Problem ①

I. L_1^R

II. $(L_1 \cup L_2)^*$

Proof: Since L_1 & L_2 are CF, $\exists$ CFGs G_1 & G_2 which generate them: Give every variable in G_1 a subscript &

Give every variable in G_2 a subscript

Take the productions $S \rightarrow S_1 S \mid S_2 S \mid \lambda$ and all productions from G_1 & G_2. This new CFG generates $(L_1 \cup L_2)^*$.

So, $(L_1 \cup L_2)^*$ is CF.

$$S \overset{*}{\Rightarrow} S_1 S_1 S_2 S_1 S_2 S_2 S_1 \underset{\lambda}{\underbrace{S}} \Rightarrow S_1 S_1 S_2 S_1 S_2 S_2 S_1 \in (L_1 \cup L_2)^*$$

III. NBDT $(L_1) = \{w \in L_1 : \text{where } |w| \% 3 \neq 0\}$

* Problem 2

I. $\{a^i b^j c^k : i \neq j \ \& \ i \neq k \ \& \ j \neq k\}$

Proof: Assum L is CF. Ogdyn's Lemma applies and gives an $M > 0$.
choose a word in language and when we pump, it doesn't work.
choose $W = a^M b^{M+M!} c^{M+2M!}$ & mark sth.

note v can contain at most one type of letter and
y can contain at most one type of letter $\Rightarrow$ one of they
must be an a.

$vy = a^l \quad a^k \cdots c^M$

$W = a^M b^{M+M!} c^{M+2M!}$

$1 \leq l \leq M$

$k + (i-1)l = M$

$i-1 = \frac{M-k}{l}$ → what if $M!$

consider $uv^i xy^i z$ where $(i-1)l = M!$ or $(i-1)l = 2M!$ where
$l = \#a'$ in vy & First, if vy misses b's & 2nd vy if
misses in either case $\notin L$ because $\#a = \#b$ in First of
$\#a = \#c$ in 2nd, $\rightarrow\leftarrow$

II. $\{w \in \{a,b,c,d\}^* : \#a = \#b = \#c \text{ or } \#d = 0\}$

Assum L is CF. Ogden's Lemma applies. Ogden's Lemma give $M > 0$
Choose $W = a^M b^M c^M d^M$ (orders of a's, b's, c's and d's doesn't matter)
and mark b's, c's and d's.

There is a splite $w = uvxyz$ where vxy has at most M
marked letters and vy has at least one marked letter.
vxy cannot contain b's & c's & d's all in the same number.

Consider $uvvxyyz$ this has a's and it has exactly b's or d's, but more than M a's are b's or c's

Becaus vy containe a marked letters. $\notin L \rightarrow\leftarrow$

III. $\{a^i b^j c^k d^l e^m : i=j \ \& \ k=l=m\}$

Assume L is CF. Ogden's Lemma applies, OL gives $M>0$.

Choose $w = a^M b^M c^{2M} d^{2M} e^{2M}$ and Mark the a's, b's, c's, d's and e's.

