The Emergence of Social Hierarchy in Prehistory

Application of Fractal Analysis on Archaeological Settlement Plans

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Abstract

In this thesis I explore the possibilities and limitations of applying fractal analysis as a methodological tool for studying levels and dynamics of social hierarchy in prehistory, using archaeological settlement plans as data source. I test the precision and reliability of the methods empirically using synthetic data, and include two case studies of well-documented settlement samples from Neolithic/Chalcolithic contexts in continental Europe: Linear Pottery settlements in the Žitava valley in south-west Slovakia, and Trypillia B-II and C-I settlements in the Syniukha basin in central Ukraine. The two case studies include settlements of widely varying sizes, from small hamlets of only a few houses, to the so-called Trypillia mega-sites of more than 2000 houses, public architecture and complex layouts, thus illustrating the wide spectrum of settlement organisations which existed in Europe in late prehistory. The social organisation and levels of hierarchy seen through their material culture have been lively debated by specialists for decades.

The use of fractal analysis is argued to be well-suited for studying hierarchical structures, and brings with it the theoretical frameworks of dynamical systems and complexity, allowing to advance beyond the false binary debate of structure versus agency as explanations for change in human societies. I argue for analytical separation between the notions of hierarchy and social inequality, which often have been confused in recent archaeological research.

Results from the case studies indicate that hierarchical structures emerge in large settlements irrespectively of cultural differences, as solutions to scalar stress and to optimise flows of information, while the ways in which these structures are played out in daily life vary widely and are historically contingent. In other words, while fractal analysis methods allow for evaluating the existence and scale of social hierarchies, the actual political systems at play, as well as levels of social inequality, are better investigated though other strands of evidence. Linear Pottery society is often considered to tend towards social inequality and lineage-based aristocracies, while Trypillia society shows more evidence of democratic institutions. Both show signs of increased hierarchy in larger settlements.

The two analytical approaches proposed in this thesis show different degrees of matureness. The distribution fitting approach and search for power laws is well documented and theoretically sound, and should be more widely applied in archaeology. The image analysis approach has been less extensively tested, and previous findings are shown here to be unreliable. How-

ever, it does show clear potential for quantifying aspects of spatial regularity and textures. In the application presented here it allows for distinguishing settlement plans with different levels of clustering, again cross-cutting cultural attribution.

Overall, fractal analysis is a methodological framework which merits much more exploration in archaeology. The applications that are presented in this thesis only represent a small portion of what is possible.

Sammendrag

I denne avhandlingen utforsker jeg muligheter og begrensninger knyttet til bruken av fraktalanalyse som metodisk verktøy for å studere sosiale hierarkier i forhistorien, med arkeologiske bosetningsplaner som datakilde. Jeg tester metodenes presisjon og pålitelighet empirisk ved bruk av sytetisk data, og gjennomfører to casestudier av godt dokumenterte bosetnings-datasett fra yngre steinalder- og kobberalderkontekster i Europa: båndkeramiske bosetninger i Žitava-dalen i sør-vestre Slovakia, og bosetninger tilhørende Trypillia B-I og B-II i Sinyukha-vassdraget i det sentrale Ukraina. De to kontekstene inneholder bosetninger av svært varierende størrelser, fra små grender med bare noen få hus, til de såkalte Trypillia mega-lokalitetene med mer enn 2000 hus, offentlig arkitektur og komplekse planmønster. Dermed illustrerer de den store bredden av bosetningsorganisering som fantes i Europa i sen forhistorisk tid. Den sosiale organiseringen og grad av hierarkisering gjenspeilet i deres materielle kultur har vært gjenstand for betydelig debatt blant spesialister i flere tiår.

Anvendelse av fraktalanalyse er argumentert for å være godt egnet til å studere hierarkiske strukturer. Metoden fører også med seg det teoretiske rammeverket kjent som systemdynamikk eller kompleksitetsteori, som jeg hevder kan bidra til å gå videre fra den falsk binære debatten om struktur mot agens som forklaring på endring i menneskelige samfunn. Jeg argumenterer for å skille analytisk mellom hierarki og sosial ulikhet, to begreper som ofte blir blandet sammen i arkeologisk forskning.

Resultatene fra casestudiene tyder på at hierarkiske strukturer oppstår i store bosetninger uavhengig av kulturforskjeller, som løsninger på skalært stress og for å optimisere informasjonsflyt, samtidig som måten disse strukturene utspiller seg i dagliglivet varierer veldig og er historisk betinget. Med andre ord, mens fraktalanalyse er velegnet til å undersøke hvorvidt

sosial hierarkisering kan spores og over hvilke størrelsesordner det er gjeldende, vil andre aspekter som politisk system og grader av sosial ulikhet bedre la seg undersøke fra andre vinkler. Båndkeramiske samfunn er ofte hevdet å tendere i retning sosial ulikhet og slektskapsbaserte aristokratier, mens Trypillia-samfunn viser mer tegn til demokratiske institusjoner. Begge viser tegn til økt hierarkisering i større bosetninger.

De to analytiske tilnærmingene som presenteres i denne avhandlingen viser ulike grader av modenhet. Fordelingsmodellmetoden med testing for potenslovfordelinger er godt dokumentert og teoretisk solid, og burde få en bredere anvendelse i arkeologi. Bildeanalysemetoden har vært gjenstand for færre studier, og tidligere funn blir her vist å være upålitelige. Samtidig viser den et klart potensial for kvantifisering av ulike aspekter ved romlig regelmessighet og teksturer. I anvendelsen som er presentert her skiller metoden ut bosetningsplaner med ulik grad av klyngegruppering, da også på tvers av kulturell tilhørighet.

Alt i alt er fraktalanalyse et metodisk rammeverk som fortjener å bli mer grundig utforsket i arkeologi. Anvendelsene som er presentert i denne avhandlingen representerer kun en liten del av hva som er mulig.

Preface

The idea for this thesis first came to me during a statistics course at Paris 1, when lecturer François Giligny mentioned in passing and with a single slide that fractal analysis could possibly have lots of applications in archaeology, but that they had remained largely unexplored (this must have been in 2015). Chance would have it that I soon after stumbled upon a dusty copy of Mandelbrot's classic *Fractals: Form, chance, and dimension* in my mother-in-law's attic, and now, in 2023, I'm editing the preface of my PhD thesis on fractal analysis in archaeology. If I had just skipped that lecture, my whole life would now be very different. Clearly, history works in non-linear ways.

Listing up those who would merit acknowledgement for supporting me in this project is one of the things I have dreaded the most. Such a list can't possibly be entirely fair, simply since I wouldn't always even be aware of the things that others have done in my favour. But here we are. I'll try to boil it down to what I perceive as the very most essential.

The spatial data used throughout this thesis was kindly shared by the GIS specialists who generated them within their respective projects, namely Nils Müller-Scheeßel and René Ohlrau (both at Kiel University) and Duncan Hale (Durham University). I fully acknowledge the immensity of the work underlying these datasets and commend their openness to sharing them for further study. All three were also very helpful and I have much enjoyed discussing spatial analysis with them.

A course in which I participated early on in my PhD period on R coding for archaeologists and the principles of Open Science, organised by the Nordic Research School in Archaeology – Dialogues with the Past, and chaired by Ben Marwick, Felix Riede, Sophie Schmidt and David Matzig, had a profound influence on the subsequent development of my thesis. I can honestly say I was in the struggling end of the class, but I was encouraged to persevere and be patient with myself. Today I can hardly imagine what this thesis would have looked like if it didn't involve coding (which was initially not intended). On that note, if there's any single group of people who has allowed me to stay more or less sane throughout this project, it is the largely anonymous coding community on forums like StackExchange and StackOverflow. Endless amounts of bugs (some of which have undoubtedly persisted), as well as useful tips and tricks, have been discussed there. In many cases my ultimate saviour has been some random blogger from years ago, who will probably never know they contributed substantially to letting a desparing PhD candidate in Norway finish his thesis on time.

The Department of Archaeology, Conservation and History at the University of Oslo allowed me to pursue this project in the best possible working environment. Meeting other PhD candidates in Norway and abroad during conferences and courses over the course of these three years, made me realise just how lucky I was to have a fully supporting group of colleagues who always would do their very best to generate ideal conditions for me and my fellow PhD candidates. This realisation could of course also imply that I wouldn't have anyone else to blame if it didn't work out well, but that kind of thinking is exactly the opposite of the general mindset at the Department. Here, it was made clear from the start that there is no such thing as failure, neither is there perfection. Just do what you do and that's good enough! (and remember to submit)

Showing confidence in others clearly empowers them to focus on what they are there to do, rather than creating barriers of pressure that so many PhDs face in universities worldwide. One thing that struck me from the onset was the open-mindedness the Department demonstrated simply by giving me a green light with this project. Fractals you say? Sure, sounds like fun. Go ahead! This of course also goes for my supervisors: Martin Furholt, who was in Oslo when I first started in 2020, now at Kiel University, Martin Hinz at Bern University, and not least, Ingrid Fuglestvedt here in Oslo. The confidence and full support they all have showed me along the way is something I do not take for granted, and for which I will truly be forever thankful.

I also had the privilege of visiting other academic institutions during the work of this thesis, which proved extremely useful. Researching the Linear Pottery and Trypillia cultures from Oslo is admittedly tricky, as most fellow archaeologists, not to speak of librarians, have hardly if ever even heard of them (that said, the university library at the UiO has gone to great lengths to finding obscure continental conference proceedings from years back for me). I was warmly welcomed for a longer stay at Kiel University in the spring of 2022, by the ROOTS Cluster

of Excellence and the CRC 1266 – Scales of Transformation, with support from Erasmus+ funding. I also got the opportunity to participate in fieldwork at Vráble in Slovakia with the latter group during three weeks of the summer of 2021 (despite all odds given the sanitary restrictions in place at the time). For all this I am greatly thankful. By naming none, I name them all.

Other shorter and more intensive stays included visits to the small but welcoming archaeology department of Bern University, as well as the UMR 8215 Trajectoires lab in Paris and the CLIOARCH research group at Aarhus University. I have also enjoyed countless fruitful encounters and discussions with colleagues from all over Europe and the world during various conferences. Silviane Scharl (Cologne University) and Christopher Prescott (University of Oslo) provided helpful guidance during my mid-way assessment. People I now consider as mentors (some of them would be surprised), include, in no particular order: Daniela Hofmann, Shumon Hussain, Jean Hubert, Isak Roalkvam, Maria Hesjedal, Tim Kerig, Mélodie Larue, Sabrina Farías-Pelayo, Caroline Hamon, Mike Ilett, Jana Anvari, Caroline Heiz and Jan-Eric Schlicht. Thanks to all of you for the inspiration and support! Even my non-academic friends, who mostly regard me as a “mad scientist”, have shown me their whole-hearted support. You know who you are.

Lastly, thanks to you, my dear Gabriel, the biggest support of all. Without you I wouldn’t even have applied for the job. Thanks for your patience, your understanding, and love.

And now sissy that walk!

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Part I

Frameworks

Chapter 1

Introduction

1.1 Background of the study

In Europe, the Neolithic is – broadly speaking – the long and messy transition period between mobile hunter-gatherer groups in the Palaeo- and Mesolithic, and the first city states appearing in the Bronze (Aegean) and Iron Ages (Mediterranean and Central Europe, Shennan 2018). Within this timespan of several millennia, early farmer societies evolved at different paces and in different directions, developing unique features, but also receiving indirect economic and technological impulses from the faraway Near East. Large variety in scale and content characterises the archaeologically defined culture groups of this period. In many phases and regions, single farmsteads and small hamlets were the dominant settlement feature – for some regions and phases, like groups within the Michelsberg, Funnel Beaker and Corded Ware horizons, there is hardly any documented settlements at all. Other phases include exceptionally large settlements, probably hosting populations of several thousand inhabitants, like at Maidanetske in central Ukraine around 3.800 BCE (see 3.3). In some settings, like in Linear Pottery society in much of continental Europe north of the Alps towards 5.100 BCE (see 3.2), the dead were buried in simple pit graves, either alone or in cemeteries, with only subtle distinction in treatment between individuals, in other phases some individuals were buried with tremendous amounts of precious goods like in Varna, Romania, or under colossal burial mounds like in Carnac, France, both in the mid-5th millennium (Jeunesse 2017).

Seen at a very large scale – across the continent and through the Holocene – the development of society from small scale and relatively egalitarian towards large scale and more hierarchical seems evident (though simplistic, see Graeber and Wengrow 2021). When we look more closely however, this evolution is anything but linear, as both population sizes and levels of hierarchical organisation seem to fluctuate considerably, sometimes over short time spans as from the Trypillia C-I to C-II, when the so-called mega-sites are abandoned and their former inhabitants regroup into much smaller settlements during a transition of maybe only one generation (Harper et al. 2021 ; Shatilo 2021; Ohlrau 2020).

In many cases, the level of social hierarchy and complexity in a given Prehistoric society is very hard for researchers to evaluate, since many indicators of such structures are either lost from the archaeological record, or were never included in the first place (Perreault 2019, see also Section 3.1). Archaeological traces that are often interpreted as signs of social complexity and hierarchy may furthermore be deceiving. Seemingly monumental structures were in many cases built through smaller additions over longer periods of time, rather than in one colossal construction campaign (e.g. Laporte et al. 2017 for the Barnenez cairn in Brittany). Furthermore, in many megalithic burial contexts, it may be impossible to estimate the proportion of the society that had access to such burial treatments.

In this thesis I explore the potential of a new methodological framework for evaluating the level of social hierarchy present in prehistoric societies. Fractal analysis is the general term for quantitative methods developed specifically for the study of highly irregular shapes, patterns or structures exhibiting self-similarity over large ranges of scale, so-called *fractals* (Falconer 2013). As is argued repeatedly throughout this thesis, pyramidal hierarchies constitute one type of such shapes, and fractal analysis is thus a suitable approach for finding hierarchical social structures reflected in material culture. The methods are strictly speaking not new, as they have been developed at least since the 1960s (Mandelbrot 1982, 1967). However, they have still only to a marginal degree been implemented in archaeological research, and thus have a high potential for use in a wide range of applications (Diachenko 2018). Inspired by recent advances within the fields of urban science and human geography (Jahanmiri and Parker 2022; Lagarias and Prastacos 2021; D'Acci 2019; Tannier and Pumain 2005; Batty 2005; Batty and Longley 1994), with this study I wish to extend the use of fractal analysis methods also into research on prehistoric social organisation.

1.2 Research question and objectives

The overall goal of the present study is to test and assess the utility of fractal analysis techniques as tools for studying hierarchical social organisation in prehistoric societies. Two methodological approaches are under special scrutiny: the distribution fitting approach and the image analysis approach (Brown and Liebovitch 2010; Brown, Witschey, and Liebovitch 2005). These are applied to architectural data series from well-preserved and documented archaeological samples within Neolithic Linear Pottery and Trypillia contexts, as well as to synthetic data series. This thesis is thus not to be considered a culture-historic study of Linear Pottery or Trypillia society, but primarily a methodological project with two prehistoric case studies. However, results from the proposed analyses of these cases do also contribute, as side-effects, to their respective fields of research.

For the distribution fitting approach, house-size distributions within settlements are modelled following a given procedure, and the retained model (the best fit) is interpreted in terms of social generating mechanisms. In particular, it is argued here that power-law distributions reflect hierarchical structure, so that the identification of these within the studied samples may indicate the presence of some hierarchical social process which warrants further interpretation.

With the image analysis approach, the spatial layouts of archaeological and synthetic settlement plans are analysed through the calculation of fractal dimension and lacunarity – summary statistics which serve as quantifications of irregular spatial patterns or image textures. I argue here that geometrically irregular settlement plans are indicative of relative independence between households, while settlements that develop within geometrically regular grids indicate stronger overarching social structures, with a continuous range of possibilities in-between. The goal here is to test to which extent quantitative measures like fractal dimension and lacunarity may help differentiating between varying degrees of planning in prehistoric settlements.

While both these methodological approaches are well developed and integrated to other disciplines, their usage in archaeology has so far remained anecdotal. A further overall goal of this thesis is to identify and explore possible constraints in the nature of archaeological data that may limit the applicability of fractal analysis methods within this discipline. For example, does fractal analysis of settlement plans require a data quality that would be practically unattainable in archaeological settings? But also, as it is impossible to explore all potential applications within the framework of one doctoral thesis, suggestions for future research are provided in the last chapter.

1.3 Defining hierarchies

The term *hierarchy* is central to the present study. Though commonly used in daily speech, defining the word is not as straight-forward as one might think, so some clarification on how it is understood here might be needed. In a volume dedicated to exploring the meanings and uses of hierarchy as a study object within a range of natural and social sciences, Pumain (2006) provides a panorama of definition nuances, but also highlighting the characteristics that are commonly found in most cases (Pumain 2006). Among these characteristics are:

- A pyramidal organisation of elements, ordered by a very unequal size distribution of a certain quality or variable, from a few large elements on top to many small elements at the bottom
- When seen as a system, the whole is constituted of sub-systems, which are again constituted of sub-sub-systems, and so forth. These can either be ordered into clearly distinguished levels (stratified), or in other cases be scaled in a continuum (branched or tree structure)
- In physical, biological and social hierarchical systems, the structure is often accompanied by a flow of energy, material, information or control in one or both directions between the top and bottom levels

Hierarchies are found in humanly constructed classification and taxonomical systems, where morphological distinctions are considered more important or fundamental at the higher end of the hierarchy, while being more detailed or specific at the lower end. Many hierarchical social systems, like religious (from Greek *hieros* – sacred, and *archê* – government), military or corporate organisations, include strongly reinforced regulations of subordination, which in modern society has led to somewhat negative connotations to both hyper-rational and despotic rule. While one prevalent explanation for the frequency of hierarchical structures in nature and society is indeed that they “represent the best solution for many optimisation problems”

(Pumain 2006, 7), that does in no way mean they need to be consciously planned. On the contrary, in most cases hierarchies seem to emerge spontaneously, often from growth processes with systems splitting into sub-systems once they reach a certain critical size limit. There is also no compulsory link between social hierarchies and despotism, as it matters little to the overall structure whether the top element is elected for a limited period or born into an inherited leading position (De Landa 2006). More detail on how hierarchical structures emerge and how they can be described as fractals, is given in Chapter 2. A further discussion on the specifics of social hierarchies is given below in Section 2.1, and on the differences between spontaneously emergent versus consciously planned structures in Chapter 7.

A possible confusion with a somewhat different meaning of the term hierarchy should however be mentioned already here. If hierarchical structures are abundant in nature in both physical (inert) and biological systems, social hierarchies on the other hand – understood as intra-species populations of individuals organised in pyramidal hierarchical, i.e. multi-level relationships to each other – seems to be almost exclusively found among human groups. At the same time, a different type of hierarchy is frequently described by biologists which is widespread among animals, namely *dominance hierarchies*, also known as pecking order (Strauss et al. 2022). These structures are hierarchical in the sense that there is difference in rank between group members, and they also seem to emerge spontaneously in the animal populations where they are described. But unlike social hierarchies, these are purely *linear*, in the sense that each group member is situated in rank above one part of the group and below the rest, so that the whole group forms a rank chain in the form A > B > C … n. The rank of an individual will typically decide their access to food and reproduction relative to the other members, and may be settled and resettled in a number of ways depending on the species and population under study, but typically involving some level of violence or threat and subordination in face-to-face encounters (Strauss et al. 2022).

A classic example – perhaps most of all in popular culture – is dominance hierarchy among wolves, led by an alpha male (e.g. Cafazzo, Lazzaroni, and Marshall-Pescini 2016; Packard 2003). Though it has been much discussed whether or not this trope model actually fits wolves (see Mech 1999; Muro et al. 2011), any reported dominance hierarchies among larger groups of wolves and stray dogs are linear rather than pyramidal, even when they are illustrated as pyramidal (e.g. Fig.1 in Rodríguez et al. 2017). Similar social organisation systems are found among a wide variety of species – mammals, birds, fish, particularly but not only among group-hunting carnivores – and are generally interpreted as an evolutionary mechanism (Strauss et al. 2022, with references). Cases of branching, multi-level social hierarchies among animals are on the other hand extremely rare, but have been reported to operate among hamadryas baboons in Ethiopia, with "clan leaders" forming relays of information flow and decision making between the "one-male units" within a total population ("band") of about 200 individuals (Schreier and Swedell 2009). Eusocial insects like ants and wasps provide a more well-known example of hierarchical organisation among animals (Shimoji and Dobata 2022), also indicating – if it should be necessary – that it is not a matter of intelligence or brain size, but rather of social function (e.g. building a hive or a village together) and population size above certain thresholds.

The reason to dwell upon this qualitative distinction between linear and pyramidal hierarchies, is that the prevalence of the former in nature is sometimes put forward as an argument for social hierarchies among hunter-gatherer groups in the Palaeo- and Mesolithic. To cite just one example, in their large-scope volume of the emergence of social inequality around the world over the Holocene, Flannery and Marcus (2012, 37–39, 58–60) apparently saw the need to explain away the dominance hierarchy seen among other great apes as something which among human foragers was made invisible by assigning the highest ranks to spirits and ancestors, only to re-emerge again in early city-states as kings took claim again to the alpha role, grounded on divine status. While it is a good point that there is no reason there couldn't be linear dominance hierarchies among small forager groups, and that such systems hardly can be described as egalitarian by those who live in them, Flannery and Marcus (2012) fail to recognise that linear (pecking order) and pyramidal (fractal) hierarchies are fundamentally different structures. And given the extreme rarity of the latter in nature, including among primates, we cannot assume that pyramidal or stratified social hierarchies have always been part of human culture, but rather that they – much like agriculture – at some point came to be as historical phenomena. Quite possibly, as suggested long ago by Leroi-Gourhan (1965), was it our capacity for language that enabled us as species to exceed the critical thresholds that would otherwise have limited our social group sizes, and organise in larger communities through a large variety of hierarchical social structures.

