1 ADMM details

We first re-parameterize $\phi_j = y - \theta_j$ so the problem is

$$\text{minimize } \rho_\tau(\phi) + \lambda||D^{(k)}(y-\phi)||_1 \tag{1}$$

We further divide ϕ order to solve smaller problems: Defining

$$\begin{align}
\phi_1 &= (\phi_{11}, \phi_{12}) \tag{2}\\
\phi_2 &= (\phi_{21}, \phi_{22}, \phi_{23}) \tag{3}\\
\phi_3 &= (\phi_{31}, \phi_{32}) \tag{4}\\
\phi &= (\phi_{11}, \phi_{12} = \phi_{21}, \phi_{22}, \phi_{23} = \phi_{31}, \phi_{32}) \tag{5}\\
& \tag{6}
\end{align}$$

Dividing y similarly, the problem then becomes

$$\begin{align}
&\text{minimize } \sum_{i=1}^{3} \rho_\tau(\phi_i) + \lambda||D^{(k)}(y_i - \phi_i)||_1 \tag{7}\\
&\text{subject to: } \phi_{12} = \phi_{21}, \quad \phi_{23} = \phi_{31} \tag{8}\\
& \tag{9}
\end{align}$$

We can further simplify by defining

$$\begin{align}
\overline{\phi} &= (\phi_{11}, \frac{\phi_{12}+\phi_{21}}{2}, \phi_{22}, \frac{\phi_{23}+\phi_{31}}{2}, \phi_{32}) \tag{10}\\
\overline{\phi_1} &= (\phi_{11}, \frac{\phi_{12}+\phi_{21}}{2}) \tag{11}\\
\overline{\phi_2} &= (\frac{\phi_{12}+\phi_{21}}{2}, \phi_{22}, \frac{\phi_{23}+\phi_{31}}{2}) \tag{12}\\
\overline{\phi_3} &= (\frac{\phi_{23}+\phi_{31}}{2}, \phi_{32}) \tag{13}
\end{align}$$

so the problem becomes

$$\begin{align}
&\text{minimize } \sum_{i=1}^{3} \rho_\tau(\phi_i) + \lambda||D^{(k)}(y_i - \phi_i)||_1 \tag{14}\\
&\text{subject to: } \phi_i = \overline{\phi_i} \tag{15}\\
& \tag{16}
\end{align}$$

The augmented Lagrangian for this problem is

$$\begin{align}
&L_\gamma(\phi_1, \phi_2, \phi_3, \overline{\phi_1}, \overline{\phi_2}, \overline{\phi_3}, \omega) = \tag{17}\\
&\sum_{i=1}^{3} \rho_\tau(\phi_i) + \lambda||D^{(k)}(y_i - \phi_i)||_1 + \omega_i^T(\phi_i - \overline{\phi_i}) + \frac{\gamma}{2}||\phi_i - \overline{\phi_i}||_2^2 \tag{18}
\end{align}$$

The ADMM updates are then given by

$$\phi_i^{k+1} = \arg\min_{\phi_i} \rho_\tau(\phi_i) + \lambda||D^{(k)}(y_i - \phi_i)||_1 + \omega_i^{kT}(\phi_i - \overline{\phi_i}^k) + \frac{\gamma}{2}||\phi_i - \overline{\phi_i}^k||_2^2 \tag{19}$$

$$\omega_i^{k+1} = \omega_i^k + \gamma(\phi_i^{k+1} - \overline{\phi_i}^{k+1}) \tag{20}$$

The ϕ_i updates can be obtained using a quadratic program solver such as Gurobi and can be obtained in parallel.

2 Simulation Metrics

Figure 1: F1 score by threshold, data size, and method (1 is best 0 is worst).

