Fractal Analysis as Method for studying Social Hierarchy in Prehistoric Settlement Plans

with case studies from the Linear Pottery and Trypillia cultures (5.500 - 3.500 cal. BCE)

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Contents

Preface

Thanks to everyone.

Remember to add abstracts

Furholt, Grier, et al. (2020), Furholt, Müller, et al. (2020), Furholt, Müller-Scheeßel, et al. (2020) are three different publications. Furholt, Grier, et al. (2020), Furholt, Müller, et al. (2020), Furholt, Müller-Scheeßel, et al. (2020) are the same three publications in the same order. gfdsdfgs , Get on with it!

Part I

Frameworks

Chapter 1

Introduction

1.1 Background of the study

- Studying social hierarchy in Prehistory through fractal analysis of settlement plans
- In Europe, the Neolithic is the long and messy transition period between mobile hunter-gatherer groups in the Palaeo- and Mesolithic, and the first city states in the Bronze (Aegean) and Iron Ages (Mediterranean and Central Europe)
- Early farming economy, influence from the Near East.
- Large variety in scale and content of archaeologically defined culture groups. Single farmsteads and small hamlets in many phases – some with hardly any settlement evidence at all **examples mid-neo, use** Shennan (2018). Other phases include exceptionally large settlements, probably hosting populations of several thousand inhabitants, like at Maidanetske in central Ukraine around 3.800 BCE (see 3.3). While in some settings, like in Linear Pottery society in much of continental Europe north of the Alps towards 5.100 BCE (see 3.2), the dead were buried in simple pit graves, with very little distinction in treatment between individuals, in other phases some individuals were buried with tremendous amounts of precious goods like in Varna, Romania, or under colossal burial mounds like in Carnac, France, both in the mid-5th millennium (Shennan 2018)**check ref**.

- Seen at a very large scale – across the continent and through the Holocene – the development of society from small scale and relatively egalitarian towards large scale and more hierarchical seems evident (though not to everyone, see Graeber and Wengrow (2021) and Section 1.5). When we look more closely however, this evolution is anything but linear, as both population sizes and levels of hierarchical organisation seem to fluctuate considerably, sometimes over short time spans as from the Trypillia **C2 to D1??, check this**, when the so-called mega-sites are abandoned and their former inhabitants regroup into much smaller settlements during a transition of maybe only **check and ref** years.

- In many cases, the level of social hierarchisation and complexity in a given Prehistoric society is very hard for researchers to evaluate, since many indicators of such structures are either lost from the archaeological record, or were never included in the first place [Perreault (2019); Section 3.1]. Archaeological traces that are often interpreted as signs of social complexity and hierarchy may furthermore be deceiving. Seemingly monumental structures were in many cases built through small additions over centuries, rather than in one colossal construction campaign [**example, Carnac alignments? Danish megaliths?**]. In many megalithic burial contexts, it may be impossible to know how large a segment of the society that had access to such inhumations (**rephrase**).

- New methods for investigating hierarchy: fractal analysis. Borrowed from other disciplines, not much tested in Archaeology. Example from human geography, use:(Batty 2005; Batty and Longley 1994; D'Acci 2019; Jahanmiri and Parker 2022; Lagarias and Prastacos 2021; Tannier and Pumain 2005) (say what fractal analysis does, don't explain what it is here)(explain just the word fractal and cite section).

1.2 Research question and objectives

The overall goal of the present study is to test and assess the utility of fractal analysis techniques as tools for studying hierarchical social organisation in prehistoric societies. Two methodological approaches are under special scrutiny: the distribution fitting approach and the image analysis approach (Brown and Liebovitch 2010; Brown, Witschey, and Liebovitch 2005). These are applied to architectural data series from well-preserved and documented archaeological samples within neolithic Linear Pottery and Trypillia contexts, as well as to synthetic data series. This thesis is thus not to be considered a culture-historic study of Linear Pottery or Trypillia society, but mainly a methodological study. However, results from the proposed analyses of these case studies may also contribute, as side-effects, to their respective fields of research.

For the distribution fitting approach, house-size distributions within settlements are modelled following a given procedure, and the retained model (the best fit) is interpreted in terms of social generating mechanisms. In particular, it is argued here that so-called power-law distributions reflect hierarchical structure, so that the identification of these within the studied samples may indicate the presence of some social hierarchisation process which warrants further interpretation.

