COMP 615 HW 2: Myhill-Nerode and Mealy-Moore Machines

For any language L, R_L is the relation on Σ^* where $R_L = \{(w_1, w_2) : \forall\ x : \ \ w_1 x \in L\ iff\ w_2 x \in L\}$.

Let $L_1 = \{w \in \{a,b\}^* : \#a\%2 = 0\ \&\ \#b\%2 = 0\}$.

Let $L_2 = \{a^i b^j : 1 \leq i \leq 3\ \&\ j < 4\}$.

Let $L_3 = \{w \in \{a,b\}^* : \ w\ contains\ exactly\ 2\ b's\}$.

Let $L_4 = \{w \in \{a,b\}^* : \ |w|\%2 = 0\ \&\ w\ contains\ exactly\ 2\ b's\}$.

Problem 1. (10 points) Let $X = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb\}$. For each element of $X \cdot X$ determine whether it is in L_1.

	λ	a	b	aa	ab	ba	bb	aaa	aab	aba	abb	baa	bab	bba	bbb
λ															
a															
b															
aa															
ab															
ba															
bb															
aaa															
aab															
aba															
abb															
baa															
bab															
bba															
bbb															

Problem 2. (10 points) Based upon the table from problem 1, you should know there are at least 4 equivalence classes in R_{L_1}. (Though you haven't proven there are only 4. Professor Noga assures you there are only 4.)

Show how the table breaks the 15 different strings into 4 different equivalence classes.

For each equivalence class, give a description of all elements in that class.

Problem 3. (10 points) Using the equivalence classes from problem 2, construct the minimum DFA which accepts L_1.

Problem 4. (10 points) Similar to problem 1: Let $X = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb\}$. For each element of $X \cdot X$ determine whether it is in L_2.

	λ	a	b	aa	ab	ba	bb	aaa	aab	aba	abb	baa	bab	bba	bbb
λ															
a															
b															
aa															
ab															
ba															
bb															
aaa															
aab															
aba															
abb															
baa															
bab															
bba															
bbb															

Problem 5. (10 points) Similar to problem 2: Show how the table in problem 4 breaks the 15 different strings into different equivalence classes.

For each equivalence class, give a description of all elements in that class.

Find the other equivalence classes (ie strings that are not equivalent to any of these) and describe them as well.

Problem 6. (10 points) Using the work from the 2 previous problems construct the minimal DFA which accepts L_2.

Problem 7. (10 points) Similar to problems 1-3 and/or 4-6. Describe the equivalence classes of R_{L_3} and construct the minimal DFA that accepts L_3.

Problem 8. (10 points) Similar to problems 1-3 and/or 4-6. Describe the equivalence classes of R_{L_4} and construct the minimal DFA that accepts L_4.

Problem 9. (10 points) Use the Myhill-Nerode Theorem to construct the minimum DFA which accepts $\{w \in \{a,b\}^* : \ w \ has \ the \ first \ two \ letters \ the \ same \ as \ the \ last \ two \ letters\}$. For example, this language includes aa,ab,aaa, and abbbbbab, but does not include aba, abbbbba, or abba.

Problem 10. (10 points) Use the Myhill-Nerode Theorem to show $\{w \in \{a,b\}^* : \#a = \#b\}$ is not regular (ie show there are infinitely many equivalence classes).

Problem 11. (10 points) Use the Myhill-Nerode Theorem to show $\{w \in \{a,b\}^* : w = w^R\}$ is not regular (ie show there are infinitely many equivalence classes).

Problem 12. (10 points) Construct a Mealy Machine that takes an input of a binary number in "littleendian" with a guaranteed zero on the right end and outputs that number +2 in binary. For example, if the input was 8 then it should output 10 except in binary: input 00010 should output 01010. For example if the input was 18 then it should output 20 except in binary: input 010010 should output 001010. For example input 31 should output 33 except in binary: input 111110 should output 100001.

Problem 13. (10 points) Construct a Moore Machine that takes an input of a binary number in "littleendian" with 2 guaranteed zeroes on the right end and outputs that number times 6. For example, if the input was 8 then it should output 48 except in binary: input 000100 should output 0000110. For example, if the input was 9 it should output 54 except in binary: input 100100 should output 0110110.

Problem 14. (10 points) Construct a Mealy Machine that that takes a string in the form $l_1 l_2 \dots l_k 00$ where each

$l_i \in \{a, b\}$ and outputs $11l_1l_2 \ldots l_k$. For example, input $abba00$ should output $11abba$.

Problem 15. (10 points) Construct a Moore Machine that takes a string in the form $l_1l_2 \ldots l_k$ and outputs the string $0l_1l_2 \ldots l_k$ except with the first 2 a's replaced with b's (if there are less than 2 a's replace all a's with b's). For example, input $abbaabba$ should output $0bbbbabba$.