In recent years, there has been a large wave of research oriented to the dynamics of social inequality over deep time, often framed more or less explicitly as a search for its "origin" in time and space and its explanation (e.g. Graeber and Wengrow 2021; Moreau 2020; Kohler and Smith 2018; Flannery and Marcus 2012; Price and Feinman 2010, 1995). Many have fallen for the temptation of conflating social inequality and hierarchy, and dealing with them as if they were the same. This, I believe has led to a number of discussions resembling semantic conundrums. Part of the problem is that words like equality or egalitarian are commonly used as the opposite of both. The difference between the concepts of social hierarchy and inequality lies in that the former relates to social and/or political organisation and the distribution of power, while the latter to economy and the distribution of wealth or income. Societies where these two are completely unrelated may be theoretically conceivable – that is, where wealth does not entail power and *vice versa* – but historically they have tended to go hand in hand, albeit not necessarily in a straightforward way. It is of course all too evident that money actually can buy power. However, to Graeber and Wengrow (2021), who have advanced heavy critiques of any form of generalising narratives of increasing social inequalities throughout the Holocene, evidence of democratic institutions such as those proposed for Trypillia mega-sites, seemingly also constitutes evidence *against* social hierarchy, as if the two were intrinsically disjoint. Again, as discussed by Pumain (2006) and De Landa (2006), among others, the fact that a structure is hierarchical does not in itself say much about the qualitative characteristics of the top element. It can be an elected leader with a temporally limited time of office, a council of elders or of delegates from various sub-units or many other configurations that are amply described in Graeber and Wengrow (2021). The "alpha-male" – to return to this terribly outdated notion – is only, if at all, one option out of many.

In sum, the related but distinct social dynamics of inequality and hierarchy should thus be studied separately and compared. Taking one variable for granted only through evidence of the other would be like assuming that the Roman Republic must have been egalitarian since it was democratic. A consequence of adopting methods specifically targeted at hierarchical structure, like fractal analysis, is that it enables us to better appreciate the variability of human social organisation, like acknowledging the existence of deeply hierarchically organised structures in economically egalitarian societies.

1.4 Main findings

The analyses which are presented this thesis allowed for a deepened understanding of how the proposed fractal analysis methods work, and what they offer to archaeology, in particular in the study of prehistoric social hierarchy.

A review of the statistical literature related to the distribution fitting approach showed that it has deep roots and well developed theoretical foundations. The link between the statistical distribution type known as the power law is to be considered a signature of hierarchical structures, and generative mechanisms like preferential attachment are known from social and natural sciences. However, the methods related to identifying them in empirical data have undergone heavy rebuilding in recent years, a development which only to a limited degree has been picked up by archaeologists. The main application of a similar approach in archaeology, which also has existed for a long time, is the so-called rank-size rule or Zipf law approach, which despite certain merits has been shown to be gravely lacking in theoretical and methodological sophistication, which also may explain why it never has been adopted into the standard analytical tool kit of archaeologists. The recent re-branding of this approach has become known as Settlement Scaling Theory, showing considerable improvements and increased reliability in detecting hierarchically scaling settlement systems. This approach is theoretically very close to the one used in this thesis for intra-site house-size analysis, but has not been pursued further here. Current applications of Settlement Scaling are largely limited to contexts from the last two millennia, mainly in the Americas, and more systematic application on European prehistoric contexts would be welcome.

The analyses of house-size distributions provided here gave clear indications firstly that power-law distributions are to be found in Neolithic settlements, which had not been firmly demonstrated previously. This in itself is a strong argument for hierarchical scaling between households within the studied Linear Pottery and Trypillia contexts. Furthermore, power laws were demonstrated only for the largest settlements within both culture groups, i.e. settlements with more than about 80 houses in their cumulative plans. Further breakdown of the data, as well as results from tests on synthetic sets, showed that this result was not a statistical artefact from artificial data aggregation. Analysis of separate spatial and temporal sub-samples of the large Linear Pottery settlement of Vráble showed that the house sizes remained power-law distributed even when the sub-sample sizes went well within the ranges of smaller non-hierarchical settlements. Though the total sample of settlements is small, the results clearly cross-cut cultural attribution, indicating that the power-law distribution of house sizes is inherently connected to settlement size, which again implies that hierarchical scaling of households emerges with population increase beyond certain thresholds, as suggested by evolutionary psychology.

The identification of such hierarchical structures is surprising both for the Linear Pottery – where multi-level social hierarchy is not widely accepted – and for the Trypillia – where current interpretations involving democratic institutions have somewhat precluded the study of hierarchy. For the Trypillia mega-sites, the identified power-law distributions went beyond the typological distinction between public and domestic buildings, and the domestic buildings which were shown to scale hierarchically, were also shown to cluster near the public buildings. These large domestic buildings have previously received little attention relative to the so-called mega-structures which are generally considered to be communal. For both cultural samples, the possibility that inhabitants of small versus large settlements lived very different lives needs to be reconsidered.

The analysis of image textures constitutes an approach which is currently on a much more experimental stage. Adapted from Mandelbrot's pioneer work in the 1970s, it has been developed and applied to contemporary urban morphology studies since the 1990s with increasingly satisfactory results. Applications to archaeological settlements have remained anecdotal and constrained to classical Mesoamerican studies. Results from analysis on synthetic data presented here indicated that much of previously published results and interpretations from archaeological settlements were highly problematic, showing signs of over-confidence to what the method could do, and to some degree limited understanding of the underlying theory. However, the same analyses on synthetic images clearly demonstrated that both fractal dimension and lacunarity (the variables calculated by the method) correlate with texture-related variables such as random noise and clustering, all other things being equal. This effect tended to be drowned by larger effects produced by methodological variables such as image size and resolution, as well as image density. It remains unclear whether these effects are fully accounted for in the cited urban studies where these methods are primarily applied. In any case, more systematic modelling on larger and carefully constructed data sets should make it possible to apply this method more efficiently on archaeological settlement plans.

The results reported here for the image analysis approach on the archaeological sample, showed moderately clear patterns. In particular, it appears that the analyses allowed for effectively differentiating between uniformly compact and loosely clustered settlements, again cross-cutting both settlement sizes and cultural attribution. While these results seem promising, it was also shown that similar results could be obtained through much simpler analytical methods, like plotting settlement size to image density on logarithmic scales. But again, it is argued that further development could strengthen the method to become an efficient tool for quantifying spatial regularity/irregularity, providing a proxy for overall planning.

1.5 Open Science framework and data availability

Because of the experimental nature of this thesis, and its focus on quantitative methodology, it was written as an open-source Rmarkdown project using the bookdown package (Xie, Allaire, and Grolemund 2018; Xie 2016), with analyses done as far as possible in the R programming language (R Core Team 2023). The data and scripts used for generating all results in this thesis can be freely consulted at the following repository:

https://github.com/hallvardbruvoll/Bruvoll_PhD_thesis

Excepted are the .shp files (house outlines) which were shared to me by the GIS specialists who created them (see Section 3.4). Maps and images of archaeological settlement plans were generated with QGIS software and can not be generated by the provided scripts.

1.6 Structure of the thesis

This thesis is structured as a monograph in four parts. In the first part the overall framework of the study is exposed, with the general introduction above, the overarching theoretical framework in Chapter 2, and the background of the study material and data in Chapter 3. Parts II and III are devoted to each their methodological approach to the material: Part II to the study of hierarchy in size distributions, and Part III to the quantification of image textures. Each of these parts consists of three chapters, the first of which – Chapters 4 and 7 – expose the theoretical and interpretative background of the applied methods and their relevance to archaeology. The following chapters – Chapters 5 and 8 – detail the technical specifics of the two approaches and their implementation in this study, and the last chapters within these parts – Chapters 6 and 9 – provide the actual analyses and summaries of results. In Part IV, the findings are summarised and further discussed. Chapter 10 gives an attempt of interpreting the results in the context of the culture-historical setting of the European Neolithic, while Chapter

11 reviews the possibilities and limitations of the fractal analysis framework in archaeology. Concluding remarks and suggestions for further study are given in Chapter 12.

Some readers might react to an apparent deviation from the academic tradition of devoting a separate chapter to research history. This is a deliberate choice, not to suggest that historiography is unimportant, but rather as a result of the fundamentally interdisciplinary scope of the study. In fact, there is very little extant history of applying fractal analysis in archaeology – the few studies that, to my knowledge, have been done in this direction are discussed primarily in Sections 4.3 and 7.2. Fractal analysis itself holds a research history of its own (see Section 2.2), and so does the study of Linear Pottery (Section 3.2) and Trypillia societies (Section 3.3), not to mention the general study of social complexity in prehistory (mainly Section 2.1). In short, instead of trying to shoehorn these parallel histories into a clearly delimited but rather hybrid chapter, I have opted for what I believe to be a more useful approach, namely to fit them in more seamlessly where they belong, in the various associated theory and methods chapters.

An additional note should be made here regarding the writing style of the different parts and chapters. It is my belief that a major obstacle for fractal analysis methods becoming more integrated into the standard tool kits of archaeological research, is the excessively technical nature of much of the associated literature. Archaeology as a discipline remains profoundly rooted in the Humanities, as seen in the inbuilt structure of teaching, research and funding institutions in most (at least European) countries. Fractal analysis is derived from pure mathematics, and most applications so far have been developed within the natural sciences. Archaeologists who are trained within a humanistic scholarly tradition cannot be expected to hold a skill level of mathematics more advanced than what is achieved in secondary school, and code programming is hardly taught at all within the walls of Humanities faculties. Technical details regarding the methods and analyses applied in this thesis are therefore – as much as possible – limited to the devoted Chapters 5 and 8, and readers who are interested in these may also refer to the online code repository for more details and reproducibility. In the rest of the thesis I have opted for a more narrative approach, in an attempt to invite a somewhat larger audience of archaeologists into the fascinating complexities of fractals.

Chapter 2

Theoretical framework: Scale, Complexity and Fractals

2.1 Micro-macro approaches in social and anthropological theory

2.1.1 Sizes of human social groups

The scalar relations between overall social structures, the individual and everything in-between is a major theme in social theory, and any discussions on the nature of social hierarchies will quickly reach in to the very core of all disciplines that deal with human societies. Here I will not attempt to do justice to such a large topic of the social sciences and humanities, but rather outline the specific questions that seem the most relevant to this thesis and that have been the most lively discussed by archaeologists and prehistorians.

The political connotations to hierarchical social structures were already mentioned in the introduction, and will be evoked repeatedly throughout this thesis. The crucial analytical distinction between political system, economic inequality and hierarchical structure is often forgotten, leading to researchers with different interests talking past each other in fruitless debates. Top-down despotic political systems are not the only possible form of hierarchy, and inversely, a search for hierarchical structure in archaeological data is not a search for top-down despotic political systems (see Furholt, Grier, et al. 2020).

A recurring theme in anthropological discussions related to hierarchy – often termed social complexity – is its purported correlation to group size. The notion of *scalar stress* was advanced in archaeology by G. A. Johnson (1982), referring to how social groups that grow to a certain size will tend to reach a limit, beyond which they can only grow by splitting (fissioning) into smaller groups, as conflict levels rise and group solidarity and integration decrease Drennan and Peterson (2008). Evolutionary psychologists have proposed more causal explanations to why this happens, with the claim that the observed thresholds for maximum group sizes are largely biologically determined within species, and that subsequent fissions at higher hierarchical levels (as well as lower level groupings) tend to happen at multiples of the same threshold (B. West et al. 2023; Zhou et al. 2005). Most famously, Robin Dunbar proposed the "social brain hypothesis" in the 1990s, claiming that ideal group sizes among primate species were determined by the average size of their neocortex, with that of humans being around 150 individuals (see Dunbar 2023 for a recent overview). This idea has since been massively criticised for being too simplistic, according excessive importance to bio-genetic factors and ignoring culture (e.g. Lindenfors, Wartel, and Lind 2021), while social media studies on very large data sets have given it more credit (e.g. Gonçalves, Perra, and Vespignani 2011).

Some of the critique of what has become widely known as "Dunbar's number" seems to stem from a certain malaise related to the linking of social complexity and brain size, especially among anthropologists (e.g. Graeber and Wengrow 2021). Ironically, while social hierarchy often is given certain negative connotations like despotism, oppression and inequality, social complexity (which to a large extent is synonymous with hierarchy, see below) is often given a positive connotation in the sense that it is indicative of highly developed systems. Such connotations are indicative of the common confusion of the terms complex and complicated. A central tenet to complexity theory is that complex phenomena, like social hierarchies, are generally not consciously planned – they *emerge*, through repeated interactions between smaller components who obey more or less simple rules. In other words, claiming that social complexity or hierarchy emerges in large human populations, and therefore generally only from late in prehistory and onwards (e.g Ross and Steadman 2017; Smith 2011), is not to say that these complex societies are in any way better or smarter than less complex (often forager) societies – they are simply different. Anthropologists who deal with comparative big history routinely express the need to point out that foragers also can be highly complex (e.g. Graeber and Wengrow 2021; Testart 2012), which is of course true in many ways, but somewhat beside the point when regarding hierarchical organisation in particular. There has never been such a thing as forager emperors (though Graeber and Wengrow 2021 do discuss some apparent exceptions).

As a small side note, some specific types of social complexity are particularly relevant for the study of forager societies. One is the phenomenon of nested hierarchies, a trait probably common to all human societies, where any single person belongs to a variety of social groups of different scales, which are made relevant in different social settings. A person can belong to a family, an extended family, a village community, an ethnic minority, a nation state, even have a continental or human identity along with many other identities at the same time. The total of all these identities form a nested hierarchy, and is indeed fractal – there are a few large ones and many small ones – but it is not the same as a hierarchically organised society. Nested social hierarchies among foragers is an interesting topic (Hamilton et al. 2007; Whitridge 2016), which is in no way in contradiction to the general framework of this thesis. Another term which has been much debated for many of the same reasons, is that of *complex hunter-gatherers* (Arnold 1996), often claimed to invalidate general narratives of increasing complexity over the Holocene and its link to agriculture. These are societies now known from many parts of the world, but originally associated with north-west coast Native American groups known for their sedentism, complex art forms, extensive food storage and competitive feasting organised by clan or house leaders (see below). To which extent they constitute exceptions of or confirmations to rules of how foragers are organised is a different debate, but it should in any case be kept in mind that they are often associated with specific eco-zones with abundant marine resources which allow them to sustain relatively large and dense populations.

Though a very general tendency of increasing levels of social hierarchy or complexity is recognised by most big history researchers, much interest has been accorded to deviations from this rule, to the extent that many authors consider such deviations to be systematically inherent in how human societies evolve over time, either in cycles (e.g. Peters and Zimmermann 2017; Zimmermann 2012b; Gronenborn et al. 2014) or in more abrupt movements (Scheidel 2017; Heitz et al. 2021; Gould 2007). In any case, it is today widely recognised that societies do not simply grow towards ever more complexity in a linear manner (Graeber and Wengrow 2021).

2.1.2 Social evolution and its critique

Another closely related approach to social hierarchies which has been subject to much debate in archaeology, is that of societal classifications, usually grounded in social evolutionary theory (A. Johnson and Earle 1987; Service 1971; Testart 2005). In general, such works have aimed to define typologies of stages of social evolution based on comparative anthropology, more or less explicitly seeing current small-scale societies as remnants of society forms that would have been representative of earlier periods in human history and prehistory Ragan (2017). The perhaps most widely (by archaeologists) cited work was done by A. Johnson and Earle (1987), who proposed a scheme of classifying small-scale societies into three main classes: those defined by the family group (without or with domestication), the local group (acephalous or “Big Man” collectivities) or regional polities. The latter class was further subdivided into chiefdoms, early or “archaic” states and nation-states with peasant economy. The focus of studies within this tradition has often been on types of political leadership, and explaining their functioning and temporal dynamics (Earle 1997, 2002). As an example, the leader type known as the Big Man – modelled on ethnographic accounts from the New Guinea Highlands – is recognised as one who bases his (male) power over the local, usually single village, community on economic surplus and prestige, but not heredity like higher level chiefs. Big Man societies are thus thought to be politically more unstable, since the leader can be challenged, and in any case at his death someone else will take the opportunity to fill his place (A. Johnson and Earle 1987, 160–93). Chiefs on the other hand, build their power on inheritance, and high level chiefs ruling over local chiefs are known to rule “complex chiefdoms”. As such, within the social evolutionary framework, the type of society is closely linked to population and scale.

While such classifications are widely used and referred to by archaeologists specialising in prehistory (perhaps especially Neolithic and Bronze Age societies), they have been subject to substantial criticisms. Specific categories have been shown to gloss over important differences between the societies they are supposed to characterise, thus making us blind to variability Kienlin and Zimmermann (2012). They have been criticised for promoting simplistic views on political power Furholt, Grier, et al. (2020). Anarchistic theory has shown that what authors like A. Johnson and Earle (1987) label as acephalous systems are not limited to small bands of mobile foragers, and systematic social difference does not necessarily involve vertical hierarchies Angelbeck (2022). Furthermore, the assumption that Palaeolithic foragers were necessarily egalitarian has been much debated, with social inequality in the so-called "princely graves" of the Gravettian being a recurrent theme Testart (1982). A somewhat ironic aspect of many of these discussions is that critics who point to the simplistic nature of societal classifications, in the ent tend to replace or add to the categories themselves. In fact, the now classical works of mid to late 20th century neo-evolutionary authors were originally also conceived as critiques of 19th century evolutionary or Marxist models (e.g. Engels 1902) perceived as too simplistic and not founded on empirical observations.

One such additional category of social structure, which has had a certain success in archaeology, was described by Claude Lévi-Strauss as *house societies* (Lévi-Strauss 1982a; Joyce and Gillespie 2000; Beck 2007). Here a house is a social unit, often centred around a material estate but also involving titles, heirlooms, land ownership, rights to hunting grounds etc. and where membership is not determined from genealogy in a systematic manner, as is the case with more regularly structured lineage or clan societies. House membership may be gained through more competitive social action, privileging those that possess the resources to engage in activities such as gift exchange and *potlatch*-style feasting (see Hayden 2014). This opens up for more complex configurations, and house membership will often entail a certain level of prestige. Inheritance may furthermore follow (male) descent or (female) affinity – that is from grand-father to grand-son via mother – in a pragmatic way depending on which option is in the best interest of the house, as long as it can be justified in more or less precise kinship-related terms Ensor (2011). A segment of society of varying size will not afford to partake in this competition, and as a result, house societies can be viewed as being in a somewhat unstable transition state between lineage and class societies. Lévi-Strauss originally pinned the term on Kwakiutl society in the American north-west coast (as described by Franz Boas in the late 19th century), but argued for its generality by associating it with the feudal system of medieval Europe. The concept has since been applied to a wide range of cases from ethnography (especially in the South-East Asian and Pacific regions), as well as to prehistoric contexts (e.g. Kuijt 2018).

The subtext of debates regarding social evolution, scale, and types of societies – as I see it – is disagreement regarding what is most important, between shared versus unique traits, or overall trends versus variation. Broad evolutionary models are systematically criticised for being too simplistic, while adding more and more nuance can make one loose sight of similarities and trends that are also real enough. In the long run, such debates inevitably take on the characteristics of false dichotomies – both points of view are correct, but largely unable to see the value in other approaches, which is why the debates do not advance much but rather seem to go in loops.

My stance in this thesis is that in order to better apprehend the complex relationships between different scales of society, and individual agency versus structure in particular, other frameworks are needed. A number of valuable approaches have indeed been proposed within social and anthropological theory precisely as attempts of bridging the gap in different ways between the micro and macro scales of society, but which have been less regularly referred to in archaeology (note that this thesis, while written at a Scandinavian university, deals with case studies and associated academic traditions in continental Europe, where neo-evolutionism is still largely dominant in late prehistoric research). These include, but are not limited to, structuralism (Lévi-Strauss 1969), world systems (Wallerstein 2011; Braudel 1976), structuration theory (Giddens 1984), actor-network theory (Latour 2005) and assemblage theory (De Landa 2006). While there is certainly not enough space here to present all of these theoretical approaches and the differences between them in detail, it can be noted that they are all mostly associated with qualitative methods. It is my conviction that the massive amounts of archaeological data which has become available in recent years (Bevan 2015), also calls for the use of quantitative methods and associated theory in research on social complexity and hierarchical structures across scales. The most well-suited framework for this purpose should be dynamical systems theory, which covers fractal analysis among other methods, and which – to the extent it is applied in the social sciences and humanities – is often labelled as complexity theory, and has strong parallels to qualitative approaches like assemblage theory. Exploring how these very separate theoretical traditions approach many of the same phenomena in the real world, would be the subject of entire theses in itself.

2.2 Complexity Theory / Dynamical Systems Theory

The study of complex dynamical systems derives from physics and the natural sciences, where such systems are characterised by multiple actors or agents – living or inert – that interact with each other following certain rules, and from which new behaviours emerge at larger scales. Such *emergence* is an overall pattern, behaviour or structure that is described as more than the sum of its parts. Common examples are galaxies, ant colonies, river systems, weather systems, flocks of birds, and so on. De Landa (2006), who in my view describes exactly such phenomena within human societies and with humanist terminology, argues that at certain points the system becomes an agent of its own. Dynamical systems theory uses a range of concepts for describing and explaining such systems, and it is widely recognised that most of them also apply to humans as well (there is no reason they should not). However, they have only to a very limited degree been integrated in the humanities, including archaeology. For a technical introduction, see Devaney (2020). A more lay-person introduction is given by G. B. West (2017). Applications in archaeology include network science, settlement scaling, cultural evolution and agent-based modelling (Daems 2021; Baden and Beekman 2016; Ross and Steadman 2017; Smith 2011; Bentley and Maschner 2008). Some central concepts in dynamical systems theory, including chaos, feedback loops, criticality and scaling, and are discussed in some more detail in Chapter 4.

Social complexity is more than social hierarchy, and all societies – whether they are classified as simple or complex within a social evolutionary framework – can arguably be studied as complex systems, since they always consist of various sub-systems and populations of individuals acting and interacting in a variety of ways (Daems 2021, 6). The sense of social complexity that is traditionally understood in archaeology, derived from social evolution theory, is somewhat narrower and relates more to the specific characteristics of organisational scale (i.e. hierarchy) than to complexity *per se*. Studying the scale of social organisation from a complex/dynamical systems perspective rather than from a social evolutionist one holds several advantages:

- The switch from discrete evolutionary stages to continuous spectra allows for more nuanced evaluation of the society in question, avoiding false binaries like simple-complex
- The complex systems approach arguably has a stronger explanatory power than the more classificatory social evolution schemes
- Complexity theory offers a better alternative for ethical reasons, as it avoids the underlying colonial and eurocentric connotations associated with classifying societies into simple and complex

As a further elaboration on the latter point, an analysis of a society from a complexity theoretical viewpoint is not a matter of establishing whether or not the society can be characterised as a complex system. When the scale of social organisation is the object of study – as in the present thesis – the word hierarchy is both more accurate and, if not neutral, at least more balanced than social complexity, since it is not obvious whether or not it is a good thing for a society to be characterised as hierarchical. The study of the dynamics of social hierarchy over time is thus not a story of progress, as the 19th century studies of social evolution too often were.

