Baseline Drift Estimation for Air Quality Data Using Quantile Trend Filtering

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1 Technical Details on the ADMM Algorithm

1.1 Convergence

Our windowed quantile trend optimization problem can be written as

$$\begin{aligned} &\text{minimize} \sum_{w=1}^{W} \left\{ \sum_{j=1}^{J} \left[\rho_{\tau_j}(\mathbf{y}^{(w)} - \boldsymbol{\theta}_j^{(w)}) + \lambda_j \|\mathbf{D}^{(k+1)} \boldsymbol{\theta}_j^{(w)}\|_1 \right] + \iota_{\mathcal{C}}(\boldsymbol{\Theta}^{(w)}) \right\} \\ &\text{subject to} \quad \boldsymbol{\Theta}^{(w)} = \mathbf{U}^{(w)} \boldsymbol{\Theta} \quad \text{for } w = 1, \ldots, W. \end{aligned} \tag{1.1}$$

Algorithm 1 has the following convergence guarantees.

Proposition 1.1. *Let* $\{\{\boldsymbol{\Theta}_m^{(w)}\}, \boldsymbol{\Theta}_m\}$ *denote the* m*th collection of iterates generated by Algorithm 1. Then (i)* $\|\boldsymbol{\Theta}_m^{(w)} - \mathbf{U}^{(w)} \boldsymbol{\Theta}_m\|_F \to 0$ *and (ii)* $p_m \to p^\star$*, where* $p^\star$ *is the optimal objective function value of* (1.1) *and* p_m *is the objective function value of* (1.1) *evaluated at* $\{\{\boldsymbol{\Theta}_m^{(w)}\}, \boldsymbol{\Theta}_m\}$.

The proof of Proposition 1.1 is a straightforward application of the convergence result presented in Section 3.2 of [1].

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2 Stopping Criteria

To terminate our algorithm, we use the stopping criteria described by [1]. The criteria are based on the primal and dual residuals, which represent the residuals for primal and dual feasibility, respectively. The primal residual at the mth iteration,

$$r^m_{\text{primal}} = \sqrt{\sum_{w=1}^{W} \|\boldsymbol{\Theta}^{(w)}_m - \mathbf{U}^{(w)}\boldsymbol{\Theta}_m\|^2_{\text{F}}},$$

represents the difference between the trend values in the windows and the consensus trend value. The dual residual at the mth iteration,

$$r^m_{\text{dual}} = \gamma\sqrt{\sum_{w=1}^{W} \|\boldsymbol{\Theta}_m - \boldsymbol{\Theta}_{m-1}\|^2_{\text{F}}},$$

represents the change in the consensus variable from one iterate to the next. The algorithm is stopped when

$$\begin{aligned} r^m_{\text{primal}} &< \epsilon_{\text{abs}}\sqrt{nJ} + \epsilon_{\text{rel}} \max_w \left[\max\left\{\|\boldsymbol{\Theta}^{(w)}_m\|_{\text{F}}, \|\boldsymbol{\Theta}_m\|_{\text{F}}\right\}\right] \\ r^m_{\text{dual}} &< \epsilon_{\text{abs}}\sqrt{nJ} + \epsilon_{\text{rel}} \sqrt{\sum_{w=1}^{W} \|\boldsymbol{\Omega}^{(w)}_m\|^2_{\text{F}}}. \end{aligned} \tag{2.1}$$

In the timing experiment plotted in Figure 5 of the main paper, Algorithm 1 was the stopped when (2.1) was satisfied, defining $\epsilon_{abs} = 0.01$ and $\epsilon_{rel} = 0.001$.

3 Simulation Results

The following tables list the simulation results that are plotted in Figures 7, 8, and 10 in the manuscript.

Table 1: RMSE and standard error for Gaussian design, by method, quantile and data size. Numbers correspond to the top panel in Fig. 7 in the manuscript.

