4/7/14

$$\frac{1}{s^2(s+3)} = \frac{A}{s+3} + \frac{B\cdot s + C}{s^2} = \frac{1}{9}\frac{1}{s+3} + \frac{-\frac{1}{9}s + \frac{1}{3}}{s^2}$$

$$1 = As^2 + (Bs+C)(s+3)$$
$$1 = As^2 + Bs^2 + 3Bs + Cs + 3C$$
$$1 = (A+B)s^2 + \cancel{Cs + 3B} + 3C$$
$$(3B+C)s$$

$\cancel{A+B=0}$ $\cancel{B=1/3}$
$\cancel{C=0}$ $\cancel{A=2/3}$
$\cancel{3B+3C=1}$ $\cancel{C=0}$

$A+B=0$ $B=-1/9$
$3B+C=0$ $A=1/9$
$3C=1$ $C=1/3$

$$\mathcal{L}\{y\} = \frac{1}{9}\frac{1}{s+3} + \frac{-\frac{1}{9}s+\frac{1}{3}}{s^2} - \frac{1}{s+3}$$

$$= \frac{1}{9}\frac{1}{s+3} - \frac{1}{9}\frac{s}{s^2} + \frac{1}{3}\frac{1}{s^2} - \frac{1}{s+3}$$

$$\mathcal{L}^{-1}\quad \mathcal{L}\{y\} = -\frac{8}{9}\frac{1}{s+3} - \frac{1}{9}\frac{1}{s} + \frac{1}{3}\frac{1}{s^2}$$

$$y = -\frac{8}{9}e^{-3t} - \frac{1}{9} + \frac{1}{3}t$$

$\frac{1}{s}$ $\mathcal{L}^{-1} \to 1$

ex $y'' + 3y' + 5y = t + e^{-t}$ $\quad y(0) = -1$
$y'(0) = 0$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 3$$

$y'' + y = \sin 2t$ $\quad y(0) = 2$
$y'(0) = 1$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + \mathcal{L}\{y\} = \frac{2}{s^2+2^2}$$

$$s^2\mathcal{L}\{y\} - 2s - 1 + \mathcal{L}\{y\} = \frac{2}{s^2+4}$$

$$\mathcal{L}\{y\}(s^2+1) = \frac{2}{s^2+4} + 2s + 1$$

$$\mathcal{L}\{y\} = \frac{2}{(s^2+4)(s^2+1)} + \frac{2s}{s^2+1} + \frac{1}{s^2+1}$$

$$\frac{2}{(s^2+4)(s^2+1)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1}$$

$A+C=0$ $A=0$
$B+D=0$ $B=-1/2$
$4C=0$ $C=0$
$A+4D=2$ $D=1/2$

$$2 = (As+B)(s^2+1) + (Cs+D)(s^2+4)$$
$$2 = As^3 + A + Bs^2 + B + Cs^3 + 4Cs + Ds^2 + 4D$$
$$2 = (A+C)s^3 + (B+D)s^2 + (4C)s + (A+4D)$$

4/1/14

$f''(x) + 5f'(x) + 6f(x) = 2e^{-x} \quad y(0) = 0 \quad y'(0) = 0$

$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 5[s\mathcal{L}\{y\} - y(0)] + 6\mathcal{L}\{y\} = \mathcal{L}\{2e^{-x}\}$

$s^2\mathcal{L}\{y\} - 0 - 0 + 5s\mathcal{L}\{y\} - 0 + 6\mathcal{L}\{y\} = \mathcal{L}\{2e^{-x}\}$

$\underbrace{(s^2+5s+6)}_{(s+2)(s+3)}\mathcal{L}\{y\} = \mathcal{L}\{2e^{-x}\} = \frac{2}{s+1}$

$$\mathcal{L}\{y\} = \frac{2}{(s+1)(s+2)(s+3)} = 2 \cdot \frac{1}{(s+1)(s+2)(s+3)}$$

$1 = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$

$1 = As^2 + 5As + 6A + Bs^2 + 4Bs + 3B + Cs^2 + 3Cs + 2C$

$1 = (A+B+C)s^2 + (5A+4B+3C)s + (6A+3B+2C)$

$A+B+C=0 \quad A = 1/2$

$5A+4B+3C=0 \quad B = -1$

$6A+3B+2C=1 \quad C = 1/2$

$\mathcal{L}\{y\} = 2\left(\frac{1}{2}\frac{1}{s+1} - \frac{1}{s+2} + \frac{1}{2}\frac{1}{s+3}\right) = \frac{1}{s+1} - 2\frac{1}{s+2} + \frac{1}{s+3}$

