

Sample output with

$$A = \begin{pmatrix} -3 & 2 & 3 \\ 5 & -7 & 11 \\ 1 & -12 & 1 \end{pmatrix}$$

$$\begin{aligned} & \left( \begin{array}{ccc|ccc} -3 & 2 & 3 & 1 & 0 & 0 \\ 5 & -7 & 11 & 0 & 1 & 0 \\ 1 & -12 & 1 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \left| \cdot \frac{5}{3} \right] \left| \cdot \frac{1}{3} \right] \\ \leftarrow + \\ \leftarrow + \end{array} \\ \Rightarrow & \left( \begin{array}{ccc|ccc} -3 & 2 & 3 & 1 & 0 & 0 \\ 0 & -\frac{11}{3} & 16 & \frac{5}{3} & 1 & 0 \\ 0 & -\frac{34}{3} & 2 & \frac{1}{3} & 0 & 1 \end{array} \right) \begin{array}{l} \left| \cdot \left(-\frac{34}{11}\right) \right] \\ \leftarrow + \end{array} \\ \Rightarrow & \left( \begin{array}{ccc|ccc} -3 & 2 & 3 & 1 & 0 & 0 \\ 0 & -\frac{11}{3} & 16 & \frac{5}{3} & 1 & 0 \\ 0 & 0 & -\frac{522}{11} & -\frac{53}{11} & -\frac{34}{11} & 1 \end{array} \right) \begin{array}{l} \leftarrow + \\ \leftarrow + \\ \left| \cdot \frac{88}{261} \right] \left| \cdot \frac{11}{174} \right] \end{array} \\ \Rightarrow & \left( \begin{array}{ccc|ccc} -3 & 2 & 0 & \frac{121}{174} & -\frac{17}{87} & \frac{11}{174} \\ 0 & -\frac{11}{3} & 0 & \frac{11}{261} & -\frac{11}{261} & \frac{88}{261} \\ 0 & 0 & -\frac{522}{11} & -\frac{53}{11} & -\frac{34}{11} & 1 \end{array} \right) \begin{array}{l} \leftarrow + \\ \left| \cdot \frac{6}{11} \right] \end{array} \\ \Rightarrow & \left( \begin{array}{ccc|ccc} -3 & 0 & 0 & \frac{125}{174} & -\frac{19}{87} & \frac{43}{174} \\ 0 & -\frac{11}{3} & 0 & \frac{11}{261} & -\frac{11}{261} & \frac{88}{261} \\ 0 & 0 & -\frac{522}{11} & -\frac{53}{11} & -\frac{34}{11} & 1 \end{array} \right) \begin{array}{l} \left| \cdot \left(-\frac{1}{3}\right) \right] \\ \left| \cdot \left(-\frac{3}{11}\right) \right] \\ \left| \cdot \left(-\frac{11}{522}\right) \right] \end{array} \\ \Rightarrow & \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{125}{522} & \frac{19}{261} & -\frac{43}{522} \\ 0 & 1 & 0 & -\frac{1}{87} & \frac{1}{87} & -\frac{8}{87} \\ 0 & 0 & 1 & \frac{53}{522} & \frac{17}{261} & -\frac{11}{522} \end{array} \right) \end{aligned}$$

Sample output with symbolic matrix

$$A = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}$$

$$\begin{aligned} & \left( \begin{array}{cc|cc} 1 & a & 1 & 0 \\ a & 1 & 0 & 1 \end{array} \right) \begin{array}{l} \left| \cdot (-a) \right] \\ \leftarrow + \end{array} \\ \Rightarrow & \left( \begin{array}{cc|cc} 1 & a & 1 & 0 \\ 0 & 1-a^2 & -a & 1 \end{array} \right) \begin{array}{l} \leftarrow + \\ \left| \cdot \frac{a}{a^2-1} \right] \end{array} \\ \Rightarrow & \left( \begin{array}{cc|cc} 1 & 0 & 1-\frac{a^2}{a^2-1} & \frac{a}{a^2-1} \\ 0 & 1-a^2 & -a & 1 \end{array} \right) \begin{array}{l} \left| \cdot \left(-\frac{1}{a^2-1}\right) \right] \end{array} \\ \Rightarrow & \left( \begin{array}{cc|cc} 1 & 0 & 1-\frac{a^2}{a^2-1} & \frac{a}{a^2-1} \\ 0 & 1 & \frac{a}{a^2-1} & -\frac{1}{a^2-1} \end{array} \right) \end{aligned}$$

Solve the equation

$$\begin{pmatrix} -1 & 4 \\ -3 & -7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\begin{aligned} & \begin{pmatrix} -1 & 4 & | & 1 \\ -3 & -7 & | & 4 \end{pmatrix} \begin{array}{l} | \cdot (-3) \\ \longleftarrow \end{array} \Big] + \\ \Rightarrow & \begin{pmatrix} -1 & 4 & | & 1 \\ 0 & -19 & | & 1 \end{pmatrix} \begin{array}{l} \longleftarrow \\ | \cdot \frac{4}{19} \end{array} \Big] + \\ \Rightarrow & \begin{pmatrix} -1 & 0 & | & \frac{23}{19} \\ 0 & -19 & | & 1 \end{pmatrix} \begin{array}{l} | \cdot (-1) \\ | \cdot (-\frac{1}{19}) \end{array} \\ \Rightarrow & \begin{pmatrix} 1 & 0 & | & -\frac{23}{19} \\ 0 & 1 & | & -\frac{1}{19} \end{pmatrix} \end{aligned}$$