Method	0.05	0.25	0.5	0.75	0.95
n=300					
detrend eBIC	0.097 (0.003)	0.065 (0.002)	0.059 (0.002)	0.063 (0.002)	0.092 (0.003)
detrend SIC	0.108 (0.003)	0.078 (0.002)	0.070 (0.002)	0.075 (0.003)	0.107 (0.004)
detrend valid	0.118 (0.005)	0.091 (0.004)	0.077 (0.004)	0.085 (0.003)	0.124 (0.005)
rqss	0.117 (0.003)	0.084 (0.002)	0.078 (0.002)	0.081 (0.002)	0.119 (0.004)
npqw	0.094 (0.003)	0.072 (0.002)	0.070 (0.002)	0.071 (0.002)	0.096 (0.003)
qsreg	0.147 (0.003)	0.107 (0.002)	0.103 (0.002)	0.106 (0.002)	0.141 (0.003)
n=500					
detrend eBIC	0.077 (0.002)	0.051 (0.001)	0.046 (0.001)	0.051 (0.001)	0.074 (0.002)
detrend SIC	0.080 (0.003)	0.058 (0.002)	0.053 (0.002)	0.057 (0.002)	0.078 (0.003)
detrend valid	0.096 (0.003)	0.069 (0.003)	0.066 (0.003)	0.066 (0.002)	0.092 (0.004)
rqss	0.097 (0.003)	0.069 (0.002)	0.063 (0.001)	0.067 (0.002)	0.097 (0.002)
npqw	0.075 (0.002)	0.060 (0.002)	0.057 (0.002)	0.059 (0.002)	0.073 (0.002)
qsreg	0.116 (0.002)	0.085 (0.002)	0.080 (0.001)	0.081 (0.001)	0.109 (0.002)
n=1000					
detrend eBIC	0.055 (0.002)	0.039 (0.001)	0.037 (0.001)	0.039 (0.001)	0.053 (0.002)
detrend SIC	0.056 (0.002)	0.040 (0.001)	0.037 (0.001)	0.039 (0.001)	0.054 (0.002)
detrend valid	0.074 (0.003)	0.057 (0.002)	0.046 (0.002)	0.055 (0.002)	0.071 (0.003)
rqss	0.074 (0.002)	0.050 (0.001)	0.047 (0.001)	0.050 (0.001)	0.073 (0.002)
npqw	0.057 (0.001)	0.046 (0.001)	0.041 (0.001)	0.045 (0.001)	0.054 (0.002)
qsreg	0.081 (0.002)	0.059 (0.001)	0.055 (0.001)	0.059 (0.001)	0.078 (0.002)

Table 2: RMSE and standard error for Mixed Normal design, by method, quantile and data size. Numbers correspond to the middle panel in Fig. 7 in the manuscript.

Method	0.05	0.25	0.5	0.75	0.95
n=300					
detrend eBIC	0.253 (0.010)	0.178 (0.006)	0.155 (0.005)	0.189 (0.007)	0.242 (0.009)
detrend SIC	0.223 (0.008)	0.179 (0.007)	0.164 (0.006)	0.188 (0.007)	0.228 (0.008)
detrend valid	0.302 (0.011)	0.253 (0.010)	0.235 (0.011)	0.233 (0.011)	0.280 (0.011)
rqss	0.280 (0.010)	0.189 (0.008)	0.164 (0.006)	0.202 (0.008)	0.275 (0.009)
npqw	0.383 (0.008)	0.243 (0.006)	0.146 (0.005)	0.250 (0.007)	0.374 (0.007)
qsreg	0.314 (0.008)	0.299 (0.008)	0.272 (0.006)	0.287 (0.009)	0.301 (0.008)
n=500					
detrend eBIC	0.180 (0.006)	0.149 (0.005)	0.120 (0.004)	0.147 (0.004)	0.186 (0.007)
detrend SIC	0.178 (0.006)	0.152 (0.006)	0.136 (0.005)	0.145 (0.005)	0.186 (0.007)
detrend valid	0.237 (0.009)	0.203 (0.010)	0.201 (0.011)	0.201 (0.010)	0.218 (0.009)
rqss	0.222 (0.007)	0.155 (0.007)	0.123 (0.004)	0.150 (0.006)	0.220 (0.008)
npqw	0.352 (0.006)	0.217 (0.005)	0.118 (0.003)	0.217 (0.004)	0.350 (0.006)
qsreg	0.245 (0.007)	0.229 (0.006)	0.226 (0.006)	0.225 (0.006)	0.248 (0.006)
n=1000					
detrend eBIC	0.135 (0.005)	0.112 (0.004)	0.095 (0.003)	0.108 (0.004)	0.137 (0.005)
detrend SIC	0.134 (0.005)	0.116 (0.004)	0.104 (0.004)	0.112 (0.004)	0.137 (0.005)
detrend valid	0.170 (0.008)	0.152 (0.007)	0.147 (0.007)	0.164 (0.009)	0.171 (0.007)
rqss	0.167 (0.006)	0.117 (0.004)	0.097 (0.003)	0.112 (0.004)	0.168 (0.006)
npqw	0.327 (0.004)	0.200 (0.004)	0.106 (0.002)	0.195 (0.004)	0.334 (0.005)
qsreg	0.168 (0.005)	0.158 (0.004)	0.162 (0.004)	0.156 (0.004)	0.169 (0.005)

Table 3: RMSE and standard error for Beta design, by method, quantile and data size. Numbers correspond to the bottom panel in Fig. 7 in the manuscript.