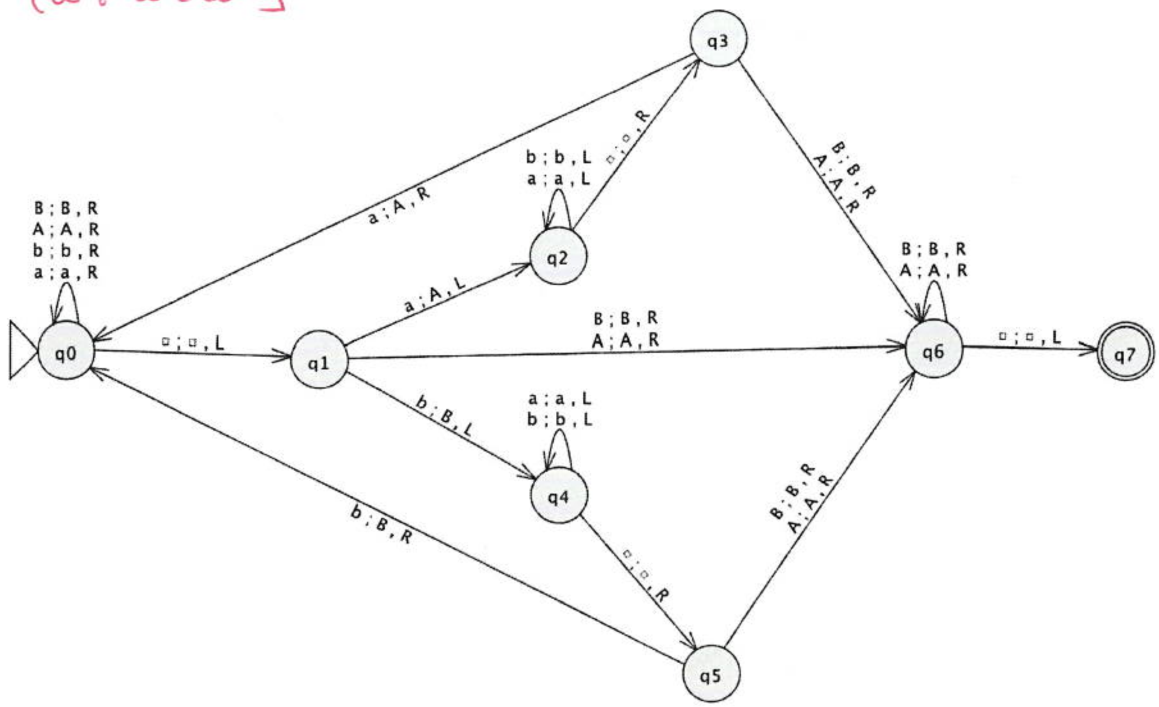
There is a split $w = uvxyz$ where vxy has at most M marked letters & vy has at least one marked letter.

vxy cannot contains a and b or c d e, because to have even one c and one e vxy would need to have 2M+2 marked letters.

Consider $uvvxyyz$, It has exactly 2M c's or e's but it has more a's or b's or d's or e's because vy contain a marked letters.

