COMP 615 HW 1: Review of Undergraduate Material - Regular

Problem 1. (10 points) Let $A = \{a, 1, b, 2\}$, $B = \{a, b, c, d\}$, and $C = \{1, 2, 3, 4\}$. Determine each of the following:

- $A \cap (B \cup C)$

- $(A \cap B) \cup C$

- $(A \cap B) \times C$

- $(A \triangle B) \setminus C$

- $2^{A \cap C}$

Problem 2. (10 points) Let $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1)\}$ be a relation over $X = \{1, 2, 3\}$. State and briefly justify whether R is

- Reflexive?

- Symmetric?

- Transitive?

- Antisymmetric?

Problem 3. (10 points) Use induction to prove $5n \leq 3^n + 2$ for all $n \geq 1$.

Problem 4. (10 points) For the DFA below: list 3 words accepted, list 3 words rejected, and give a simple description of the language (ie the set of words that are accepted).
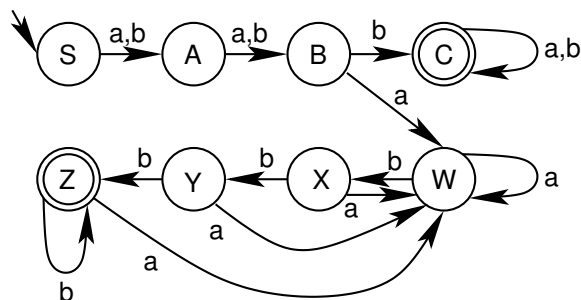
Problem 5. (10 points) Create (draw or give a mathematical description) of a DFA that accepts the language $L = \{a^i b^j : i \neq 1 \ AND \ j \neq 1\}$.

Problem 6. (10 points) For the NFA below: list 3 words accepted, list 3 words rejected, and give a simple description of the language (ie the set of words that are accepted).
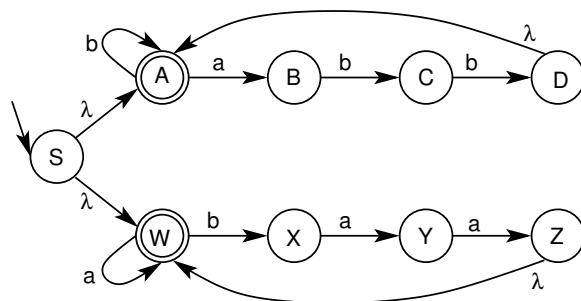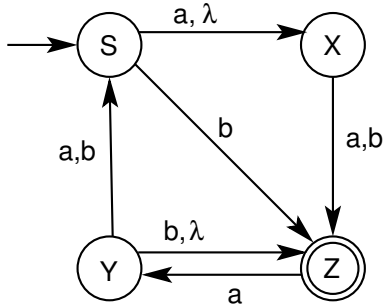
Problem 7. (10 points) Create (draw or give a mathematical description) of an NFA that accepts the language $\{w \in \{a,b\}^* : w \text{ contains } aaa \text{ and } bbb\}$ (note the 2 substrings could occur in either order).

Problem 8. (10 points) Convert the following NFA into a DFA that accepts the same language.


Problem 9. (10 points) Give regular expressions for both of the languages below.

$\{w \in \{a,b\}^* : w \text{ contains } aaa \text{ and } bbb\}$

$\{a^i b^j : i \cdot j \geq 5\}$

Problem 10. (10 points) Draw an NFA that accepts the language described by the regular expression $(aa+ba)(a+b)^*(aa+ab) + (a+b)a(a+b) + aa$ (ie convert the reg exp into an NFA).

Problem 11. (10 points) Create a regular expression that describes the language accepted by the DFA below (ie convert the DFA into a regular expression).
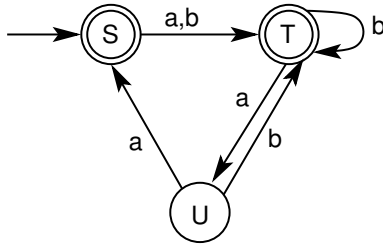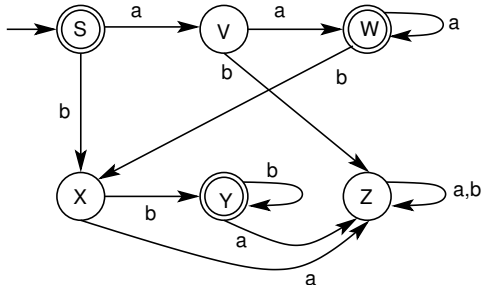

Problem 12. (10 points) Give a simple description for the language generated by the following regular grammar.

$$
\begin{aligned}
S &\rightarrow aaB \mid baB \mid abA \mid bbA \\
A &\rightarrow aA \mid bA \mid aa \mid ab \\
B &\rightarrow aB \mid bB \mid ba \mid bb
\end{aligned}
$$

Problem 13. (10 points) Give a regular grammar for the language generated by the DFA below.

Problem 14. (10 points) Draw an NFA that accepts the language generated by the regular grammar below.

$$
\begin{aligned}
S &\rightarrow aaB \mid baB \mid abA \mid bbA \mid ab \mid ba \\
A &\rightarrow aA \mid bA \mid aa \mid ab \\
B &\rightarrow aB \mid bB \mid ba \mid bb
\end{aligned}
$$

Problem 15. (10 points) Use the pumping lemma (for regular languages) to show that $\{w \in \{a, b\}^* : \#a > \#b\}$ is not a regular language.

Problem 16. (10 points) For any language L define $\mathrm{Two}(L) = \{w_1 w_2 : w_1, w_2 \in L\}$. In other words, $\mathrm{Two}(L)$ is the language consisting of all strings created by concatinating two words in L.

Show that if L is regular then $\mathrm{Two}(L)$ is regular.