Method	0.05	0.25	0.5	0.75	0.95
n=300					
detrend eBIC	0.020 (0.000)	0.025 (0.001)	0.032 (0.001)	0.041 (0.001)	0.056 (0.002)
detrend SIC	0.016 (0.001)	0.030 (0.001)	0.042 (0.001)	0.048 (0.002)	0.065 (0.002)
detrend valid	0.022 (0.001)	0.033 (0.002)	0.042 (0.002)	0.052 (0.002)	0.072 (0.002)
rqss	0.020 (0.001)	0.034 (0.001)	0.047 (0.001)	0.054 (0.001)	0.073 (0.002)
npqw	0.044 (0.001)	0.035 (0.001)	0.038 (0.001)	0.048 (0.001)	0.060 (0.002)
qsreg	0.025 (0.001)	0.040 (0.001)	0.053 (0.001)	0.063 (0.001)	0.087 (0.002)
n=500					
detrend eBIC	0.018 (0.000)	0.020 (0.001)	0.025 (0.001)	0.031 (0.001)	0.044 (0.001)
detrend SIC	0.015 (0.001)	0.020 (0.001)	0.028 (0.001)	0.034 (0.001)	0.046 (0.001)
detrend valid	0.018 (0.001)	0.028 (0.001)	0.035 (0.001)	0.040 (0.001)	0.059 (0.002)
rqss	0.017 (0.001)	0.029 (0.001)	0.038 (0.001)	0.043 (0.001)	0.060 (0.001)
npqw	0.039 (0.001)	0.030 (0.001)	0.029 (0.001)	0.036 (0.001)	0.046 (0.001)
qsreg	0.017 (0.001)	0.031 (0.001)	0.040 (0.001)	0.049 (0.001)	0.067 (0.001)
n=1000					
detrend eBIC	0.014 (0.000)	0.015 (0.000)	0.019 (0.001)	0.025 (0.001)	0.035 (0.001)
detrend SIC	0.012 (0.000)	0.015 (0.001)	0.020 (0.001)	0.025 (0.001)	0.037 (0.001)
detrend valid	0.012 (0.001)	0.020 (0.001)	0.025 (0.001)	0.032 (0.001)	0.043 (0.001)
rqss	0.011 (0.000)	0.021 (0.001)	0.028 (0.001)	0.034 (0.001)	0.047 (0.001)
npqw	0.032 (0.001)	0.022 (0.001)	0.023 (0.001)	0.030 (0.001)	0.037 (0.001)
qsreg	0.011 (0.000)	0.021 (0.001)	0.029 (0.001)	0.037 (0.001)	0.048 (0.001)

Table 4: RMSE and standard error by method, quantile, and data size for peaks design. Numbers correspond to Fig. 8 in the manuscript.