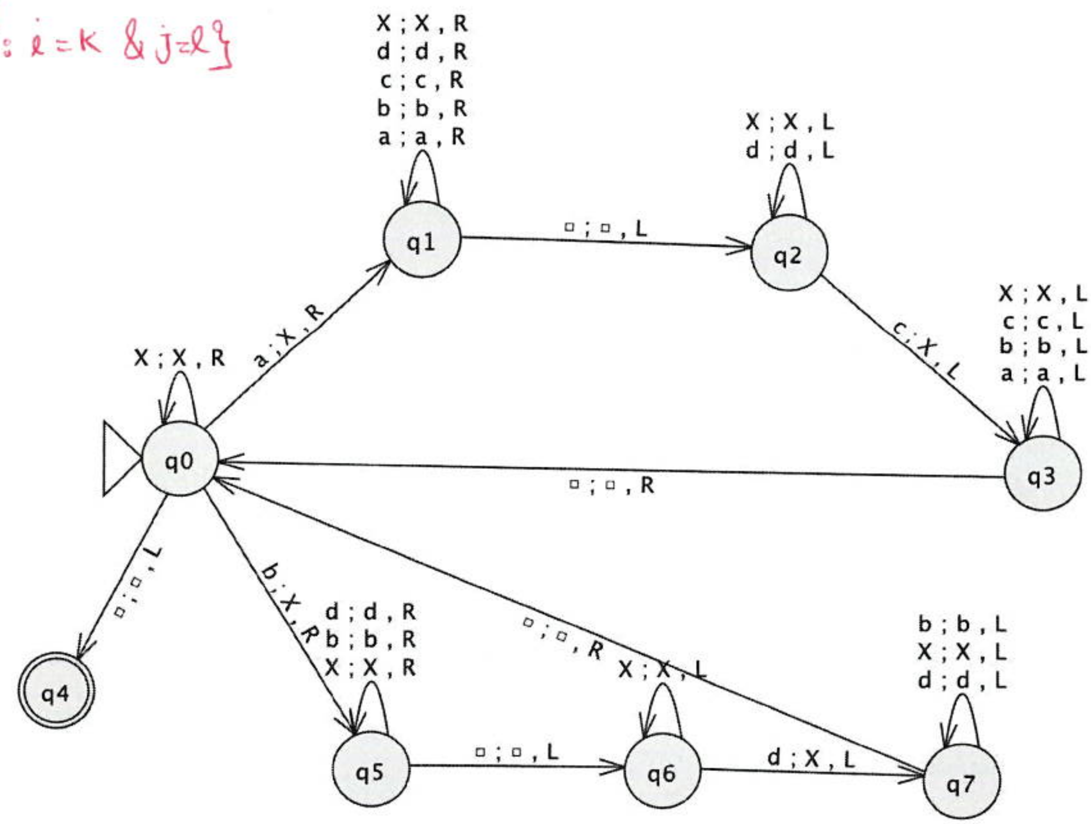
Problem 3

a. $\{\omega : \omega = \omega^R\}$



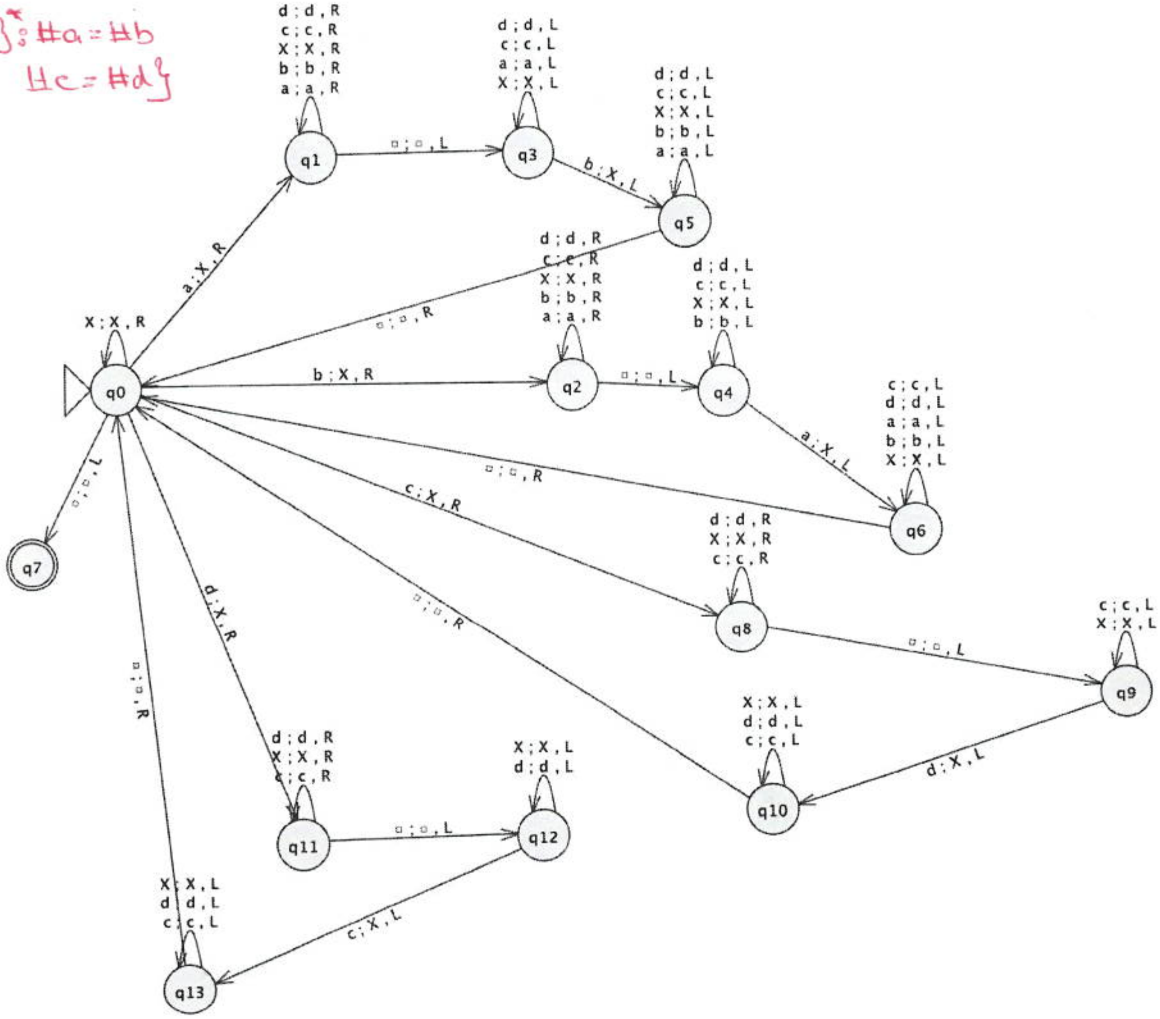
b.