With the image analysis approach, archaeological and synthetic settlement plans are analysed through the calculation of fractal dimension and lacunarity – summary statistics which serve as quantifications of irregular spatial patterns or image textures. I argue here that geometrically irregular settlement plans are indicative of relative independence between households, while settlements that develop within geometrically regular grids indicate stronger overarching social structures, with a continuous range of possibilities in-between. The goal here is to test to which extent quantitative measures like fractal dimension and lacunarity may help differentiating between varying degrees of planning in prehistoric settlements.

While both these methodological approaches are well developed and integrated to other disciplines, their usage in Archaeology have so far remained anecdotal (Diachenko 2018). A further overall goal of this thesis is to identify and explore possible limitations in the nature of archaeological data that may limit the applicability of fractal analysis methods within this discipline. For example, does fractal analysis of settlement plans require a data quality that would be practically unattainable in archaeological settings? But also inversely, as it is impossible to explore all potential applications within the framework of one doctoral thesis, suggestions for future research are provided in the last chapter.

1.3 Defining hierarchies

The term *hierarchy* is central to the present study. Though commonly used in daily speech, defining the word is not as straight-forward as one might think, so some clarification on how it is understood here might be needed. In a volume dedicated to exploring the meanings and uses of hierarchy as a study object within a range of natural and social sciences, Denise Pumain provides a panorama of definition nuances, but also highlighting the characteristics that are commonly found in most cases (Pumain 2006). Among these characteristics are:

- A pyramidal organisation of elements, ordered by a very unequal size distribution of a certain quality or variable, from a few large elements on top to many small elements at the bottom
- When seen as a system, the whole is constituted of sub-systems, which are again constituted of sub-sub-systems, and so forth. These can either be ordered into clearly distinguished levels (stratified), or in other cases be scaled in a continuum (branched or tree structure)
- In physical, biological and social hierarchical systems, the structure is often accompanied by a flow of energy, material, information or control in one or both directions between the top and bottom levels

Hierarchies are found in humanly constructed classification and taxonomical systems, where morphological distinctions are considered more important or fundamental at the higher end of the hierarchy, while being more detailed or specific at the lower end. Many hierarchical social systems, like religious (from Greek *hieros* – sacred, and *archê* – government), military or corporate organisations, include strongly reinforced regulations of subordination, which in modern society has led to somewhat negative connotations to both hyper-rational and despotic rule(**pumain2006:5-6?**). While one prevalent explanation for the frequency of hierarchical structures in nature and society is indeed that they "represent the best solution for many optimisation problems" (*ibid.*:7), that does in no way mean they need to be consciously planned. On the contrary, in most cases hierarchies seem to emerge spontaneously, often from growth processes with systems splitting into sub-systems once they reach a certain critical size limit. There is also no compulsory link between social hierarchies and despotism, as it matters little to the overall structure whether the top element is elected for a limited period or born into an inherited leading position. More detail on how hierarchical structures emerge and how they can be described as fractals, is given in Chapter 2. A further discussion on the specifics of social hierarchies is given in Chapter 4, and on the differences between spontaneously emergent versus consciously planned structures in Chapter 7.

A possible confusion with a somewhat different meaning of the term hierarchy should however be mentioned already here. If hierarchical structures are abundant in nature in both physical (inert) and biological systems, social hierarchies on the other hand – understood as intra-species populations of individuals organised in pyramidal hierarchical, i.e. multi-level relationships to each other – seems to be almost exclusively found among human groups. At the same time, a different type of hierarchy is frequently described by biologists which is common among animals, namely *dominance hierarchies*, also known as pecking order (Strauss et al. 2022). These structures are hierarchical in the sense that there is difference in rank between group members, and they also seem to emerge spontaneously in the animal populations where they are described. But unlike social hierarchies, these are purely *linear*, in the sense that each group member is situated in rank above one part of the group and below the rest, so that the whole group forms a rank chain in the form A > B > C … n. The rank of an individual will typically decide their access to food and reproduction relative to the other members, and may be settled and resettled in a number of ways depending on the species and population under study, but typically involving some level of violence or threat and subordination in face-to-face encounters (Strauss et al. 2022).