	Method	0.01	0.05	0.1
	detrend eBIC	0.134 (0.005)	0.102 (0.005)	0.100 (0.005)
	detrend SIC	0.310 (0.012)	0.249 (0.014)	0.233 (0.015)
	detrend valid	0.188 (0.014)	0.166 (0.011)	0.217 (0.014)
n=500	detrend Xing	0.315 (0.011)	0.187 (0.012)	0.137 (0.010)
	rqss	0.242 (0.007)	0.280 (0.012)	0.241 (0.014)
	npqw	0.196 (0.012)	0.185 (0.012)	0.192 (0.012)
	qsreg	0.372 (0.009)	0.326 (0.011)	0.314 (0.012)
	detrend eBIC	0.125 (0.003)	0.092 (0.003)	0.092 (0.003)
	detrend SIC	0.306 (0.010)	0.241 (0.012)	0.229 (0.012)
	detrend valid	0.147 (0.006)	0.178 (0.010)	0.237 (0.012)
n=1000	detrend Xing	0.289 (0.009)	0.134 (0.008)	0.110 (0.006)
	rqss	0.256 (0.006)	0.295 (0.010)	0.238 (0.012)
	npqw	0.212 (0.008)	0.196 (0.008)	0.207 (0.008)
	qsreg	0.265 (0.005)	0.247 (0.006)	0.255 (0.007)
	detrend eBIC	0.118 (0.002)	0.093 (0.002)	0.092 (0.002)
	detrend SIC	0.288 (0.008)	0.210 (0.009)	0.198 (0.009)
	detrend valid	0.141 (0.004)	0.159 (0.008)	0.229 (0.010)
n=2000	detrend Xing	0.250 (0.008)	0.109 (0.005)	0.094 (0.003)
	rqss	0.259 (0.004)	0.287 (0.007)	0.217 (0.008)
	npqw	0.193 (0.008)	0.190 (0.008)	0.197 (0.008)
	qsreg	0.166 (0.002)	0.163 (0.004)	0.175 (0.004)
	detrend eBIC	0.117 (0.002)	0.092 (0.002)	0.092 (0.001)
	detrend SIC	0.265 (0.005)	0.188 (0.006)	0.178 (0.006)
	detrend valid	0.132 (0.002)	0.168 (0.006)	0.228 (0.006)
n=4000	detrend Xing	0.215 (0.007)	0.093 (0.002)	0.089 (0.002)
	rqss	0.262 (0.003)	0.257 (0.007)	0.196 (0.006)
	npqw	0.189 (0.012)	0.182 (0.012)	0.185 (0.011)
	qsreg	0.137 (0.005)	0.102 (0.001)	0.109 (0.002)

Table 5: Class averaged accuracy when the threshold is 0.9, by data size, and method (1 is best 0.5 is worst). Numbers correspond to top panel in Fig. 10 in manuscript.

	Method	0.01	0.05	0.1
	detrend eBIC	0.759 (0.029)	0.758 (0.030)	0.720 (0.029)
	detrend SIC	0.557 (0.024)	0.543 (0.024)	0.537 (0.024)
	detrend valid	0.748 (0.029)	0.660 (0.028)	0.565 (0.025)
n=500	detrend Xing	0.544 (0.023)	0.613 (0.028)	0.645 (0.028)
	rqss	0.626 (0.026)	0.505 (0.021)	0.522 (0.023)
	npqw	0.628 (0.028)	0.612 (0.028)	0.576 (0.026)
	qsreg	0.524 (0.022)	0.484 (0.020)	0.472 (0.019)
	detrend eBIC	0.860 (0.013)	0.853 (0.013)	0.818 (0.013)
	detrend SIC	0.655 (0.015)	0.644 (0.015)	0.633 (0.015)
	detrend valid	0.850 (0.014)	0.730 (0.017)	0.624 (0.015)
n=1000	detrend Xing	0.658 (0.015)	0.782 (0.016)	0.783 (0.015)
	rqss	0.708 (0.014)	0.563 (0.012)	0.614 (0.014)
	npqw	0.685 (0.014)	0.672 (0.015)	0.632 (0.014)
	qsreg	0.707 (0.014)	0.616 (0.012)	0.580 (0.011)
	detrend eBIC	0.882 (0.003)	0.876 (0.004)	0.848 (0.004)
	detrend SIC	0.701 (0.010)	0.700 (0.011)	0.682 (0.011)
	detrend valid	0.874 (0.004)	0.776 (0.011)	0.645 (0.010)
n=2000	detrend Xing	0.728 (0.010)	0.845 (0.007)	0.832 (0.006)
	rqss	0.725 (0.007)	0.593 (0.007)	0.653 (0.010)
	npqw	0.765 (0.005)	0.738 (0.007)	0.693 (0.006)
	qsreg	0.844 (0.004)	0.756 (0.006)	0.696 (0.006)
	detrend eBIC	0.881 (0.002)	0.869 (0.003)	0.836 (0.003)
	detrend SIC	0.712 (0.007)	0.713 (0.007)	0.694 (0.008)
	detrend valid	0.877 (0.002)	0.749 (0.009)	0.631 (0.007)
n=4000	detrend Xing	0.762 (0.008)	0.858 (0.004)	0.829 (0.003)
	rqss	0.711 (0.004)	0.624 (0.008)	0.667 (0.007)
	npqw	0.812 (0.003)	0.784 (0.003)	0.745 (0.003)
	qsreg	0.878 (0.002)	0.837 (0.003)	0.782 (0.003)

Table 6: Class averaged accuracy when the threshold is 1.0, by data size, and method (1 is best 0.5 is worst). Numbers correspond to second panel in Fig. 10 in manuscript.

