Two Sample Paired t Test - Burning Times of Two Chemical Flare Formulations

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Source: Montgomery, D. C. (2013). *Design and analysis of experiments* (8th ed.). Wiley.

Diameter Measurements from Two Caliper Types

The diameter of a ball bearing was measured by 12 inspectors, each using two different kinds of calipers. The results were

Instructor	Caliper1	Caliper 2
1	0.265	0.264
2	0.265	0.265
3	0.266	0.264
4	0.267	0.266
5	0.267	0.267
6	0.265	0.268
7	0.267	0.264
8	0.267	0.265
9	0.265	0.265
10	0.268	0.267
11	0.268	0.268
12	0.265	0.269

- Is there a significant difference between the means of the population of measurements from which the two samples were selected? Use $\alpha = 0.05$.
- Find the p-value for the test.
- Construct a 95 percent confidence interval on the difference in mean diameter measurements for the two types of calipers.

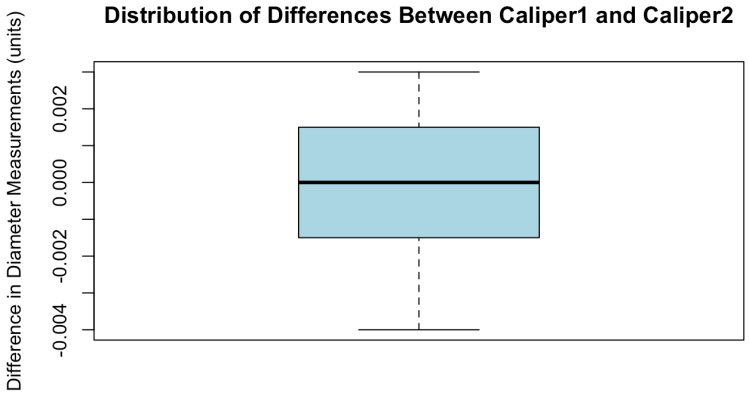
R Code

```
# Data
Caliper1 <- c(0.265, 0.265, 0.266, 0.267, 0.267, 0.265, 0.267, 0.267, 0.265, 0.2638, 0.268, 0.265)
Caliper2 <- c(0.264, 0.265, 0.264, 0.266, 0.267, 0.268, 0.264, 0.265, 0.265, 0.267, 0.268, 0.269)

# Normality Test
Differences <- Caliper1 - Caliper2
boxplot(Differences,
        main = "Distribution of Differences Between Caliper1 and Caliper2",
        ylab = "Difference in Diameter Measurements (units)",
        col = "lightblue")
```

```
shapiro.test(Differences)

# Test for Equality of Mean
t.test(Caliper1, Caliper2, paired = TRUE)
```



Normality Check

As seen from the boxplot of the differences in diameter measurements between Caliper 1 and Caliper 2, the distribution appears roughly symmetric. Although the median is slightly above the center, indicating a mild left skew, the deviation is minimal. These observations suggest that the normality assumption is reasonably met. To further validate these observations, we apply the Shapiro-Wilk test.

- **Hypotheses**
  - H_0 : The data follows a normal distribution.
  - H_a : The data does not follow a normal distribution.
- **Level of Significance:** $\alpha = 0.05$
- **Test Statistic**

Data	W statistic	p-value
Differences in Measurements	0.90163	0.1665

- **Decision Rule:** Reject H_0 if $p < \alpha = 0.05$
  - Since $p = 0.1665 > \alpha = 0.05$, we fail to reject H_0.
- **Interpretation/Conclusion**
  - At the 95% confidence level, there is sufficient statistical evidence to conclude that the data is normally distributed and is suitable for **Paired T-Test**.

(a) Test for Equality of Means

- **Hypotheses**
  - H_0 : There is no significant difference in the mean caliper diameter measurements.
  - H_a : There is a significant difference in the mean caliper diameter measurements.
- **Level of Significance:** $\alpha = 0.05$
- **Test Statistic**

t statistic	DF	p-value
-0.15617	11	0.8787

- **Decision Rule:** Reject H_0 if $p < \alpha = 0.05$
  - Since $p = 0.8787 > \alpha = 0.05$, we fail to reject H_0.
- **Interpretation/Conclusion**
  - At the 95% confidence level, there is sufficient statistical evidence to conclude that there is no significant difference in the mean caliper diameter measurements.

(b) p - VALUE

- From the Paired t-Test table above, we get a p-value of 0.8787.

(c) 95% Confidence Interval

- A 95% confidence interval on the difference in mean diameter measurements for the two types of calipers is [-0.001509318, 0.001309318].