

```

In[1]:= ClearAll["Global`*"];
D1u[x_, u_, v_] = v;
D2u[x_, u_, v_] = 2 * u * v;
D3u[x_, u_, v_] = 2 * (v^2 + 2 * u^2 * v);
D1v[x_, u_, v_] = 2 * u * v;
D2v[x_, u_, v_] = 2 * (v^2 + 2 * u^2 * v);
D3v[x_, u_, v_] = 8 * u * v * (2 * v + u^2);
sn0 = 0.5;
sn1 = 0.1;
x0 = 0;
xf = 1;
gamma = 1;
n = 4;
phin0 = 1;
phin1 = 1;
x = Table[j, {j, 1, n + 1}];
u1 = Table[j, {j, 1, n + 1}];
v1 = Table[j, {j, 1, n + 1}];
u2 = Table[j, {j, 1, n + 1}];
v2 = Table[j, {j, 1, n + 1}];
h = (xf - x0) / n;
u10 = 0.5;
u20 = 0.5;

```

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In[24]:= While[phin1 > 0.005,
  v10 = sn0;
  x[[1]] = x0;
  u1[[1]] = u10;
  v1[[1]] = v10;
  For[i = 1, i < n + 1, i++,
    {
      u1[[i + 1]] = u1[[i]] + h * D1u[x[[i]], u1[[i]], v1[[i]]] +
        h^2/2 * D2u[x[[i]], u1[[i]], v1[[i]]] + h^3/6 * D3u[x[[i]], u1[[i]], v1[[i]]];
      v1[[i + 1]] = v1[[i]] + h * D1v[x[[i]], u1[[i]], v1[[i]]] +
        h^2/2 * D2v[x[[i]], u1[[i]], v1[[i]]] + h^3/6 * D3v[x[[i]], u1[[i]], v1[[i]]];
      x[[i + 1]] = x[[i]] + h;
    }];
  datau1 = Transpose[{x, u1}];
  datav1 = Transpose[{x, v1}];

  v20 = sn1;
  x[[1]] = x0;
  u2[[1]] = u20;
  v2[[1]] = v20;
  For[i = 1, i < n + 1, i++,
    {
      u2[[i + 1]] = u2[[i]] + h * D1u[x[[i]], u2[[i]], v2[[i]]] +
        h^2/2 * D2u[x[[i]], u2[[i]], v2[[i]]] + h^3/6 * D3u[x[[i]], u2[[i]], v2[[i]]];
      v2[[i + 1]] = v2[[i]] + h * D1v[x[[i]], u2[[i]], v2[[i]]] +
        h^2/2 * D2v[x[[i]], u2[[i]], v2[[i]]] + h^3/6 * D3v[x[[i]], u2[[i]], v2[[i]]];
      x[[i + 1]] = x[[i]] + h;
    }];
  datau2 = Transpose[{x, u2}];
  datav2 = Transpose[{x, v2}];

  phin0 = Abs[u1[[n + 1]] - gamma];
  phin1 = Abs[u2[[n + 1]] - gamma];
  sn2 = sn1 - ((sn1 - sn0) / (phin1 - phin0)) * (phin1);
  sn0 = sn1;
  sn1 = sn2;
]

```

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In[25]:= u1
v1
u2
v2

```

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Out[25]= {0.5, 0.572967, 0.670133, 0.805921, 1.00885}
```

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Out[26]= {0.255857, 0.333939, 0.454251, 0.65346, 1.01799}
```

```
Out[27]= {0.5, 0.571951, 0.667717, 0.801387, 1.0007}
```

```
Out[28]= {0.252312, 0.329237, 0.447496, 0.642697, 0.998316}
```