Homework 1

Encryption

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Task 1: Affine cipher in $\mathbb{Z}_{97}$.

(a) Encryption and Decryption functions with key $k = (a, b)$ (the message is m and the resulting ciphertext is c):

$$E(k, m) = a \cdot m + b \bmod 97$$
$$D(k, c) = a^{-1} \cdot (c - b) \bmod 97$$

(b) The number of possible meaningful and different keys is

$$|a| \cdot |b|$$

Where $|a|$ is the possibilities for a and $|b|$ is the different possibilities for b. b can have 97 different values, and a can have as many as there are co-primes below 97 with 97. This is given by Euler's Totient function: $\phi(97) = 96$ (this is because 97 is prime).
In total the number of different keys is:

$$96 \cdot 97 = 9312$$

(c) We have an intercepted ciphertext-plaintext pair: (DOG)=(28 83 43), and a ciphertext with the same key: (78 23 33).
To figure out the key we can use two letters from the known ciphertext-plaintext pair with either the Encryption or the Decription functions: (I choose the decryption with letters (DO) = (28 83))

$$D(k, 28) = a^{-1} \cdot (28 - b) \bmod 97 = (D)3$$
$$D(k, 83) = a^{-1} \cdot (83 - b) \bmod 97 = (O)14$$

Thus we get:

$$a^{-1} \cdot (28 - b) \bmod 97 = 3$$
$$a^{-1} \cdot (83 - b) \bmod 97 = 14$$

Rewriting the equations we get:

$$b = 28 - 3a \bmod 97$$
$$b = 83 - 14a \bmod 97$$

Making the two equal each other:

$$28 - 3a = 83 - 14a \bmod 97$$

With rearranging we get:

$$55 = 11a \bmod 97$$

We need the multiplicative inverse of 11 in mod97, which is 53. (Using the Extended Euclidean Algorithm this is fairly straightforward.) With multiplication we get:

$$a = 55 \cdot 53 \bmod 97 = 2915 \bmod 97 = 5$$

Substituting back to one previous equation we get:

$$b = 28 - 3 \cdot 5 \bmod 97 = 28 - 15 \bmod 97 = 13$$

Meaning the key is: $k = (5, 13)$
For decrypting the second message we firstly need the multiplicative inverse of $5 \pmod 9)7$ which is 39. Writing this into the decryption function we get:

$$D(k, 78) = 39 \cdot (78 - 13) \bmod 97 = 13(N)$$
$$D(k, 23) = 39 \cdot (23 - 13) \bmod 97 = 2(C)$$
$$D(k, 33) = 39 \cdot (33 - 13) \bmod 97 = 4(E)$$

The encrypted message is (NCE).

Task 2 My choice of English plaintext is from "The Hobbit":
In a hole in the ground there lived a hobbit. Not a nasty, dirty, wet hole, filled with the ends of worms and an oozy smell, nor yet a dry, bare, sandy hole with nothing in it to sit down on or to eat: it was a hobbit-hole, and that means comfort
This is the original frequency analysis of the text:

