COMP 615 (Fall 2021): HW 2
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Section: Mon (7:00 pm – 9:45 pm)

COMP 615 HW 2: Myhill-Nerode and Mealy-Moore Machines

Problem 1. (10 points)

Let X = {λ, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb}
For each element of X·X determine whether it is in L1.
L1 = {w ∈ {a, b}* : #a%2 = 0 & #b%2 = 0}

It means the language L_1 has even number of a's **and** even number of b's.

	λ	a	b	aa	ab	ba	bb	aaa	aab	aba	abb	baa	bab	bba	bbb
λ	✓	✗	✗	✓	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗
a	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗	✓	✗	✓	✓	✗
b	✗	✗	✓	✗	✗	✗	✗	✗	✓	✓	✗	✓	✗	✗	✓
aa	✓	✗	✗	✓	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗
ab	✗	✗	✗	✗	✓	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
ba	✗	✗	✗	✗	✓	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
bb	✓	✗	✗	✓	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗
aaa	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗	✓	✗	✓	✓	✗
aab	✗	✗	✓	✗	✗	✗	✗	✗	✓	✓	✗	✓	✗	✗	✓
aba	✗	✗	✓	✗	✗	✗	✗	✗	✓	✓	✗	✓	✗	✗	✓
abb	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗	✓	✗	✓	✓	✗
baa	✗	✗	✓	✗	✗	✗	✗	✗	✓	✓	✗	✓	✗	✗	✓
bab	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗	✓	✗	✓	✓	✗
bba	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗	✓	✗	✓	✓	✗
bbb	✗	✗	✓	✗	✗	✗	✗	✗	✓	✓	✗	✓	✗	✗	✓

Problem 2. (10 points)

Based upon the table from problem 1, you should know there are at least 4 equivalence classes in R_{L1}. (Though you haven't proven there are only 4. Professor Noga assures you there are only 4.)
Show how the table breaks the 15 different strings into 4 different equivalence classes.
For each equivalence class, give a description of all elements in that class.

class 1:

$\llbracket \lambda \rrbracket = \llbracket aa \rrbracket = \llbracket bb \rrbracket = \{w \in \{a,b\}^* : \#a \% 2 = 0 \ \& \ \#b \% 2 = 0\}$

class 2:

$\llbracket a \rrbracket = \llbracket aaa \rrbracket = \llbracket abb \rrbracket = \llbracket bab \rrbracket = \llbracket bba \rrbracket = \{w \in \{a,b\}^* : \#a \% 2 = 1 \ \& \ \#b \% 2 = 0\}$

class 3:

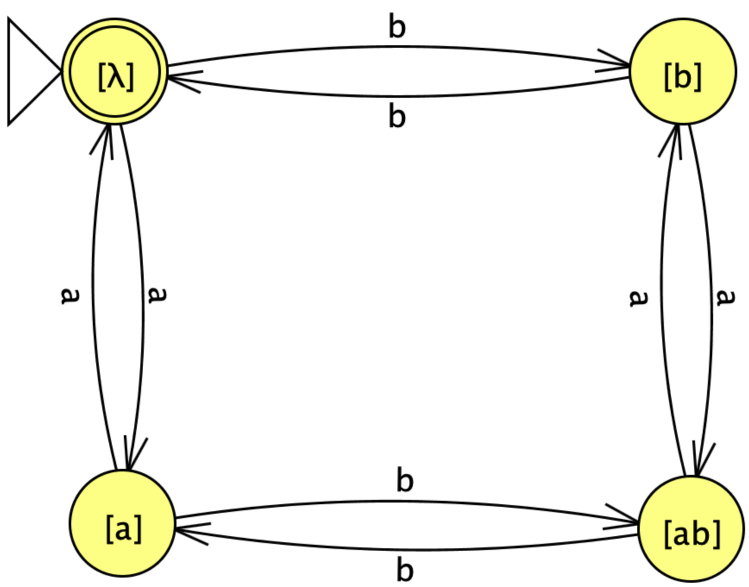
$\llbracket b \rrbracket = \llbracket aab \rrbracket = \llbracket aba \rrbracket = \llbracket baa \rrbracket = \llbracket bbb \rrbracket = \{w \in \{a,b\}^* : \#a \% 2 = 0 \ \& \ \#b \% 2 = 1\}$

class 4:

$\llbracket ab \rrbracket = \llbracket ba \rrbracket = \{w \in \{a,b\}^* : \#a \% 2 = 1 \ \& \ \#b \% 2 = 1\}$

Problem 3. (10 points)

Using the equivalence classes from problem 2, construct the minimum DFA which accepts L1.