A classic example – perhaps most of all in popular culture – is dominance hierarchy among wolves, led by an alpha male (e.g. Cafazzo, Lazzaroni, and Marshall-Pescini 2016; Packard 2003). Though it has been much discussed whether or not this trope model actually fits wolves (see Mech 1999; Muro et al. 2011), any reported dominance hierarchies among larger groups of wolves and stray dogs are linear rather than pyramidal, even when they are illustrated as pyramidal (e.g. Fig.1 in Rodríguez et al. 2017). Similar social organisation systems are found among a wide variety of species – mammals, birds, fish, particularly but not only among group-hunting carnivores – and are generally interpreted as an evolutionary mechanism (Strauss et al. 2022, with references). Cases of branching, multi-level social hierarchies among animals are on the other hand extremely rare, but have been reported to operate among hamadryas baboons in Ethiopia, with "clan leaders" forming relays of information flow and decision making between the "one-male units" within a total population ("band") of about 200 individuals (Schreier and Swedell 2009). Eusocial insects like ants and wasps provide a more well-known example of hierarchical organisation among animals (Shimoji and Dobata 2022), also indicating – if it should be necessary – that it is not a matter of cognitive abilities or brain size, but rather of social function (e.g. building a hive or a village together) and population size above a certain threshold.

The reason to dwell upon this qualitative distinction between linear and pyramidal hierarchies, is that the prevalence of the former in nature is sometimes put forward as an argument for social hierarchies among hunter-gatherer groups in the Palaeo- and Mesolithic [find precise reference here –> Graeber, if else strawman]. While it is a good point that there is no reason there couldn't be linear dominance hierarchies among small forager groups, and that such systems hardly can be described as egalitarian by those who live them, we cannot assume that pyramidal social hierarchies have always been part of human culture, but rather that they – much like agriculture – at some point were established **rewrite this**. HERE.

Bottom line hierarchy: pecking order/linear dominance hierarchy may be prevalent in any part of human Prehistory, but could also be very hard to study from material remains. Branching (i.e. fractal) hierarchies as well as social stratification should leave material traces, and are most probably related to population size (see Section 4, and thus less likely to be found within pre-Neolithic societies. They should be regarded as historically situated phenomenons – much like agriculture, pottery, writing or the steam engine – justifying the search for and explanation of their possible origins (*contra* e.g. Graeber and Wengrow 2021).

- Inequality Kohler and Smith (2018), Midlarsky (1999), Price and Feinman (2010), Price and Feinman (1995) NA

1.4 What is social hierarchy?

- Political assumptions – all hierarchical social structures are not despotic top-down rule. Democracies can also be very hierarchical. Matter of scale rather than political system Graeber and Wengrow (2021). But, tendencies? Use Pumain (2006), also Furholt, Grier, et al. (2020). Use Redhead and Power (2022): status and leadership, multiple overlapping networks. Emergence of hierarchies: "Positions in the dominance hierarchy is determined by a combination of attributes of individuals, stochastic processes, and social context" Strauss et al. (2022).

- Nested and non-nested social hierarchies, hierarchical hunter-gatherers? Hamilton et al. (2007), Whitridge (2016) (or use in Chapter 11?)

- Biologically defined thresholds to group size? Dunbar's number and controversies, Dunbar (2022), West et al. (2023) (add published papers). Scalar stress Johnson (1982), Alberti (2014), Zhou et al. (2005). Also Carneiro (1986)

- Temporal dynamics of social hierarchies: cyclicity (Peters and Zimmermann 2017), saw-tooth waves (Scheidel 2017), punctuated equilibrium [Gould (2007); zimmermann?]. Archaeo. example South Sweden Neo/BA: Nordquist (2001). Transitions villages to urban: Birch (2014)

1.5 Social typologies and their critique

- Tools for classifying societies, or evolutionary model? Discussion of Johnson and Earle (1987), Testart (2005), Service (1971), Earle (1997).

  - Lineage and Chiefdom societies Earle (2002), Sahlins (2020)

- Anti-evolution critique in Yoffee (1993), Yoffee (2005), Fontijn and Brück (2013), Kienlin and Zimmermann (2012), Lund, Furholt, and Austvoll (2022), Furholt, Grier, et al. (2020). Anarchistic critique/heterarchy: Crumley (1995), Haude and Wagner (2019), Graeber and Wengrow (2021)

1.6 Main findings here?

Maybe leave to the end, and fill in, like abstract.