2.2.1 Fractals and Fractal Analysis

Fractal analysis relates to complex systems/dynamical systems in that it is the methodological framework which allows for studying the complex patterns that they tend to generate (Mandelbrot 2021; Falconer 2013).

The term *fractal* was pinned by the French-American mathematician Benoît Mandelbrot (1924-2010), from the latin word *fractus* meaning broken or irregular, to describe patterns that because of their apparent limitless complexity defied concise description within the frameworks of Euclidean geometry and classical calculus (Falconer 2013, 116–20). Such patterns – both theoretical and empirical – had been described and analysed by mathematicians and researchers within other disciplines since the end of the 19th century, but were mostly regarded as curiosities and exceptions, and Mandelbrot was the first to link all these previous studies within a unified theoretical framework (Mandelbrot 1977, 2021).

In one influential paper, drawing on previous work by mathematician Lewis Fry Richardson, Mandelbrot (1967) argued that a rugged linear feature like a coastline could not be fully described through traditional geometry with a set of line segments, since this would result in a curve of infinite complexity. More importantly, he showed that the traditional measure of lines – the length – will inevitably depend on the scale of observation when applied to a coastline. If measured in kilometres, a coastline will always appear shorter in total length, than if it is measured in metres, since smaller bays and inlets can then also be accounted for. But this phenomenon continues seemingly without limit, since the same coastline measured in centimetres will appear much longer, and in millimetres far longer again, and so on (Figure 2.1). Length as a measure of rugged linear features thus seems inadequate, which may become a problem in practical settings when comparing coastlines between countries that operate with different measurement units and procedures. The same problem occurs when describing irregular patterns in the plane (like island or continent outlines) with area or in three-dimensional space (like clouds or galaxies) with volume. As a solution, Mandelbrot proposed the use of the *fractal dimension* as a descriptive tool for characterising such patterns (see Chapter 7 for definition).

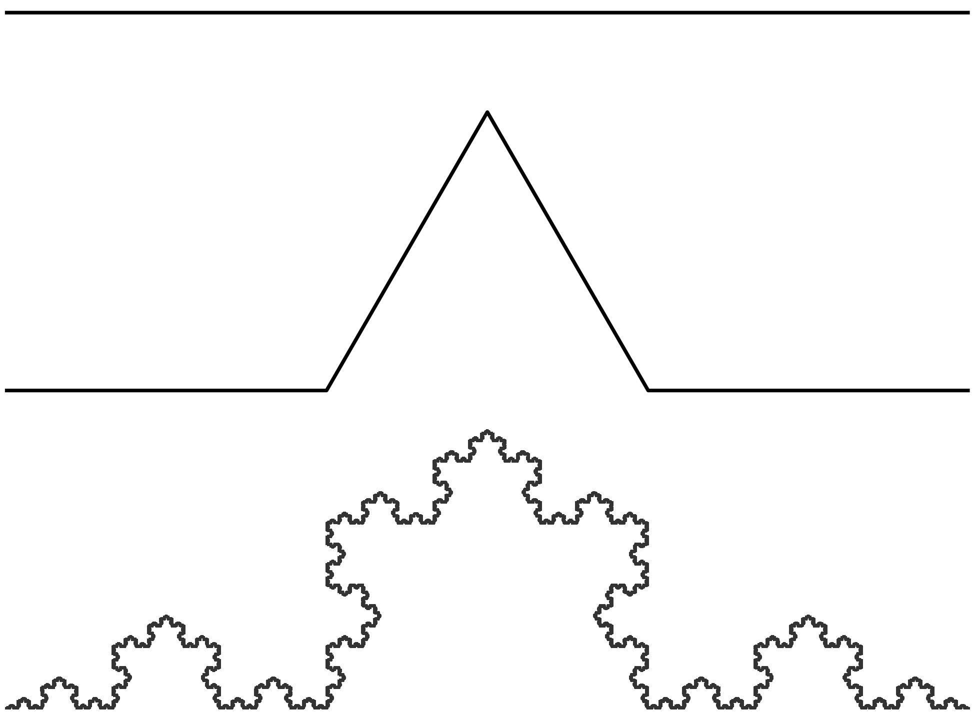

Figure 2.1: *Construction of mathematical fractals, illustrated by the von Koch curve, with its generator (top), iterator (middle) and limit set as iterations approach infinity (bottom). For each segment, the middle third is replaced by two segments forming sides of an equilateral triangle. In theory, the von Koch curve has infinite length, and is nowhere differentiable (has no tangent). All figures by author unless stated otherwise*

Mandelbrot furthermore defined fractals as patterns, shapes or structures that exhibit *self-similarity* and *scale invariance*, meaning that their overall features consist of copies of themselves in ever smaller scales. At smaller scales, these copies also become ever more numerous, so that their size distribution follow a *power law* (see Chapter 4). In this sense, they are geometrical expressions of hierarchies.

Fractal shapes are found almost everywhere in nature – plants, geological features, clouds, river systems and galaxies, inside our bodies in our nervous systems, lungs and blood vessels (Barnsley 1993) – but also in many humanly constructed features, like cityscapes, settlement systems, road and cable networks, waste deposits and so on, and can be used to model more abstract structures like demographic patterns and decisional hierarchies (Batty 2005; Diachenko 2018). Many processes and mechanisms have been shown to produce fractals, and they often include dynamical growth, emergence and chaos (these processes are further discussed in the following chapters). The jargon in fractal analysis literature often borrows terminology from biology and fluid dynamics to allude to fractals in nature, like cascading bifurcations and confluences (indicating splitting or merging), tree structure or arborescence for branching, and turbulence for chaotic and irregular movement or behaviour (Figure 2.2).

Figure 2.2: *Fractal shapes are ubiquitous in nature – here represented by branching trees*

Some common features are seen in most settings in nature and society where fractal patterns are found: the involved processes are un-planned, they emerge spontaneously from many iterations of simple rules that are followed by many smaller parts that each follow independent trajectories. These features have some important implications. While the small independent agents (e.g. water molecules) obey simple deterministic rules, outcomes are largely unpredictable. If a raindrop falls from a cloud over the Alps – at a critical point – infinitely small variations in wind direction and intensity can decide if its final destination will be in the Adriatic or the North Sea. Such systems are both deterministic *and* unpredictable – they are *chaotic*. The concept of *critical* or *tipping point* is expressed in daily speech with expressions like "the straw that breaks the camel's back". Small additions of randomness to entirely regular and mathematically defined fractals can generate highly complex shapes, effectively imitating shapes we see around us in nature (Figure 2.3)

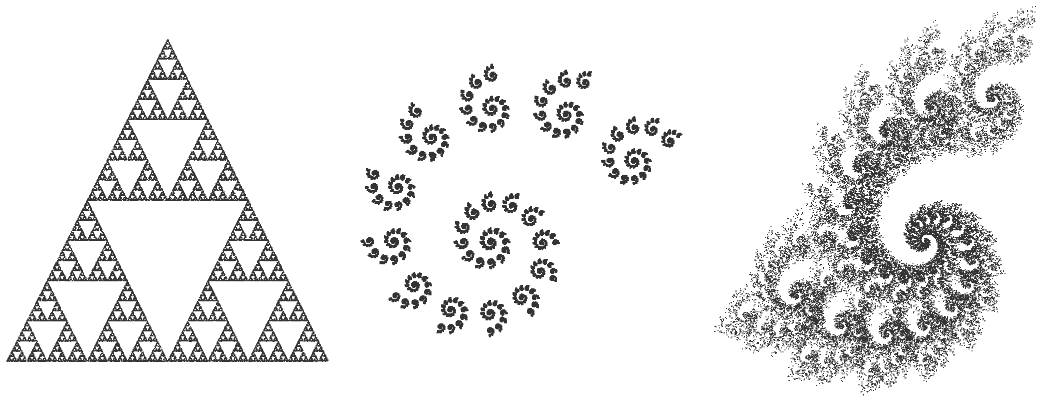

Figure 2.3: *Examples of fractal shapes: the Sierpinski triangle (left), a regularly self-similar spiral (middle), and interference between two self-similar spirals starting from different positions, generating a turbulent pattern (right). These shapes – as well as the von Koch curve above – were generated by chaos games, a type of Markov Chain Monte Carlo algorithm (Falconer 2013, 17–29), and plotted in R using* ggplot *functions (Wickham 2016). While highly complex, the code for generating them is remarkably simple (see online repository for details)*

In addition to being embedded in space, fractal structures can be seen in more abstract levels:

- Time series: earthquakes, finance (Mandelbrot 1997)
- Networks and abstract hierarchies: organisations and companies, income distributions, word counts, 1/f or pink noise, the World Wide Web (see Chapter 4 for details)
- Pure mathematics: Julia and Mandelbrot sets, strange attractors (Falconer 2013)

Fractal analysis is developed for studying irregular phenomena (methods are described in more detail Chapter 5 and (**ref?**)(images-methods)), and thus constitutes a tool box for quantitative empirical research. After a wave of apparent over-optimism to the ubiquitous nature of fractals and the possibilities offered by their analysis in the 1990s and early 2000s – as seen in publication titles like *Fractals everywhere* (Barnsley 1993) and *How Nature Works*

(Bak 2013) – it is now more widely recognised that this framework, like anything else, has its limits. Searching for these limits when methods are applied to archaeological settlement data is the overall topic of this thesis.

Chapter 3

Material and data: Neolithic settlement plans

3.1 Studying social hierarchy in archaeology and prehistory

Grave goods, monuments and hoards are frequently used proxies for social hierarchies in prehistory (e.g. Jeunesse 2017). In some cases, like the famous Varna cemetery in Bulgaria, or the Carnac mounds in Brittany, both from the mid – 5^{th} millennium BCE – the wealth associated with a few burials is so striking that it is hard to argue for anything but deep social inequalities combined with hierarchical governance. However, these proxies may also be deceiving, as it is often hard to estimate the proportion of the total population that is represented in such burial complexes, the duration of construction of large monuments, and the perceived value of the included grave goods. In many contexts, burials are almost entirely missing from the archaeological record (life for much of the Trypillia), making it hard to argue either for or against hierarchies.

For this project I opted for the use of houses and built environments as proxy for social hierarchy. In this way I hope to largely avoid the denominator problem associated with burials. While it is true that for many archaeological culture groups in late prehistory habitats are poorly preserved and hard to discover, leaving us still with a limited understanding of them (the case in many Michelsberg, Corded Ware and Bell Beaker groups, only to mention a few), in groups where habitats are well preserved, there is little reason to suspect that the available record would not cover the whole range of social statuses if these societies were hierarchical.

Unlike burials, every individual in a sedentary society – with few exceptions like homeless persons in more recent urban contexts – will normally have at least one fixed place to stay overnight, and these homes will in most cases be constructed within the same fundamental framework of techniques and building materials, depending more on culture specific traditions and environmental factors than on social status. As an example, in a society where mudbrick is the main building material, like in the Neolithic Near East and Anatolia, nearly all constructions are made in mudbrick, regardless of the social status of the inhabitants. In continental Europe north of the Alps, wattle-and-daub construction was the almost exclusive building technique for any architectural feature from the early Neolithic until the Roman conquest, and well into the Middle Ages north of the *limes*. One can of course enumerate exceptions, but more importantly houses are in any stratified society also a marker of social status, which can be exhibited in a range of ways, from decorations, use of more precious raw materials as well as size. That is precisely the reason for using houses as a proxy for social status and hierarchy in archaeological settings. But the point here is that there should be little taphonomic differentiation between groups of high and low status within a given archaeological context, at least in prehistory, and at least not as much as can be expected for burials, meaning that we can expect to find samples of houses that are representative of the social structure of the archaeological culture in question. On sites where there is taphonomical loss of architectural structures, as long as the overall building tradition is homogeneous, there is no reason this loss should affect one segment of the society more or less than others.

Some caveats do remain, however, for the use of houses as proxy of social status. Firstly, there may be a documentation bias favouring larger houses, since they may be easier to discover both during excavation and in remote-sensing surveys. In samples from originally very skewed house-size distributions, there may also be a further taphonomic bias towards large houses, since smaller houses – being far more numerous – are statistically in greater risk of being affected by post-depositional disturbances. Both of these biases are hard to evaluate empirically, though computer modelling could potentially give indications of their importance. This, however, is not within the scope of the present study.

A second, and maybe more important issue, is that of the contemporaneity of houses. When the goal is to investigate the social structure of a settlement as reflected in its architecture – be it through the size distribution of buildings or their spatial layout – all the analysed features should ideally have been in use at the same point in time. This is however very hard to achieve in most archaeological settings, and many researchers choose to either ignore the issue, or to accept a temporal resolution that is far wider than what their research questions should logically allow for (Perreault 2019). One way of limiting this problem is to select study samples with little to no stratigraphic overlap, which might indicate short occupation span, though as shown below this indicator can also be deceiving. For both study areas selected for this thesis – the Linear Pottery in south-west Slovakia and the B2/C1 Trypillia of central Ukraine – settlement plans show very little overlap between houses, even though some of them probably developed over more than three centuries, as shown by radiocarbon dates and modelling (see below). Such settlement plans may be impossible to differentiate into separate coeval time samples without precise dating of construction and abandon of every individual house, counting in the thousands on the Trypillia mega-sites (an alternative method is presented only for the Linear Pottery settlements in Section 3.2). On the other hand, the fact that there is so little overlap between houses despite temporal differences clearly illustrates how these settlement plans emerged over time, not as a *tabula rasa* in each generation but rather with new constructions respecting the location and orientation of older ones long after abandonment. Though such practices are indeed interesting, it is not at all obvious, however, to which extent they may reflect or even relate to social factors such as hierarchy. With settlement types with much higher degree of stratigraphic overlap, like the tell sites of the Balkan and Near Eastern Neolithic traditions, it may be easier to distinguish more or less coeval occupation phases, but they are again harder to document extensively – because of the high density of archaeological finds and features, excavation surfaces typically cover only very small portions of the settlement, while remote sensing performs less well and not allowing for temporal disentanglement of constructed feature. In any case, the issue of temporal resolution of the data and its influence on analytical results is crucial, and will be discussed repeatedly throughout this thesis, with a summary in Chapter 11.

In the following I propose a brief overview of the general archaeological contexts of the two cases analysed in this thesis, namely the Linear Pottery and the Trypillia cultures. Both have been extensively studied and enjoy long and rich research histories, and any attempt at a comprehensive overview here would be futile. In stead, I focus on the aspects that are the most relevant for the present study – in particular of architecture, settlement layout and social organisation – and address the interested readers to more complete reviews. In the final sections of this chapter I provide some discussion on the nature and limits of the data material used here, which consists mainly of geomagnetic imagery with expert interpretation of archaeological features, as well as synthetic data for simulations.

3.2 The Linear Pottery culture complex

The Linear Pottery culture complex – often referred to as the LBK from the German designation *Linearbandkeramik* – represents the initial transition to agriculture and permanent settlements in continental Europe north of the Alps. It is recognised through a number of defining traits, seen primarily through pottery style (the name alludes to the commonly seen decorative patterns of incised linear bands on pottery vessels), but also through architecture, subsistence economy, burial practices and various aspects of technology, showing a remarkable coherence across a very wide geographical range between northern France and western Ukraine (Bickle and Whittle 2013a; Shennan 2018). As hamlet and village-dwelling farmers using already fully domesticated plant and animal species, including wheat and barley which do not occur in wild varieties in central Europe, the Linear Pottery phenomenon stands archaeologically in stark contrast to the preceding Mesolithic foragers in the same regions. It was therefore posited early on that it was driven primarily by migration from south-east Europe via the Carpathian Basin along the Danube (e.g. Childe 1929; Whittle 2022). This assumption has later been confirmed by a number of proxies, including pottery seriations and ^{14}C dating showing that the central characteristics (pottery, burials, architecture) were formed in the Lake Balaton region/Transdanubia (Hungary) around 5600-5500 BCE (Bánffy and Höhler-Brockmann 2020), before spreading progressively to Slovakia, Lower Austria, Moravia (Czech Republic), and from there westwards through Bavaria, to the Rhine and Seine basins, as well as north to Saxony and eastwards along the northern slopes of the Tatras and Carpathians through southern Poland to western Ukraine and Moldova (Saile 2020; Bickle and Whittle 2013a). More recently, genetic evidence has shown that the populations of Linear Pottery settlements were closely related and were almost exclusively of south-east European, and ultimately to a large extent Near-Eastern/Caucasian ancestry – there was in other words very little if any genetic mixing with local forager groups (Szécsényi-Nagy et al. 2015). This also corresponds to the origin of the domesticated animals, some of which could in theory have been domesticated locally (cattle from aurochs and pig from wild boar).

More subtle evidence from Linear Pottery finds tends to link them, to varying degrees depending on the regions, to local forager populations, and there has been considerable discussions over the years regarding the exact nature of such farmer-forager contacts. These traces include the persistence of some Mesolithic bead types and associated material use throughout central Europe (Rigaud, D'Errico, and Vanhaeren 2015), regular inclusions of the very contrasting Limbourg and La Hoguette potteries in the westernmost settlements, or similarities in lithic production and technology (see Bickle and Whittle 2013a, 5–6 for a more detailed overview). However all in all the Linear Pottery culture is today seen primarily as the archaeological remains of people who spread across the fertile loess soils along river valleys in central continental Europe, driven by demic diffusion – i.e. gradual rather than abrupt spread (though rapidly in archaeological time scales, and in leapfrog steps between favourable areas), with new homesteads established a few kilometres further up or down the valley for every generation, due to population pressure (Dubouloz 2008; Shennan 2018; Bocquet-Appel 2008; Ammerman and Cavalli-Sforza 1973), possibly also involving competition for the best cultivation spots (Shennan 2008a) and social prestige in establishing new settlements (Frirdich 2005). While slash-and-burn cultivation was earlier assumed to be an explanatory factor for the Linear Pottery expansion (e.g. by Gordon Childe), more recent archaeobotanical studies have pointed to "intensive garden cultivation" as a main cultivation strategy, involving long-term field maintenance spanning several generations and considerable work investment, possibly motivating claims to land ownership and inheritance (Bogaard 2012).

More cultural variability is seen towards the end of the Linear Pottery time period, for which a number of regional sub-groups have been defined, primarily though not exclusively based on diversified pottery styles (Želiezovce in the south-east, Šarka in the north-east, Blicquy/Villeneuve-Saint-Germain in the west), though often with strong continuity in settlement patterns with the more "classic" earlier phases. In most areas the end of the last phase, dated approximately to 5050-4950 depending on regional variations, is associated with a more abrupt rupture in material culture, followed after some time by new groups with more markedly different material culture and/or settlement systems (Lengyel, Hinkelstein, Großgartach, Cerny) in some cases after an apparent depopulation in the region (Peters and Zimmermann 2017; Gronenborn et al. 2014; Riedhammer 2018). Furthermore, over the last 30 years or so, there has been a steady increase in evidence of considerable social tensions leading up to this apparent disintegration of the Linear Pottery culture. Large enclosures, possibly for defensive purpose, were constructed around villages across the culture zone, and a number of mass graves within the enclosure ditches dating from this phase have been identified. At Herxheim (near Karlsruhe in Rheinland) Boulestin et al. (2009) have reported on the remains of up to 1000 individuals of various origins (judging from the accompanying ceramics), having been violently dismembered and in many cases showing clear traces of cannibalism. At Asparn/Schletz north of Vienna, a smaller massacre deposit containing 67 individuals is interpreted as the remains of a single-event slaughter of the entire village. The age-sex ratios of the deceased are representative of expected values for a Neolithic village community, apart from the absence of young adult females, leading Teschler-Nicola (2012) to the conclusion of mass abduction by the attackers. Other sites from this phase which include human remains disposed of *en masse* are known from Germany, Hungary and France, but not all show clear signs of violence (e.g. Meyer et al. 2014). Yet an example is the large settlement of Vráble in Slovakia, presented below, of which the human remains are currently under study (Nils Müller-Scheeßel et al. 2021; Furholt, Müller-Scheeßel, et al. 2020). Preliminary results point to a diversity in treatment of the deceased, and again clear traces of direct peri-mortal violence are largely lacking, adding nuance to earlier claims of societal crisis and warfare leading to the end of the Linear Pottery culture (e.g. Farruggia 2002). Nevertheless, the concentration of these events – however diverse – towards the end of the period, combined with their wide geographical distribution, has reanimated earlier debates regarding the internal dynamics of Linear Pottery society, and the possibility that increased social inequality lead to the observed social instability and its ultimate decline (Augereau 2021; Jeunesse 2022).

3.2.1 Sampled Linear Pottery settlements: the Žitava valley

The nine Linear Pottery settlements sampled for analysis in this thesis are located within the small river valley of the Žitava in south-west Slovakia (Figures 3.2 and (**ref?**)(fig:03-plans)). They have been extensively documented through an ongoing collaborative research project run by German and Slovak archaeologists (Furholt, Müller, et al. 2020). One already published result is the collection of high-quality geomagnetic images of the complete or near-complete settlement plans (Nils Müller-Scheeßel et al. 2020). The Vráble settlement is by far the largest, with a total 313 houses grouped in three major clusters (labelled neighbourhoods by the research team), and is dated to a timespan from about 5250 to 5000 cal.

BCE through Bayesian modelling of available ^{14}C dates, corresponding well with the pottery finds attributed primarily to the Želiezovce group (Meadows et al. 2019). The remaining settlements, which are all sensibly smaller, are currently under study in the ongoing project. However, they have been modelled to having been occupied within the same time range, with Vráble growing to a dominant position within the valley only after the first 100 years, possibly as a result of a micro-regional agglomeration process (Nils Müller-Scheeßel et al. 2020). Among the published results, is the finding of a double enclosure ditch surrounding the southwest neighbourhood of Vráble (not included in Figure 3.1), with at least five entrance passages facing away from the two other neighbourhoods of the site. The ditch system has been dated to around 5050 BCE, towards the last phase of occupation, and in the infill deposits a large quantity of human remains have been uncovered, showing complex patterns of post-mortem treatments (Furholt, Müller-Scheeßel, et al. 2020; Nils Müller-Scheeßel et al. 2021).