	Method	0.01	0.05	0.1
	detrend eBIC	0.757 (0.030)	0.728 (0.029)	0.688 (0.028)
	detrend SIC	0.550 (0.024)	0.532 (0.023)	0.527 (0.023)
	detrend valid	0.747 (0.029)	0.638 (0.028)	0.546 (0.024)
n=500	detrend Xing	0.538 (0.023)	0.599 (0.027)	0.622 (0.027)
	rqss	0.616 (0.026)	0.500 (0.021)	0.511 (0.022)
	npqw	0.624 (0.028)	0.598 (0.027)	0.564 (0.025)
	qsreg	0.516 (0.021)	0.480 (0.019)	0.470 (0.018)
	detrend eBIC	0.855 (0.013)	0.820 (0.013)	0.781 (0.013)
	detrend SIC	0.639 (0.014)	0.627 (0.014)	0.615 (0.014)
	detrend valid	0.845 (0.014)	0.706 (0.016)	0.608 (0.014)
n=1000	detrend Xing	0.648 (0.015)	0.755 (0.016)	0.748 (0.014)
	rqss	0.691 (0.014)	0.557 (0.012)	0.598 (0.013)
	npqw	0.673 (0.014)	0.649 (0.015)	0.612 (0.014)
	qsreg	0.689 (0.014)	0.601 (0.011)	0.569 (0.010)
	detrend eBIC	0.880 (0.003)	0.846 (0.004)	0.809 (0.004)
	detrend SIC	0.681 (0.010)	0.677 (0.010)	0.659 (0.010)
	detrend valid	0.874 (0.005)	0.748 (0.011)	0.624 (0.009)
n=2000	detrend Xing	0.714 (0.011)	0.813 (0.007)	0.792 (0.006)
	rqss	0.704 (0.006)	0.582 (0.006)	0.634 (0.009)
	npqw	0.754 (0.006)	0.707 (0.007)	0.660 (0.006)
	qsreg	0.827 (0.004)	0.723 (0.006)	0.668 (0.005)
	detrend eBIC	0.875 (0.003)	0.836 (0.003)	0.795 (0.003)
	detrend SIC	0.691 (0.007)	0.688 (0.007)	0.667 (0.007)
	detrend valid	0.872 (0.003)	0.722 (0.008)	0.614 (0.006)
n=4000	detrend Xing	0.747 (0.009)	0.823 (0.004)	0.787 (0.003)
	rqss	0.691 (0.004)	0.611 (0.007)	0.644 (0.006)
	npqw	0.807 (0.003)	0.750 (0.003)	0.707 (0.003)
	qsreg	0.873 (0.003)	0.801 (0.003)	0.740 (0.003)

Table 7: Class averaged accuracy when the threshold is 1.1, by data size, and method (1 is best 0.5 is worst). Numbers correspond to third panel in Fig. 10 in manuscript.