1.7 Research ethics

- Social complexity and evolution. Are less complex societies simple? Is that a bad thing?
- Open science and open-source scripts
- Terminology and spelling (British English for text. For geographical place names, Slovak special characters are kept as far as possible, even though it can be a pain in the xxx to render in Rmarkdown on Windows OS, and the 2010 Ukranian National transliteration system with only ASCII characters and no soft sign)
- Abstracts in Slovak and Ukranian (and not only in Norwegian)

1.8 Structure of the thesis

This thesis is structured as a monograph in four parts. In the first part the overall framework of the study is exposed, with the general introduction above, the overarching theoretical framework in Chapter 2, and the background of the study material in Chapter 3. Parts II and III are devoted to each their methodological approach to the material: Part II to the study of hierarchy in size distributions, and Part III to the quantification of image textures. Each of these parts consists of three chapters, the first of which – Chapters 4 and 7 – exposes the theoretical and interpretative background of the applied methods and their relevance to Archaeology. The following chapters – Chapters 5 and 8 – detail the technical specifics of the two approaches and their implementation in this study, and the last chapters within these parts – Chapters 6 and 9 – provide the actual analyses and summaries of results. In Part IV, the findings are summarised and further discussed. Chapter 10 gives an attempt of interpreting the results in the context of the culture-historical setting of the European Neolithic, while Chapter 11 reviews the possibilities and limitations of the fractal analysis framework in Archaeology. Concluding remarks and suggestions for further study are given in Chapter 12.

Some readers might react to an apparent deviation from the academic tradition of devoting a separate chapter to research history. This is a deliberate choice, not to suggest that historiography is unimportant, but rather as a result of the fundamentally interdisciplinary scope of the study. In fact, there is very little extant history of applying fractal analysis in Archaeology – the few studies that, to my knowledge, have been done in this direction are discussed primarily in Sections 4.3 and 7.3. Fractal analysis itself holds a research history of its own (see Section 2.2), and so does the study of Linear Pottery (Section 3.2) and Trypillia societies (Section 3.3), not to mention the general study of social complexity in Prehistory (mainly Section 3.1). In short, instead of trying to shoehorn these parallel histories into a clearly delimited but rather hybrid chapter, I have opted for what I believe to be a more useful approach, namely to fit them in more seamlessly where they belong, in the various associated theory and methods chapters.

An additional note should be made here regarding the writing style of the different parts and chapters. It is my belief that a major obstacle for fractal analysis methods to become more integrated into the standard tool kits of archaeological research, is the excessively technical nature of much of the associated literature. Archaeology as a discipline remains profoundly rooted in the Humanities, as seen in the inbuilt structure of teaching, research and funding institutions in most (at least European) countries. Fractal analysis is derived from pure mathematics, and most applications so far have been developed within the natural sciences (**ref necessary?**). Archaeologists who are trained within a humanistic scholarly tradition cannot be expected to hold a skill level of mathematics more advanced than what is achieved in high school, and code programming is hardly taught at all within the walls of Humanities faculties. Technical details regarding the methods and analyses applied in this thesis are therefore – as much as possible – limited to the devoted Chapters 5 and 8, and readers who are interested in these may also refer to the online code repository for more details and reproducibility **cite repository**. In the rest of the thesis I have opted for a more narrative approach, in an attempt to invite a somewhat larger audience of archaeologists into the fascinating complexities of fractals.

END Chapter

Here I skipped a line.