$y = e^{-x} - 2e^{-2x} + e^{-3x}$

$\boxed{\text{Discussion}}$ 6.1 Laplace Transform 4/8/14

2.9 Find $f(t)$ for $F(s) = \mathcal{L}\{f(t)\}$

$F(s) = \frac{1-2s}{s^2+4s+5} = \frac{1-2s}{(s+2)^2+1} = \frac{1-2(s+2)+4}{(s+2)^2+1}$

$F(s) = \frac{5-2(s+2)}{(s+2)^2+1} = 5\underbrace{\left[\frac{1}{(s+2)^2+1}\right]}_{F_1(s+2)} - 2\underbrace{\left[\frac{s+2}{(s+2)^2+1}\right]}_{F_2(s+2)}$

$F(s-c) = \mathcal{L}\{e^{ct}f(t)\}$,

$F(s) = \mathcal{L}\{f(t)\} \quad F_1(s) = \mathcal{L}\{\sin t\} \quad F_2(s) = \mathcal{L}\{\cos t\}$

$5\mathcal{L}\{e^{-2t}\sin t\} \,/\, -2\mathcal{L}\{e^{-2t}\cos t\}$

$F(s) = \mathcal{L}\{\underbrace{5e^{-2t}\sin t - 2e^{-2t}\cos t}_{f(t) =}\}$

4/8/14

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \int_\pi^{2\pi} e^{-st}(t-\pi)\,dt$

$\mathcal{L}\{u_c(t) f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$

$\qquad\qquad\qquad f(t)=t$

$f(t) = u_\pi(t-\pi) - u_{2\pi}(t - 2\pi + \pi)$

$f(t) = u_\pi(t-\pi) - u_{2\pi}(t-2\pi) - \pi u_{2\pi}$

$\mathcal{L}\{f(t)\} = e^{-\pi s}\mathcal{L}\{t\} - e^{-2\pi s}\mathcal{L}\{t\} - \pi\mathcal{L}\{u_{2\pi}\}$

$\qquad = e^{-\pi s}\left(\frac{1}{s^2}\right) - e^{-2\pi s}\left(\frac{1}{s^2}\right) - \pi / e^{-2\pi s}\mathcal{L}\{1\}$

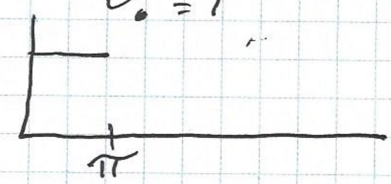
$\left(\frac{1}{s^2}\right)\left[e^{-\pi s} - e^{-2\pi s}\right] - \pi e^{-2\pi s}\left(\frac{1}{s}\right) = \mathcal{L}\{f(t)\}$

---

24 Find Laplace transform of $\mathcal{L}\{y(t)\} = Y(s)$ $\qquad y(0)=1$, $y'(0)=0$

3 $y'' + 4y = \begin{cases} 1, & 0 \le t < \pi \\ 0, & \pi \le t \end{cases} = 1 - u_\pi$

$u_0 = 1$



$\mathcal{L}\{u_c f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$

$y'' + 4y = 1 - u_\pi$

$\mathcal{L}\{y''\} + 4\mathcal{L}\{y\} = \mathcal{L}\{1 - u_\pi\}$

$s^2 Y - sy(0) - y'(0) + 4[Y] = \mathcal{L}\{1\} - \mathcal{L}\{u_\pi\}$

$s^2 Y - s\cdot 1 + 0 + 4Y = \frac{1}{s} - e^{-\pi s}\mathcal{L}\{1\}$

$s^2 Y - s + 4Y = \frac{1}{s}\left(1 - e^{-\pi s}\right)$

$Y(s^2+4) = s + \frac{1}{s}\left(1 - e^{-\pi s}\right)$

shifting

$Y(s) = \frac{s}{s^2+4} + \left(1 - e^{-\pi s}\right)\left[\frac{1}{s(s^2+4)}\right]$

$\mathcal{L}\{\cos 2t\}$

$\frac{1}{s^2+4} = \frac{A}{s} + \frac{Bs + \cancel{C}}{s^2+4} = \frac{A(s^2+4) + B(s) + Cs}{s(s^2+4)}$

