# COMP 615 HW 5
### Due: Never

**Problem 1.** (10 points) Let $L_1$ and $L_2$ be context free languages. Prove each of the following is context free.

- $L_1^R$

- $(L_1 \cup L_2)^*$

- $NDBT(L_1) = \{w \in L_1 : \ where \ |w|\%3 \neq 0\}$

**Problem 2.** (10 points) Use Ogden's lemma to show that each of the following languages are not context free.

- $\{a^i b^j c^k : i \neq j \ \& \ i \neq k \ \& \ j \neq k\}$

- $\{w \in \{a, b, c, d\}^* : \#a = \#b = \#c \ OR \ \#d = 0\}$

- $\{a^i b^j c^k d^\ell e^m : i = j \ \& \ k = \ell = m\}$

**Problem 3.** (10 points) Create a TM that decides each of the following languages.

- $\{w : w = w^R\}$

- $\{a^i b^j c^k d^\ell : i = k \ \& \ j = \ell\}$

- $\{w \in \{a, b, c, d\}^* : \#a = \#b \ \& \ \#c = \#d\}$

- $\{w \in \{a, b, c, d\}^* : \#a = 0 \ OR \ \#b = \#c = \#d\}$

**Problem 4.** (10 points) Create a TM that acts a transducer and converts the given input to the given output.

- Input: $w \in \{a, b\}^*$          Output: $w^R$

- Input: $w \in \{a, b\}^*$          Output: $a^i b^j$ where $i = \#a$ in $w$ and $j = \#b$ in $w$

- Input: $a^i b^j$          Output: $a^j b^i$

**Problem 5.** (10 points) Prove that each of the following languages are undecidable.

- RunForever $= \{(M; w) : M(w) = \nearrow\}$

- Finite $= \{M : \ |L(M)| < \infty\}$

- Opposite $= \{(M_1, M_2) : L(M_1) = \overline{L(M_2)}\}$

**Problem 6.** (10 points) Give an unrestricted grammar for each of the following

- $\{ww : w \in \{a, b\}^*\}$

- $\{a^i b^j c^k d^\ell : i = k \ \& \ j = \ell\}$

- $\{w \in \{a, b, c, d\}^* : \#a = \#b \ \& \ \#c = \#d\}$

- $\{a^i b^j c^k : i > j > k\}$

- $\{w \in \{a, b, c, d\}^* : \#a = 0 \ OR \ \#b = \#c = \#d\}$

**Problem 7.** (10 points) What language is generated by the following grammar? Briefly explain.

$$
\begin{aligned}
S &\rightarrow X \mid Y \\
X &\rightarrow bX \mid cX \mid dX \mid \lambda \\
Y &\rightarrow AY \mid AZ \\
Z &\rightarrow BCDZ \mid \lambda \\
AB &\rightarrow BA \\
BA &\rightarrow AB \\
AC &\rightarrow CA \\
CA &\rightarrow AC \\
AD &\rightarrow DA \\
DA &\rightarrow AD \\
BC &\rightarrow CB \\
CB &\rightarrow BC \\
BD &\rightarrow DB \\
DB &\rightarrow BD \\
CD &\rightarrow DC \\
DC &\rightarrow CD \\
A &\rightarrow a \\
B &\rightarrow b \\
C &\rightarrow c \\
D &\rightarrow d
\end{aligned}
$$

**Problem 8.** (10 points) What language is generated by the following grammar? Briefly explain.

$$
\begin{aligned}
S &\rightarrow KX \\
X &\rightarrow AXC \mid BXD \\
BA &\rightarrow AB \\
DC &\rightarrow CD \\
KA &\rightarrow aK \\
KB &\rightarrow bL \\
LB &\rightarrow bL \\
KX &\rightarrow M \\
LX &\rightarrow M \\
MC &\rightarrow cM \\
MD &\rightarrow dN \\
ND &\rightarrow dN \\
M &\rightarrow \lambda \\
N &\rightarrow \lambda
\end{aligned}
$$