Problem 4. (10 points)

Similar to problem 1: Let X = {λ, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb}
For each element of X·X determine whether it is in L_2.
Let L2 = { $a^i b^j$: $1 \leq i \leq 3$ & $j < 4$}

L_2 = { a, aa, aaa, ab, aab, aaab, abb, aabb, aaabb, abbb, aabbb, aaabbb}

	λ	a	b	aa	ab	ba	bb	aaa	aab	aba	abb	baa	bab	bba	bbb
λ	✗	✓	✗	✓	✓	✗	✗	✓	✓	✗	✓	✗	✗	✗	✗
a	✓	✓	✓	✓	✓	✗	✓	✗	✓	✗	✓	✗	✗	✗	✓
b	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
aa	✓	✓	✓	✗	✓	✗	✓	✗	✗	✗	✓	✗	✗	✗	✓
ab	✓	✗	✓	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗
ba	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
bb	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
aaa	✓	✗	✓	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✓
aab	✓	✗	✓	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗
aba	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
abb	✓	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
baa	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
bab	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
bba	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
bbb	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

Problem 5. (10 points)

Similar to problem 2: Show how the table in problem 4 breaks the 15 different strings into different equivalence classes.
For each equivalence class, give a description of all elements in that class.
Find the other equivalence classes (ie strings that are not equivalent to any of these) and describe them as well.

class 1: $[\![\lambda]\!] = \emptyset$

class 2: $[\![a]\!] = \{a\}$

class 3: $[\![aa]\!] = \{aa\}$

class 4: $[\![aaa]\!] = \{aaa\}$

class 5: $[\![ab]\!] = [\![aab]\!] = [\![aaab]\!] = \{a^i b : 1 \leq i \leq 3\}$

class 6: $[\![abb]\!] = [\![aabb]\!] = [\![aaabb]\!] = \{a^i b^2 : 1 \leq i \leq 3\}$

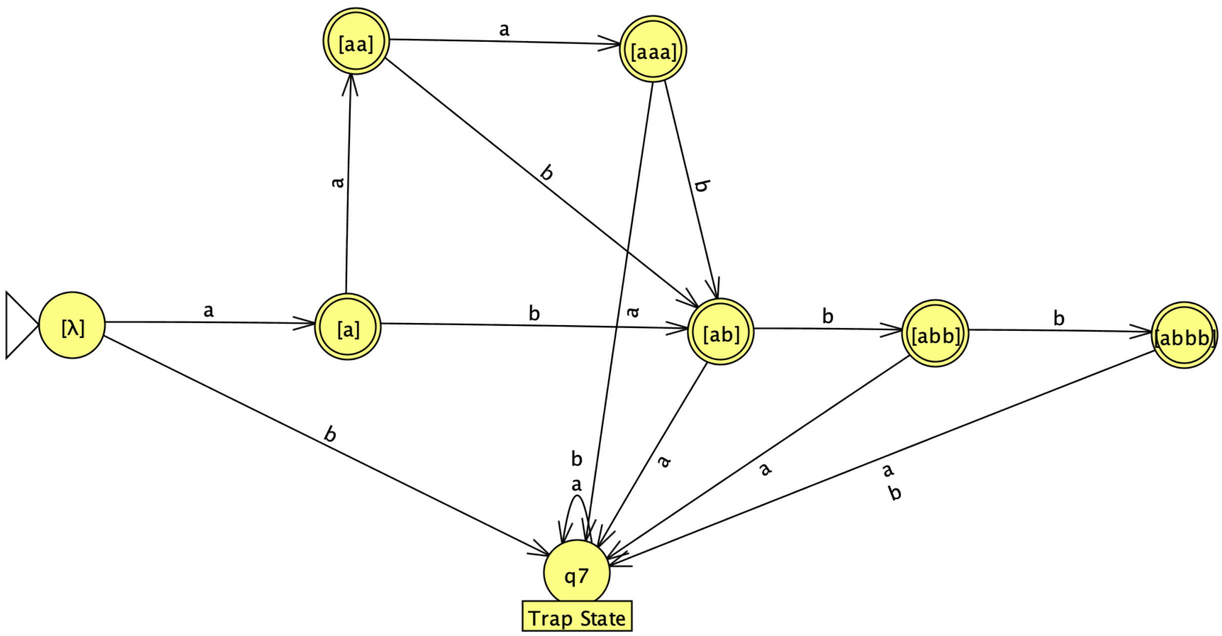
class 7: $[\![abbb]\!] = [\![aabbb]\!] = [\![aaabbb]\!] = \{a^i b^3 : 1 \leq i \leq 3\}$