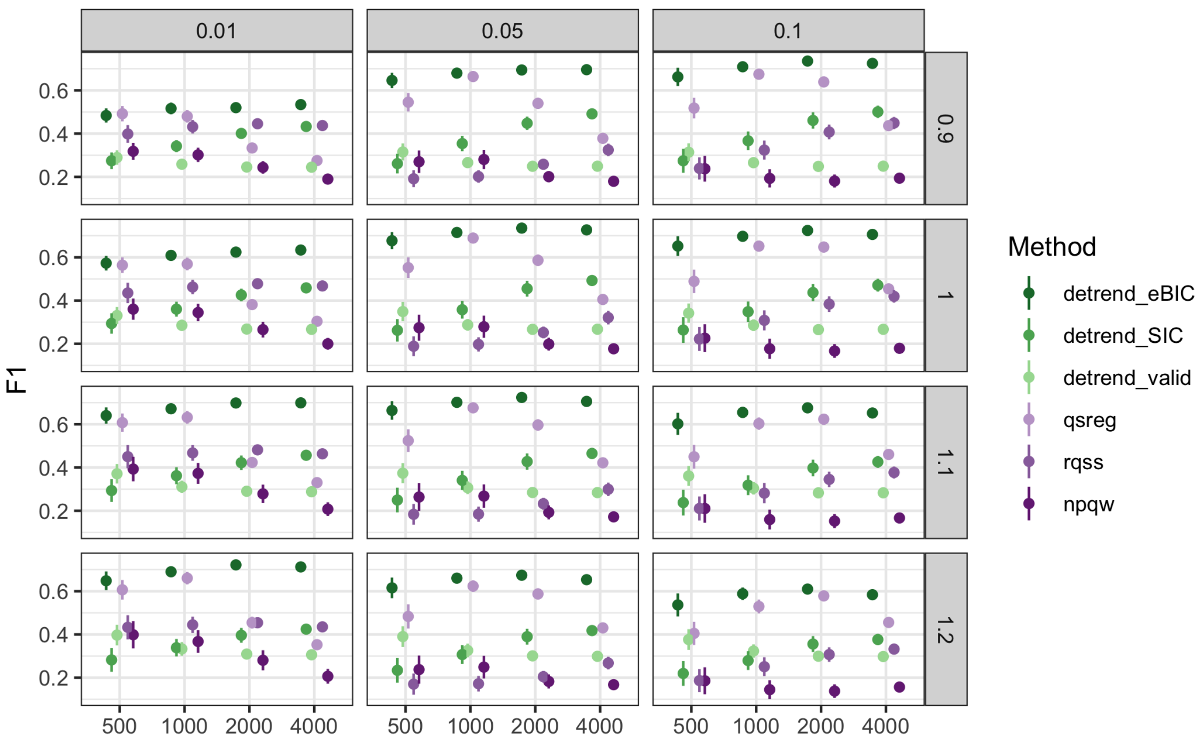

Figure 2: Precision by threshold, data size, and method (true positive over true positives + false positives).