$\{a^i b^j c^k d^l : i = k \ \& \ j = l\}$



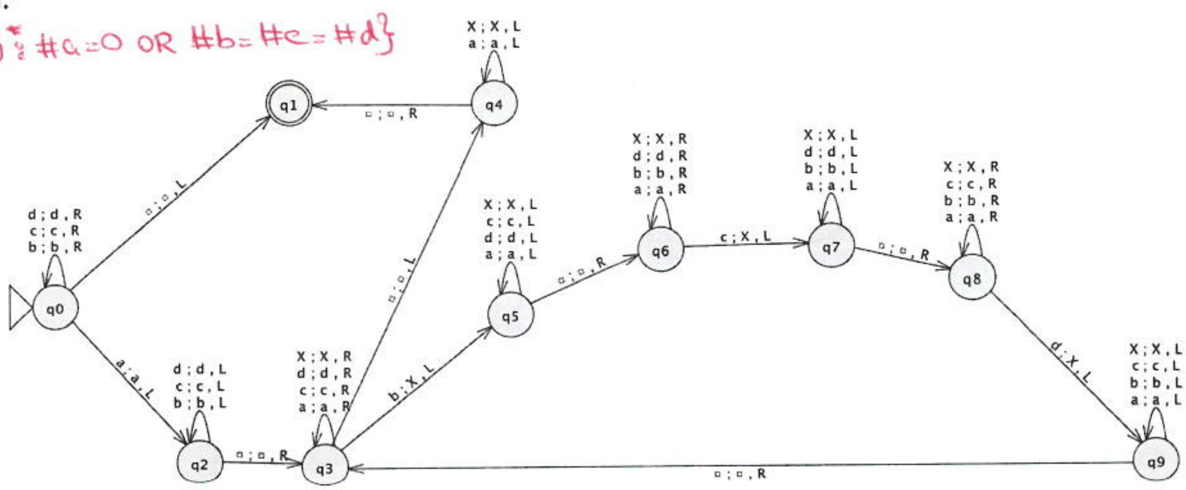
c.

$\{w \in \{a,b,c,d\}^*: \#a = \#b, \#c = \#d\}$



d.