$[\![b]\!] = [\![bb]\!] = [\![ba]\!] = [\![bab]\!] = \{u_1 b u_2 : u_1, u_2 \in \{a,b\}^*\}$

trap state

Problem 6. (10 points)

Using the work from the 2 previous problems construct the minimal DFA which accepts L2.



Problem 7. (10 points)

Similar to problems 1-3 and/or 4-6. Describe the equivalence classes of RL3 and construct the minimal DFA that accepts L3.

L3 = {w ∈ {a, b}*: w contains exactly 2 b's}

For the language L_3, the Regular Expression is: **RE: a* b a* b a***

class ①:

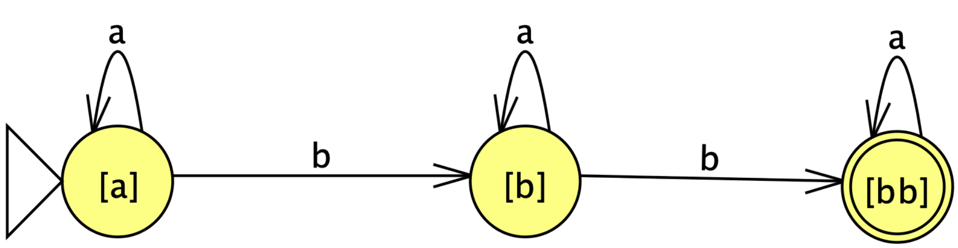
$[\![\lambda]\!] = [\![a]\!] = [\![aa]\!] = [\![aaa]\!] = \{w \in \{a,b\}^* : w$ contains NO b's$\}$

class ②:

$[\![b]\!] = [\![ab]\!] = [\![aab]\!] = [\![aaab]\!] = \{w \in \{a,b\}^* : w$ contains Only one b's$\}$

class ③

$[\![bb]\!] = [\![abb]\!] = [\![abaab]\!] = \{w \in \{a,b\}^* : w$ contains exactly 2 b's$\}$



Problem 8. (10 points)

Similar to problems 1-3 and/or 4-6. Describe the equivalence classes of R_{L4} and construct the minimal DFA that accepts L_4.

$L_4 = \{w \in \{a, b\}^*: |w|\%2=0$ & w contains exactly 2 b's$\}$

⟦λ⟧

⟦a⟧

⟦aa⟧ = ⟦λ⟧

⟦b⟧

⟦bb⟧

⟦ab⟧

⟦aab⟧

⋮

λ.bb → acc
aa.bb → acc

λ.b → rej
a.b → rej
aa.b → rej
b.b → acc

λ.ba → X
a.ba → X
aa.ba → X
b.ba → X
ab.ba → ✓
bb.ba → X

λ.bb → acc
a.bb → rej

λ.bba → rej
a.bba → acc
aa.bba → rej

λ.λ → rej
a.λ → rej
aa.λ → rej
bb.λ → acc
b.λ → rej

λ.b → X
a.b → X
aa.b → X
ab.b → X
b.b → ✓
aab.b → ✓

As you can see above, this language has infinite number of equivalence classes, so we have infinite number of states, in results this language is not regular and has no DFA.

Problem 9. (10 points)

Use the Myhill-Nerode Theorem to construct the minimum DFA which accepts $\{w \in \{a, b\}^*$: w has the first two letters the same as the last two letters$\}$.