Here I added a backslash.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

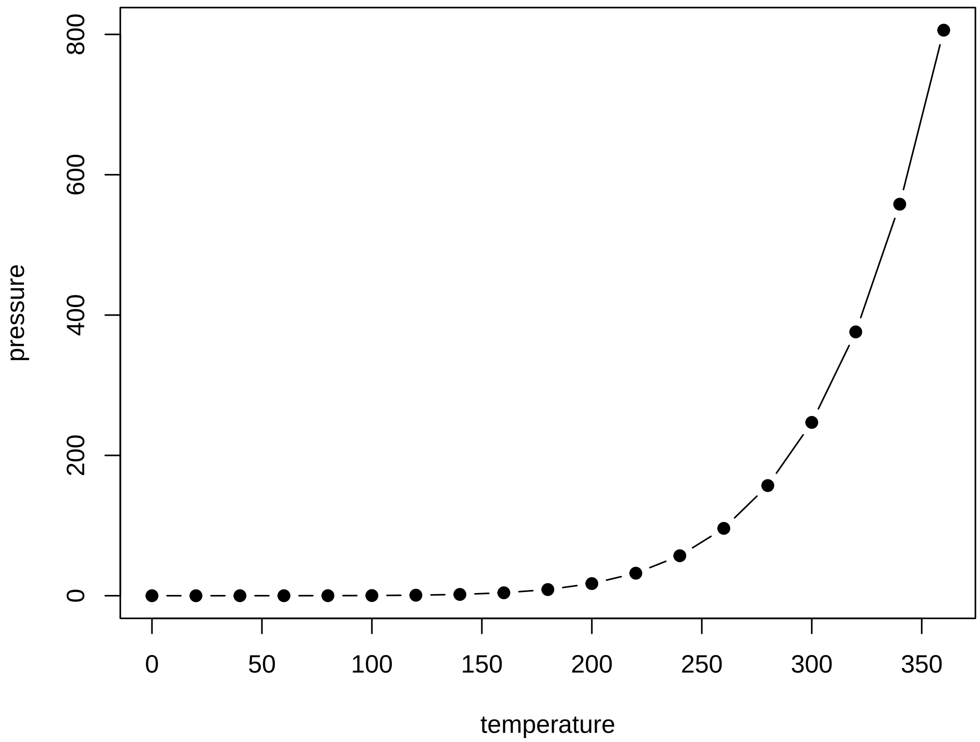

Figure 1.1: Here is a nice figure!