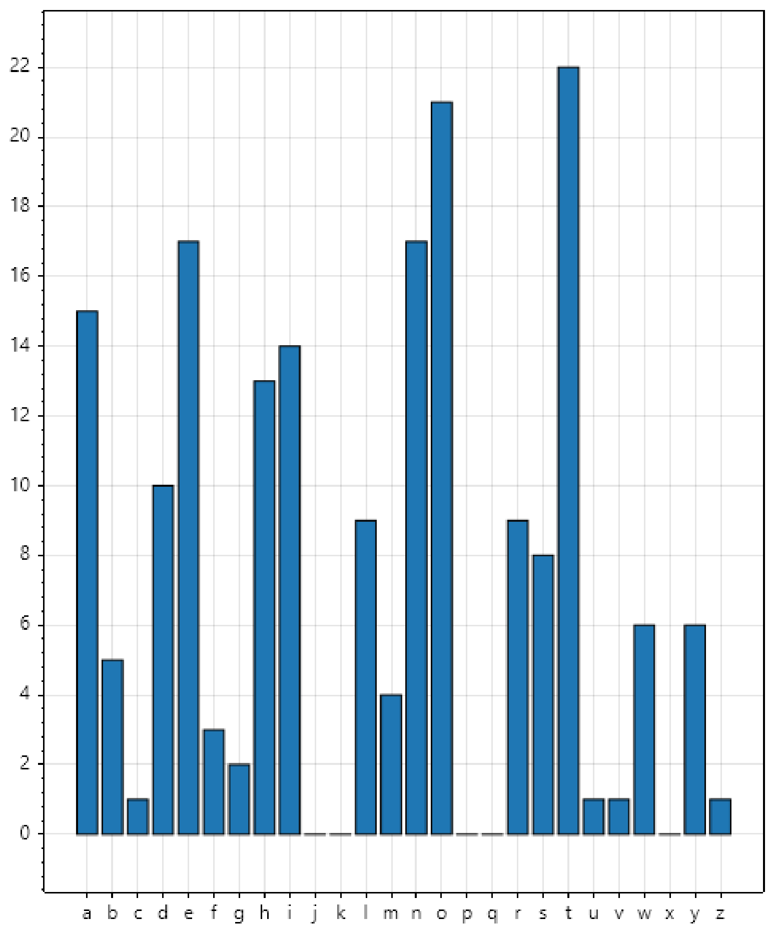

Figure 1: The plaintext histogram

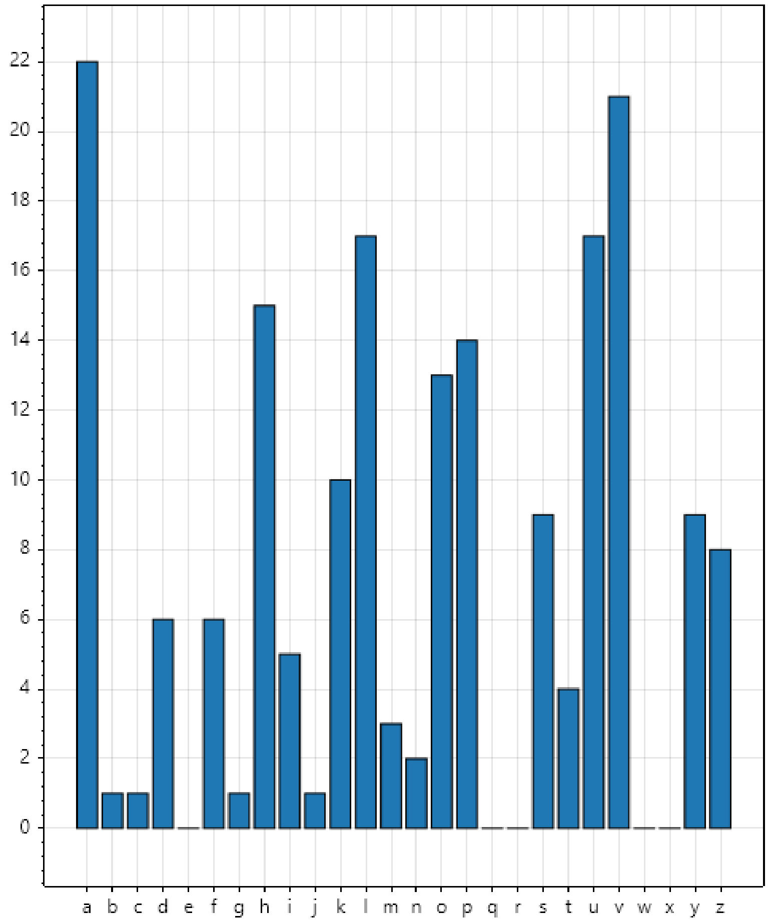

Figure 2: Ciphertext histogram after shift cipher

Then I generated a ciphertext using shift cipher with key $k = 7$.
On the histogram 2 the bars' height does not change compared to the original text, only their position moves with the key. This is the key giveaway for the shift cipher.

