COMP 615 HW 4: Context Free Review

Problem 1. (10 points) Create an CFG that accepts $\{a^i b^j c^k d^\ell : i > 2j \ \& \ k > 3\ell\}$.

Problem 2. (10 points) What language is generated by the CFG below.

$$\begin{aligned} S &\rightarrow aB \mid aS \mid bA \mid \lambda \\ A &\rightarrow aS \mid bAA \\ B &\rightarrow aBB \mid bS \end{aligned}$$

Problem 3. (10 points) Modify the grammar below to remove all λ-productions without changing the language generated.

$$\begin{aligned} S &\rightarrow SAS \mid CaA \mid b \\ A &\rightarrow BaB \mid CC \mid a \\ B &\rightarrow AaC \mid BA \mid b \\ C &\rightarrow aSa \mid SaACB \mid a \mid \lambda \end{aligned}$$

Problem 4. (10 points) Modify the grammar below to remove all unit-productions without changing the language generated.

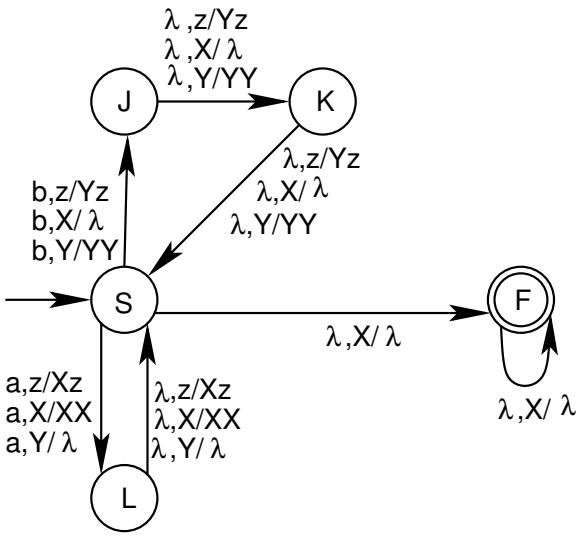
$$\begin{aligned} S &\rightarrow aB \mid aS \mid bA \mid cA \mid cB \mid c \\ A &\rightarrow aS \mid bAA \mid C \\ B &\rightarrow aBB \mid bS \mid C \\ C &\rightarrow cS \mid aBC \mid aAC \mid S \end{aligned}$$

Problem 5. (10 points) Convert the grammar below into Chomsky Normal Form (ie give a grammar in CNF that still generates the same language).

$$\begin{aligned} S &\rightarrow SAS \mid CaA \mid b \\ A &\rightarrow BaB \mid CC \mid a \\ B &\rightarrow AaC \mid BA \mid b \\ C &\rightarrow aSa \mid SaACB \mid a \end{aligned}$$

Problem 6. (10 points) Create an NPDA that accepts $\{a^i b^j c^k d^\ell : i > j \ \& \ k > \ell\}$.

Problem 7. (10 points) What language is accepted by the NPDA below.



Problem 8. (10 points) Use the pumping lemma to prove that $L = \{a^i b^j c^k : i = j \cdot k\}$ is not context free

Problem 9. (10 points) Use the Pumping Lemma for CFL to show that $L = \{w \in \{a, b, c\}^* : \#a \cdot \#b = \#c\}$ is not context free.

Problem 10. (10 points) Given any 2 languages L_1 and L_2 define INTERLACE(L_1, L_2) as the set of strings that can be created by taking any string $x = x[1]x[2]\dots x[k] \in L_1$ and $y = y[1]y[2]\dots y[\ell] \in L_2$ and performing the following process

```
int i=0, j=0;
String str="";
while (i<k || j<l) {
   if (i==k)
      str += y[++j];
   else if (j==l)
      str += x[++i];
   else {
      str += x[++j];
      str += y[++i];
   }
}
```

Prove: if L_1 and L_2 are context free then INTERLACE(L_1, L_2) is also context free.