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 1.1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 1.1.

```
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package (**R-bookdown?**) in this sample book, which was built on top of R Markdown and **knitr** (Xie 2015).

Table 1.1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

Test: Morgan (1965).

Test: Mandelbrot (2021). jløkj fdsasdfadfasdfadf new text

Chapter 2

Theoretical framework: Complexity and Fractals

2.1 Very short introduction to Complexity Theory / Dynamical Systems Theory

Lit. use Daems (2021), Baden and Beekman (2016), Ross and Steadman (2017), Smith (2011) and Bentley and Maschner (2008).

For Dynamical Systems, use Devaney (2020), but don't go into detail.

Describe complexity, dynamical systems, chaos, feedback loops, criticality, emergence. Scale West (2017) NA

Social complexity

Mention the most common applications in Archaeology: ABMs, **and what**? check in literature.

2.2 Very short introduction to Fractals and Fractal Analysis

Lit. use Mandelbrot (2021), Falconer and Falconer (2013) (general), also Brown and Liebovitch (2010), Brown et al. (2005) and Diachenko (2018) (for Archaeology)

The term *fractal* was pinned by the French-American mathematician Benoît Mandelbrot (1924-2010), from the latin word *fractus* meaning broken or irregular, to describe patterns that because of their apparent limitless complexity defied concise description within the framework of Euclidean geometry (Falconer and Falconer 2013:116–20). Such patterns – both theoretical and empirical – had been described and analysed by mathematicians and researchers within other disciplines since the end of the 19th century, but were mostly regarded as curiosities and exceptions, and Mandelbrot was the first to link all these previous studies within a unified theoretical framework (Mandelbrot 1975, 2021).

In one influential paper, drawing on previous work by mathematician Lewis Fry Richardson, Mandelbrot (1967) argued that a rugged linear feature like a coastline cannot be fully described through traditional geometry with a set of line segments, since this would result in a curve of infinite complexity. More importantly, he showed that the traditional measure of lines – the length – will inevitably depend on the scale of observation when applied to a coastline. If measured in kilometres, a coastline will always appear shorter in total length, than if it is measured in metres, since smaller bays and inlets can then also be accounted for. But this phenomenon continues seemingly without limit, since the same coastline measured in centimetres will appear much longer, and in millimetres far longer again, and so on. Length as a measure of rugged linear features thus seems inadequate, which may become a problem in practical settings when comparing coastlines between countries that operate with different measurement units and procedures. The same problem occurs when describing irregular patterns in the plane (like island or continent outlines) with area or in three-dimensional space (like clouds or galaxies) with volume. As a solution, Mandelbrot proposed the use of the *fractal dimension* as a descriptive tool for characterising such patterns.

Fractals as hierarchy

Self-similarity and scale invariance

Processes/mechanisms that produce fractals:

- Cascading bifurcations and confluences (splitting or merging - tree structure/arborescence/branching, and relation to size. Terminology borrowed from biology and fluid dynamics (including turbulence/turbulent flow)

The role of randomness - tidy and messy fractals (romanesco broccolis are not more fractal than regular broccolis, only more regular.

The relationship with (self-organised) criticality and chaos: deterministic *and* unpredictable

Fractals embedded in

- Space (hence "fractal geometry"): geomorphology, plants, ocean and wind currents, galaxies, also human constructed features (see Chapter xx for details)
- Time series: earthquakes, finance (not applied in this study, though should be done later)
- Networks/abstract: hierarchical organisations, income distributions, word counts, 1/f or pink noise, www. Barabási and Albert (1999) etc. (see Chapter xxx for details). Fractal social networks: West et al. (2023).
- Pure mathematics: Julia and Mandelbrot sets, strange attractors (don't go into details!)

No, not everything is fractal: e.g. Central Limit Theorem

Fractal analysis for studying irregular phenomena (methods described in more detail Chapters xxx), and thus as a tool for quantitative empirical research.

2.3 Very short introduction to micro-macro approaches in social theory

- Lévi-Strauss and Structuralism
- Giddens and Structuration Theory
- Delanda and Assemblage Theory

For social hierarchies, he refers to Weber's classification of legitimation strategies, as being founded on sacred tradition, personal charisma or rational bureaucracy (see also Graeber and

Wengrow 2021, pp.). Delanda furthermore places Bourdieu's concept of *habitus* on the mid-range between micro and macro processes, as an explanation of social action that is not entirely individual dependent, but not emergent properties of society-as-a-whole either.

- Latour and Actor-Network Theory

What these approaches all have in common, is that they are entirely qualitative (**check**).

- That's not a problem in itself.
- Quant approach is both possible for the stated purpose, and desirable for reasons of comparative analysis.
- Data deluge (refer to chap. on geomagn data). The goal here is to establish a quantitative framework for studying social complexity and hierarchy in archaeological/prehistoric settings. Further articulating fractal analysis with existing social theoretical approaches is not the primary goal here, as it could constitute a separate research project. In the present thesis, bla bla.