	Method	0.01	0.05	0.1
	detrend eBIC	0.738 (0.029)	0.693 (0.028)	0.653 (0.027)
	detrend SIC	0.540 (0.023)	0.522 (0.023)	0.517 (0.022)
	detrend valid	0.727 (0.029)	0.614 (0.026)	0.533 (0.023)
n=500	detrend Xing	0.528 (0.022)	0.580 (0.026)	0.598 (0.026)
	rqss	0.603 (0.025)	0.497 (0.021)	0.506 (0.021)
	npqw	0.613 (0.027)	0.580 (0.026)	0.548 (0.024)
	qsreg	0.506 (0.021)	0.476 (0.019)	0.467 (0.018)
	detrend eBIC	0.832 (0.013)	0.781 (0.013)	0.742 (0.013)
	detrend SIC	0.624 (0.013)	0.609 (0.013)	0.597 (0.013)
	detrend valid	0.826 (0.014)	0.680 (0.015)	0.593 (0.013)
n=1000	detrend Xing	0.633 (0.014)	0.725 (0.015)	0.713 (0.014)
	rqss	0.669 (0.013)	0.549 (0.011)	0.584 (0.013)
	npqw	0.652 (0.014)	0.627 (0.014)	0.591 (0.013)
	qsreg	0.664 (0.013)	0.584 (0.011)	0.559 (0.010)
	detrend eBIC	0.861 (0.004)	0.808 (0.004)	0.765 (0.004)
	detrend SIC	0.659 (0.009)	0.653 (0.009)	0.636 (0.009)
	detrend valid	0.857 (0.005)	0.719 (0.010)	0.606 (0.008)
n=2000	detrend Xing	0.695 (0.010)	0.776 (0.007)	0.749 (0.006)
	rqss	0.682 (0.006)	0.572 (0.005)	0.614 (0.008)
	npqw	0.730 (0.007)	0.672 (0.007)	0.633 (0.006)
	qsreg	0.800 (0.005)	0.691 (0.005)	0.642 (0.005)
	detrend eBIC	0.852 (0.003)	0.793 (0.003)	0.750 (0.003)
	detrend SIC	0.669 (0.006)	0.663 (0.006)	0.642 (0.006)
	detrend valid	0.851 (0.003)	0.693 (0.007)	0.598 (0.006)
n=4000	detrend Xing	0.725 (0.008)	0.781 (0.004)	0.743 (0.003)
	rqss	0.668 (0.004)	0.596 (0.006)	0.622 (0.006)
	npqw	0.784 (0.004)	0.712 (0.004)	0.671 (0.003)
	qsreg	0.850 (0.003)	0.759 (0.003)	0.702 (0.003)

Table 8: Class averaged accuracy when the threshold is 1.2, by data size, and method (1 is best 0.5 is worst). Numbers correspond to bottom panel in Fig. 10 in manuscript.

	Method	0.01	0.05	0.1
	detrend eBIC	0.703 (0.028)	0.658 (0.027)	0.620 (0.025)
	detrend SIC	0.529 (0.022)	0.514 (0.022)	0.509 (0.022)
	detrend valid	0.696 (0.028)	0.595 (0.025)	0.519 (0.022)
n=500	detrend Xing	0.519 (0.022)	0.564 (0.025)	0.576 (0.024)
	rqss	0.585 (0.024)	0.491 (0.020)	0.497 (0.021)
	npqw	0.595 (0.026)	0.561 (0.025)	0.530 (0.023)
	qsreg	0.499 (0.020)	0.473 (0.019)	0.464 (0.018)
	detrend eBIC	0.797 (0.013)	0.745 (0.013)	0.704 (0.012)
	detrend SIC	0.605 (0.012)	0.592 (0.012)	0.581 (0.012)
	detrend valid	0.798 (0.014)	0.654 (0.014)	0.576 (0.012)
n=1000	detrend Xing	0.615 (0.013)	0.695 (0.014)	0.680 (0.013)
	rqss	0.646 (0.013)	0.543 (0.011)	0.571 (0.012)
	npqw	0.628 (0.013)	0.605 (0.013)	0.576 (0.012)
	qsreg	0.637 (0.012)	0.573 (0.010)	0.550 (0.009)
	detrend eBIC	0.829 (0.004)	0.764 (0.004)	0.724 (0.004)
	detrend SIC	0.638 (0.008)	0.632 (0.008)	0.617 (0.008)
	detrend valid	0.826 (0.006)	0.688 (0.009)	0.591 (0.007)
n=2000	detrend Xing	0.674 (0.010)	0.736 (0.007)	0.710 (0.005)
	rqss	0.658 (0.006)	0.561 (0.005)	0.597 (0.007)
	npqw	0.698 (0.007)	0.644 (0.006)	0.607 (0.005)
	qsreg	0.763 (0.005)	0.662 (0.005)	0.622 (0.005)
	detrend eBIC	0.816 (0.003)	0.751 (0.003)	0.710 (0.003)
	detrend SIC	0.644 (0.005)	0.638 (0.005)	0.620 (0.005)
	detrend valid	0.817 (0.004)	0.665 (0.006)	0.584 (0.005)
n=4000	detrend Xing	0.698 (0.008)	0.739 (0.004)	0.703 (0.004)
	rqss	0.646 (0.003)	0.582 (0.005)	0.603 (0.005)
	npqw	0.750 (0.004)	0.675 (0.004)	0.641 (0.004)
	qsreg	0.816 (0.003)	0.718 (0.003)	0.667 (0.003)

References

[1] Stephen Boyd, Neal Parikh, Eric Chu, Borja Peleato, Jonathan Eckstein, et al. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends® in Machine learning*, 3(1):1–122, 2011.