

```
In[1]:= ClearAll["Global`*"];
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In[2]:= f[x_, u_, v_] := v;  
g[x_, u_, v_] := v - 2 u + x;  
u10 = 0;  
v10 = 1;  
x0 = 0;  
xf = 1;
```

```
In[8]:= n = 1000;  
x = Table[j, {j, 1, n + 1}];  
u1 = Table[j, {j, 1, n + 1}];  
v1 = Table[j, {j, 1, n + 1}];  
x[[1]] = x0;  
u1[[1]] = u10;  
v1[[1]] = v10;  
h = (xf - x0) / n;  
For[i = 1, i < n + 1, i++,  
{  
k1 = h * f[x[[i]], u1[[i]], v1[[i]]];  
l1 = h * g[x[[i]], u1[[i]], v1[[i]]];  
k2 = h * f[x[[i]] + 0.5 * h, u1[[i]] + k1, v1[[i]] + l1];  
l2 = h * g[x[[i]] + 0.5 * h, u1[[i]] + k1, v1[[i]] + l1];  
k3 = h * f[x[[i]] + 0.5 * h, u1[[i]] + k2, v1[[i]] + l2];  
l3 = h * g[x[[i]] + 0.5 * h, u1[[i]] + k2, v1[[i]] + l2];  
k4 = h * f[x[[i]] + h, u1[[i]] + k3, v1[[i]] + l3];  
l4 = h * g[x[[i]] + h, u1[[i]] + k3, v1[[i]] + l3];  
x[[i + 1]] = x[[i]] + h;  
u1[[i + 1]] = u1[[i]] + (1 / 6) * (k1 + 2 * k2 + 2 * k3 + k4);  
v1[[i + 1]] = v1[[i]] + (1 / 6) * (l1 + 2 * l2 + 2 * l3 + l4);  
}];
```

```
In[17]:= datau1 = Transpose[{x, u1}];  
datav1 = Transpose[{x, v1}];
```

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In[19]:= u20 = 1;  
v20 = 2;
```

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In[21]:= u2 = Table[j, {j, 1, n + 1}];
v2 = Table[j, {j, 1, n + 1}];
u2[[1]] = u0;
v2[[1]] = v0;
h = (xf - x0) / n;
For[i = 1, i < n + 1, i++,
{
k1 = h * f[x[[i]], u2[[i]], v2[[i]]];
l1 = h * g[x[[i]], u2[[i]], v2[[i]]];
k2 = h * f[x[[i]] + 0.5 * h, u2[[i]] + k1, v2[[i]] + l1];
l2 = h * g[x[[i]] + 0.5 * h, u2[[i]] + k1, v2[[i]] + l1];
k3 = h * f[x[[i]] + 0.5 * h, u2[[i]] + k2, v2[[i]] + l2];
l3 = h * g[x[[i]] + 0.5 * h, u2[[i]] + k2, v2[[i]] + l2];
k4 = h * f[x[[i]] + h, u2[[i]] + k3, v2[[i]] + l3];
l4 = h * g[x[[i]] + h, u2[[i]] + k3, v2[[i]] + l3];
x[[i + 1]] = x[[i]] + h;
u2[[i + 1]] = u2[[i]] + (1/6) * (k1 + 2 * k2 + 2 * k3 + k4);
v2[[i + 1]] = v2[[i]] + (1/6) * (l1 + 2 * l2 + 2 * l3 + l4);
}];

```

```

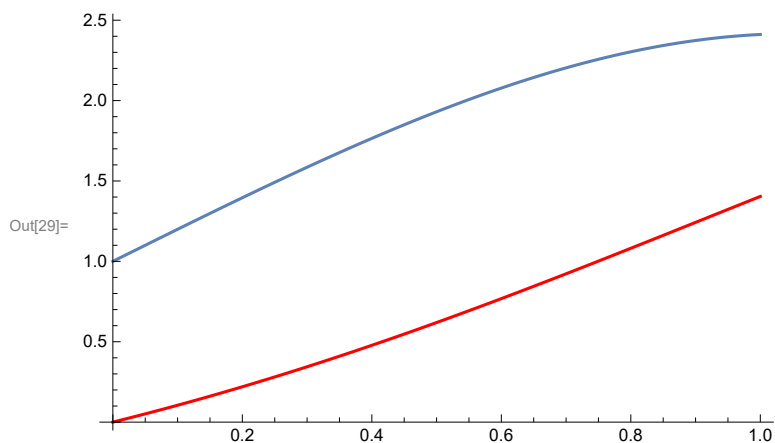
In[27]:= datau2 = Transpose[{x, u2}];
datav2 = Transpose[{x, v2}];