END chapter.

Math can be added in body using usual syntax like this

p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt(\frac{p(1-p)}{n}) \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

You can also use math in footnotes like this[1].

We will approximate standard error to 0.027[2]

[1] where we mention $p = \frac{a}{b}$

[2] p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt(\frac{p(1-p)}{n}) \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

Chapter 3

Material and data: social complexity in the European Neolithic

3.1 Studying social complexity in Archaeology and Prehistory.

- Grave goods
- Burial monuments
- The denominator problem
- The use of ethnography
- Other approaches (osteological, isotopes **refs**)
- This project: house-size distributions and settlement layouts (details in subsequent chapters), just very short argumentation.
- Comparative approach: Neolithic technology (not bronze axes), wood and wattle-and-daub architecture, (near) complete settlement plans/extensive documentation

3.2 The Linear Pottery culture complex

- General intro to the culture
- Linear Pottery architecture and house construction
- Organisation of Linear Pottery society: egalitarian or hierarchic?
- The Žitava valley and research project

3.3 The Cucutení-Trypillia culture complex

- General intro to the culture
- Trypillia architecture and house construction
- Trypillia social organisation: current debate

Side note on Varna: Lichardus (1991; also Gimbutas), as discussed in Kadrow (2013), proposed that the social inequality observed at Varna in the mid-5th millennium, was already then adopted from pastoral North Pontic steppe cultures (*check this and delete if unsure*); cf. Chapman et al. (2006) – three social levels interpreted from grave goods, but not clear if these are really discrete or constructed on a continuum. The term *fractal* is here used purely as a metaphor for complex personhood structures, rather in opposition to the described social hierarchy.

- The B2/C1 and the mega-sites of the Southern Bugh – Dnipro interfluve

3.4 Reading site plans from geomagnetic imagery

- Caveats: fill in here.

3.5 Synthetic data

- And why I'm not (this time) relying on ethnographic data.
- Don't go into technicalities here, just the reasoning.

Part II

Size distributions

Chapter 4

House sizes and social meaning

4.1 Possible reasons for house-size difference

Often underlying assumptions: household wealth or size? Or both?

- Kinship and households: who lives in a house? Sahlins (2013), Ensor (2013), Ensor, Irish, and Keegan (2017). Blanton (1994). Carpenter and Prentiss (2022)
- Do clan leaders have bigger houses? check Haude and Wagner (2019), check Carsten and Hugh-Jones (1995)

Discuss some archaeo references

- Schiesberg 2010 2016, go through refs in Zotero, family size and houses for the LBK

Functional difference:

- Ethnography of initiation houses, communal/assembly houses, ritual houses, including Barley (2011), Godelier (1986), Wilk (1983), Fraser (1968), Haude and Wagner (2019)
- Caveats: building materials and constraints, climate (heating), mobility, multi-floored, see Porčić (2012) ++

4.2 Interpreting distribution types and their underlying mechanisms

- Power-law distributions, hierarchy and scale invariance.
- Normal distributions and the Central Limit Theorem.
- Exponential distributions and growth rates.
- Combinations: Log-normal, stretched exponential, parabolic fractal
- Notes on terminology: Power law, Pareto and Zipf, Newman (2005)
- A law (distribution) is not a law (of nature), see Grove (2011) for review of the long-lasting confusion in Archaeology (e.g. Hodder (1979)), also "rank-size rule"

4.3 Fitting heavy-tailed distributions in Archaeology

- Lit. use Strawinska-Zanko et al. (2018), Crabtree et al. (2017), Maschner and Bentley (2003), Grove (2011) ++
- Zipf law and Settlement Scaling theory, Bettencourt (2021), Gomez-Lievano, Youn, and Bettencourt (2012), Lobo et al. (2020). Connection with Central Place Theory, e.g. Müller-Scheeßel (2007), Chen (2011). Why I'm not doing settlement scaling in this study.
- Not fitting distributions in Archaeology, just assuming they are heavy-tailed, or avoiding the question: ex. Brink (2013) (could include lots more!)

END chapter

Chapter 5

Methods: Distribution fitting

5.1 Heavy-tailed distributions, testing for power laws

- Technical characteristics of power laws, Newman (2005), do I need more?
- Old style distribution fitting (which is used in Brown and Liebovitch (2010) and Brown et al. (2005) and check. Mitzenmacher (2004), Harrison (1981)
- New style presented in Clauset, Shalizi, and Newman (2009), Stumpf and Porter (2012), implemented in R with the poweRlaw package Gillespie (2015), and used in Strawinska-Zanko et al. (2018) and Crabtree et al. (2017). More recent?

5.2 Methodological procedure

- Reminder of main goal for this part of the study: identify power-law structures in the house-size distributions of the Linear Pottery and Trypillia samples.
- Synthetic data genaration:
  - Why?
    1. Process: how long/much does in take for a normal distribution to become power-law? And reverse sense?

2. Temporal resolution issue: does the temporal palimpsest of several phases with e.g. log-normal distributions produce false power-law signals?

  – How?

    * Random number generation and iterated multiplicative (1.) or additive (2.) sequences, with K-S testing (Gillespie 2015) at each stage. Report when the distributions become power laws.

- Present data set with categories (settlements, quarters/neighbourhoods, time samples for Vráble)