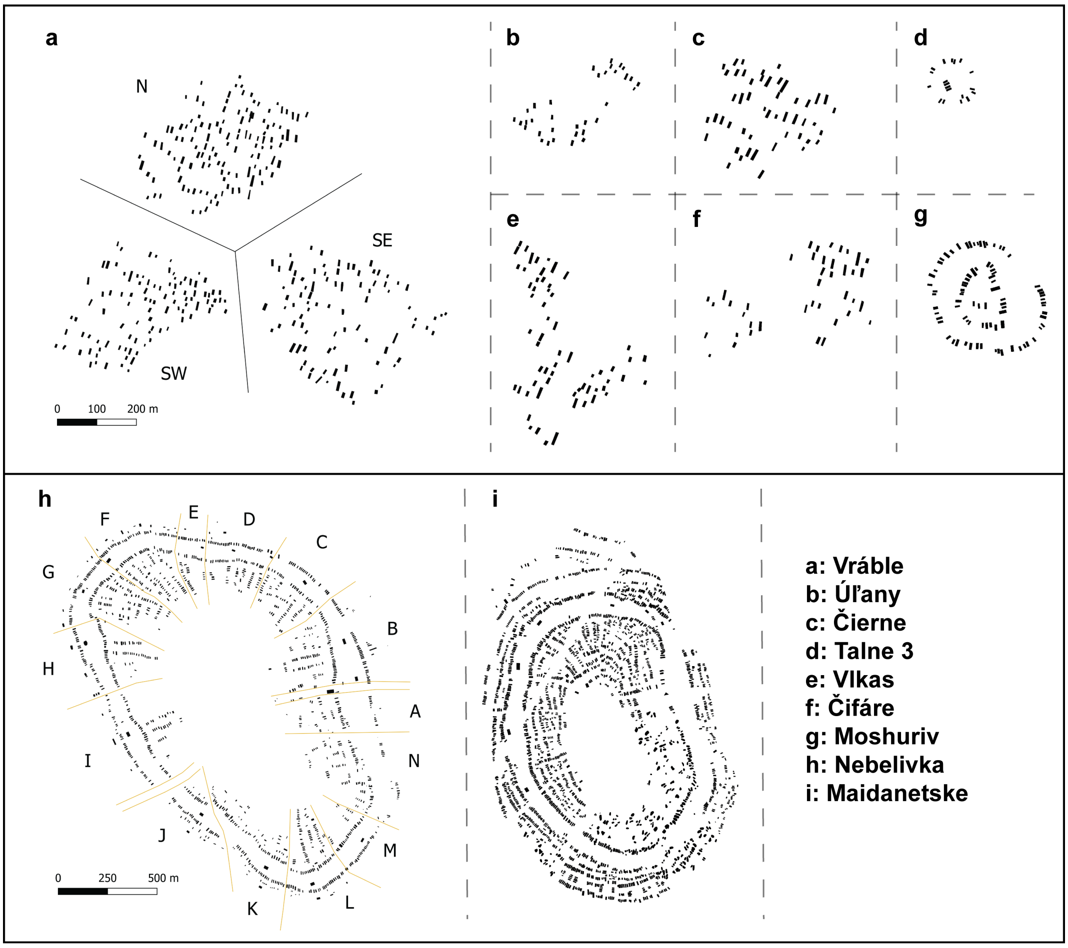

Figure 3.1: *Settlement plans sampled for this study. Circular layouts represent Trypillia settlements, while grid layouts represent Linear Pottery settlements. Adapted from Nils Müller-Scheeßel et al. (2020) (a-c, e, f), Ohlrau (2020) (d, g, i) and Hale (2020) (h)*

In the following, central aspects of Linear Pottery society as seen trough architecture and burials is discussed, in order to provide some background for interpreting the analytical results given in the following chapters.

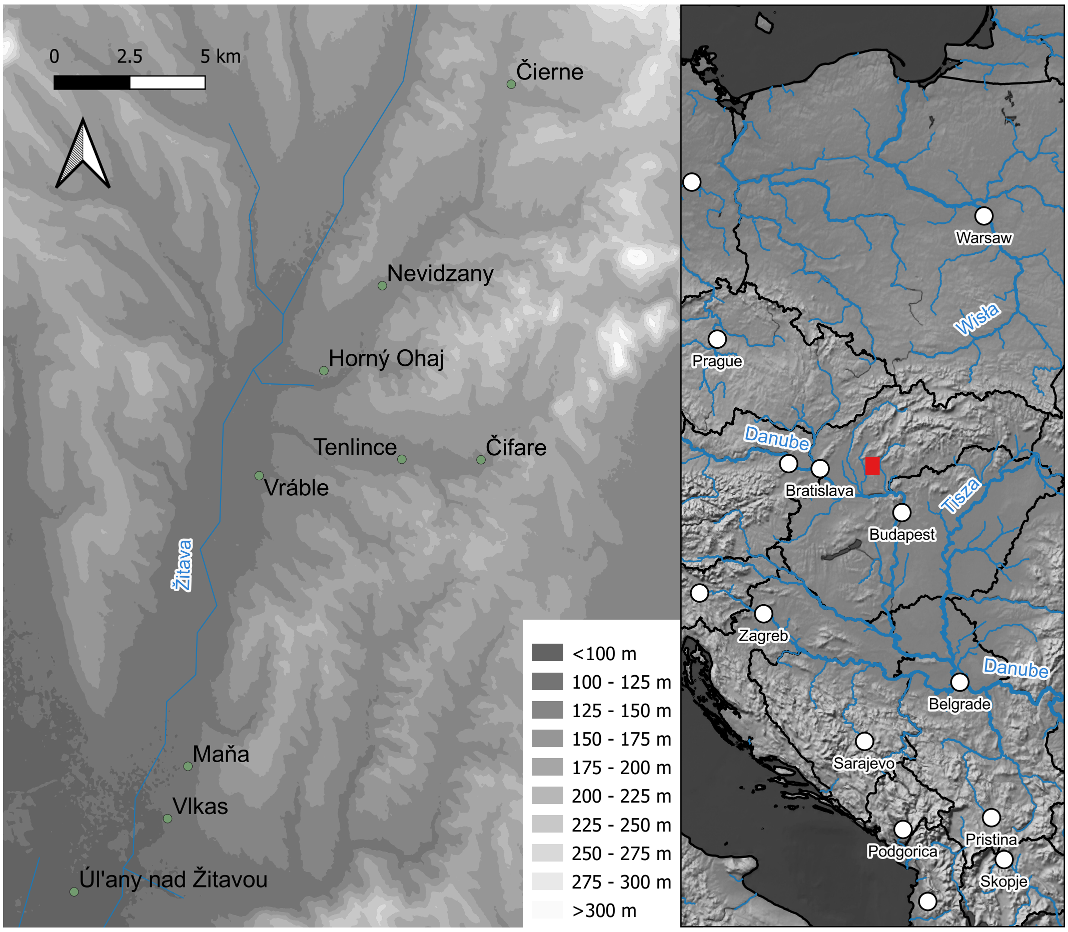

Figure 3.2: *Location of the Linear Pottery settlements analysed in this thesis, in the river valley of the Žitava, a small left tributary (via the Nitra and Váh rivers) of the Danube in south-west Slovakia. The large Linear Pottery burial site at Nitra is located about 22 km to the west of Vráble. Terrain model data in public domain, courtesy of the U.S Geological Survey (https://earthexplorer.usgs.gov)*

3.2.2 Linear Pottery houses and households: current perspectives

One of the most characteristic aspects of Linear Pottery material culture next to its ceramics, is the house architecture (Coudart 2015, 1998; Last 2015). Since these Neolithic settlements were founded following largely the extent of fertile loess soils in what are now areas of either intensive agriculture or dense populations, original ground levels are not preserved, and what is known about Linear Pottery architecture – indeed about the settlements alltogether – comes from below-ground features like post holes and pits. Linear Pottery houses are recognised by sequences of groups of three post holes, varying in numbers and distances between groups. Through numerous experimental reconstructions, these features have been shown to map the roof-bearing posts of longhouses, with walls being made in lighter wattle-and-daub construction. House outlines are furthermore typically demarcated by large lateral extraction pits for daub – in other words, houses were constructed with material available on the spot, yet a reason to select locations with silty loess soil fit for the purpose. Once the house was constructed, the lateral pits were often used for domestic waste disposal, and these features usually provide the richest material for interpreting household organisation (e.g. Hamon and Gomart 2021; Gomart et al. 2015; Hachem and Hamon 2014).

A typology of Linear Pottery houses, based on the spatial organisation of post holes and occasional wall ditches, was first proposed by Modderman (1970, 3:100–120) based on excavations at the Dutch settlement sites of Elsloo and Stein as well as other plans that were available at the time, such as the large settlement at Bylany (Czech Republic), and it still holds as the basis for understanding their internal layouts. Focussing on the presence and number of narrow passages (two rows of three post holes), he defined three main types of houses, composed of either 3 (*Großbauten*), 2 (*Bauten*) or 1 (*Kleinbauten*) consecutive sections in systematic order. The middle-section is common to all three types, in the *Bauten* type an additional section is added in (presumably) the back, and in the *Großbauten* type a third section is added to the front (he labelled these types 1, 2 and 3 from largest to smallest. An additional variable was defined for the presence and extent of a wall ditch surrounding the house: *a* for entirely circumscribed houses, *b* for a ditch surrounding the back section only (so by definition not possible for *Kleinbauten*), and *c* for no surrounding ditch. In Modderman's corpus types *1b* and and *2b* – that is, large and medium sized houses with a ditch surrounding the back section – were seen as the most common. The front section in type 1 houses (*Großbauten*) usually have double posts, and are then interpreted as holding raised platforms for granaries (see also Schiesberg 2010). Though longhouse orientation varies from region to region, and at a different scale within single settlements (further discussed below), the strong tendency for Linear Pottery houses in the Limburg and lower Rhine area is a south-east to north-west orientation, with the front section and gable wall – and presumably the main entrance – oriented south-east.

A larger quantitative study was later added by Coudart (1998), on a much larger sample spanning almost the whole Linear Pottery culture area (not covering the easternmost regions), which largely confirmed the overall trends presented by Modderman, while also adding more nuance regarding regional and temporal variations. Notably, she showed that there was a clear correlation between house type and size, but that the single most important variable affecting overall house size was the length of the back section (Coudart 1998, 37–51). Expanding on this, and focussing specifically on the perhaps most amply documented sub-set of Linear Pottery houses and settlements – the Aldenhoven plateau between Cologne and Aachen – Schiesberg (2010) added that the presence and extent of a wall ditch (*Wandgrab*, Modderman's *a* and *b* criteria) was the most decisive variable for house size, at least within that region. She noted that type 1 houses (*Großbauten*) without ditch (i.e. 1c) could be as small as type 3 houses (*Kleinbauten*). However, admitting that the largest houses are almost always of Modderman's type 1a (*Großbau* with entirely surrounding wall ditch), she argued against earlier interpretations of this type according it special social functions, e.g. analogous to Melanesian Men's houses (Milisauskas 1972) or a chief's residence (Velde 1990). The argument presented by Velde (1990) – sparking a debate on the possibility of Linear Pottery social hierarchies which is still ongoing (e.g. Jeunesse 2022; Augereau 2021; *contra* Coudart 2015) – rested on the observation that there was only one type 1a house at a settlement at any given time, as well as a small number of type 1b houses. Furthermore, larger houses had been shown – Velde (1990) argued – to yield more presumably prestigious items in the associated longitudinal pits, like polished stone adzes. Using simulations, Schiesberg (2016, 2010) countered that these higher frequencies of prestige finds were simply quantitative rather than qualitative, and that they did not exceed what would be expected from larger houses. To her, Linear Pottery house sizes were simple reflections of family size that would vary randomly and tend to be lightly skewed as a result of post-marital residence patterns (more discussed below).

A recurring cycle in Linear Pottery research is the recognition of trends seen in a regional sample of material, attempts at generalising them to the entire area and time span of the culture arguing from the clear overall homogeneity in material culture, and subsequent critiques from others pointing out that other regions show somewhat different characteristics (Bickle and Whittle 2013a). This is also the case for research on architecture and social organisation. Settlements like Cuiry-lès-Chaudardes at the westernmost part of the Linear Pottery area in northern France, which date to later phases of the culture, typically lack three-sectioned *Großbauten* houses, so that the number of post rows in the back section of type 2 houses become the decisive factor for house size (Hachem 2000). Comparing the largest houses from several regions, D. Hofmann and Lenneis (2017) found that there was no single function connected to these across the whole area, but at the same time pointed to a trend of economic differentiation between the largest and smaller houses. In conclusion they kept open both possibilities of special communal functions as well as special social status of the inhabitants, but admitting that these interpretations remain hard to substantiate. Again based on evidence from Cuiry-lès-Chaudardes and detailed analysis of the distribution of various finds categories across the site, Gomart et al. (2015) showed that house size was correlated to the level of social integration of the dwellers in the village community (see also Hamon and Gomart 2021; Hachem and Hamon 2014). More specifically, they showed that small houses tended to represent newly established small households (either scissioned from larger local ones or arriving from other settlements), probably consisting of a nuclear family, which were economically dependent on other households to meet their needs of subsistence. Larger houses on the other hand, were argued to represent households which had been established at the settlement for the longest time, and thus having grown in number of individuals (extended families) and strengthening their productive capacities. As such, large houses represented more independent supplier households, while small houses reflected more dependent demander households. Furthermore, while Schiesberg (2016, 2010) has argued for a clean slate mechanism where new houses would be built routinely for each new generation, and where house size would be a linear reflection of number of sons in the (patrilocal) household, Hamon and Gomart (2021) argue that house size would evolve with the household lineage over multiple generations, thus reflecting the time since the household was first established at the village. In their view, the level of prestige and decisional power of the various households would be unstable over time, and also be reset to the lowest level when the household size reached some upper limit leading to fission (which is inherent in their model). While apparently contradicting, these two interpretations may not be mutually exclusive, but they may also simply reflect social practices that were different in different regions within the Linear Pottery area. While Schiesberg's model does not directly preclude interpretations of social inequality, a disadvantage of it is that it does not account for population growth and demic expansion, which is now largely accepted as the main driver of the spread of the Linear Pottery culture (see above). Hamon and Gomart (2021) and Whittle and Bickle (2013) on the other hand, explicitly refer to Lévi-Strauss' concept of "house societies" (1982b), suggesting that the social life in Linear Pottery villages could be marked by competition of status between different more-or-less rigidly defined lineages over time.

Much of the above discussions regarding social structure and architecture rely on some underlying assumptions as to the coeval existence of houses. This, however, has repeatedly been shown to be very difficult to establish in many cases (Bickle and Whittle 2013a; Meadows et al. 2019). Temporal dynamics of Linear Pottery settlements have traditionally been evaluated through very detailed analyses and seriation of pottery styles, but more recent studies involving also pottery manufacture techniques have shown that stylistic traits do not necessarily bear temporal significance only, but may also reflect the spatial movements of the potters during their lifetimes Shatilo (2021). The interpreted number of coeval houses in a village has potentially great influence on both population estimates and the associated social system (further discussed in Chapter 4). In her large-scope study on Linear Pottery houses and settlement plans, Coudart (1998) proposed relatively low house count estimates, with a mean of five coeval houses (standard deviation of 3), but high numbers of inhabitants (between 4 and 7 m^2 per inhabitant depending on total house size). By modelling of a number of ethnographic analogies she furthermore suggested typical population numbers to around 150-260 coeval inhabitants, though fewer (up to around 80 people) for the smallest hamlets of 3 houses (1998, 91). Coudart's population model is somewhat problematic however – to find population values representative of the whole range of Linear Pottery house sizes (from about 40 to 230 m^2) she combined mean values of a range of comparable documented societies (1998, 78–81). Large Linear Pottery houses were thus compared to the mean inhabitants per m^2 in Tlingit houses (British Columbia) while small Linear Pottery houses were compared to mean values of Iatmul (New Guinea) and Jivaro (Peru) houses. Regarding residence pattern and house size, Schiesberg pointed out that Linear Pottery houses are in the higher size range, which has earlier constituted an argument for matrilocality, but at the same time not being strictly above the range documented for patrilocal societies (Schiesberg 2016, 2010; Murdock 1949). She argued that the Iroquois longhouses (which have been one of the favourite ethnographic parallels to German scholars, e.g. Rück 2007), were significantly larger, and that their internal layouts reflected the various matri-clan segments in ways that are not easily transferred to Linear Pottery house plans.

Largely as a result of the difficulty of establishing exact construction dates and duration of Linear Pottery houses from ^{14}C dates and pottery seriations, several competing models of the spatial organisation and layout principles of settlements have been proposed. The most widely cited and accepted model is the so-called “yard model” (*Hofplatzmodell*), developed from observations made at the extensively documented Langweiler 8 settlement on the Aldenhoven plateau in the 1970s and 80s (Zimmermann 2012a; Petrasch 2012). It stipulates that new houses were largely constructed in the close vicinity of the previous house of the household lineage, over time forming (archaeological) clusters of primarily non-coeval houses on the cumulative settlement plans. It follows that households then had some level of claim or at least connection to the land slot (the yard) they were using. Estimating the average duration of a house to approximately 25 years, i.e. one average human generation, the seriation of houses at Langweiler 8 was divided into 13-15 house generations, thus covering the settlement span of some 300 years given by ^{14}C dates. Houses from within the same house generation were considered coeval, thus opening the possibility of constructing coeval settlement plans. A competing model – the “row settlement model” (*Zeilensiedlungsmodell*) – was proposed by Rück Rück (2007), who pointed to the frequently observed tendency of houses being constructed in loosely defined rows across settlements, thus adhering more to overall settlement planning and possibly forming street-like features on coeval plans. In order to defend this interpretation however, houses needed to be allowed a much longer duration, up to about 80-100 years, an assumption which is not well supported by radiocarbon nor typological evidence (Petrasch 2012). Since both models have good arguments for and against, neither seems entirely satisfactory, and some have argued for combinations of the two (Link 2012) or advocated consideration of (again) regional variations (Lenneis 2012). In any case, the general scarcity of stratigraphic overlaps of houses from different phases in Linear Pottery settlements indicates that the inhabitants remembered previous houses long after they were abandoned, and took them into consideration whenever new houses were constructed. In this thesis, a more recent technique for evaluating the contemporaneity of Linear Pottery houses, proposed by Nils Müller-Scheeßel et al. (2020) and based on house orientation, is tentatively applied in order to construct coeval settlement plans and investigate temporal developments (detailed at the end of this chapter).

Regarding intra-regional settlement structure, it has been recognised for some time that there seems to be some sort of hierarchy within areas like small river valleys, again first argued from observations in the Aldenhoven/Merzbachtal excavations in the 1970s and 80s, where Langweiler 8 was argued to represent a Central Place (*sensu* Christaller 1966) in a regional network of smaller hamlets, orienting exchange flows notably in flint material and tools (Claßen 2005).

While the exact functional relations between small and larger sites within a region have proven hard to establish beyond doubt, similar settlement systems have since been reported in Lower Bavaria (Pechtl 2012), Lower Austria (Pieler 2012) and the Aisne valley in northern France (Dubouloz 2012). The above presented sample from the Žitava valley in Slovakia seems to be yet an example. The gradual emergence of such patterns seems to be well predicted and explained by the demic diffusion models described by Shennan (2008a) and Dubouloz (2008) – the earliest settlements in such micro-regions do seem to be the ones that become the largest over time, and the lineage of the earliest settlers would possibly have certain advantages from retaining the best spots for cultivations (Hamon and Gomart 2021). However, while any interpretation of these systems in terms of regional polities or complex chiefdoms would seem unfounded, given the lack of any sign of administration, public buildings etc. for this period, other plausible interpretations – including seasonal occupation and economic specialisation – do not seem to have been fully explored as of yet. Also, to my knowledge no serious attempts of settlement scaling have been done on Linear Pottery settlement systems to quantify formally the relationships between smaller and larger sites (in the way proposed by Lobo et al. 2020).

3.2.3 Burial practices and social organisation

A last note on social organisation must be made concerning the rich material known from Linear Pottery cemeteries. In recent years much evidence has come from analyses of burials, based on isotope and aDNA as well as grave goods inventories, applied on large samples (Augereau 2021; Bickle and Whittle 2013b). Linear Pottery burials are typically individual, with the deceased lying on their side in a crouched position, and in about 60% of cases accompanied with artefacts, primarily pottery, ochre, animal bones and beads, but occasionally also tools including flint blades, arrowheads, grindstones and polished adzes. In a large study on a sample of several hundred graves for which the biological sex of the deceased could be reliably determined, clear gender patterns emerged in the distribution of artefact types as well as work-related pathologies, e.g. linking archery to male activities and cereal grinding to female activities (Augereau 2021). Analysis of aDNA (Szécsényi-Nagy et al. 2015; Rasteiro and Chikhi 2013; Rasteiro et al. 2012), as well as strontium isotope analyses (Bentley et al. 2012; Whittle and Bickle 2013), has shown regular gendered differences in mobility, with

Linear Pottery women generally being more likely to move far from their birth places during their lifetimes, and the relatively few men buried with polished stone adzes usually being born in the same area where they were buried. These results have to some degree concluded the long-standing debates on residence patterns in favour of patrilocality (see however Brück 2021; Ensor 2021a; Hrnčíř, Vondrovský, and Květina 2020; Ensor, Irish, and Keegan 2017). Any unique patrilinear decent rule has for now been more difficult to demonstrate. They have also generated a renewed interest in the socio-symbolic role of polished adzes in Linear Pottery society. For decades, the cemetery at Nitra (Slovakia) was considered the archetype of Linear Pottery burial grounds. The principal excavator, Pavúk (1972) argued that the nine graves including adzes (*Schuhleistenkeile*), of a total of 74, did not represent persons with any particular status, but that the adzes were simply utilitarian woodworking tools and that the cemetery reflected an essentially egalitarian society. Jeunesse (1996) pointed out that almost all these graves contained older males, but also some infants, concluding that they represented clan leaders with inherited status. The comparatively local strontium isotope signatures of these men-with-adzes constitute an additional argument for them having inherited a high status from early settlers at the site, and it is generally believed that only a selected segment of Linear Pottery society received any (archaeologically visible) burial at all (e.g. Hamon and Gomart 2021; Whittle and Bickle 2013).