For example, this language includes aa, ab, aaa, and abbbbbab, but does not include aba, abbbbba, or abba.

class ①: ⟦λ⟧ = {λ}

class ②: ⟦a⟧ = {a}

class ③: ⟦b⟧ = {b}

class ④: ⟦aa⟧ = ⟦aaa⟧ = ⟦aaaa⟧ = {aa, aaa} ∪ {aa w aa : w ∈ {a,b}*}

class ⑤: ⟦bb⟧ = ⟦bbb⟧ = ⟦bbbb⟧ = {bb, bbb} ∪ {bb w bb : w ∈ {a,b}*}

class ⑥: ⟦ab⟧ = ⟦abab⟧ = ⟦abaabab⟧ = {ab w ab : w ∈ {a,b}*}

class ⑦: ⟦ba⟧ = ⟦baba⟧ = ⟦baaabba⟧ = {ba w ba : w ∈ {a,b}*}

class ⑧: ⟦aab⟧ = ⟦aabb⟧ = ⟦aabbbab⟧ = {aa w b : w ∈ {a,b}*}

class ⑨: ⟦aaba⟧ = ⟦aaabba⟧ = ⟦aabbaba⟧ = {aa w ba : w ∈ {a,b}*}

class ⑩: ⟦aba⟧ = ⟦ababa⟧ = ⟦abba⟧ = {ab w a : w ∈ {a,b}*}

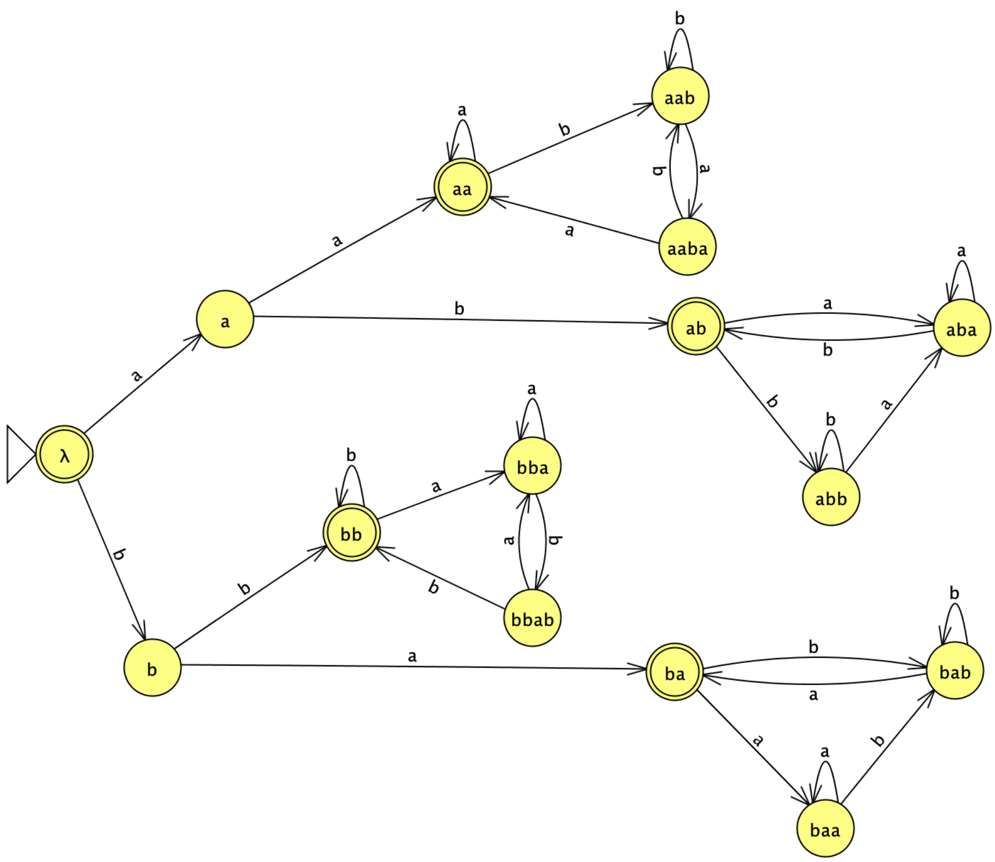
class ⑪: ⟦abb⟧ = ⟦abba⟧ = ⟦abbbaa⟧ = {ab w b : w ∈ {a,b}*}

class ⑫: ⟦bba⟧ = ⟦bbba⟧ = ⟦bbbaa⟧ = {bb w a : w ∈ {a,b}*}

class ⑬: ⟦bbab⟧ = ⟦bbaab⟧ = ⟦bbaba⟧ = {bb w ab : w ∈ {a,b}*}

class ⑭: ⟦bab⟧ = ⟦babb⟧ = ⟦bababb⟧ = {ba w b : w ∈ {a,b}*}

class ⑮: ⟦baa⟧ = ⟦baaa⟧ = ⟦baabaa⟧ = {ba w a : w ∈ {a,b}*}



Problem 10. (10 points)

Use the Myhill-Nerode Theorem to show $\{w \in \{a, b\}^*: \#a = \#b\}$ is not regular (ie show here are infinitely many equivalence classes).