- Parameter settings: dist. types, xmin, testing the pl hypothesis etc. Minimal house-count cutoff (min. sample size). Isolating top house.

5.3

END chapter

Chapter 6

Results: Distribution fitting

6.1 Synthetic distributions

Fill in analysis for

1. Multiplicative process
2. Additive process

6.2 Settlements

House-size distributions of whole settlements (both Linear Pottery and Trypillia)

6.3 Quarters/neighbourhoods

House-size distributions of separate quarters (Nebelivka) and neighbourhoods (Vráble). Can I include separate Nebelivka neighbourhoods as well? (prob. too small, but check)

6.4 Temporal samples (Vráble)

If time, I can add more Žitava sitesre, but not necessary. I can also analyse temporal samples of Vráble neighbourhoods separately.

6.5 Summary of findings

END Chapter

Part III

Settlement Plans

Chapter 7

Village planning in Prehistory

7.1 Settlement layout and social structure

Or the social organisation of village layout. Research background:

Lit. use Furholt (2016), Fraser (1968), Ensor et al. (2017) (not the correct reference, I should ask him!) Artursson et al. (2010) (Bronze Age, descriptive/interpretive approach)

Use the Trypillia volumes. Also Müller-Scheeßel (2019), Trebsche, Müller-Scheeβel, and Reinhold (2010)

Transition from village to urban (again): Birch (2014).

Factors affecting village layout:

- Political structure (but, as with hierarchy, an organised layout does not necessarily equate top-down despotic decision making).
- Kinship, matrimonial and locality structures
- Cosmology (e.g. Linear Pottery house orientations)
- Economic and ritual functions of village elements (constructed and non-constructed)
- Local landscape setting (to be factored out)

7.2 The geometries of conscious planning vs. emergent behaviour

- I need to find some references here!
- Euclid: grids, lines, circles – how humans think in shapes. Social settings: architect/planner, strong common institutions/ideals (examples?)
- Mandelbrot: irregular, self-similar, scale independent (i.e. fractal) shapes – emergent, not consciously preconceived. Self-organisation. Does the "no pattern" case exist? Emergence from repetitive sequences of simple choices/mechanisms. Examples.
- Binary or continuum? Needs to be studied empirically.

7.3 Fractal image analysis in archaeology

END Chapter

Chapter 8

Methods: Fractal image analysis

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8.1 Fractal dimension and lacunarity

- Box-counting, lit. Mancuso (2021), Li, Du, and Sun (2009), Klinkenberg (1994)
- Gliding-box algorithmsc, Allain and Cloitre (1991), Hingee et al. (2019), Cheng (1997), Plotnick et al. (1996)
- Caveats:
  - Fractional box-counting dimension does not equal self-similarity in a simple way
  - My summary L is not equal to the one used in *FracLac* and thus by Farías-Pelayo (2017)

8.2 Image preparation

- Procedure for archaeological samples, same as in article
- Procedure for synthetic sample

END Chapter

Chapter 9

Results: Image analysis

9.1 Synthetic settlement plans

Relationship between image density (built-up area) and fractal dimension, evaluated in Thomas, Frankhauser, and De Keersmaecker (2007), where they show that these two parameters, under certain conditions (constant observation window, prefactor values close to 1), are exponentially correlated, which is also what I found in my article. They show that observation window size and shape, as well as centroid placement, have little influence on *D*, while they have more influence on density when the pattern is not homogeneous. They do show, however, that images with the same density may have quite much variation in *D*, which is reflected in the layouts. Judging from their examples, more clustered layouts give higher *D* values, while more dispersed or dusty layouts give lower *D*, when density is constant. I ignore the use of prefactor values. According to Thomas et al. (2007), density is a crude measure of the overall intensity of the pattern, while fractal dimension is characterises the morphological structure, though it is not directly descriptive.

9.2 Settlements

9.3 Quarters/neighbourhoods

9.4 Temporal samples (Vráble)

9.5 Summary of findings

END Chapter

Part IV

Synthesis

Chapter 10

Discussion: Social complexity in Linear Pottery and Trypillia settlements

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END Chapter

Chapter 11

Discussion: Fractal Analysis and Archaeological data

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Also mention here fractals or related concepts used as a metaphor, with no mathematics involved (e.g. Chapman et al. 2006; Sherratt 2004; Sindbæk 2022; Whitridge 2016).

END Chapter

Chapter 12

Conclusion and Outlook

12.1 Things I would like to have done, but that didn't fit into this study

- Ethnoarchaeology: Measure house sizes and settlement layouts in contemporary settings, and relate to social organisation (largely overlooked by ethnographers)
- Test distributions and settlement layout analysis on other settings: Lake dwellings, later/historic periods, other materials (e.g. megaliths)… Add more complex distribution models, add observation windows on images. Try on remote sensing imagery.
- Settlement Scaling on Neolithic settings
- Time series: Hurst exponent and scale invariance in temporal development of e.g. regional settlement or population
- Integrate – bridge the gap – between opposite theoretical (nat. and soc./hum.) approaches to the same phenomena
- Chaos and strange attractors in Archaeology
- More?

12.2 Concluding remarks

END Thesis

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Appendix A

This is my first appendix

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Appendix B

This is my second one

Bla bla bla