$\{w \in \{a,b,c,d\}^*: \#a = 0 \text{ OR } \#b = \#c = \#d\}$

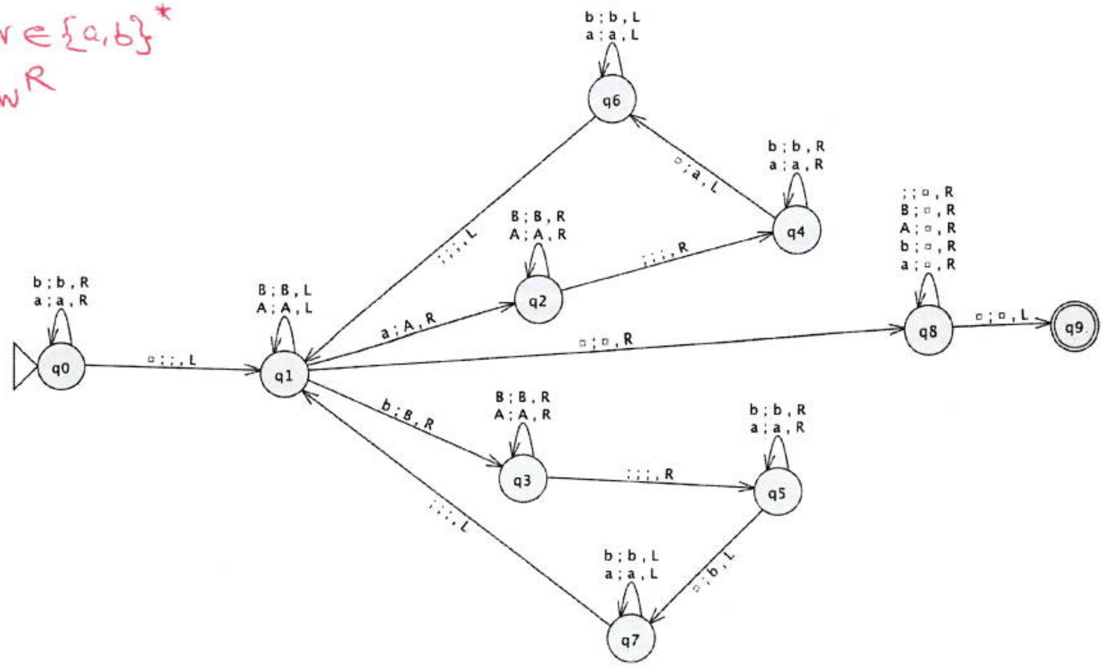


Problem 4

4-1:

input = $w \in \{a,b\}^*$

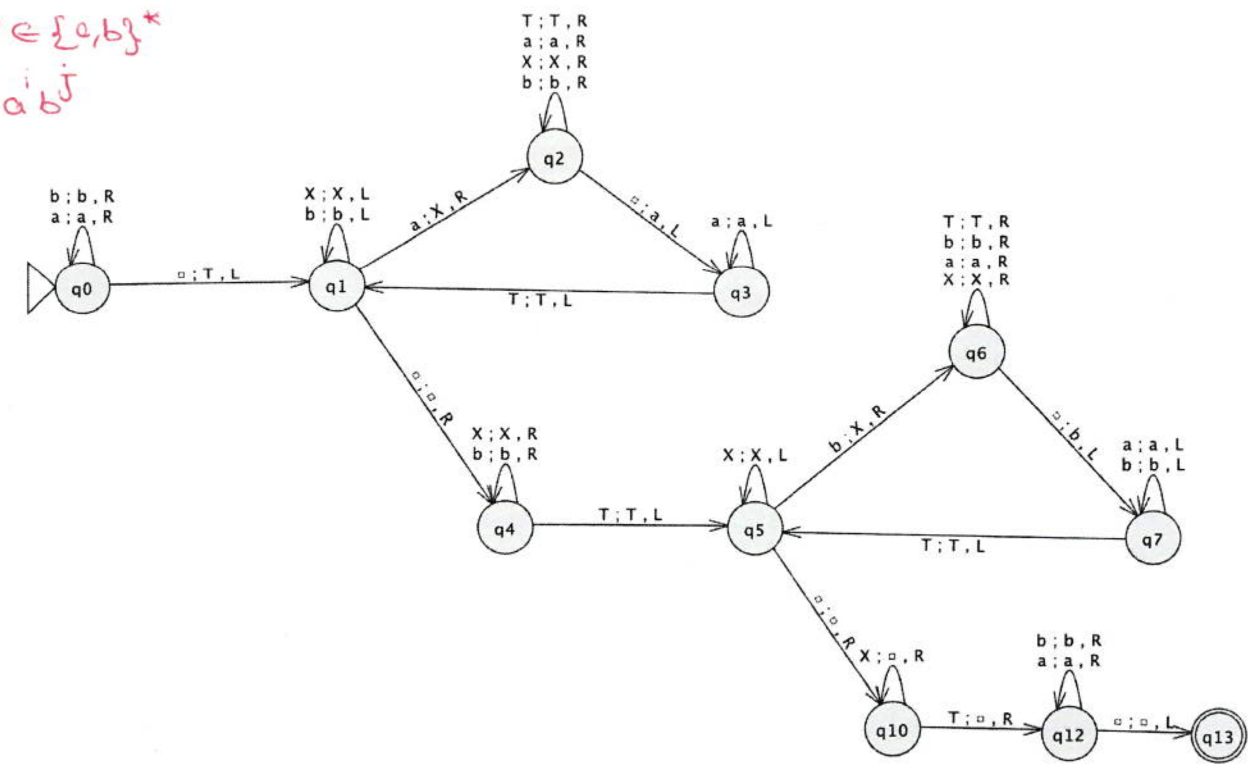
output = w^R



4-2:

