sim Ber(p) \leftrightarrow P(X = 1) = p \wedge P(X = 0) = 1 - p$$

Interpretation: 1 trial with success or failure

- Mean: $E[X] = p$
- Variance: $Var(X) = p(1 - p)$

### Binomial Distribution

$$Y = X_1 + X_2 + ... + X_n$$
$$Y \sim Bin(n, p) \leftrightarrow P(Y = k) = \binom{n}{k}\ p^k(1 - p)^{n-k}$$

Interpretation: number of successes in $n$ trials

- Mean: $E[X] = np$
- Variance: $Var(X) = np(1 - p)$

### Poisson Distribution

$$X \sim Pois(\lambda) \leftrightarrow P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

$$e^{\lambda} = \Sigma_{k=0}^{\infty} \frac{\lambda^k}{k!} = 1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6} + ...$$

Poisson Limit Theorem

Let $Y_n \sim Bin(n, \frac{\lambda}{n})$, $X \sim Pois(\lambda)$. Then $lim_{n\to\infty} P(Y_n = k) = P(X = k)$

- Mean: $E[X] = \lambda$
- Variance: $Var(X) = \lambda$

### Discrete Uniform Distribution

$$P(X = x_i) = \frac{1}{n}$$

- Mean: $E[X] = \frac{1}{n}\Sigma_{i=1}^{n} x_i$
- Variance: $Var(X) = E[X^2] - E[X]^2 = \frac{1}{n}\Sigma_{i=1}^{n} x_i^2 - (\frac{1}{n}\Sigma_{i=1}^{n} x_i)^2$

### Geometric Distribution

$$X \sim Geom(p) \leftrightarrow P(X = k) = p(1 - p)^{k-1}$$
$$\text{Tail probability: } P(X \geq k) = (1 - p)^{k-1}$$

Interpretation: waiting time for 1st success

- Memoryless: $P(X = i + k \mid X = k) = P(X = i)$
- Mean: $E[X] = \frac{1}{p}$
- Variance: $Var(X) = \frac{1-p}{p^2}$

### Negative Binomial Distribution

(don't remember me)

$$X \sim NB(r, p) \leftrightarrow P(X = n) = \binom{n-1}{r-1}\ p^r(1 - p)^{n-r}$$

Sum of $r$ iid Geom(p) random variables

Interpretation: waiting time for $r$ successes

- Mean: $E[X] = \frac{r}{p}$
- Variance: $Var(X) = \frac{r(1-p)}{p^2}$

### Hypergeometric Distribution

(don't remember me)

$$X \sim H(n, N, m) \leftrightarrow P(X = i) = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$$

Interpretation: number of successes in choosing $n$ of $N$, where $m$ of $N$ give success and *(N-m)* of $N$ give failure (without replacement!)

- Mean: $E[X] = n \cdot \frac{m}{N}$
- Variance: $Var(X) = \frac{N-n}{N-1} \cdot n \cdot \frac{m}{N}(1 - \frac{m}{N})$

## 3. Continuous Prob Distributions

### Uniform Distribution

$$X \sim U(a, b)\ \leftrightarrow\ f_X(x) = \frac{1}{b-a} 1_{[a,b]}(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in (a, b) \\ 0 & \text{if } x \notin (a, b) \end{cases}$$

- Mean: $E[X] = \frac{a+b}{2}$
- Variance: $Var(X) = \frac{(b-a)^2}{12}$

### Exponential Distribution (rmb pdf)

$$X \sim Exp(\lambda)\ \leftrightarrow\ f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

Intepretation: waiting time for one event to happen

- $\lambda$ is rate at which 'clock' ticks
- Similar to *geometric distribution* (limit as time interval $\to 0$)
- Memoryless: $P(X \geq s + t \mid X \geq t) = P(X \geq s)$
- Mean: $E[X] = \frac{1}{\lambda}$
- Variance: $Var(X) = \frac{1}{\lambda^2}$
- Tail probability: $P(X > t) = e^{-\lambda t}$

### Gamma Distribution (no rmb pdf)

$$X \sim \Gamma(\alpha, \lambda)\ \leftrightarrow\ f_X(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Interpretation: waiting time for $\alpha$ events to happen

- $\Gamma(x) = \int_0^{\infty} x^{\alpha-1} e^{-x}\ dx$
- Sum of $\alpha$ independent $Exp(\lambda)$ RVs
- Similar to negative binomial
- Mean: $E[X] = \frac{\alpha}{\lambda}$
- Variance: $Var(X) = \frac{\alpha}{\lambda^2}$

### Normal Distribution (rmb pdf!)

$$X \sim N(\mu, \sigma^2)\ \leftrightarrow\ f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
$$Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$$

- Mean: $E[X] = \mu$
- Variance: $Var(X) = \sigma^2$