The picture which has been emerging in recent years, when considering together multiple subtle strands of evidence – possible hierarchies in house types and sizes (Velde 1990; D. Hofmann and Lenneis 2017), inheritance of land ownership indicated by cultivation techniques (Bogaard 2012) and social status by rich child graves (Jeunesse 1996), economic dominance of the most long-lasting household lineages within villages (Hamon and Gomart 2021; Gomart et al. 2015; Shennan 2008a), as well as higher social status of older local men (Bentley et al. 2012; Jeunesse 2017), patrilocality (Whittle and Bickle 2013) and possible mass abductions of young women (Teschler-Nicola 2012) – is one of a perhaps not so peaceful nor egalitarian society (e.g. Coudart 2015, 1998), and much less matriarchal (Gimbutas 1991). Augereau (2021, 221) sees the Nitra cemetery as an early sign of emerging patriarchy, and Jeunesse (2017) reads the late Linear Pottery as the beginnings of the European warrior aristocracies that are usually associated with much later periods. In any case, it is safe to say the question of social structure and hierarchy in Linear Pottery research is unstable, and interpretations have varied widely over the recent decades, which makes any impetus from new methods to the available data ever more welcome.

3.3 The Cucuteni-Trypillia culture complex

The Cucuteni-Trypillia culture complex (CTCC) is also a large and well-defined archaeological culture, geographically somewhat less extensive than the Linear Pottery, but found over a longer chronological range of almost 2000 years from the early 5th millennium to around 3000 BCE, a period referred to as the Eneolithic or Chalcolithic/Copper Age within its regional context (Harper et al. 2021, 2023). Covering a large area within modern Ukraine, Moldova and Romania from the foothills of the Carpathians in the west to the middle Dnieper in the east – though inland from the coast of the Black Sea – it was independently identified in the late 19th centrury at the eponymous sites of Trypillia on the right bank of the Dieper (Kyiv Oblast in Ukraine, then within the Russian Empire) and in Romania at Cucuteni (Iaşi County near the border to Moldova). Over much of the 20th century it has been debated whether they should be considered as two separate cultures, and today they are generally treated as one, or at least as closely related, as seen through their highly similar pottery traditions (Shatilo 2021, 13 ff.). One significant trait that does distinguish Cucuteni and Trypillia settlements, is a tendency of the former to form irregular grid layouts, while the latter form radial layouts of concentric circles. The CTCC settlements analysed in this study belong to the Trypillia group, and all show this characteristic settlement layout (further descriptions below). Trypillia sites are today most well known both among archaeologists and the wider public for their so-called "mega-sites" reaching up to about 3000 houses (Maidanetske) and covering areas up to 340 ha (Taljanki), settlement sizes that are otherwise unheard of within the European Neolithic, and have spawned a vivid debate as to whether they should be considered the earliest cities (more discussed below). Note that the spelling of Trypillia is transliterated to English from the Ukrainian toponyme, while it often – especially in older literature – is transliterated from the Russian equivalent Tripolje.

The near absence of stratified settlements within the Trypillia culture and the low availability of radiocarbon dating for Soviet and post-Soviet archaeologists, has lead to the elaboration of a highly complex sequence of relative chronology based on pottery typology. The general periodisation of Trypillia pottery and material culture – with periods A, B, C and sub-divisions – was originally made by Tatyana Passek in the 1930s and 40s, and still forms the backbone of the culture's chronology (Passek published only in Russian, but detailed English summaries with references are now available, e.g. Shatilo 2021, 18–26; Ryzhov 2012). Besides the temporal sequence, Trypillia material culture has subsequently been further divided into spatial sub-groups, notably to the main groups of the so-called Western and Eastern Trypillia cultures. The former (abbreviated to WTC in specialised literature) being primarily characterised by the prevalence of polychrome painted pottery (similar to Cucuteni pottery) while the Eastern group (ETC) is characterised by pottery with primarily incised and fluted decorations, building on traditions from the initial Precucuteni/Trypillia A phase. There is also a general long-term trend of migration and pioneer settlement from the south-west towards the north-east, and the ETC – representing the earliest settled farmers in central Ukraine – first appear after the initial stages of WTC and Cucuteni development (Harper et al. 2023, 646–47). Combining temporal and spatial dynamics of pottery styles, a number of geographically more-or-less stable local traditions have been proposed, termed "local groups" or "lines of development" for both the WTC and the ETC (Ryzhov 2012; Diachenko and Menotti 2012; Diachenko 2012). These are generally thought to represent somewhat coherent population groups, changing the location of their settlements at a relatively rapid rate, often in small steps but from time to time in larger leaps, leading to considerable geographical overlaps and mixing between different traditions (e.g. the WTC settlements presented below are actually further *east* than most of the ETC settlements).

It is clear that for much of its research history and until today, this richly detailed partitioning of the Trypillia into small local groups has gone hand-in-hand with a theoretical framework which has been largely rejected by anglophone archaeologists for a long time, namely the equation of pottery styles with ethnic or tribal groups (Shatilo 2021, 44–49). While this academic tradition is still strong in Ukraine, as in much of south-east Europe more generally, it is increasingly being nuanced with more critical archaeological theory, e.g. by taking into account the numerous transition zones between groups of pottery styles, and acknowledging that other sources of material culture do not follow the pottery periodisation in a systematic and homogeneous way. Also, the substantial efforts made over the past decade or so of combining the existing Trypillia typo-chronology with modern AMS radiocarbon dates is currently leading to a more precise mapping of the spatio-temporal dynamics of specific pottery traits (Harper et al. 2023, 2021; Rassamakin 2012).

Trypillia settlements in Ukraine and Moldova are to a large extent located within the forest-steppe eco-region – a transition zone between the temperate forests further north and west, and the dry Pontic-Caspian steppe to the south and east. The forest steppe has a humid to semi-arid continental climate, and sustains naturally an open mixed oak forest. Recent archaeobotanical and pedological studies have shown a temporal coincidence – at least in the mega-site region – between Trypillia occupation and the transition from more dense forest to open forest steppe, and Kirleis and Dreibrodt (2016) suggest the initial formation of this "cultural steppe" with its associated Chernozem soil might result from deforestation induced by Trypillia settlers, which could also explain some of the frequently observed instability in settlement location over the period.

Despite the very long period of development of the Trypillia culture, subsistence economy is thought to have remained relatively stable. Animal husbandry was based mainly on cattle, pig and sheep/goat livestock, with some variation in proportions throughout the phases (Kirleis and Dal Corso 2016). Domestic species were supplemented with hunted game, with species depending on the local environment – red deer and boar in woodland dominated areas, and occasionally horse nearer the steppe region. There is no evidence of horse domestication from Trypillia contexts. Hunting was most prevalent in the early phase (Trypillia A, up to almost 50% of faunal remains in some cases), and decreased gradually over time as husbandry became more prevalent. Plant cultivation included varieties of wheat and barley – with the introduction of millet from Trypillia C – as well as pulses (pea and lentil, bitter vetch in Trypillia A). Wild plants like hazelnut and various fruits were also consumed, some of which – like apricot – were of exotic origin and might have been cultivated.

It should also be noted that both of the eponymous sites are located toward the extremities of the area defined by CTCC sites, and though there are certain geographical biases in the documented material, it seems clear that the core area of settlement and population density for much of the culture's duration was in and near the river valleys of the Dniester and Southern Bug (mainly in present-day Moldova and south-western Ukraine). The Trypillia settlements included in this study are attributed to the B-II and C-I phases of the WTC and are located within a small micro-region in the Southern Bug-Dnieper interfluve, around the limit between the Cherkassy and Kirovograd oblasts, some 30-40 km east of Uman (Figure 3.3). Based mainly on pottery style, one settlement – Nebelivka – is associated with the eponymous Nebelevskaya local group (phase B-II), while the three other settlements are associated with the related but slightly later Tomashovka group (phase C-I. See Shatilo 2021, 41–44 for a condensed overview of WTC local groups; Ryzhov 2012; Diachenko 2012). This small area has been the focal point of intense research over the last 15 years, and has – among many other remarkable results – led to the publication of complete or near-complete settlement plans, identified through geomagnetic imagery with unprecedented quality (Rassmann et al. 2014, 2016).

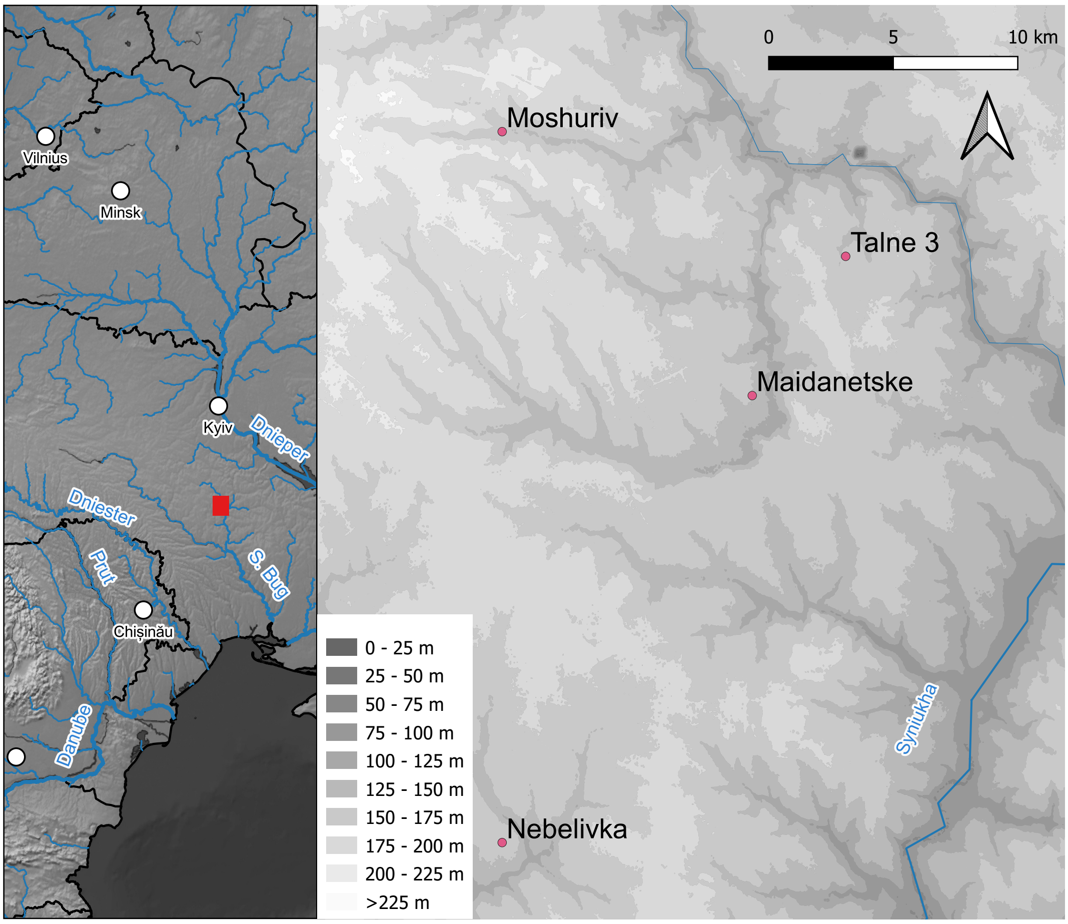

Figure 3.3: *Location of the Trypillia settlements analysed in this thesis, within the drainage basin of the Syniukha river, a small left tributary of the Southern Bug in central Ukraine. Terrain model data in public domain, courtesy of the U.S Geological Survey (https://earthexplorer.usgs.gov)*

3.3.1 Sampled Trypillia settlements: The Syniukha basin

The four Trypillia settlements that were sampled and analysed in this thesis had been known by Ukrainian and Sovjet archaeologists for some time, having been subject to documentation through aerial photography and small-scale excavations mainly in the second half of the last century (see Ohlrau 2020, 36–60, 241–44; Kruts 2012), but their settlement plans were first extensively documented through geomagnetic imagery from 2009 and onwards, as part of two collaborative research projects, one German-Ukrainian (Maidanetske, Moshuriv and Talne 3, Ohlrau 2020, 61–65; Rassmann et al. 2014) and one British-Ukrainian (Nebelivka, Hale 2020; J. Chapman, Gaydarska, et al. 2014). This sample is admittedly very small, and other settlements from the same period have been documented in the area (Taljanki, Apolianka, Dobrovody and others), but with larger gaps in their settlement plans due to lacking access for the surveyors or less optimal site preservation. These four settlements are furthermore selected for their extreme variation in size – from 3.1 ha at Talne 3 to 235 ha at Nebelivka – despite being dated within a relatively short time period. The Trypillia B-II/C-I phases are currently broadly estimated to the range 4100-3500 BCE, and more precisely to 3950-3750 for the Nebelevskaya local group (Nebelivka) and 3850-3600 for the Tomashovskaya local group (Maidanetske, Moshuriv and Talne 3, Harper et al. 2021).

All four settlements are spatially organised according to the circular-radial layout characteristic to Trypillia settlements (see settlement plans on Figure 3.1). The two very large settlements – Nebelivka and Maidanetske – show some additional traits that have come to be known as characteristic for the so-called "mega-sites" (Hale 2020; Ohlrau 2020), namely:

- Two or more concentric house circles separated by some space, creating a circular "main street"
- A large open space in the centre of the settlement, entirely void of constructed features
- A rectangular main plaza containing an exceptionally large building toward the east end of the settlement, inside the inner house circle (only a small portion of this building is preserved at Maidanetske, but its original presence is assumed by analogy to the other mega-sites)
- Secondary large buildings located at highly visible spots at regular intervals within the main street (these, as well as the largest building, are generally termed "mega-structures" by the German team and "assembly houses" by the British team, see below)
- Regular domestic houses constructed both on the outside and the inside of the main street, to varying degrees forming secondary streets (especially in Nebelivka) and plazas (Maidanetske)
- Varying degrees of clustering of houses into quarters and neighbourhoods (Nebelivka terminology), presumably also reflected by the spatial distribution of assembly houses/mega-structures

- Exceptionally large settlement size (Ohlrau 2020, 27 proposes a threshold value of ca. 30 ha for all Trypillia phases)

- Location within the forest-steppe, at the border to the steppe

The smallest settlement analysed here, Talne 3, consists of only a single circle, while the Moshuriv settlement has two. Both of them also have a single row of houses in the centre, contrarily to the mega-sites, wile Moshuriv also has its largest house placed in the eastern part of its inner house circle, similarly to the mega-sites. There is still much uncertainty regarding the internal temporal development of Trypillia settlements, as well as to their total durations (more discussion on this below). Based on the layout of smaller settlements like Talne 3 and Moshuriv – seemingly growing outwards like year-rings in a tree – it is hypothesised that this also was the case for mega-sites, i.e. that the inner circle would be established first, and the rest of the settlement plan emerging over some time (Ohlrau 2022; Müller et al. 2016). In any case, it must be noted that mega-sites constitute an exception rather than the rule for Trypillia settlements, as they are only found within a very small region and over a relatively short range of the whole CTCC.

The further brief overview of house construction, architecture and social organisation is focussed mainly on the settlements within the Syniukha basin, though many of the traits are widely shared within larger parts of the Trypillia area.

3.3.2 Trypillia house construction, architecture and social organisation

As with Linear Pottery houses, Trypillia domestic architecture has been amply documented through more than a century of numerous excavations. However, there are also three additional factors that have contributed to an even more detailed understanding of these latter houses:

- The very common practice in Trypillia settlements of burning houses entirely at the end of their use life

- Better post-depositional preservation than most comparable West-European soils, due to less intensive use of modern agro-industrial machinery, leaving architectural remains including living floors most of the time *in situ*, and

- Numerous finds of detailed miniature clay models of Trypillia houses

Combined, these favourable conditions allow for a relatively precise archaeological reconstruction of how people built their houses and lived in them within the Trypillia culture (see Chernovol 2012 for a detailed overview in English; A. G. Korvin-Piotrovskiy, Chabanyuk, and Shatilo 2012). On the other hand, Trypillia burials are very rare, and none have been found within the Syniukha basin, which has left large blind spots in the documentation regarding gender, kinship and family structures, in stark contrast with the situation in Linear Pottery research. It is generally assumed that one domestic house was used primarily by a single family unit, because of its layout and interior organisation (see below).

The main feature of house remains – which is usually very visible on geomagnetic images – is a compact clay floor of about 5 to 12 cm thickness interpreted as the main living space of the inhabitants. Imprints of wooden beams are regularly found on the downward side of floor fragments, and it is widely accepted that these floors were constructed on a raised platform of some height, leaving an open space below possibly for storage or even housing for livestock. This configuration is also regularly seen on clay house models, which also hint at the use of arched ceilings and gables. Walls were made mainly of wattle-and-daub, also seen in wattle imprints on fired daub fragments, but there is also occasional evidence of walls made from horizontal wooden beams – possibly this technique was applied for wall foundations (Chernovol 2012, 183). The main living floor on the raised platform was presumably reached by a ladder from the outside, since openings are not found on the platform floors.

The interior organisation of rooms and installations was very homogeneous across the Trypillia complex, to the degree that Chernovol speculates that there were taboos associated with deviating from the strict norm (2012, 200). For settlements in the Tomashovka local group (see above), the vast majority of houses were rectangular and about 3 to 4 times longer than wide, with the upper level composed of an entrance porch on one end, leading through a doorway to the main room. There, a clay oven was located immediately to the right of the entrance, while a trough and long podium followed the left side wall. By the short end opposite the entrance, there was a special installation usually interpreted as a domestic altar – a clay disc about 1.6–2 m in diameter and 10–15 cm thick, often decorated with red and yellow paint and linear incised patterns. Tableware is regularly found at different parts of the house during excavation, notably near the altar and entrance, and the question of whether items were placed there in a meaningful way prior to house burning has been a matter of much discussion (Chernovol 2012, 198). Some more rare variations on this rather strict norm of house organisation include secondary altars either in the main room or the porch, as well as an additional annex – possibly for craft activities – appended to the front porch.

Loom weights are generally found within regular houses, most often on the raised platform near the altar, suggesting that weaving was a domestic activity. This is not the case for remains associated with flint knapping, which must have taken place at least outside regular domestic areas, and possibly within specialised workshops, some cases of which have been suggested within CTCC settlements (Chernovol 2012, 198–200). The unusually fine (for the European Neolithic) Trypillia pottery seems to largely have been produced at sophisticated double-chamber kilns which are found across the mega-sites, indicating that this activity was either organised at a communal (quarter/neighbourhood) level or performed by specialised artisans – or some combination of the two (A. Korvin-Piotrovskiy et al. 2016; Costin 1991). Interestingly, A. Korvin-Piotrovskiy et al. (2016) showed that these kilns were evenly distributed at sites like Taljanki and Petreni, while at Maidanetske they were more clustered and not corresponding to the distribution of mega-structures. Metal finds are very rare in Trypillia contexts, and it remains unclear to which extent metallurgy was actively practised or whether objects were imported from neighbouring regions in the lower Danube or the Carpathian basin. In any case, objects such as copper chisels/axes were probably not the commonly used tools for woodworking (polished stone adzes are more widely documented), at least not before the latest stages of the Trypillia culture (Gaydarska and Chapman 2020a, 454–55). A single small gold spiral, interpreted as a hair ornament, was found within the largest structure at Nebelivka, and is apparently the only gold find recorded in any Trypillia context (Gaydarska, Chapman, et al. 2020).

Similarly to many South-East European Neolithic/Chalcolithic settlement contexts, Trypillia house remains are often – or even usually – heavily burnt at the end of their use-life. While earlier interpretations tended towards catastrophic site-ranging accidents or warfare (invasion from the steppe populations), more careful excavation and interpretation of house remains, combined with some experimental programs, have led to a consensus that the houses were intentionally set on fire individually as they fell out of use (A. G. Korvin-Piotrovskiy, Chabanyuk, and Shatilo 2012; Johnston 2020). In the recent high-resolution geomagnetic images of Trypillia mega-sites, house remains were recorded as either "burnt" or "unburnt"/"eroded", depending on the intensity of the signal and thus how visible the remains were (Rassmann et al. 2016, 2014). For the mega-sites the unburnt category was assigned to 21% of the houses at Maidanetske (610 of 2932 houses, Ohlrau 2020, 61–64) and 26% at Nebelivka (368 of 1445, Hale 2020, 124–29). More recently this categorisation has been questioned, since excavated building remains labelled as unburnt also showed clear signs of firing at high temperatures. Pickartz et al. (2019) suggested that the so-called unburnt houses were simply constructed differently, using less clay and daub, and thus leaving less material after burning that would easily be identified through geomagnetics. Whether there was any functional difference between these two house categories remains an open question.

The special category of much larger houses, generally referred to as "mega-structures" and primarily known from the "mega-sites", was first discovered in 2009 through geomagnetic imagery, and has received much attention since (comparative overview in R. Hofmann et al. 2019; see also Nebbia et al. 2018; J. Chapman, Gaydarska, and Hale 2016; Rassmann et al. 2016, 2014; Burdo and Videiko 2016; Müller, Hofmann, and Ohlrau 2016; J. Chapman, Videiko, et al. 2014). These are recognised by a number of architectural features that clearly distinguish them from the regular domestic houses described above (R. Hofmann et al. 2019, 35 ff.). Firstly, they usually have twice the floor area or more compared to regular houses – buildings referred to as "mega-structures" range in sizes from about 100 m^2 up to 1.200 m^2 for the largest known examples (in Dobrovody and Nebelivka) – and are spatially distributed in characteristic ways within the settlements. They are not constructed on raised platforms, and in many cases they are only partially if at all roofed (e.g. with a large walled front yard or atrium). Only a few of these buildings have yet been excavated – R. Hofmann et al. (2019) enlist four: one each in Maidanetske, Nebelivka and Dobrovody (all representing Trypillia B-II/C-I mega-sites in the Syniukha basin, central Ukraine) as well as one in Baia (Precucuteni, north-east Romania). The excavated mega-structures in Dobrovody and Nebelivka both represented the largest structures within their settlement, and while the former was interpreted as an entirely unroofed rectangular enclosure, several key aspects of the internal architecture of the latter – the existence of an upper floor, supporting vertical posts and galleries, number of use phases, burning events – have been subject to highly conflicting views (Gaydarska, Nebbia, et al. 2020; J. Chapman, Gaydarska, and Hale 2016; Burdo and Videiko 2016; J. Chapman, Videiko, et al. 2014). The excavated mega-structure at Maidanetske was not the largest at the site, but both its size and positioning clearly distinguished it from domestic houses. It was interpreted by the excavators as being sectioned in two almost equally sized parts, one roofed and one walled open yard (R. Hofmann et al. 2019, 13–29).