input $w \in \{a,b\}^*$

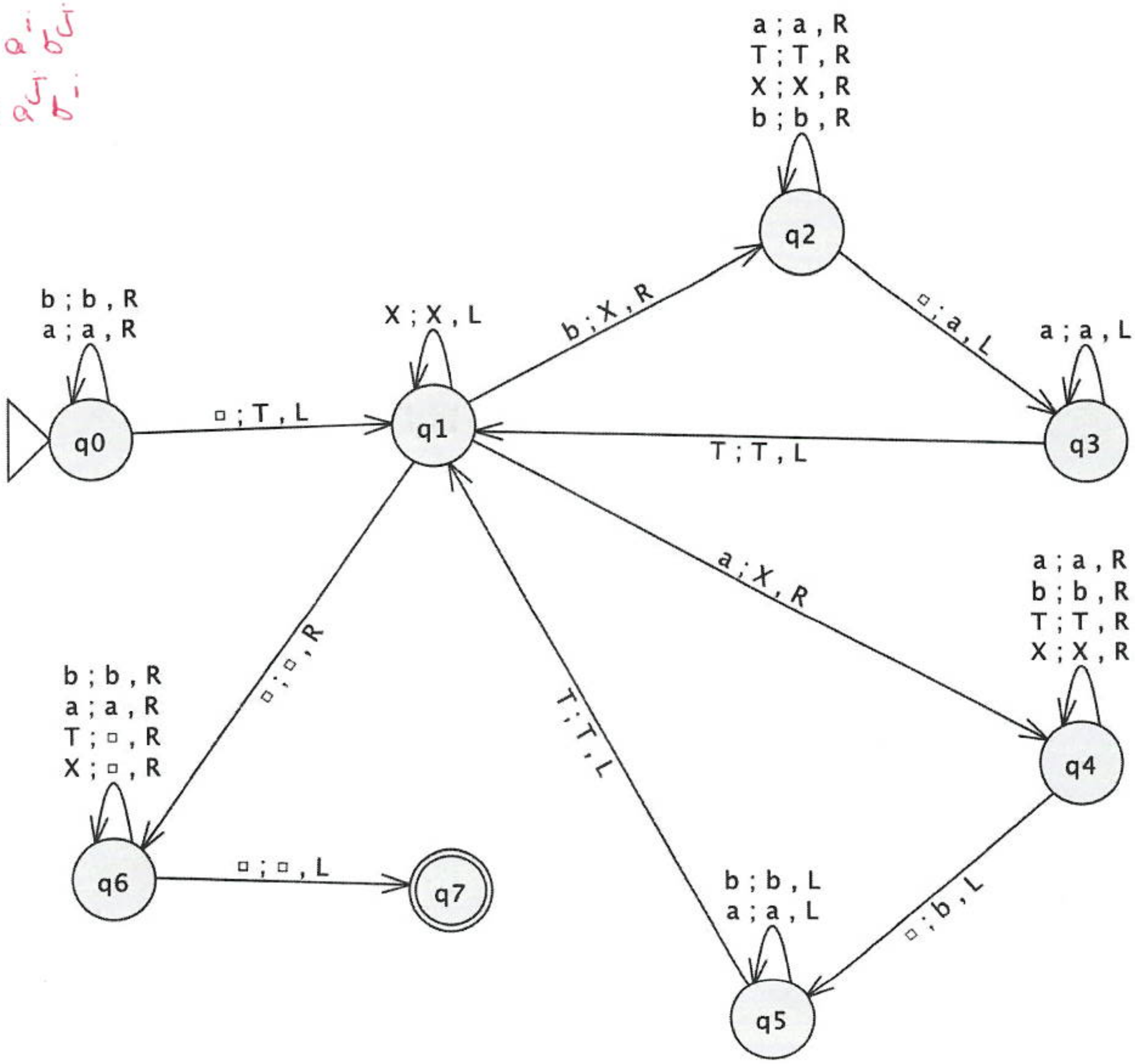
output $a^i b^j$



4.3

input : $a^i b^j$

Output : $a^j b^i$



Problem 5

5.1 : RunForever = $\{(M;w) : M(w) = \uparrow\}$

5.2 : Finite = $\{M : |L(M)| < \infty\}$

5.3. Opposite $= \{ (M_1, M_2) : L(M_1) = L(\overline{M_2}) \}$

Problem (6)

6_1 : $\{ww : w \in \{a,b\}^*\}$

$S \to FME$

$F \to FA \mid FB$

$FA \to aFX$

$FB \to bFY$

$XA \to AX$ $\quad YA \to AY$

$XB \to BX$ $\quad YB \to BY$

$XM \to MX$ $\quad YM \to MY$

$XE \to AE$

$YE \to BE$

$FM \to Z$

$ZA \to aZ$

$ZB \to bZ$

$ZE \to \lambda$

6_2 : $\{a^i b^j c^k d^l ; i=k \;\&\; j=l\}$

$S \to KX$

$X \to AXC \mid BXD$

$BA \to AB$ $\quad KX \to M$

$DC \to CD$ $\quad LX \to M$

$KA \to aK$ $\quad MC \to cM$

$KB \to bL$ $\quad MD \to dN$

$LB \to bL$ $\quad ND \to dN$

$M \to \lambda$

$N \to \lambda$

6-3: $\{w \in \{a,b,c,d\}^* : \#a = \#b \ \& \ \#c = \#d\}$

$S \to SAB \mid SCD \mid \lambda$

$AB \to BA$ $\quad$ $CA \to AC$

$AC \to CA$ $\quad$ $CB \to BC$

$AD \to DA$ $\quad$ $CD \to DC$

$BA \to AB$ $\quad$ $DA \to AD$

$BC \to CB$ $\quad$ $DB \to BD$