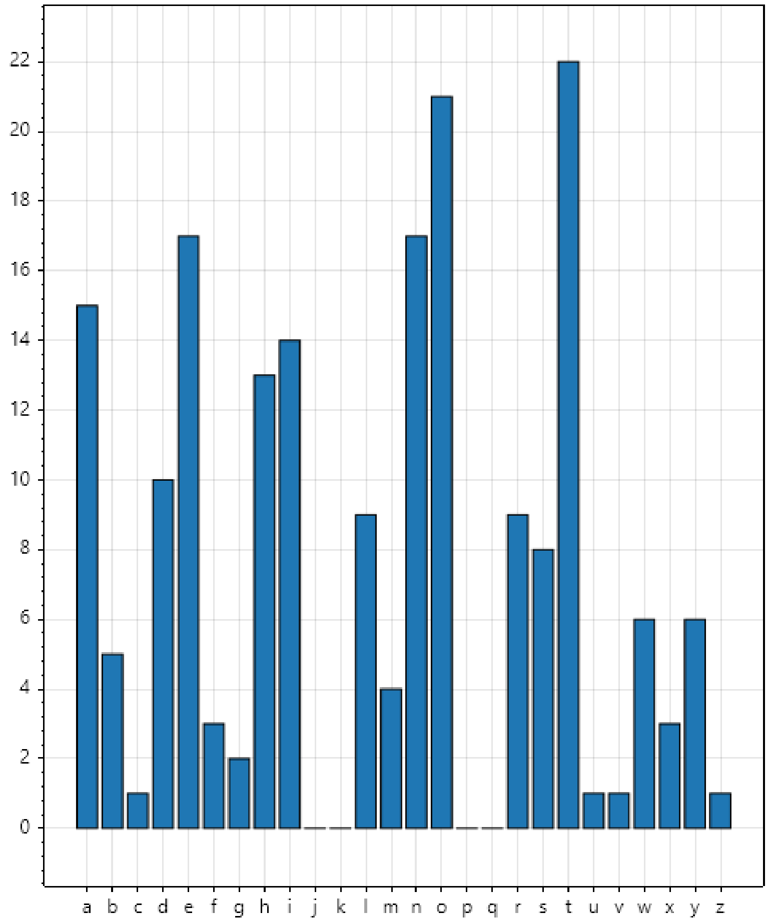

Figure 3: Ciphertext histogram after permutation cipher

Then I generated a ciphertext with permutation cipher using the key $(7, 2, 5, 3, 8, 4, 1, 6)$, meaning the length of the key is 8. I denoted the first element with 1 in the key.
The histogram 3 shows that all the letter frequencies remained the same, except for x, which I used to pad out the last segment of the plaintext. This is the main property for permutation ciphers, the letter frequencies do not change, only the individual letters' position.

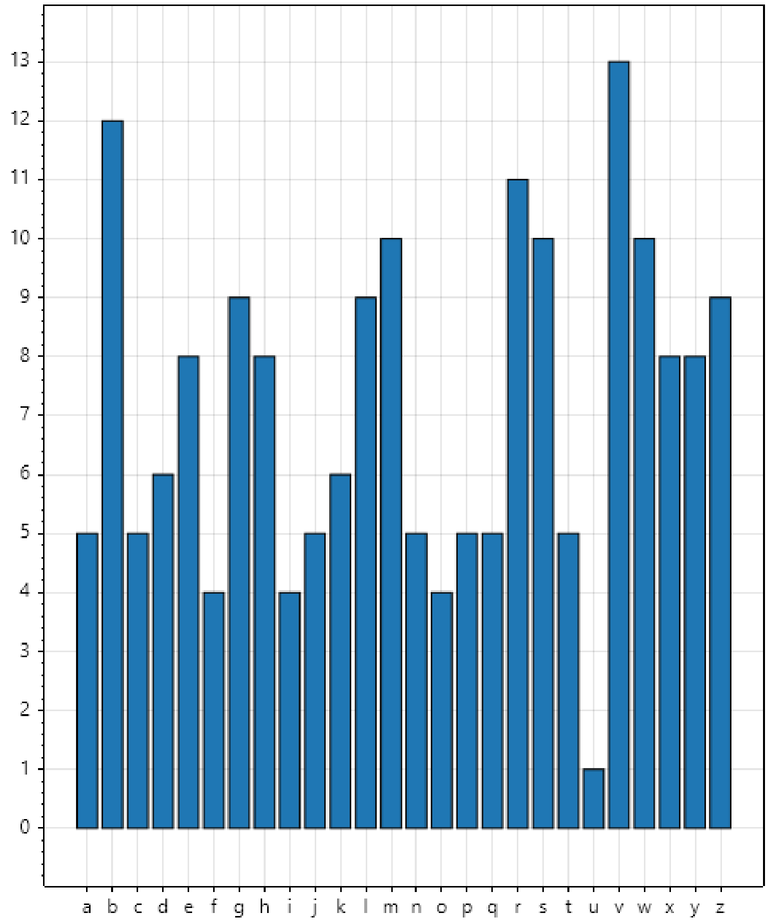

Figure 4: Ciphertext histogram after Vigenère cipher

The last ciphertext is generated with Vigenère cipher, using the key (*tolkien*) (I felt this to be appropriate for the plaintext).
The resulting histogram 4 shows a much more evenly distributed letter frequency. This is the result of each individual letter being encoded with a different individual key. The more even histogram indicates a Vigenère cipher.
For each cipher I wrote the code using C#. The histograms are generated using ScottPlot, which is a free tool for .NET. The code is available (publicly) on GitHub. Program code

Task 3

Task 4

Task 5