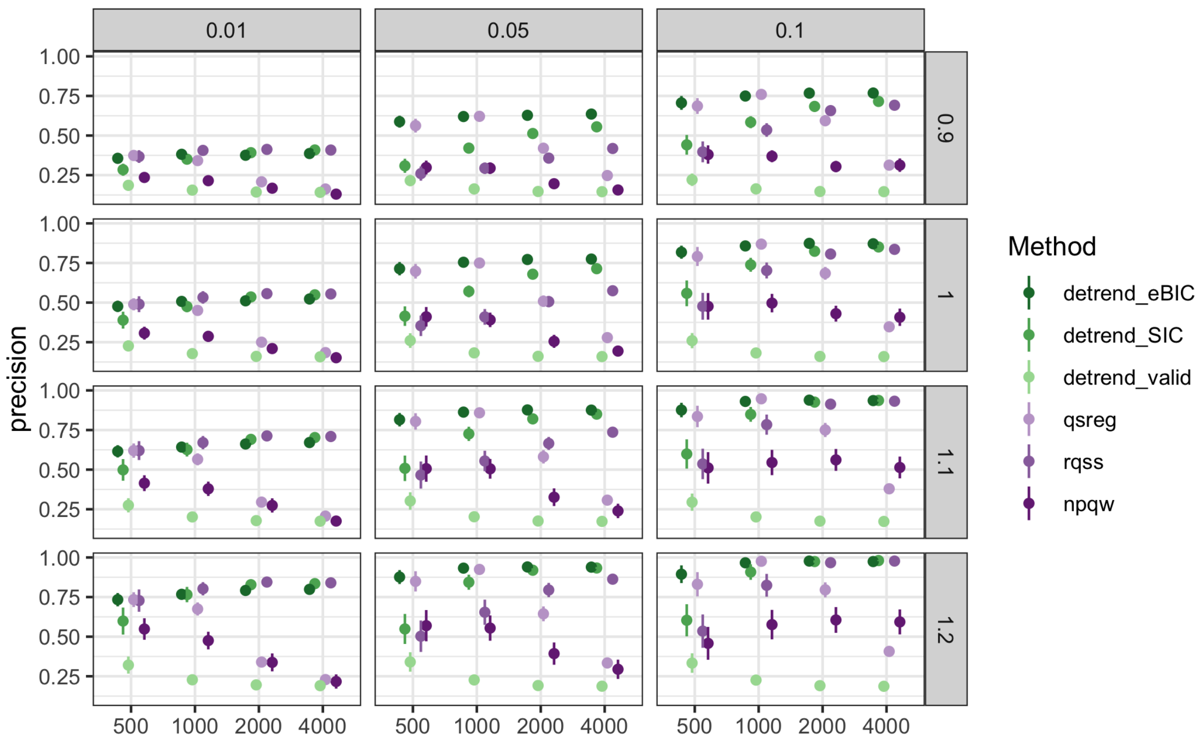

Figure 3: Recall by threshold, data size, and method (true positive over true positives + false negatives).

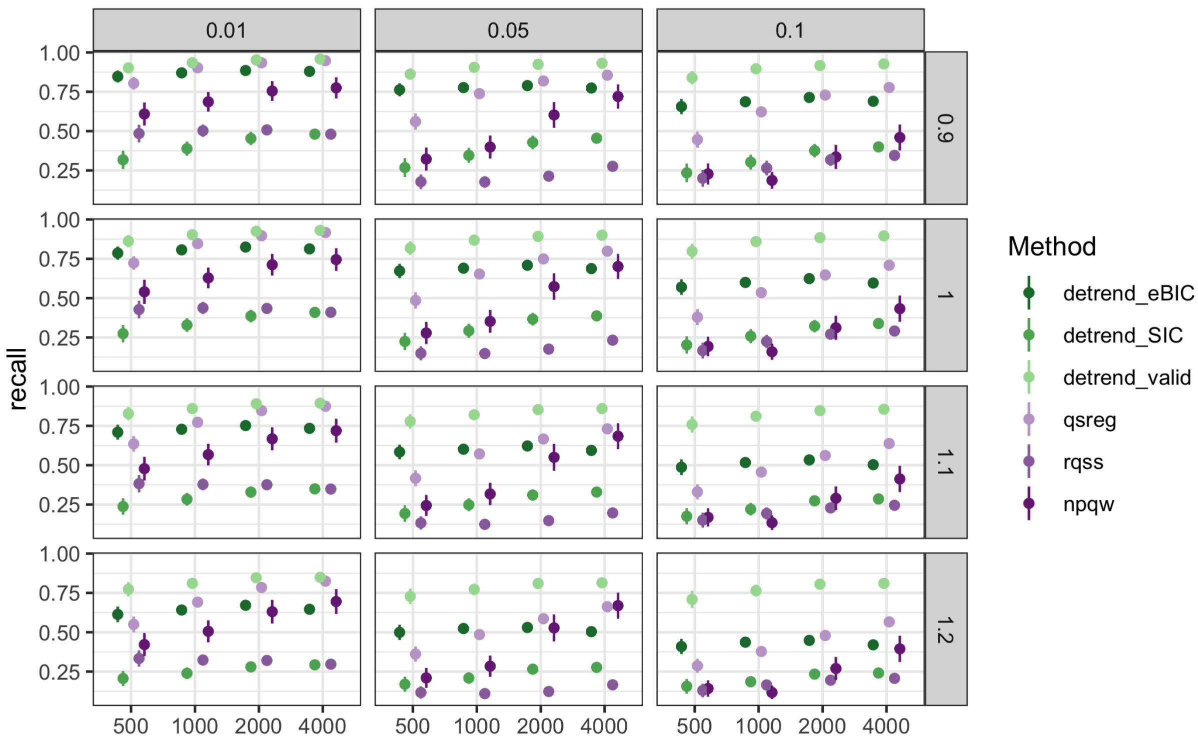

Figure 4: Miss-classification rates by threshold, data size, and method, values above the upper limit (npqw) not shown.