$[\![\lambda]\!] = \{\lambda\}$

1 $[\![a]\!]$

2 $[\![b]\!]$

3 $[\![aa]\!]$

4 $[\![bb]\!]$

5 $[\![ab]\!] = [\![ba]\!]$

6 $[\![aaa]\!]$

7 $[\![bbb]\!]$

8 $[\![abb]\!] = [\![bab]\!] = [\![bba]\!]$

$[\![aab]\!] = [\![aba]\!] = [\![baa]\!]$

⋮

*We can do all these infinitely

1
λ.b → rej
a.b → acc

2
λ.a → rej
a.a → rej
b.a → acc

3
λ.bb → rej
a.bb rej
b.bb → rej
aa.bb → acc

4
λ.aa → rej
a.aa → rej
b.aa → rej
aa.aa → rej
bb.aa → acc

5
λ.ba → rej
a.ba → rej
b.ba → rej
aa.ba → rej
bb.ba → rej
ab.ba → acc

6
λ.bbb → rej
a.bbb rej
b.bbb rej
aa.bbb rej
bb.bbb rej
ab.bbb rej
ba.bbb rej
aaa.bbb acc

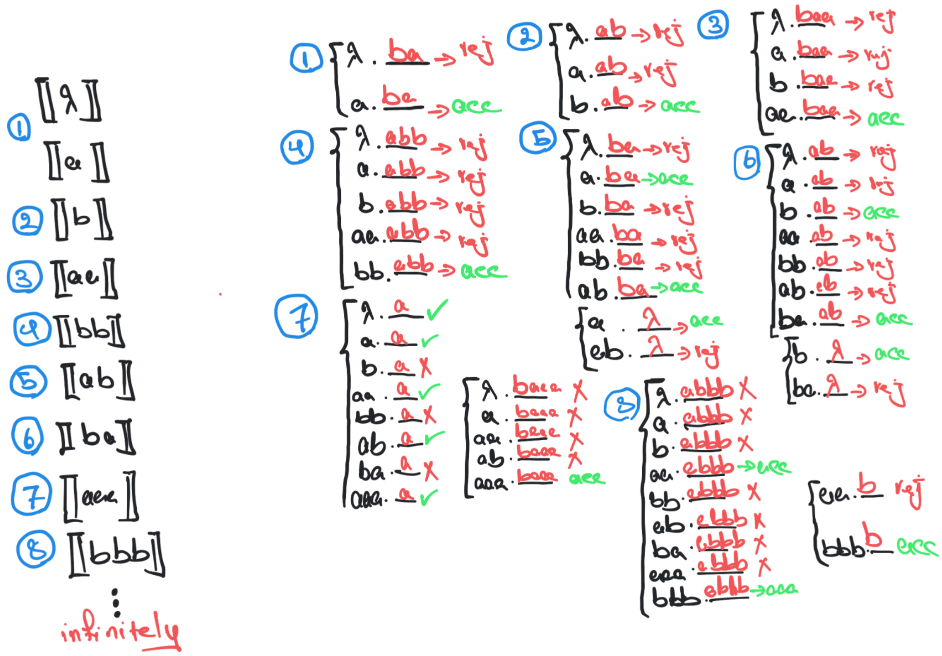
7
λ.aaa → rej
a.aaa → rej
b.aaa → rej
aa.aaa → rej
bb.aaa → rej
ab.aaa → rej
aaa.aaa → rej
bbb.aaa → acc

8
λ.baa → rej
a.baa → rej
b.baa → rej
aa.baa → rej
bb.baa → rej
ab.baa → rej
aaa.baa → rej
bbb.baa → rej
abb.baa → acc

There is an infinite number of equivalence classes, so there is infinite number of states. Then we cannot make a DFA for this class and it’s not a Regular language.

Problem 11. (10 points)

Use the Myhill-Nerode Theorem to show $\{w \in \{a, b\}^*: w = w^R\}$ is not regular (ie show there are infinitely many equivalence classes).