All of the three excavated mega-structures in the Syniukha basin yielded rather regular domestic purpose finds, but in much lower quantities than what would be expected from their sizes if they were permanently inhabited. This, along with the apparent absence of installed ovens, has led the involved research teams to interpret them as being non-domestic in function, i.e. that they did not serve as the homes of ruling-class elites or chiefs. However, what exactly *did* take place there remains a more open question. The Nebelivka mega-structure housed as much as seven cruciform clay platforms analogous to the "altars" of domestic houses, but some of them being much larger. The Ukrainian part of the excavation team at Nebelivka has advocated an interpretation of a two-storey building, with the inner room of the upper floor resembling a dramatically up-scaled version of a domestic house, suggesting that this part of the structure could be well suited for permanent occupation (Burdo and Videiko 2016, 111), while the mentioned altar platforms – showing clear traces of fire-related use – would be located on the lower floor. The authors do not hesitate to characterise the structure as a whole as a communal temple. The British part of the Nebelivka excavation team has favoured an interpretation where only the middle part of the structure would be roofed, and where no rooms would be suited for year-round occupation (J. Chapman, Gaydarska, and Hale 2016). While they are more prudent regarding the interpretation of the raised clay platforms as altars (they prefer the more neutral term "Raised Area"), they in turn do not hesitate to designate the complex as an "Assembly House", an interpretive term first suggested for the Trypillia mega-structures by Cahokia specialist Timothy Pauketat (Hale 2020, 123).

Somewhat curiously, while the two parts of the excavation team largely disagree in their architectural and social interpretations of this exceptional building, they do seem to agree on the one hand that it was not the residence of an elite leadership, while on the other hand it holds several characteristics of such, including its monumentality in size, elevation and visibility, restricted access, the presence of large storage vessels and stamp seals ("tokens"), as well as special finds and long-distance imports not found elsewhere on site (or hardly within the entire CTCC), like a small gold spiral and graphite-decorated pottery (Gaydarska, Nebbia, and Chapman 2019, 107–9; Gaydarska, Chapman, et al. 2020). One could almost ask if the same structure found in a later context, like the Bronze Age Aegean, would not be immediately recognised as the residence of a *wanax*. The finds are certainly somewhat scarce in quantity, but considering that the ritual (non-accidental) destruction of the building at the end of its use-life is beyond reasonable doubt, this can easily be explained either by a prior removal of administratively important inventory as well as valuable items, leaving only the objects required for the burning ceremony, either by the building's inhabitants themselves or pillaged by others if the elites were removed from office. If the structure was really used communally for seasonal feasts and ceremonies, one would normally expect far more broken tableware and feasting deposits (Hayden 2014). The main recurring argument against a top-down rule at Nebelivka and other Trypillia mega-sites seems to be the lack of evidence from monumental graves (e.g. Nebbia et al. 2018). However, the evidence of burial practices is almost entirely lacking from the Trypillia as a whole, and in particular within the mega-site region, so it seems clear that this is a part of their cultural practice that is currently far from well understood. Though I do admittedly not have first-hand field experience with Trypillia settlements, I am inclined to hold that the burden of proof in this case should be on those who claim that Trypillia mega-sites reflect an "egalitarian society" (Nebbia et al. 2018, 51; Gaydarska and Chapman 2020a; Graeber and Wengrow 2021, 288–97).

The smaller secondary mega-structure (placed within the "main street" of the settlement) excavated by the German-Ukrainian project at Maidanetske did not yield special exotic finds, but was similarly characterised by a striking paucity of more standard finds categories (R. Hofmann et al. 2019, 13–29). Similarly to the large Nebelivka mega-structure, this structure was also seemingly partitioned in a roofed and an open section, but with entrances from the outside to both parts. The activities taking place within the structure, judging from the finds inventory, were more of household-economic rather than ritual or administrative character, like weaving, cereal processing and storage, but also food preparation and meat consumption connected to an open fireplace. The absence of ovens and the generally lighter architecture compared to domestic houses were seen as further indications of economic rather than domestic function, and arguing from numerous ethnographic analogies the research team proposed that these large main street buildings were polyvalent community structures for collective use. Their regular placement within the mega-sites serves as a strong argument for a settlement organisation of socio-economically separate quarter communities, integrated together at the site-level in some way or another through the largest building and its main plaza. R. Hofmann et al. (2019) furthermore noted that the potential use-groups of each secondary mega-structure (number of households) by any measure would largely exceed the numbers seen in ethnographic analogies, arguing that there could well be also a third level of social integration reflected through house clusters (the single household being a fourth level). However, the numbers of coeval houses at Maidanetske and other mega-sites are yet far from being securely established.

3.3.3 Are the mega-sites early cities?

As a last note on Trypillia social organisation, mention must be made of the ongoing debate about whether the mega-sites should be considered as cities rather than large villages. If so – and given that they pre-date the earliest cities in Mesopotamia by several centuries – they should be recognised as the earliest cities in the world. Definitions of cities or urban settlements are numerous but generally follow that of Wirth (1938), who listed their characteristic criteria as being large, dense and permanently settled by heterogeneous individuals – i.e. exhibiting some socio-economic variability and thus exchange of services and resources between the inhabitants, as well as a functional complementarity with surrounding rural areas. For the Trypillia mega-sites, the debate has thus been largely focussed on population estimates, craft specialisation, economic diversity and hierarchical administration, as well as their functional relations with the hinterland. Population estimates have varied widely, from some hundreds to tens of thousands living coevally at the sites (see e.g. Gaydarska and Chapman 2020a, 433 for an overview of estimates for Maidanetske). Surprisingly, the researchers who have advocated for the lowest coeval populations (the "minimalist view" in the terms of Gaydarska and Chapman 2020b) are also those who have insisted the most on labelling the sites as urban. Arguing from their evidence of an apparently limited human impact on the surrounding environment (e.g. degrees of erosion, deforestation, pollen records) over the duration of Nebelivka, Gaydarska and Chapman (2020a) have proposed that the site would likely not be permanently occupied (see also Gaydarska and Chapman 2020b; Gaydarska, Nebbia, and Chapman 2019; Nebbia et al. 2018). They developed three main alternatives for explaining this paradox of a high population and low environmental impact (summarised in Gaydarska and Chapman 2020b, 35; Gaydarska, Nebbia, and Chapman 2019, 111–13):

- The Assembly model (colloquially referred to among the excavators as the "Burning Man" model), of a yearly gathering of disparate communities for a short duration, to partake in ceremonial exchanges of information, goods, partners etc.

- The Pilgrimage model, where the location of the settlement would have some great religious significance to the population of a much wider region
- The Distributed Governance model, where each quarter community would take turns on a yearly basis of the responsibility for acquiring necessary resources for the others from a wider economic region

According to the two first models, only a small "caretaker" population would be permanently settled at the mega-site. The third model would posit some middle-ground allowing for a somewhat larger coeval population, but with the organisation of substantial resource imports from elsewhere. All three models have some support from ethnographic analogies (amply discussed in Graeber and Wengrow 2021), but have also been criticised for being fanciful narratives lacking grounding in archaeological evidence (e.g. Ohlrau 2020, 274–76; 2022). The rather agrarian character of finds inventories from Nebelivka and other mega-sites – typical of what is usually labelled as Neolithic or Chalcolithic societies – as well as the low settlement density following from the models of only partial occupation, led the researchers to interpret the Trypillia mega-sites as examples of *low-density urbanism*, a term promoted by Fletcher (2011) to describe agrarian dispersed cities, primarily within the tropics, like Ankor and Tikal (J. Chapman and Gaydarska 2016).

The German-led investigations at the mega-sites have on the one hand proposed higher coeval population estimates, higher densities and generally refuted the seasonality hypotheses suggested by the British team – also questioning the environmental evidence serving as its main argument, e.g. Ohlrau (2022), p. 85 – and on the other hand been more reluctant to characterise them as urban (Müller, Hofmann, and Ohlrau 2016; Müller and Pollock 2016; Müller et al. 2016). To Müller and Pollock (2016), typically urban traits that are seen at late 4th millennium Uruk in Mesopotamia, like the spatial separation between a religious-administrative and domestic districts as well as clear signs of division of labour, are not present at the Trypillia mega-sites. The perhaps most "maximalist" views have been held by Ukranian specialists who, grounded in the typo-chronological tradition of following the movements of population groups at the regional scale, with added results from modelled population dynamics, have pointed to the large scale (largely coeval) but short-livedness and apparent instability of the mega-sites, holding that the very large groups of people could not be sustained by the environment for a long time at the same location, while unoccupied land suited for resettlement was readily available (Shatilo 2021; Diachenko and Menotti 2017, 2012; Diachenko and Zubrow

2015; Diachenko 2012).

While these studies have tackled the uniqueness of the mega-site phenomenon differently, inn all cases it seems clear that for a number of reasons these settlements do not fit classic definitions of cities in a straight-forward way. More recently, Sindbæk (2022) proposed the term *anomalocivitas* to characterise various types of settlements known through history and prehistory for which the urban definition is seen as problematic. Acknowledging that efforts to shoehorn archaeological material into inadequate definitions are often based on teleological fallacies, preventing researchers from seeing the settlements for what they really are, he constructed a framework of definitions allowing for more variability and connecting the "lack" of certain urban features with specific socio-economic functions (Sindbæk 2022, 21–22). In this sense, Trypillia mega-sites are similar to Iron Age *oppida* like Bibracte and Manching, or Viking trade *emporia* like Ribe and Kaupang, in that they all have low values of one variable central to urbanism – low size in the case of *emporia*, low density in the case of *oppida*, and low heterogeneity in the case of the mega-sites. The low socio-economic heterogeneity is interpreted (e.g. Ohlrau 2022; Müller and Pollock 2016) as a result of the agglomeration of essentially similar village populations from within a small region – populations who had not already developed a high degree of economic specialisation unlike in the Near East Chalcolithic. R. Hofmann et al. (2019) see a gradual tendency towards centralisation and monopolisation of power within the Trypillia mega-sites over the period of their duration (secondary mega-structures becoming fewer and smaller, the spatial distribution of pottery kiln becoming increasingly clustered), and argue that this, rather than ecological degradation, caused their ultimate disintegration in the Trypillia C-i/C-II transition. As for now, the most unresolved question regarding the Trypillia mega-site anomaly, seems to be why they appeared and grew so large in the first place.

3.4 Reading site plans from geomagnetic imagery

The unprecedented quality of the recently published geomagnetic plans of Linear Pottery and Trypillia settlements (Rassmann et al. 2016, 2014; J. Chapman, Gaydarska, et al. 2014; Nils Müller-Scheeßel et al. 2020), as well as the theoretical and culture-historical considerations presented at the beginning of this chapter, make these two study areas ideal as cases for testing the performance of the fractal analysis methodology for quantifying levels of social hierarchy.

For this thesis, spatial data in the form of shape-files were generously shared by the GIS specialists of the different research projects: Niels Müller-Scheeßel (Kiel university, Nils Müller-Scheeßel et al. 2020) for the Linear Pottery sites in the Žitava valley, René Ohlrau (Kiel university, Ohlrau 2020) for Maidanetske, Moshuriv and Talne 3, and Duncan Hale (Durham university, Hale 2020) for Nebelivka. The house outlines were thus identical to those presented in the cited publications. However, this also means that they were drawn by (at least) three different people, which may have lead to certain biases between the data sets. Furthermore, Linear Pottery and Trypillia houses are not identified on geomagnetic images in the same way: the former are seen as "empty" spaces between the longitudinal pits which are generally well visible (Winkelmann et al. 2020), while the latter are seen more directly by the burnt clay platform forming the floor area (visible to varying degrees, as already discussed above, Pickartz et al. 2021; Pickartz et al. 2019). Both culture groups are known to allow for very little overlap between houses – an issue which would be far more problematic e.g. with Balkan or Anatolian Neolithic villages. The data to be extracted from these files were house sizes for Chapter 6 and the house outlines themselves as images for Chapter 9. Both the extraction of house (polygon) sizes and settlement layout raster images were done using standard functions in QGIS 3.20.1 (see Section 8.1 for more details on image preparation). For the mega-site of Maidanetske, the south-eastern section of the settlement is now badly preserved, but house features are known from earlier surveys. These were drawn by Ohlrau (2015) based on older site plans, but the corresponding house outlines are therefore not as precise as the remainder of houses, which are drawn from the recent geomagnetic data. These less precise features were included in the image analysis, but excluded from the size-distribution analysis.

3.5 Time samples of the Vráble settlement

Separating an archaeological settlement plan into samples with temporally coeval structures only is a demanding exercise, and more so the larger the settlement. The best and most reliable method is to date every single structure in the settlement, but this is usually impossible both in theory, since not all structures yield datable material, and in practice when considering the costs involved. The more realistic approach is to sample a smaller amount of structures throughout the settlement for dating and modelling, and then extrapolate the results to get a more or less rough overview of how much, proportionally, of the site was occupied at the same time. This has been done for the Trypillia mega-sites, indicating that the maxima of coeval habitation would be at approximately 33% of houses at Nebelivka (Millard 2020, 253–56; Müller et al. 2022) and 52% at Maidanetske (Ohlrau 2020, 233–35; 2022, 86–88). Even though some general tendencies of the spatio-temporal development of these settlement plans have been proposed (Shatilo 2021, 247; Müller et al. 2022, 218–19), it is for now impossible to extract precise temporal samples of coeval house-size distributions here. This also goes for the image analysis presented in Chapter 9 (see also Bruvoll, n.d.). The smaller sites of Moshuriv and Talne 3 are only dated through relative chronology of surface finds and thus attributed as a whole to their chronological phase, though their small sizes and ordered layout would suggest that most houses could be coeval (Ohlrau 2020, 241–44).

For houses in Linear Pottery settlements, a special dating proxy was recently developed, based on the observation that within this cultural context houses were constructed to be parallel to pre-existing ones, but with a slow and gradual counter-clockwise shift in orientation of approximately 1.3 degrees per decade (Nils Müller-Scheeßel et al. 2020). This shift was hypothesised to be too slow for any conscious intention among the house builders, but rather a result of so-called pseudoneglect, or the tendency of a slight leftwards bias in the perception of parallel lines, mostly among the right-handed. Nils Müller-Scheeßel et al. (2020) observed this gradual shift in the orientation of 17 houses in Vráble for which construction dates had been estimated based on Bayesian modelling of ^{14}C dates. Their linear regression model for all these houses only gave a weak correlation however, and they argued for basing the model on the eight houses in the SW neighbourhood only, where the sampling strategy had been the most systematic. This gave a regression model defining house orientation as $0.129x - 651.016$ with correlation coefficient $r = 0.84$, x being the modelled construction year BCE, which when applied inversely (solving for x) gives modelled construction year as $\frac{orientation+651.016}{0.129}$. Use-life of houses in Vráble was modelled by Meadows et al. (2019) to a median of 27.5 years, a number which according to the same authors has varied greatly throughout the long history of Linear Pottery research. Extrapolating the model for construction year based on house orientation and adding this median duration onto all known houses in Vráble gives a settlement occupation span from 5297 to 4975 cal. BCE, which is in good agreement with the currently available and modelled ^{14}C dates (though admittedly a few decades earlier, see Meadows et al. 2019). For the distribution fitting analysis in Chapter 6 I defined a set of

16 sample dates spaced 20 years apart from 5290 to 4990 within this timespan, and assigned houses to each sample accordingly. Since the duration between time samples is a few years shorter than the duration of houses, some houses were assigned to more than one sample. Furthermore, following the method requirements defined in Chapter 5, samples with 10 or less houses in them were filtered out, mainly since the distribution fitting method then becomes too unreliable (this is already a quite generous allowance), but also since it makes little sense to speak of hierarchy in a group of less than 10.

It must be noted that there are several caveats with this procedure of selecting out houses to be considered coeval. Firstly, the dating proxy method proposed by Nils Müller-Scheeßel et al. (2020) is recent and not yet well established. It is based on linear regression of very few data points (8 houses in only one part of the settlement), all of which are not unique certain values but dates that are themselves also modelled. Though the publication does not specify it explicitly, it seems their model was fitted by ordinary least squares (OLS) regression, while systematic measurement uncertainty in the independent variable (the modelled ^{14}C dates) should warrant more robust methods like orthogonal regression or probabilistic Monte Carlo methods. My use of a single value for house duration rather than the probability distribution that it really is, is also a simplification that could affect the results of the analysis. In general, longer house durations lead to more of the houses being coeval throughout the timespan of a settlement, while inversely shorter house durations lead to fewer coeval houses. There could also potentially be systematic relationships between house size and duration, e.g. that larger houses were occupied by more temporally stable households for several generations, putting in greater efforts to maintenance. These questions are not further pursued here however, and the results of this analysis must be considered preliminary.

3.6 Ethnographic and synthetic data

A challenge with applying new methods onto any material with the goal of using the obtained results as proxies for some unknown or little known variable (in this case the level of social hierarchy), is that this use of the method must in some way be validated by also applying it on material where the variable is known. At the onset of this PhD project my intention was to compare results obtained from the archaeological samples with results from ethnographic samples. Using the settlement plans of villages and settlements where the actual degree of social hierarchy had been analysed and documented by ethnographers would have been a valuable contribution to this study. However, it quickly became clear that such settlement plans, with the necessary degree of detail and accuracy, are hard to come by. For comparability with Neolithic societies, the ethnographic sample should ideally consist of settlements having been minimally transformed by modern industrialisation – archaeologists specialising in Neolithic Europe tend to prefer comparisons with Sub-Saharan African or Austronesian village societies, preferably documented before the second half of the 20^{th} century. Unfortunately, while such documentation is abundant, in the vast majority of cases settlement layouts and house sizes are either entirely missing, or documented in a highly approximate manner, inhibiting any meaningful comparison with the sampled archaeological settlement plans. Ethnographers have simply not been very interested in mapping villages by precise measurement in the past. However, although not directly transferable to the quantitative analyses proposed in this thesis, relevant ethnographic literature is to some extent reviewed in Sections 4.1 and **??**.

An alternative approach was therefore adopted, using synthetically generated data (i.e. computer-simulated), both for size distributions and spatial layouts (Chapters 5 and 8). The advantage of this approach to the ethnographic one, is that it allows for setting parameter values entirely freely and thus testing the methods over wide and regular ranges of data. The disadvantage is that the connection to actual real-world social systems remains somewhat uncertain, as the synthetic data simulates the material culture (the settlement plans) and not the social system that generated it. This gap between the data and interpretation can to some extent be filled by considering theoretical perspectives. While the theory of size distributions and their generative social mechanisms is well developed (Section 4.2), this is much less the case for the theory regarding spatial layouts and textures of social organisation (Section 7.1).

Part II

Size distributions

Chapter 4

House sizes and social meaning

4.1 Interpreting house-size differences

In archaeology there are two recurrent and seemingly contradicting assumptions underlying interpretations of house-size differences within a society. In studies where the goal is to provide population size estimates, this is often calculated from living area, where the square metres per inhabitant is modelled from ethnographic analogies. The population of a village is then found by summing together those of every house (e.g. Coudart 1998, 79–80). A simpler and probably more simplistic version of this is to consider a m^2/inhabitant proportion that is constant no matter the size, and thus calculate the population directly from the total living area of the village. This mean value is generally obtained from multiple ethnographic parallels. With this assumption – that every inhabitant requires a similar amount of living space – the size of a house effectively reflects its number of inhabitants. The second assumption, which is more frequently seen in studies focussing on wealth inequality and social stratification, is that house-size differences are expressions of differences in some sort of wealth or power. In this view, a larger house would have belonged to a wealthier household, capable of procuring more raw materials and activating a larger labour force for its construction and maintenance, in which case there would be significant disparities in living space per individual. Even though there is not necessarily any contradiction between these two interpretations from an anthropological point of view, most archaeologists seem to be unable to consider both possibilities simultaneously (Wilk 1983). From a methodological point of view, each of these assumptions will tend to mask our ability to see traces in the archaeological data relating to the other assumption – that is, with existing methods we cannot convincingly provide estimations of both population size and level of social inequality from the same data, even though house sizes frequently form the basis of both argumentations. Far from proposing a solution to this issue, my argument here is that a variety of social institutions known from ethnography and historical sources can explain some level of correlation between the two variables. Dowry and bride price are geographically and temporally widespread practices that link number of offspring (daughters, sons or both, depending on cultural context) with wealth. Clan leaders may draw upon kinship ties and dependencies in order to obtain the workforce needed to construct a larger house (A. Johnson and Earle 1987). In agricultural societies, land ownership is often seen to correlate with household size, since land owners tend to attach workers and servants to them, people who themselves in turn tend to come from landless households (Wilk and Rathje 1982, 629; Netting 1982). Nevertheless, if the notion of wealth is at all to be applied meaningfully to non-capitalistic societies, it should designate cases where there is significant and persistent material disparities between members of a population, and not simply point to different household sizes where wealth is proportional. House sizes could thus potentially reflect a somewhat more complex culture-specific interplay between household size and wealth, meaning that for a given house size, one could assume a range of possible values of the two parameters (Figure 4.1). This relationship between household size, wealth and house size should be studied more in detail empirically through the available ethnographic data, rather than reducing its complexity to a mean surface area per inhabitant for the entire population. Such a study, however, lies beyond the scope of the present thesis. Here I will largely leave aside the question of population and household size, focussing on distribution types of house-size data, arguing that the most unequal distribution type considered here – the power-law distribution – is unlikely to emerge only from random differences in household size and standard marital patterns, favouring thus interpretations relating to systemic wealth and/or power differences.

