Group 14
Week 5 Lambda Calculus

Exercise 1:
$\lambda x.\, x$It can be used to map a function to another input for example $(\lambda x.\, x)\ (y)$will map to y so we can curry the function in a way.

Exercise 2:
1. b
2. b
3. b

Exercise 3:
1. Normal form: y
2. Normal form: x x
3. Divergent
4. Normal form y y

Exercise 4:
1. $(\lambda y.\, zy)\ a$
   $z\, y\, [y := a]$
   $z\, a$
2. $(\lambda x.\, x)(\lambda x.\, x)$
   $`x\, [x := (\lambda x.\, x)]$
   $(\lambda x.\, x)(\lambda x.\, x)$
   Diverges
3. $(\lambda x.\, xy)(\lambda x.\, xx)$
   $x\, y\, [x := (\lambda x.\, xx)]$
   $(\lambda x.\, xx)\, y$
   $x\, x\, [x := y]$
   $y\, y$
4. $(\lambda z.\, z)(\lambda a.\, aa)(\lambda z.\, zb)$
   $z\, [z := (\lambda a.\, aa)](\lambda z.\, zb)$
   $(\lambda a.\, aa)(\lambda z.\, zb)$
   $aa[a := (\lambda z.\, zb)]$
   $(\lambda z.\, zb)\ (\lambda z.\, zb)$
   $z\, b\, [z := (\lambda z.\, zb)]$
   $(\lambda z.\, zb)\ b$
   $zb\, [z := b]$
   bb

Exercise 5:

1. λx.zx
   z
2. $\lambda x.\, xz$
   z
3. $(\lambda x.\, bx)(\lambda y.\, ay)$
   ba

Exercise 6:
1, 2 and 3

Exercise 7:

λf.(λx.f(xx))(λx.f(xx))g

(λx.f(xx))(λx.f(xx)) [f := g]

(λx.g(xx))(λx.g(xx))

g(xx) [x := (λx.g(xx))]

g((λx.g(xx))(λx.g(xx)))

Diverges:

g(Yg) = g(g(Yg)) = g(g(g(Yg)))...

Exercise 8:

1. Not False:
    (\x. IF x FALSE TRUE) FALSE
    IF x FALSE TRUE [x:=FALSE]
    IF FALSE FALSE TRUE
    (λbtf.b t f) FALSE FALSE TRUE
    b t f [b:= FALSE , t:= FALSE, f:= TRUE]
    FALSE FALSE TRUE
    λxy.y FALSE TRUE
    y [y:=TRUE]
    TRUE

2. IF (OR TRUE FALSE)
    \btf. b t f (OR TRUE FALSE)
    b t f [b := OR, t := TRUE, f:=FALSE]
    OR TRUE FALSE
    \xy. IF x TRUE y (TRUE FALSE)
    IF x TRUE y [x:=TRUE, y:=FALSE]
    IF TRUE TRUE FALSE
    \btf. b t f (TRUE TRUE FALSE)
    b t f [b := TRUE, t := TRUE, f:=FALSE]
    TRUE TRUE FALSE
    \xy. x (TRUE FALSE)
    x [x:=TRUE, y:=FALSE]
    TRUE

3. IF (AND TRUE TRUE)
    \btf. b t f (AND TRUE TRUE)
    b t f [b:=AND, t:=TRUE, f:=TRUE]
    AND TRUE TRUE
    \xy. IF x y FALSE (TRUE TRUE)
    IF x y FALSE [x:=TRUE, y:=TRUE]
    IF TRUE TRUE FALSE
    \btf. b t f (TRUE TRUE FALSE)
    TRUE TRUE FALSE
    \xy. x (TRUE FALSE)
    x [x:=TRUE, y:=FALSE]
    TRUE