Exercise 2

1. Implement and plot following fuzzy membership functions over x. Plot each function with three different sets of other parameters. Do not use premade membership functions if such are available.

$$\text{triangular}(x,a,b,c) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c \\ 0 & c \le x \end{cases} \qquad a \le b \le c \qquad (1)$$

$$\text{trapezoid}(x,a,b,c,d) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{d-c} & c \le x \le d \\ 0 & d \le x \end{cases} \qquad a \le b \le c \le d \qquad (2)$$

$$\text{bell}(x,a,b,c) = \frac{1}{1+\left(\frac{x-c}{a}\right)^{2b}} \qquad a > 0 \qquad (3)$$

$$\text{gaussian}(x,c,\sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \qquad \sigma > 0 \qquad (4)$$

2. Let x^* be a set of points $\{x_1^*, x_2^*, \dots, x_N^*\}$ which satisfy $\mu_A(x_i^*) = max_x \mu_A(x)$ for all $x \in \mathbb{R}$ for some fuzzy set A.

   Implement following defuzzification methods, and print their results with a fuzzy set trapezoid (x,10,30,50,90).

$$\text{Smallest-of-max} = \min x^* \qquad (5)$$

$$\text{Largest-of-max} = \max x^* \qquad (6)$$

$$Z_{COA}^* = \frac{\sum_{i=1}^{n} \mu_A(x_i) x_i}{\sum_{i=1}^{n} \mu_A(x_i)} \qquad (7)$$

$$Z_{MOM}^* = \frac{\sum_{i=1}^{N} x_i^*}{N} \qquad (8)$$

3. A single-input and single-output Mamdani fuzzy inference system is described as follows:

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IF X is small THEN Y is small.
IF X is medium THEN Y is medium.
IF X is large THEN Y is large.
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The three membership functions for **inputs** small, medium and large are

trapezoid(*x*,-20,-15,-6,-3),

trapezoid(*x*,-6,-3,3,6) and

trapezoid(*x*,3,6,15,20), respectively.

The three membership functions for **outputs** small, medium and large are

trapezoid(*x*,-2.46,-1.46,1.46,2.46),

trapezoid(*x*, 1.46,2.46,5,7) and

trapezoid(*x*,5,7,13,15), respectively. Note that these are different from input's membership functions.

Calculate defuzzified outputs for inputs x = {-8,-5,0,5,8} (five different outputs) using centroid defuzzification strategy shown in Equation 7. Use same methods presented in lecture notes (check course Moodle page).