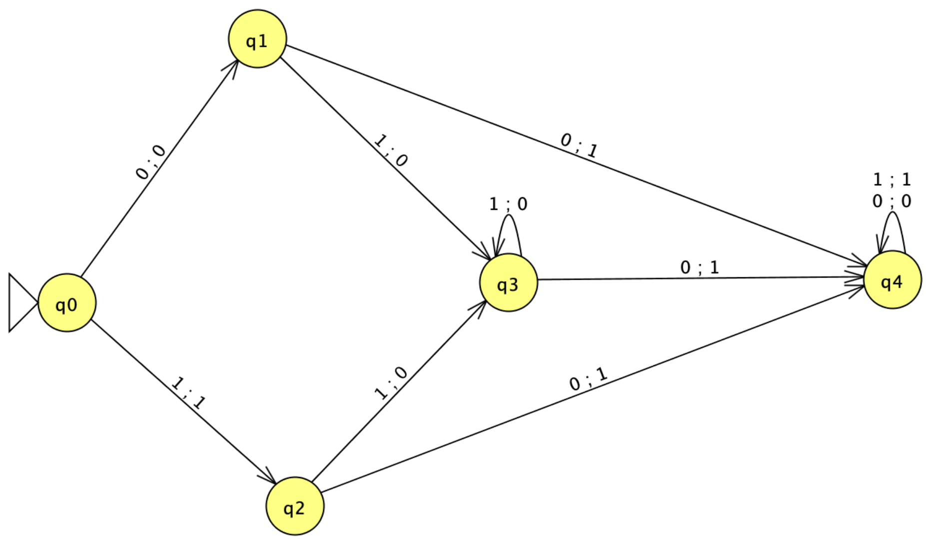


There is an infinite number of equivalence classes, so there is infinite number of states. Then we cannot make a DFA for this class and it’s not a Regular language.

Problem 12. (10 points)

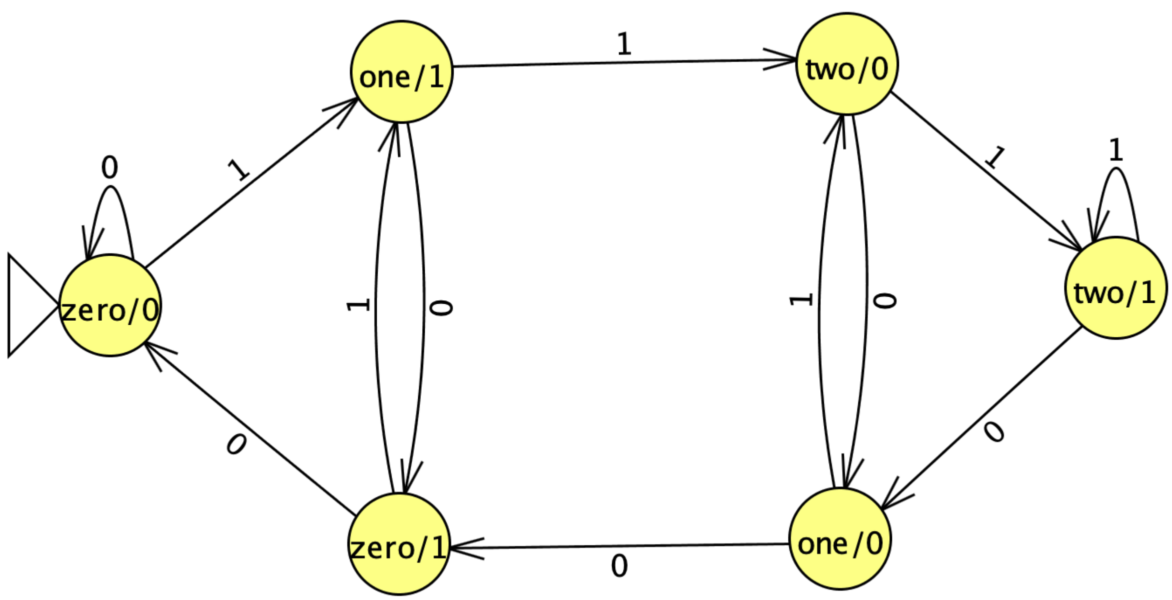
Construct a Mealy Machine that takes an <u>input of a binary number in “little-endian"</u> with a guaranteed zero on the right end and outputs that number +2 in binary.