$BD \to DB$ $\quad$ $DC \to CD$

$A \to a$

$B \to b$

$C \to c$

$D \to d$

6-4: $\{a^i b^j c^k ; i > j > k\}$

$S \to ABCS \mid ABT$

$T \to ABT \mid AU$

$U \to AU$

$BA \to AB$

$CB \to BC$

$CA \to AC$

$CU \to Uc$

$BU \to Vb$

$AV \to Za$

$AZ \to Za$

$Z \to \lambda$

$\{a^i b^j c^k : i = j+1 = k+1\}$

$S \to SABC \mid \lambda$

$BA \to AB$

$CA \to AC$

$CB \to BC$

$SA \to aaa$

$aA \to aaaa$

$aB \to abb$

$bB \to bbb$

$bC \to bc$

$cC \to cc$

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if $\{a^i b^j c^k : i \geq j \geq k\}$

$S \to ABCS \mid ABS \mid AS$

⋮

6-5: $\{W \in \{a, b, c, d\}^*: \#a = 0 \text{ OR } \#b = \#c = \#d\}$

$S \to X \mid Y$

$X \to dX \mid bX \mid cX \mid \lambda$

$Y \to AY \mid ZY$

$Z \to BCDZ \mid \lambda$

$AB \to BA$
$AC \to CA$
$AD \to DA$
$BA \to AB$
$BC \to CB$
$BD \to DB$

$CA \to AC$
$CB \to BC$
$CD \to DC$
$DA \to AD$
$DB \to BD$
$DC \to CD$

$A \to a$
$B \to b$
$C \to c$
$D \to d$

Problem 7. $\{W \in \{a, b, c, d\}^*: \#a = 0 \text{ or } \#b = \#c = \#d\}$

Problem 8. $\{a^i b^j c^k d^l : i = k \ \& \ j = l\}$