Even though the goal here is not to investigate household sizes but to focus on the material aspect of house-size distributions, some fundamental issues of terminology should be addressed. The use of the word *house* (in the wider material sense rather than the Lévi-Straussian sense, see Section 2.1) is indicative of an underlying assumption that the building in question was in use primarily for domestic purposes – essentially, a fixed architectural unit where someone would spend their nights at least most of their time, and in many cases also cook and eat their

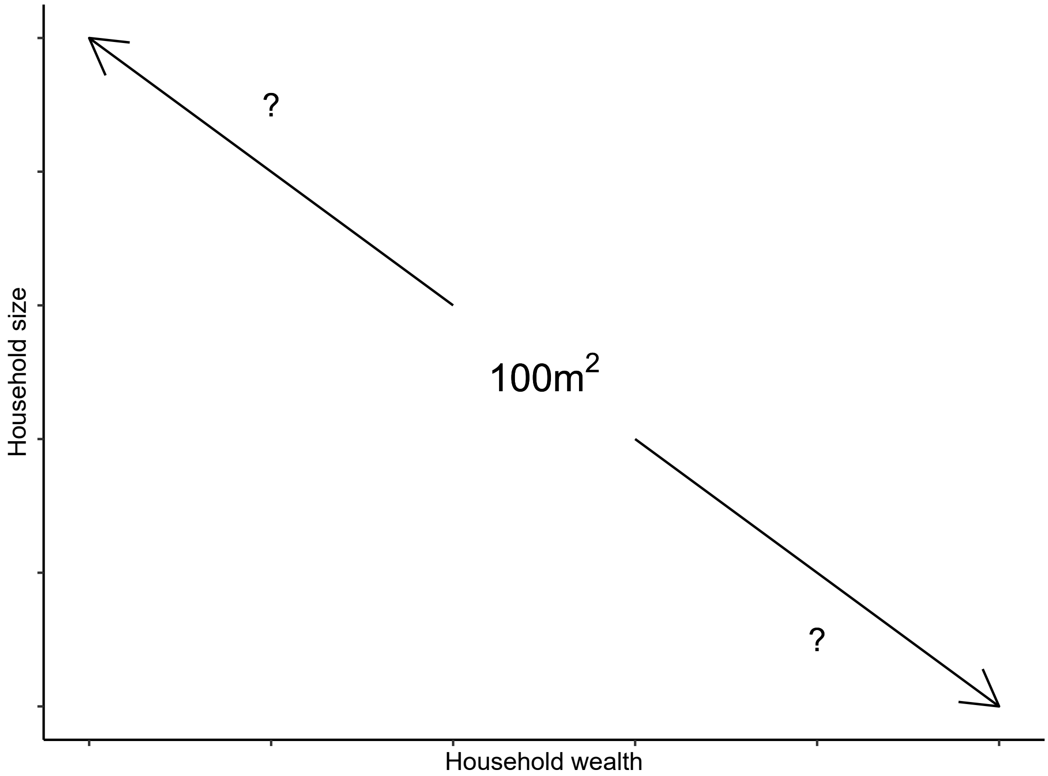

Figure 4.1: *House size can be interpreted in terms of number of inhabitants (household size) or material wealth of the inhabitants, but the exact relationship between these two variables remains poorly understood and is probably both complex and culturally contingent. Any given house size within a distribution can thus result from a combination of effects from the two. In many cultural contexts household size and wealth may furthermore be directly correlated*

main meal during the day – though proving this directly is not always straightforward in archaeology. For the cultural contexts discussed here – the Linear Pottery and Trypillia groups in the Neolithic-Chalcolithic – there is however little evidence for buildings with specific non-domestic (e.g. economic, religious or administrative) purposes, with the probable exception of the so-called mega-structures or assembly houses in the Trypillia mega-sites, which are discussed more in detail below. This lack of evidence does not imply that there was no specialised economic, religious or administrative activity in these societies, as ethnography and history clearly shows that such activity must reach a certain degree of specialisation before it materialises in distinct buildings devoted exclusively for these functions. Artisans, shamans and chiefs could be specialised to some extent but still perform their activity at their domestic home or more diffusely outdoors or without any fixed location (e.g. Costin 1991, 25; Kahn 2021). It is in any case of common usage to speak of houses when discussing architectural units in Neolithic Europe and other prehistoric contexts, maybe because of a lack of a better generic word, but this usage should not prevent archaeologists from recognising other non-domestic functions of buildings whenever there is evidence for it. The *household* is furthermore the designation of all the people, genealogically related or not, usually (though not necessarily) living under the same roof or within the same architectural unit, constituting a functional whole economically and socially, and potentially including more than a single family unit, as well as servants or slaves, depending on the context (Wilk and Rathje 1982, 620). The emphasis for defining a household is thus more on its economic and social function than on co-residence and kinship relations, which are somewhat more variable. In the following, I go into some more detail as to how household organisation as well as wealth are known to influence house size.

4.1.1 House size and household organisation

The focus on houses and households in archaeology – as opposed to larger units of analysis like whole settlements, cultures and periods – started to attract momentum by the end of the 1970s, with the work of Wilk and Rathje (1982) often cited as the original manifesto of its validity and importance, pinning "Household Archaeology" as an independent genre of study. In their view, the household could be understood as the most abundant activity group in any society, despite considerable variations as to its importance relative to other types and scales of social groupings. They defined it by their *social*, *material* and *behavioural* constituent elements, that is, its members and their relations, the dwelling and other possessions, and the activities it performs. The material element is of course the only one directly accessible to archaeologists, and the task for researchers would be to reconstruct the social and behavioural elements from the material. This realisation led to ambitious comparative projects of mapping the variability of houses and their occupants in different parts of the world and socio-economical contexts, with inferences onto prehistoric contexts built upon existing ethnographic literature (e.g. Murdock 1949) as well as new ethno-archaeological observations (e.g. Wilk 1983; Blanton 1994).

One of the main characteristics of households is their *size*, which, according to Wilk and Rathje (1982), to some extent is determined by the scale of the production activities that fall within their organisational sphere. In societies where large scale complex tasks necessitating the simultaneous cooperation of many hands are organised at the household level, the optimal household size will be accordingly large. Such activities can include agricultural tasks like irrigation or terracing, as well as house construction. However, when large-scale activities are necessary only once or twice a year, as with seasonal large game hunting or intensive fisheries, they tend to be organised at a community level by many households working together temporally. In such contexts, households can be smaller as their daily activities can be performed by a smaller number of people (Wilk and Rathje 1982, 623; see also Hamilton et al. 2007). Furthermore, ethnography has repeatedly shown that large households performing complex activities often need a head coordinator for the activities to run smoothly. However, the set of activities organised at the household level in a society and thus determining the optimal household size, will affect all households similarly unless there is some economic differentiation between them. In societies where households are economically more or less self-sufficient, their size differences should be expected to be random (i.e. normally distributed, see Section 4.2.1).

Kinship studies within anthropology have shown over the last decades that in most societies, contrarily to common misconception, kin affiliation is *not* simply a matter of biological relatedness. In an attempt of grouping together all possible justifications for kinship ties, Marshall Sahlins (Sahlins 2013) defined kinship as "mutuality of being" – that some real or imagined substance is shared, and that this substance is not necessarily genes, as is mostly the case in modern Western societies (with adoption as the main exception). A variety of non-genetic foundations of kinship are widely accepted in different cultures, like sharing of name, time or place of birth or childhood, food source, shared experiences, blood ties, and so on. At the same time, archaeology as a whole has arguably been very slow in taking this diversity of kinship configurations into account, far too often taking for granted the modern Western (especially post-war 20th century) ideal of patrilineal nuclear families as the default configuration for all of human history (Ensor 2021b). That being said, anthropological kinship studies such as that represented by Sahlins typically shows little concern with material culture, and are mostly silent on the question of who – at the end of the day, quite literally – sleeps under the same roof, making it hard to interpret domestic architecture in terms of kinship structure and social organisation. The recent comeback of kinship studies in archaeology has been far more focussed on linking isotope and aDNA data with social structures, largely fuelled by the rapid developments of the related methodologies.

4.1.2 House size and wealth

Wilk and Rathje pointed to the important role of households in inter-generational transmission of wealth in many societies (1982, 627–31). Specifically, following Murdock (1949) and other ethnographers, they argued that as populations grow, land tenure tends to institutionalise at the household – and later individual – level around the moment when agricultural land becomes more scarce than labour. Before this – in pioneer phases of agricultural development – land is readily available and if there is any concept of land ownership at all, it usually lays at the community level. Once the agricultural land in a region is saturated, rights to use it will tend to be transferred within households, and children from households with extensive rights have a greater incentive to stay within or close to it, while children from households with less land rights are more likely to emigrate. Further population pressure will tend to limit partition of inheritance between siblings, so that land ownership over time is transmitted within a smaller and smaller fraction of the population. This situation is also shown to entail stratified (and parentally arranged) marriage, further entrenching social stratification.

Kahn (2021) describes an example from eastern Polynesia of agricultural expansion and intensification developing over 400 years from the initial colonisation before social stratification starts to become materialised in differential architecture. In later phases, she links the appearance of specialised buildings devoted to communal assemblies, rituals as well as residential and ritual buildings for elites with the emergence of complex chiefdoms. Though quantitative measures of house size are not presented in the study, it clearly demonstrates that elite architecture was consistently larger than that of commoners in the prehistoric Society Islands. However, she also identifies types of houses of special economic function like workshops, and points out that these can be hard to distinguish from common residential houses archaeologically simply by looking at size (Kahn 2021, 90–91). In such cases – where there is a range of non-residential building types – it may be of little use to analyse house-size distributions as single blocks. It may then be necessary to first group buildings into functional types based on internal structure and finds inventories, given that this information is available. This issue concerning the prerequisites of data input for the analyses done in this thesis, is further discussed in Chapter 11. It should also be noted that interpretations regarding the social structure of pre-contact Polynesian societies are greatly aided by near-contemporary written accounts from the first European explorers in the region, and house floor levels are often well-preserved due to the relatively recent dates of the structures (10^{th} to 18^{th} centuries CE) – both factors being in stark contrast to the central European Neolithic, where the spatial organisation and sizes of houses are often nearly the only information available.

Regarding the domestic architecture of clan leaders and chiefs, several studies have highlighted how elites both have the material means and the socio-economic incentives to build houses that are more monumental than those of commoners (e.g. Řídký et al. 2019). Others have warned that this should not lead archaeologists to systematically interpret large buildings as evidence of elite-based top-down social hierarchies, since there are also ethnographic examples of monumental buildings being constructed collectively by more egalitarian communities for assemblies or other communal activities. Referencing a range of ethnographic and archaeological examples, Goodale, Quinn, and Nauman (2021) point out that the act of constructing a monument may be organised and to some extent coerced by a leader, or equally by a collective group. In both cases the intended function of the finished building will serve in the interest of the one or those who initiated the project. This is furthermore valid at all social scales – the constructuion of a residential building is initiated and driven by those who wish to live in it, and the construction of a ritual building for the village like an assembly house is initiated and driven by the entire village. Furthermore, the construction of a palace in a state society is initiated by the leader and financed through the tribute collected throughout their effective territory. Consequently, buildings should be expected to somewhat reflect the scale of their social importance through the level of effort put into their construction, so that in a hierarchically organised society (be it top-down coercive or bottom-up collective), this effort should also be hierarchically distributed in its buildings. I argue here that building size is the best (and often the only) available proxy to this construction effort (see Section 4.2.4 below for more discussion on this point).

P. Květina and J. Řídký point out both architecture (construction, size, orientation) and settlement layout as possible distinctive features between Big Men (achievement-based) and Chief societies, arguing that the former type may be recognised by a dispersed intra-settlement layout combined with uniform architecture, while the latter type would tend towards more regular settlement layout and more marked differences in architecture (Květina and Řídký 2019, 13).

Single communal houses within villages that are markedly larger than regular domestic houses have been extensively documented by ethnographers in a range of societies and geographical contexts. Such communal structures may be economic, function as initiation houses, assembly houses, ritual houses, or, probably most often, a mixture of many functions Adler and Wilshusen (1990). Distinguishing such structures from chief or clan leader houses may of course be challenging if judging by size alone.

Some factors other than household size and wealth have been suggested and empirically reported to systematically influence house size (Wilk 1983, 101). Among these are:

- Mobility – Seasonally mobile groups tend to build smaller dwellings than more sedentary groups (Porčić 2012)
- Post-marital residence – Houses in matrilocal societies tend to be larger on average than in patrilocal societies (Hrnčíř et al. 2020; Porcic 2010)
- Climate – Houses are smaller in cold circumpolar or mountainous regions because of the cost of heating
- Duration of residence – Households that have been established at a location for a long time or are well integrated in the community tend to have larger houses (Wilk 1983)
- Material use and technology – All building materials have associated constraints and costs, and innovations can lead to larger constructions at lower costs

Of these factors, only the duration of residence will have a direct influence on house-size difference *within* communities. Mobility, residence patterns and climate are more constant factors affecting entire communities and will thus affect the average house size, but not the level of inequality. Access to building materials and technology on the other hand may be differentiated in stratified societies, and lead to unequal house sizes as a materialisation of wealth. A related factor that may lead to a certain under-representation of the wealthiest households, is the construction of multi-floored houses, unless this feature is recognised archaeologically and included in the calculation of house sizes. For the case studies presented in this thesis however, building materials and techniques are not noticeably differentiated between small and large houses, and the possibility of multi-floored houses is only a minor issue that has been discussed in Chapter 3.

Lastly, in his ethnographic comparison between egalitarian and more ranked contemporary Maya village communities in Belize, Wilk (1983, 111–14) also pointed to social norms potentially *preventing* differences in public display of wealth even when such differences existed.

He linked such norms to the openness of the village economies – in closed self-sufficient villages there was greater mutual dependence between households, and wealth display was strongly discouraged, potentially sanctioned with witchcraft, whereas in villages with more open economies household wealth was more readily displayed through house size and the quality of construction materials.

4.2 Distribution types and their underlying mechanisms

The main question in this part of the thesis is on the nature of house-size differences in the studied contexts, and to explain them either as being due to random fluctuations or as a material expression of a more structural inequality, or something in-between. In statistical terms, this is a question of distributions. A distribution is a mathematical model of the spread of data along a variable or axis, and it can be modelled on empirical data as a succinct description, or used to predict unobserved data (e.g. future developments, or, in the case of archaeology, data that is lost to taphonomy). In theory, there is an infinite number of possible distributions, as there is no limit to how many parameters one can include to fit the data. It is however generally considered good practice in statistics to limit the number of parameters and identify the simplest possible model that gives a good fit, since a larger number of parameters can often lead to a better fit, but at the same time be harder to explain in terms of underlying mechanisms. Adding many parameters only to achieve a marginally better fit is referred to as *overfitting*, and for most real-world contexts there is a limited number of model types that can be considered reasonable candidates. In cases where we are interested in inequality or uneven distributions of data (so-called *skewed* distributions), the most likely distribution models are those that can be described as *heavy-tailed*, meaning that on typical graphical representations (like density/PDF plots or histograms) they will show a characteristic stretch of some of the data towards the right end of the x-axis, while most of the data remains on the left side (Figure 4.2). The opposite orientation is also possible in theory, in which case the distribution can be referred to as left-tailed. It is important to keep in mind however, that not all distributions are heavy-tailed, and that other distributions also model the spread of data across a variable, but result from very different underlying mechanisms. Identifying the most likely distribution model for a data series is therefore crucial for understanding how the data could have been generated. And though it is true that one model type can have multiple different explanations

– in an archaeological setting for example, many different social behaviours can lead to the same material outcome, an issue known as *equifinality* – excluding one or more model types for the observed data can help limiting the number of plausible interpretations considerably.

Distributions can be represented graphically in a number of ways, most commonly in a coordinate system with the variable on the x-axis and its density, or *probability density function* (PDF), on the y-axis (Figure 4.2). Often, and in many of the sources cited in this part of the thesis (e.g. Clauset, Shalizi, and Newman 2009), the PDF is denoted by $p(x)$, reading as *the probability of x*, or simply $f(x)$ (*the function of x*). It gives the probability for a drawn sample (X) of falling within a given arbitrarily short range of the distribution, written $Pr(x \leq X < x+dx)$. By definition, the area between the PDF curve and the x-axis sums to 1. For reasons that are further discussed in Chapter 5, heavy-tailed distributions, and power laws in particular, are instead often represented with their *cumulative distribution function* (CDF), which is the integral of the PDF (inversely the PDF is the derivative of the CDF; Figure 4.3). Similarly to the PDF, the CDF is often denoted as $P(x)$, depending on disciplinary tradition. It indicates the probability of a random sample value being equal to or lower than the function value, or $Pr(X \leq x)$. Furthermore, the specific version of the CDF used for plotting heavy-tailed distributions, is the *complementary* or upper tail CDF (sometimes referred to as the *survival function*, or denoted cCDF), which is $1 - CDF$ or $P(X \geq x)$, indicating the probability that a random sample is higher than the function value. Both axes on such cCDF plots are traditionally set on logarithmic scales, usually $\log_{10}$ for readability. To avoid confusion, in this thesis I refer to PDF and cCDF for density and distribution functions respectively (except in equations, where I use $p(x)$ and $P(x)$ respectively), and all PDFs are plotted on linear scales and cCDFs on $\log_{10}$ scales, unless otherwise stated. Apart from scales on plots, whenever I refer to logarithms (i.e. in calculations), I imply natural logarithms, that is ln or $\log_e$ where the base number $e \approx 2.718$.

In the following, I present briefly the main distribution types that will be discussed further in the following chapters, with special focus on the *power-law distribution*, which is the model type associated with fractals and structural hierarchy. All of these distributions are modelled on continuous univariate data series (see N. L. Johnson 1994 for more detailed presentations). Even though their mathematical definitions may seem complicated to non-initiated readers, most of the distribution types discussed in this thesis are readily implemented in standard statistical software, including Microsoft Excel, allowing for a more straight-forward use of

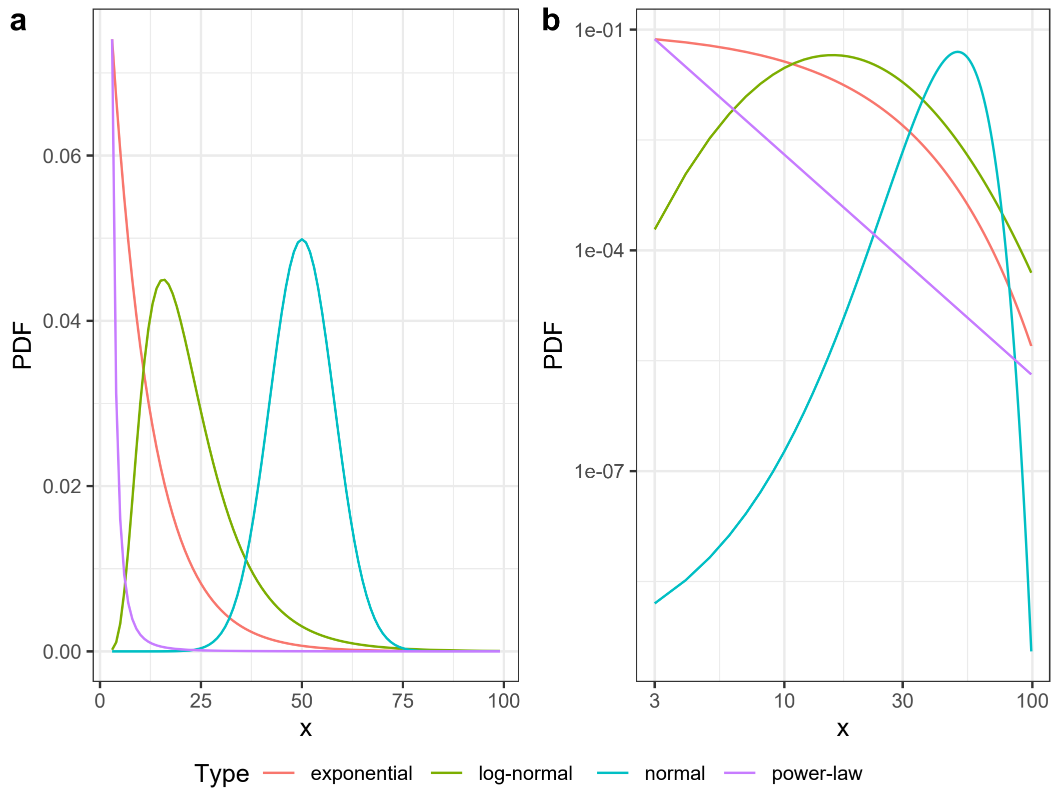

Figure 4.2: *Example curves of the probability density function (PDF) of four common distribution types: normal (blue, $\mu = 50$, $\sigma = 8$), exponential (red, $\lambda = 0.1$), log-normal (green, $\mu = 3$, $\sigma = 0.5$) and power-law (purple, $\alpha = 3$, $x_{min} = 1$), in linear (a) and logarithmic scales (b). Parameter values are arbitrary and x-axis is truncated at $2 < x < 100$ for readability. The power-law distribution is the only to form a straight line when both scales are logarithmic*

them. For this thesis, I used base *R* functions for calculating PDFs and cCDFs, and for random number generation for normal, log-normal and exponential distributions (R Core Team 2023), and equivalent functions from the *poweRlaw* package for power-law and Weibull/stretched exponential distributions (Gillespie 2015).