$\frac{(A+B)s^2 + Cs + 4A}{s(s^2+4)}$ $\Big/$ $A + B = 0 \quad C = 0 \quad 4A = 1 \quad A = 1/4 \quad B = -1/4$

$\frac{1}{s^2+4} = \frac{1}{4}\left(\frac{1}{s}\right) + \frac{-1}{4}\left(\frac{s}{s^2+4}\right)$

$\qquad = \frac{1}{4}\mathcal{L}\{1\} - \frac{1}{4}\mathcal{L}\{\cos 2t\}$

4/9/14

$$\frac{e^{-\pi s}}{s(s^2+2s+2)}$$

t polynomial

of

thal

en

$$\frac{1}{s(s^2+2s+2)} \overset{\text{Partial}}{=} \frac{\frac{1}{2}}{s} - \frac{\frac{1}{2}s+1}{s^2+2s+2}$$

$$= \frac{1/2}{s} - \frac{\frac{1}{2}s+1}{(s+1)^2+1}$$

$$= \frac{1/2}{s} - \frac{\frac{1}{2}(s+1)+\frac{1}{2}}{(s+1)^2+1}$$

$$= \frac{1/2}{s} - \frac{1}{2}\frac{s+1}{(s+1)^2+1} - \frac{1/2}{(s+1)^2+1}$$

Don't un shift

$$= \frac{1/2}{s} - \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{1}{s^2+1} \xrightarrow{\mathcal{L}^{-1}} \frac{1}{2} - \frac{1}{2}\cos t - \frac{1}{2}\sin t$$

$$\frac{1}{2} + e^{-t}\left(\cancel{1} - \frac{1}{2}\cos t - \frac{1}{2}\sin t\right)$$

$$\frac{1}{2}\cancel{\tfrac{1}{2}}e^{-(t-\pi)}(\cos(t-\pi)+\sin(t-\pi))$$

$-\cos(t) - \sin(t)$

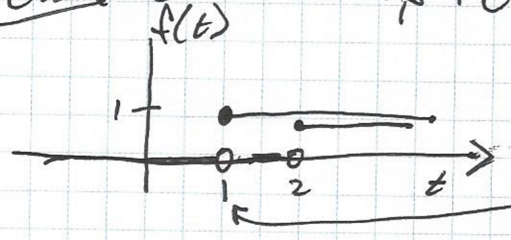
shift by π

03689

147

147	52
2	5
294	260

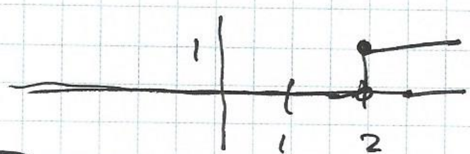
$\boxed{GE}$ 6.3 Step Function 4/9/14



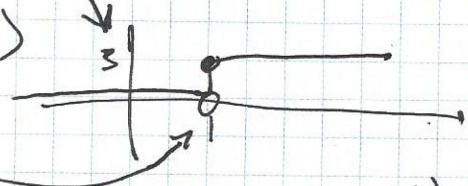
Step Function

$f(t) = u_1(t)$

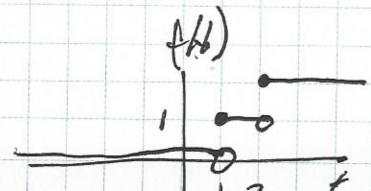
$f(t) = u_2(t)$



$f(t) = 3u_1(t)$



$f(t) = u_1(t) + u_2(t)$



4/9/14

$$\left(\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}\right) s(s^2+1)$$

$$1 = A(s^2+1) + Bs^2 + Cs \qquad A+B=0$$

$$1 = As^2 + A + Bs^2 + Cs \qquad C=0$$

$$1 = (A+B)s^2 + Cs + A \qquad A=1 \quad B=-1$$

$$\frac{1}{(s^2+1)s} = \frac{1}{s} + \frac{-s}{s^2+1} = \frac{1}{s} - \frac{s}{s^2+1}$$

$$\mathcal{L}\{y\} = \frac{1}{s(s^2+1)} - e^{-3\pi s}\left(\frac{1}{s(s^2+1)}\right) + \frac{1}{s^2+1}$$

$$\mathcal{L}\{y\} = \frac{1}{s} - \left(\frac{s}{s^2+1}\right) - e^{-3\pi s}\left[\frac{1}{s} - \frac{s}{s^2+1}\right] + \frac{1}{s^2+1}$$