```

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In[29]:= Show[{ListLinePlot[datau1, PlotStyle -> Red]},
{ListLinePlot[datau2]}, Mesh -> All, AxesOrigin -> {0, 0}, PlotRange -> Automatic]

```



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In[30]:= u1[[n + 1]]
v1[[n + 1]]

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Out[30]= 1.40372

Out[31]= 1.60724

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In[32]:= u2[[n + 1]]
v2[[n + 1]]

```

Out[32]= 2.41126

Out[33]= 0.199298

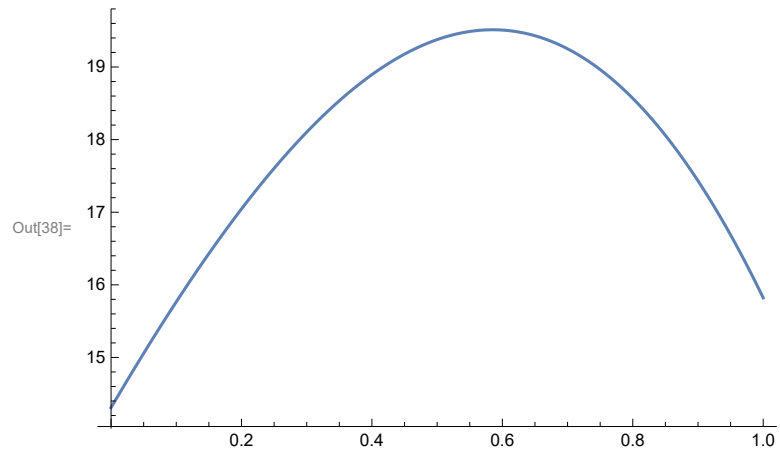
```
In[34]:= sol = Flatten[
  Solve[m * u1[[n + 1]] + (1 - m) * u2[[n + 1]] + m * v1[[n + 1]] + (1 - m) * v2[[n + 1]] == -E, m];
```

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In[35]:= lambda = m /. %[[1]]
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Out[35]= -13.3088
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In[36]:= u = lambda * u1 + (1 - lambda) * u2;
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```
In[37]:= datau = Transpose[{x, u}];
ListLinePlot[datau]
```



```
In[39]:= ClearAll["Global`*"];
sol = DSolve[{y''[x] - y'[x] + 2*y[x] == x, y[0] - y'[0] == -1, y[1] + y'[1] == -E}, y, x]
Plot[y[x] /. sol, {x, 0, 1}]
```

```
Out[40]= { {y -> Function[{x}, - $\frac{1}{8\sqrt{e}} \left( \sqrt{7} \cos\left[\frac{\sqrt{7}}{2}\right] - \sin\left[\frac{\sqrt{7}}{2}\right] \right)$ 
```

$$\left(-2\sqrt{7}e^{\frac{1}{2}} \cos\left[\frac{\sqrt{7}}{2}\right] - 4\sqrt{7}e^{\frac{1}{2}}x \cos\left[\frac{\sqrt{7}}{2}\right] + 4\sqrt{7}e^{1+\frac{x}{2}} \cos\left[\frac{\sqrt{7}}{2}x\right] + 5\sqrt{7}e^{x/2} \cos\left[\frac{\sqrt{7}}{2}x\right] + \right.$$

$$3\sqrt{7}e^{\frac{1}{2}+\frac{x}{2}} \cos\left[\frac{\sqrt{7}}{2}\right] \cos\left[\frac{\sqrt{7}}{2}x\right] + 2\sqrt{e} \sin\left[\frac{\sqrt{7}}{2}\right] + 4\sqrt{e}x \sin\left[\frac{\sqrt{7}}{2}\right] +$$

$$9e^{\frac{1}{2}+\frac{x}{2}} \cos\left[\frac{\sqrt{7}}{2}x\right] \sin\left[\frac{\sqrt{7}}{2}\right] + 4e^{1+\frac{x}{2}} \sin\left[\frac{\sqrt{7}}{2}x\right] + 5e^{x/2} \sin\left[\frac{\sqrt{7}}{2}x\right] -$$

$$\left. \left. 9e^{\frac{1}{2}+\frac{x}{2}} \cos\left[\frac{\sqrt{7}}{2}\right] \sin\left[\frac{\sqrt{7}}{2}x\right] + 3\sqrt{7}e^{\frac{1}{2}+\frac{x}{2}} \sin\left[\frac{\sqrt{7}}{2}\right] \sin\left[\frac{\sqrt{7}}{2}x\right] \right) \right]] }$$