For example, if the input was 8 then it should output 10 except in binary: input 00010 should output 01010. For example, if the input was 18 then it should output 20 except in binary: input 010010 should output 001010. For example, input 31 should output 33 except in binary: input 111110 should output 100001.



Problem 13. (10 points)

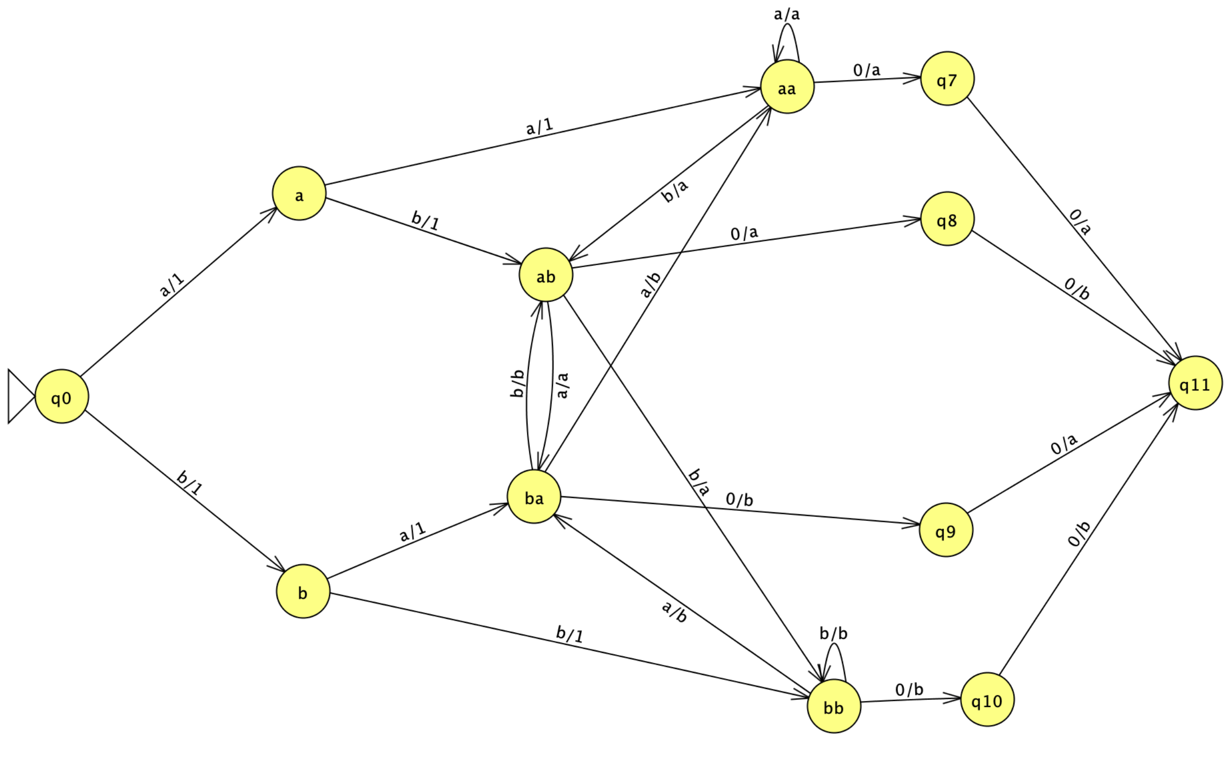
Construct a **Moore Machine** that takes an input of a binary number in "little-endian" with 2 guaranteed zeroes on the right end and outputs that number times 6. For example, if the input was 8 then it should output 48 except in binary: input 000100 should output 0000110. For example, if the input was 9 it should output 54 except in binary: input 100100 should output 0110110.



Problem 14. (10 points)

Construct a **Mealy Machine** that takes a string in the form $l_1 l_2 \ldots l_k 00$ where each $l_i \in \{a, b\}$ and outputs $11\, l_1 l_2 \ldots l_k$.

For example, input abba00 should output 11abba.



Problem 15. (10 points)

Construct a **Moore Machine** that takes a string in the form $l_1\ l_2 \ldots l_k$ and outputs the string $0l_1\ l_2 \ldots l_k$ except with the first 2 a's replaced with b's (if there are less than 2 a's replace all a's with b's).

For example, input abbaabba should output 0bbbbabba.