#6

problem $\Rightarrow$ Use Thm 6.3.1

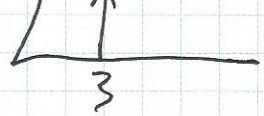
$$y = 1 - \cos t - U_{3\pi}(t)(1 - \cos(t - 3\pi)) + \sin t$$

$\boxed{6.5}$ Impulse Function

delta

$$f(c) = \delta(t-c)$$

$\delta(t-3)$



$\Rightarrow$ look at #17

ex

$$y'' - y = -20\delta(t-3) \qquad y(0) = 1 \quad y'(0) = 0$$

$$\mathcal{L}\{y''\} - \mathcal{L}\{y\} = -20\mathcal{L}\{\delta(t-3)\}$$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) - \mathcal{L}\{y\} = -30\mathcal{L}\{\delta(t-3)\}$$

$$s^2\mathcal{L}\{y\} - s - \mathcal{L}\{y\} = -20\mathcal{L}\{\delta(t-3)\}$$

$$s^2\mathcal{L}\{y\} - s - \mathcal{L}\{y\} = -20e^{-3s}$$

$$\mathcal{L}\{y\} = \frac{-20e^{-3s}}{s^2-1} + \frac{s}{s^2-1} = \frac{s}{s^2-1} - \frac{20e^{-3s}}{s^2-1}$$

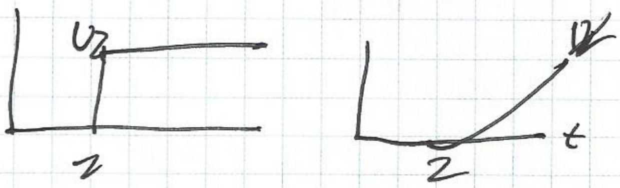
Use #8

#13

4/4/14

3.13 Find $\mathcal{L}\{f(t)\}$

$f(t) = \begin{cases} 0 & t<2 \\ (t-2) & t>2 \end{cases} \quad u_2(t-2)^2$



$\mathcal{L}\{f(t)\} = \mathcal{L}\{u_2(t-2)^2\}$ | $c=2$, $f(t)=t^2$

$\left[\mathcal{L}\{u_c f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}\right.$

$\mathcal{L}\{f(t)\} = e^{-cs}\mathcal{L}\{t^2\} = e^{-2s}\dfrac{2}{s^3}$

$\left[\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}\right.$

3.27 $F(s) = \dfrac{2s+1}{4s^2+4s+5} = \dfrac{2s+1}{(2s+1)^2+4} = \dfrac{2s+1}{4(s^2+s+\frac{5}{4})}$

$= \dfrac{2s+1}{2^2(s+\frac{1}{2})^2+2^2} = \dfrac{1}{4}\left[\dfrac{2s+1}{(s+\frac{1}{2})^2+1}\right]$

$= \dfrac{1}{4}\left[\dfrac{2(s+\frac{1}{2})}{(s+\frac{1}{2})^2+1}\right] = \dfrac{1}{2}\left[\dfrac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+1}\right]$ $\qquad \dfrac{s}{s^2+1} = \mathcal{L}\{\cos t\}$

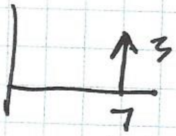
$F_1(s+\frac{1}{2})$

$= \mathcal{L}\{e^{ct}f_1(t)\} = F_1(s-c)$

$= \dfrac{1}{2}F_1(s+\dfrac{1}{2}) = \mathcal{L}\{e^{-t/2}\cos t\}$

$-\frac{1}{2}$

4/14/14

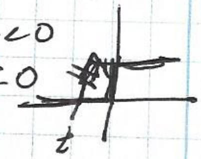


$3\delta(t-7)$

$\int_0^5 3\delta(t-7)\,dt = 3$

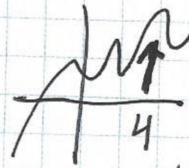
$= u_0(t)$

$f(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = \begin{cases} 0 & t<0 \\ 1 & \geq 0 \end{cases}$



$\int_0^\infty \delta(t-4) f(t)\,dt$

$= f(4)$



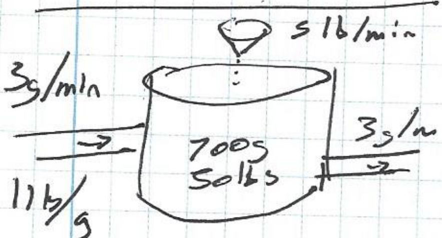
$\delta(4) \cdot 1_m^{mass}$

$\mathcal{L}\{\delta(t-c)\} = \int_0^\infty e^{-st}\delta(t-c)\,dt = e^{-sc}$

$\mathcal{L}\{\delta(t)\} = e^{-0\cdot s} = 1$

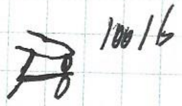
#17

5 lb/min from t = 5 to 10



t = 30 dump 100lb salt in tank

$\frac{dy}{dt} = 3\cdot 1 - \frac{3y}{700} + 5(u_5 - u_{10}) + 100\delta(t-30)$



$y(0) = 50$

$y(0)$

$s\bar{Y} - 50 = \frac{3}{s} - \frac{3}{700}\bar{Y} + 5\frac{e^{-5s}}{s} - \frac{5e^{-10s}}{s} + 100e^{-30s}$

$\bar{Y} = \frac{50}{s+3/700} + \frac{3}{s(s+3/700)} + \frac{5e^{-5s}}{''} - \frac{5e^{-10s}}{''} + \frac{100e^{-30s}}{s+3/700}$

$\frac{100e^{-30s}}{s+3/700}$ $\quad u_{30}\, 100 e^{-\frac{3(t-30)}{700}}$

$\downarrow \qquad \uparrow$

$\frac{100}{s+3/700}$ $\quad e^{-\frac{3t}{700}}\, 100$

$\downarrow \qquad \uparrow$

$\frac{100}{s} \longrightarrow 100$

10.1 Eigen Values and Eigen Function 4/14/14

~~$y'' + \lambda y = 0$~~

$y'' + 2y = 0$ $y(0) = 1$, $y(\pi) = 0$] Boundary Condition

$r^2 + 2 = 0$

$r = \pm\sqrt{2}\, i$

$y = C_1 \cos\sqrt{2}\, x + C_2 \sin\sqrt{2}\, x$

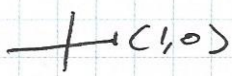
$1 = C_1 \cos 0 + C_2 \sin 0$ $C_1 = 1$

$0 = C_1 \cos\sqrt{2}\,\pi + C_2 \sin\sqrt{2}\,\pi$

$0 = \cos\sqrt{2}\,\pi + C_2 \sin\sqrt{2}\,\pi$ $C_2 = \dfrac{-\cos\sqrt{2}\,\pi}{\sin\sqrt{2}\,\pi} = -\cot\sqrt{2}\,\pi$

$y'' + 2y = 0$ $y(0) = 0$, $y(\pi) = 0$

$r^2 + 2 = 0$

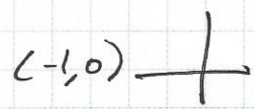


$y = C_1 \cos\sqrt{2}\, x + C_2 \sin\sqrt{2}\, x$

$0 = C_1 \cos 0$ $C_1 = 0$

$0 = C_2 \underbrace{\sin\sqrt{2}\,\pi}_{\text{cannot be } 0}$ $C_2 = 0$

$y = 0$ ← trivial soln, bad



$y'' + y = 0$ $y(0) = 0$, $y(\pi) = 0$

$r^2 + 1 = 0$

$r^2 = -1$

$r = \pm\sqrt{-1} = \pm i$

$y = C_1 \cos x + C_2 \sin x$

$0 = C_1$

~~$0 = C_2 \sin\pi$~~

~~$C_2 = 0$~~

$y = 0$

$0 = C_1 \overset{-1}{\cos\pi} + C_2 \sin\pi$

$0 = -C_1$ $C_1 = 0$

$y = C_2 \sin x$

$y'' + \lambda y = 0$ $y(\pi) = 0$, $y(0) = 0$ λ is unknown constant

$r^2 + \lambda = 0$

$r = \pm i\sqrt{\lambda}$

Cases ① $\lambda > 0$ ② ~~$\lambda = 0$~~ ③ $\lambda < 0$

~~λ~~ $\lambda = 0$ $y'' = 0$ $r^2 = 0$, $r = 0, 0$ $y = C_1 e^{0t} + C_2 t e^{0t} = C_1 + C_2 t$

$0 = C_1 = 0$

$0 = C_1 + C_2\pi$ $C_2 = 0$

10.1 4/14/11

$y'' + \lambda y = 0$ $\quad y'(0) = 0$, $y(\pi) = 0$ Find eigenvalues and eigenfunctions

i) $\lambda = 0$

$r^2 = 0$
$r = 0, 0$

$y = c_1 e^{0t} + c_2 t e^{0t} = c_1 + c_2 t$

$0 = c_1 + c_2 \pi$

$y' = c_2$

$y'(0) = 0 = c_2$ $\qquad c_2 = 0$, $c_1 = 0$

ii) $\lambda < 0$ $\quad \lambda = -\mu^2$

$\int e^{ax} = \frac{e^{ax}}{a}$

$y'' + \lambda y = 0$
$y'' - \mu^2 y = 0$
$r^2 - \mu^2 = 0$
$r^2 = \mu^2$
$r = \pm\mu$

$y = c_1 e^{\mu t} + c_2 e^{-\mu t}$

$y' = \mu c_1 e^{\mu t} - \mu c_2 e^{-\mu t}$

$y'(0) = \mu c_1 - \mu c_2 = 0 \quad c_2 = c_1$

$y(\pi) = c_1 e^{\mu\pi} + c_2 e^{-\mu\pi} = 0$

$0 = c_1 e^{\mu\pi} + c_1 e^{-\mu\pi}$

$0 = c_1(e^{\mu\pi} + e^{-\mu\pi}) = 2c_1\left(\frac{e^{\mu\pi} + e^{-\mu\pi}}{2}\right)$

$= 2c_1 \underbrace{\cosh(\mu\pi)}_{\neq 0} \Rightarrow c_1 = 0 = c_2$

iii) $\lambda > 0$ $\quad \lambda = \mu^2$

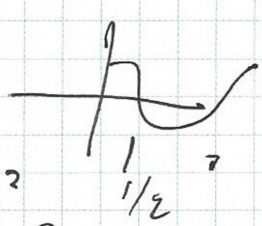
$y'' + \lambda y = 0$
$y'' + \mu^2 y = 0$
$r^2 + \mu^2 = 0$
$r^2 = -\mu^2$
$r = \pm i\sqrt{\mu}$

$y = c_1 \cos \mu x + c_2 \sin \mu x$ $\qquad y'(0) = 0$, $y(\pi) = 0$

$y' = c_1 \mu \sin \mu x + c_2 \mu \cos \mu x$

$0 = -c_1 \mu \sin \mu \cdot 0 + c_2 \mu \cos \mu 0$

$0 = c_2 \mu$ $\qquad c_2 = 0$



$\cos\left(\frac{2n-1}{2}\pi\right)$

$0 = c_1 \cos \mu\pi + c_2 \sin \mu\pi$

$\lambda = \mu^2 = \left(\frac{2n-1}{2}\right)^2$ ~ Eigenvalue

$0 = c_1 \cos \mu\pi$

$y = c_1 \cos\left(\left(\frac{2n-1}{2}\right)x\right)$ ← Eigenfunction

Laplace Transform, Gamma Function 4/15/14

Dirac $D(t_a = \delta(t-t_a)$

$t_a > 0$, $\mathcal{L}\{\delta(t-t_4)\} = \int_0^\infty e^{-st}\delta(t-t_4)\,dt = 0$

$t(a,b)$ $\int_a^b f(t)\,\delta(t-t_1) = f(t_1)$

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$y'' + 4y = \delta(t-\pi) - \delta(t-2\pi) \qquad y(0)=0 \quad y'(0)=0$

$\mathcal{L}\{y''\} + 4\{y\} = \mathcal{L}\{\delta(t-\pi)\} - \mathcal{L}\{\delta(t-2\pi)\}$

$s^2\bar{y} - \cancel{sy(0)} - \cancel{y'(0)} + 4\bar{y} = e^{-\pi s} - e^{-2\pi s}$

$(s^2+4)\bar{y} = e^{-\pi s} - e^{-2\pi s} \qquad \mathcal{L}\{u_c f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$

$y = (e^{-\pi s} - e^{-2\pi s})\dfrac{1}{s^2+4}$

$= (e^{-\pi s} - e^{2\pi s})\underbrace{\left(\dfrac{2}{s^2+2^2}\right)}_{\mathcal{L}\{\sin 2t\}}\left(\dfrac{1}{2}\right)$

$y(s) = \left(\mathcal{L}\{u_\pi \sin[2(t-\pi)]\} - \mathcal{L}\langle u_{2\pi}\sin[2(t-2\pi)]\rangle\right)\cdot\frac{1}{2}$

$y(s) = \frac{1}{2}\left[\mathcal{L}\langle u_\pi \sin(2t - \cancel{2\pi}) - u_{2\pi}\sin(2t - \cancel{4\pi})\rangle\right.$

↑ Periodic so set it off

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$y'' + y = u_{\pi/2} + 3\delta(t - 3\pi/2) - u_{2\pi} \qquad y(0) = y'(0) = 0$

$s^2\bar{y} - \cancel{sy(0)} - \cancel{y'(0)} + \bar{y} = \dfrac{e^{-\frac{\pi}{2}s}}{s} + 3e^{-3\pi/2\cdot s} - e^{-2\pi s}/s$

$(s^2+1)\bar{y} = \dfrac{e^{-\pi/2\cdot s} - e^{-2\pi s}}{s} + 3e^{-\frac{3\pi}{2}s}$

$y(s) = 3e^{-3\pi s/2}\underbrace{\left(\dfrac{1}{s^2+1}\right)}_{\mathcal{L}\langle \sin t\rangle} + \left(e^{-\pi s/2} - e^{-2\pi s}\right)\left[\dfrac{1}{s(s^2+1)}\right]$

$\dfrac{1}{(s^2+1)s} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+1} = \dfrac{1}{s} - \dfrac{s}{s^2+1} = \dfrac{s^2+1-s^2}{s(s^2+1)}$

$\bar{y}(s) = 3e^{-3\pi/2\cdot s}\,\mathcal{L}\langle \sin t\rangle + (e^{-\frac{\pi}{2}s} - e^{-2\pi s})\left[\frac{1}{s} - \frac{s}{s^2+1}\right]$

$\mathcal{L}\langle 1\rangle - \mathcal{L}\langle \cos t\rangle$

$\bar{y} = 3\mathcal{L}\langle u_{3\pi}\sin(t - \frac{3\pi}{2})\rangle + (e^{-\frac{\pi}{2}s} - e^{-2\pi s})\,\mathcal{L}\langle 1 - \cos t\rangle$

4/15/14

$\Gamma(n+1) = n(n-1)\ldots 2\Gamma(2)$

$\Gamma(1+1) = 1\cdot\Gamma(1) = 1\cdot 1$

$\Gamma(n+1) = n(n-1)\ldots 2\cdot 1 = n!$

$\mathcal{L}\{t^n\} = \frac{\Gamma(n+1)}{s^{n+1}} = \frac{n!}{s^{n+1}}$

.2 $y'' + 4y =$

$y'' + y = \cos + \delta(t-2\pi)$ $\quad y(0) = 0 \quad y'(0) = 1$ $\quad 2\pi \in (0,\infty)$

$\mathcal{L}\langle y''\rangle + \mathcal{L}\langle y\rangle = \mathcal{L}\langle \cos t\,\delta(t-2\pi)\rangle$

$s^2\bar{y} - sy(0) - y'(0) + \bar{y} = \int_0^\infty \underbrace{e^{-st}\cos t}\,\delta(t-2\pi)\,dt$

$\in(a,b)$ $\int_a^b f(t)\,\delta(t-t_0)\,dt = f(t_0)$ $\quad f = e^{-2\pi s}\cdot 1$

$= e^{-2\pi s}\cos(2\pi)$

$s^2\bar{y} - \cancel{sy(0)} - 1 + y = e^{-2\pi s}$

$(s^2+1)\bar{y} = 1 + e^{-2\pi s}$

$y = (1+e^{-2\pi s})\left(\frac{1}{s^2+1}\right)$ $\quad e^{-cs}\mathcal{L}\{f(t)\} = \mathcal{L}\langle u_c f(t-c)\rangle$

$\mathcal{L}\langle \sin t\rangle$

$y(s) = \mathcal{L}\langle \sin t\rangle + \mathcal{L}\langle u_{2\pi}\sin(t-2\pi)\rangle$

$= \mathcal{L}\langle \sin t\,(1+u_{2\pi})\rangle$

$y(t) = \sin t\,(1+2u_{2\pi}(t))$