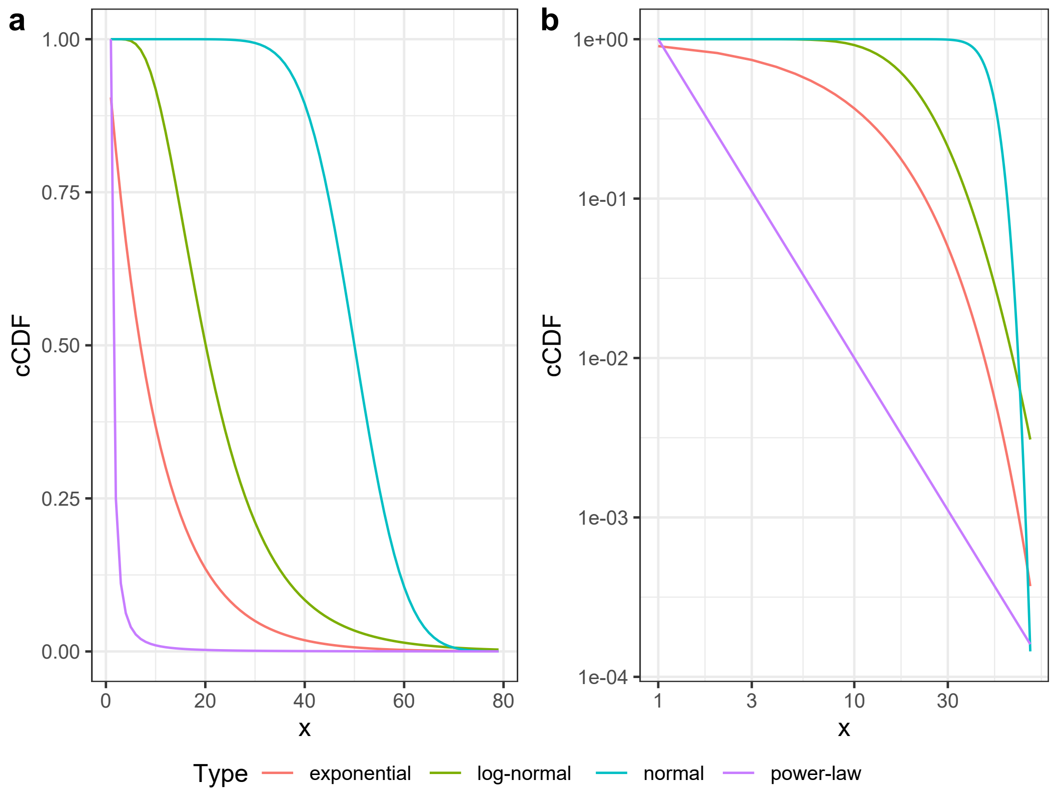

Figure 4.3: *The same distributions as in Figure 4.2, but with the complementary (right-tail) cumulative distribution function (cCDF), in linear (a) and logarithmic scales (b), with* $0 < x < 80$ *for readability*

4.2.1 Normal distributions and the Central Limit Theorem

One of the distribution models that are the most commonly referred to and well-known, is the *normal distribution*, also known as the "Bell Curve" due to the characteristic bell shape of its PDF (Figure 4.2a), or Gaussian after mathematician C.F. Gauss who contributed to its exploration in the early 19$^{\text{th}}$ century. Its characterising parameters are the *mean* (denoted μ, *mu*) and *standard deviation* (σ, *sigma*), and the PDF is defined mathematically as

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \tag{4.1}$$

where e (*Euler's number*) and π (*pi*) are mathematical constants (N. L. Johnson 1994, Vol. 1:80–88). This distribution is centred around its mean, with dwindling amounts of data spread outwards to either side. The mean – commonly known and widely used in daily speech – is the sum of all observations divided by number of observations, or

$$\mu = (\sum_{i=1}^{n} x_i)\,\frac{1}{n}. \tag{4.2}$$

The spread of the data from the mean is defined by the standard deviation, which is the square root of the mean of all squared deviations from the overall mean, or

$$\sigma = \sqrt{(\sum_{i=1}^{n}(x_i - \mu)^2)\,\frac{1}{n}}. \tag{4.3}$$

Mathematically, the square of the standard deviation, σ^2, or *variance*, is simpler, but since it is also less intuitive, I refer here to the former whenever possible. The standard deviation can be thought of as the mean of all deviations, positive or negative, from the overall mean.

The great importance the normal distribution has to a wide range of phenomena is explained through the *Central Limit Theorem* (CLT), according to which the sum of random variables tends to a normal distribution as the number of variables increases towards infinity, under certain conditions (N. L. Johnson 1994, Vol. 1:85–88). More specifically, if $X_1, X_2, \ldots, X_n$ are independently drawn and identically distributed (condition referred to as *i.i.d*) random variables or samples, their sum will be normally distributed in the limit as n tends to infinity, or

$$\lim_{n\to\infty} \sum_{i=1}^{n} X_i = \mathcal{N}. \tag{4.4}$$

The sum may be standardized in some way to avoid infinite numbers, but in practice n will always be limited, so the resulting distribution will always also be approximately normal at best. The original distribution of the random variables (i.e. that of the *population*) does not need to be normal for the theorem to hold, but can be any distribution as long as its variance is finite. The normal distribution (referred to as the *sample distribution* in this context) is related to the population distribution of the random variables (samples), in that the mean of the population equals the mean sum of the sample distribution divided by n, and the standard deviation of the sample distribution, referred to as the standard error s, equals the standard deviation of the population divided by the square root of n, or $s = \frac{\sigma}{\sqrt{n}}$, meaning that s is reduced with higher n (i.e. larger sample size). A common variant of the CLT is that the distribution of sample means, rather than sums, are normally distributed, in which case the mean of the sample distribution equals the mean of the population distribution.

In more laypersons' terms, the total weight of a box of strawberries is the sum of the weights of the strawberries it contains (subtract the weight of the box itself, and assume the same number of berries per box for the sake of argument). In the packaging facility, a large population of strawberries are continuously distributed randomly into boxes of the same size. The berries in the factory (the population) have weights that follow some distribution with limited variance – some are bigger than others, but there are upper and lower limits to how much a strawberry can weigh – and all the berries in the boxes are drawn from this same population (so identically distributed). New berries are shipped to the facility all the time, and berries are not sorted according to size but randomly mixed, so if one box by chance gets filled with only very large berries, that does not affect the weight of subsequent boxes (each box is the sum of n independently drawn sample berries X). Under these circumstances, the weight of boxes of n berries (i.e. the sum of the berries' weights, the sample distribution) will be normally distributed by the Central Limit Theorem.

Some details here are crucial: the distribution of the strawberry weights themselves will, with higher n, tend towards that of the population, which is not necessarily normal. The theorem is only valid for summary measures like sums or means, and not for the observations directly. If the strawberries at the facility are mostly large (heavy) and with only smaller proportions of small berries, this skewed distribution will be reflected in strawberry boxes of a certain size (n), but since the influence of this distribution is the same on all boxes of the same size, their overall weights (or mean weights) will be normally distributed. Furthermore, if we draw some boxes from one producer and some from another producer who has significantly larger or smaller berries, the samples are then drawn from different populations and are thus not identically distributed, and the sample distribution will not necessarily be normal (colloquially referred to as "comparing apples and oranges"). Similarly, if there is an overall trend of boxes becoming heavier over the season, then samples from across the season will not be normally distributed. If however, we compare the mean box weights from multiple seasons, these will again be normally distributed, unless there is also a multi-year trend of strawberries becoming larger or smaller. In other words, the i.i.d. condition of the CLT means that the samples are drawn at random, with no important underlying trends. As a side note, there is a substantial body of research on the conditions under which normal distributions can emerge *without* the i.i.d. condition being met (see N. L. Johnson 1994, Vol. 1:87–88 for details and further references).

Also important to note is that the number of samples X_n (boxes of sample size n in the example above) does not matter to the shape of the sample distribution, other than to the resolution of the curve or binwidth of the histogram when plotting, and the weight of a single sample can be modelled as a probability following a distribution. The normal curve of the sample distribution can be entirely defined by the population μ and σ, and sample size n. In many practical settings however, the parameter values of the population distribution (and even its type of distribution) are unknown, in which case more samples are needed in order to model a sample distribution and from there infer the parameter values of the overall population. This is the case when statistical tests like Student's t-test or ANOVA are applied for examining the relations between samples and populations.

In other cases, n (sample size) can be unknown, but assumed to be approximately the same for all X_n (samples), like in the somewhat more abstract cases where each of the n variables contained in X_n are of different nature – or stated otherwise, when the size of X_n is the sum of many different and independent causes. In the case of a normal house-size distribution within a given cultural setting (like a village), one can assume that the same number of causes affect the size each house takes when constructed, but to varying degrees (colloquially we often say *factors* for such causes, though in this setting one should strictly speaking prefer *terms*, *summands* or *addends*, since they add up rather than multiply). Say that house size in a given context results from the cumulative effects of household size, inherited wealth, soil quality, exposure to sun, wind or flooding, artisan specialisation, raw material availability in the year of construction and many more variables. Each of these may have separate probability distributions – e.g. the wealth distribution may be heavy-tailed while household size may be normal and symmetric – but as long as the overall population, so to speak, of contributing causes is distributed in the same way to all households, and that none of the variables dominates the effect of the others, and the value of each variable is independent from the values of the other variables, their sums expressed in house sizes should be normally distributed by the CLT. Such a distribution is then an expression of *random difference*, and in the case of house sizes, there would be no particular reason as to why a few houses would be bigger than all the others, just as a few houses would also be smaller, while most would be centred around the mean size. But again, if samples are drawn from *different* populations, e.g. houses from different villages or cultural contexts – where the probability distributions of the underlying variables are categorically different – the i.i.d. condition is not met, and the distribution of house sizes is unlikely to be normal. Likewise if the population contains grades, i.e. is grouped, or where the different variables are correlated between them, e.g. if wealthier households are also larger households and have more access to raw material and better quality soils, and so on, their house sizes may also become disproportionately large and deviate from normal expectations.

A final caveat for normal house-size distributions that may be mistaken for skewed ones, is the case when there are no significant differences between house sizes, except for one that has a clearly different function – as in the ethnographic cases of community houses and men's houses discussed above. Then it makes little sense to interpret the resulting slight skewness of the distribution as a sign of social inequality in itself (notwithstanding gender inequalities). As a rule of thumb, to avoid such misinterpretations, it is useful to test whether isolating the single largest house changes the retained model when performing distribution fitting.

4.2.2 Exponential distributions and constant rates of growth and decay

The exponential distribution is – next to the normal – one of the most widely applicable distribution models. In its simplest form, it is a function of a positive random variable x where some base number (usually $e \approx 2.718$, for compound continuous growth) is raised to the power of $-x$, in other words when x has the probability density

$$p(x) = e^{-x}.$$

This simple form is called the *standard* exponential distribution, and in most practical applications there will also be a *rate* parameter λ (*lambda*), so that the density function becomes

$$p(x) = \lambda e^{-\lambda x}. \tag{4.5}$$

In the case of the standard version, $\lambda = 1$ and can be left out. The negative rate in the exponent is the actual rate that determines the shape of the distribution, whereas the rate multiplier to the base is a *normalising constant* which assures that the area under the curve adds up to 1, and thus that the values shown on the y-axis are probabilities. This constant can be though of as the y-intercept of a linear model, since at $x = 0$ the function gives $\lambda e^0 = \lambda\, 1$. This is seen more clearly if we take the logarithm of the exponential density function, $p(log(x)) = log(e)\,(-\lambda x) + log(\lambda) = -\lambda x + log(\lambda)$, which is a linear model with slope $-\lambda$ and $log(\lambda)$ as y-intercept. As a rule of thumb, an exponential distribution can thus also be recognised as a straight line on a plot with one linear and one logarithmic axis. A wide variety of more complex forms have been formulated, and the distribution type is furthermore generalisable to both the Gamma and Weibull distributions (N. L. Johnson 1994, Vol. 1:494–99). Note that the simple form presented here may also have more complex notations in specialised statistical literature, *cf.* Eq. 19.1 in N. L. Johnson (1994), which is equivalent to the density function above given that $\sigma^{-1} = \lambda$ and $\theta = 0$ (see also N. L. Johnson 1994, Vol. 1:522–23; Clauset, Shalizi, and Newman 2009, 664).

Exponential distributions have the highest probability (so the most data) at low values of x, and ever lower probabilities towards the right end of the curve (Figures 4.2a and (**ref?**)(fig:04-cCDF)a), with the rate of decrease determined by λ – the higher the rate value the steeper the curve falls off from left to right, and inversely low λ values give more heavy-tailed distributions. The type of setting which is most commonly modelled as an exponential distribution, is that of "events recurring at random in time" (N. L. Johnson 1994, Vol. 1:494). If x represents the the duration in time between events (or duration of single events) that occur continuously and independently from each other, with a constant average rate of λ events per unit of time, it can be modelled as an exponential distribution. An important feature of the underlying process (a so-called Poisson point process), is that it is memoryless, meaning that the duration between events X_1 and X_2 does in no way affect the duration between X_2 and X_3, as all durations are drawn independently from the distribution with the same average rate λ. The example of such a process that may be the most familiar to archaeologists, is the radioactive decay of the ^{14}C isotope with its average decay rate $\lambda \approx 0.00012$ or 0.012% per year. With a rate that low and using years as the time unit, it is more useful and intuitive to work with the *half-life* measure, or the time it will take on average for the initial quantity to be halved, which for ^{14}C is about 5730 years. The half-life (the median of the distribution) is given by $\log(2)/\lambda$, solving for x in the cCDF function $e^{-\lambda x} = 1/2$, where the normalising constant in the PDF is replaced with 1 (and therefore omitted in multiplication) for the initial quantity or y-intercept. If we replace $1/2$ with the remaining proportion of ^{14}C in the organic material of an ancient artefact, compared to the expected amount in the same material when alive, we can use the same equation to solve for the approximate year when the organic material died, which is the principle behind radiocarbon dating. In a radioactive decay process, a total amount of individual unstable isotopes are present from the start (here the death of an organic material), and their individual lifetimes until decay are exponentially distributed. As an additional metric of exponential random variables, the mean μ or *expected value* is given as the reciprocal of λ, so that $\mu = 1/\lambda$ and $\lambda = 1/\mu$. The mean is larger than the half-life, and for ^{14}C decay, this corresponds to $\mu \approx 1/0.00012 \approx 8267$ years.

The exponential distribution can also model many processes that are closer to an everyday human scale than ^{14}C decay. Expanding from the example used for normal distributions, let λ be the average risk for a strawberry of being harvested within a week x. As the weeks go by (as x increases), the cumulative risk of being picked grows exponentially, so that there are very few berries that are older than a few weeks, and the mean age of the berries in the field is $1/\lambda$. The berries are furthermore picked by a harvesting machine that is unable to aim for a certain size category, and the berries grow linearly (which may be a rather poor approximation, but for the sake of argument). The harvested berries are also continuously replaced by new berries which start growing at the same pace, so the field is always renewed. Under these circumstances, the lifetime of a strawberry during which it grows is exponentially distributed, and the probability of surviving x weeks follows Equation (4.5). The example illustrates how exponential functions model repeated multiplication or multiplicative processes, since the x in the exponent means "multiplied x number of times". The rate λ (technically $e^{-\lambda}$ in the case of continuous decay) is multiplied with itself x number of times (keep in mind that $(a^b)^c = a^{bc}$). Multiplying the rate for each new step in time means that the value of λ is applied to the current value of x rather than to the initial value. For example, let λ be $1/5$ or 0.2, so that there is a 20% risk of being harvested within a week, and thus 80% chance of being left in the field. The expected lifetime of a strawberry is then 5 weeks, and the probability of surviving 6 weeks or more is $P_X(6) = e^{-(1/5)6} \approx 0.301$ or 30,1%, while that of surviving 7 weeks or more is $P_X(7) = e^{-(1/5)7} \approx 0.247$ or 24,7%, corresponding to a relative decrease in probability of $\frac{(0.301-0.247)\,100}{0.301} \approx 18$ %, equal to $1 - e^{-1/5}$. Note that using the rate directly as the base, rather than as exponent of e, will give the same results as a (discrete) geometric distribution, in which case the reduction would be exactly the rate between each period. The difference lies in what is termed compound and simple interest in economics. For all the purposes discussed in this thesis, the continuous exponential distribution (with $e^{-\lambda}$ as base to x) is deemed more appropriate than the discrete geometric distribution.

From an archaeological point of view, the essential point from the strawberry example above is that this process will also materialise in the size distributions of each single harvest, of all the strawberries at the depot, as well as the strawberries in a finished box for sale, all of which will be exponentially distributed (unless there is some additional sorting process involved). Of course, this does not change the fact that the box weights, as well as the mean strawberry size per box will be normally distributed, as previously shown. When it comes to house sizes, several scenarios involving growth could explain an exponential distribution. Let house size (x) be directly dependent on household size, so that each inhabitant has a constant average number of m^2 of roofed space. Say that the households grow exponentially at some rate, that is they increase in size by a factor of λ_1 each cycle of some unit length (y). At the same rate, the households also extend their houses proportionally to their growth, or replace their house altogether with a bigger house, and lastly, for each new cycle a new household is added to the village with newcomers, so that the total number of households follows y linearly. If all new households start at some minimal size x_{min}, the size x of a house after y cycles will equal $x_{min}e^{\lambda_1 y}$, and we can solve for its age (time since establishment of the household) as $y = \frac{\log(x)-\log(x_{min})}{\lambda_1}$. Here I use x_{min} for θ in N. L. Johnson (1994), p. 494, following Clauset, Shalizi, and Newman (2009), p. 664. In other words, this context would generate an exponential house-size distribution of coeval houses in a village, where the largest houses are those of the households which were the first to settle in the village.

This model is of course not very realistic. For example, households cannot grow without limit, so the larger they become, the higher the probability that they split into two or more factions (see Alberti 2014 and Johnson 1982, and Section 2.1). The splitting of households itself can in fact also generate an exponential distribution. Let the households in a village grow linearly – say they each have a constant surplus of 1 person per year (persons who arrive or are born - persons who leave or die = 1) – but they also run a risk of λ_2 (e.g. of 5%) of disintegrating and being replaced by a minimal sized household (x_{min}) each year (y), no matter their current size. House sizes are then linearly correlated with their age (or time since establishment of the household) so that $y = x - x_{min}$ (disregarding the constant of m^2/inhabitant). However, over time, the probability that a household continues to exist without splitting, will decrease exponentially, and we can write the survival function for households (and therefore their sizes) as $P(X > x) = e^{-\lambda_2 y}$, or $e^{-\lambda_2 (x - x_{min})}$.

But again this model is not very satisfactory, particularly since it assumes purely linear population growth, which is unlikely in most cases. It would also seem likely that the probability of splitting of households would not be the same across the range of sizes, but rather be concentrated around some upper threshold, determined by the social structure between the inhabitants or by material or ecological constraints (or a combination). Common for both of the models above, is that one aspect is well described as exponential, but this aspect is only one out of many in the complex process which may underlie the house-size distribution of a settlement. Intuitively, it would also seem strange to have the whole range of houses in a settlement to scale exponentially, since it would imply that single house sizes would be ever closer and closer to each other all the way down to the smallest house in the village. Adding the x_{min} parameter does change this situation though, as it could then apply to cases within some upper class in which household size and/or wealth would increase to some fixed rate over time, not affecting the size of the main part of the settlement's households, which could well be normally distributed. Or the two models above could be combined to one, pulling exponentially in opposite directions (note that the two rates, λ_1 and λ_2 are positive and negative respectively). In both cases, the resulting house-size distribution would no longer be exponential, and these common combination distributions – the log-normal and the power law, both of which are heavy-tailed – are presented in more detail below.

Another issue with exponential distributions resulting from growth or decay in time, from the archaeological point of view, is that the time-averaging that so often infiltrates our analyses because of the difficulty of distinguishing temporally coeval data sets, may very well influence the observed data distribution. This influence is much more challenging to evaluate theoretically however, so in this thesis it is instead addressed empirically through simulation in Section **??**.

4.2.3 Log-normal distributions and Gibrat's law

One of the main candidate models for heavy-tailed continuous distributions is the log-normal, defined as a variable x of which the logarithm is normally distributed. Adapted from Equation (4.1), its density function can be written (following the notation in Mitzenmacher 2004, 229) as

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-(\log(x)-\mu)^2/2\sigma^2}. \tag{4.6}$$

The μ (mean) and σ (standard deviation) parameters are usually understood as the equivalent values associated with the normal distribution of $\log(x)$. It can be thought of as a combination of the normal and the exponential distributions, and like these, it has been shown to apply well to a wide range of natural and social phenomena, from the growth of organisms in biology to the pricing of options in finance N. L. Johnson (1994). One implication of $\log(x)$ being normally distributed, is that the density curve of x will appear as a normal bell curve when plotted with a logarithmic scale on the x-axis (Figure 4.5). With linear scales, the curve is skewed with the mode to the left and a tail of high values to the right. More technically, the (natural) logarithm of a variable (x) is the variable of exponents that may raise e to the values of x. A linear increment in a variable of exponents – say from 1 to 2 – will, with the same base, correspond to an exponential increment in powers (the result of exponentiation), as $e^1 \approx 2.718$ and $e^2 \approx 7.389$. Thus, if the exponents of e that correspond to the values of x are normally distributed, then x itself will resemble an exponentially stretched normal distribution, which is a log-normal distribution.

Since exponents represent repeated multiplication and normal distributions result from random additive processes (see above), log-normal distributions may be most easily understood as resulting from random multiplicative processes. The product rule of logarithms states that the logarithm of a product of numbers equals the sum of the logarithms of those same numbers. This can be expressed as $\log(ab) = \log(a) + \log(b)$. Then, since the sums of many random numbers are normally distributed according to the Central Limit Theorem, the logarithm of the product of many random numbers (that is, these numbers multiplied together) should also be normally distributed (see e.g. Newman 2005, 347–48 for more elaboration).

The product of many random numbers is typically the result of a growth or decay process where the rate fluctuates randomly. A convenient example, following Newman (2005, 348), is that of a financial investment. If an initial value (a) is invested in stocks that generate a return (e^λ) which fluctuates randomly from year to year with a finite variance, the return y after x years will follow a wiggly exponential curve, or $y = ae^{\lambda x}$ (Figure 4.4). After some years, the value of y will follow a log-normal probability distribution (Figure 4.5). If several persons start investing the same amount in stocks at the same time, then after a period, say of 10 years, most of them will have earned returns of comparable size, centred around some mean return, while the earnings of the top investor may be several orders of magnitude higher, simply by chance. This assumes however that everyone invests randomly in the stock market, which is rarely the case. More scrupulous investment strategies may reduce the effects of chance and thus the spread of final returns, but this effect may again be countered by the risk-willingness of investors. In either case the resulting distribution after a given time period will be log-normal, which explains why this model is a central tool in financial analyses (e.g. see Mitzenmacher 2004, 236 for its use in the Black-Scholes option pricing model).

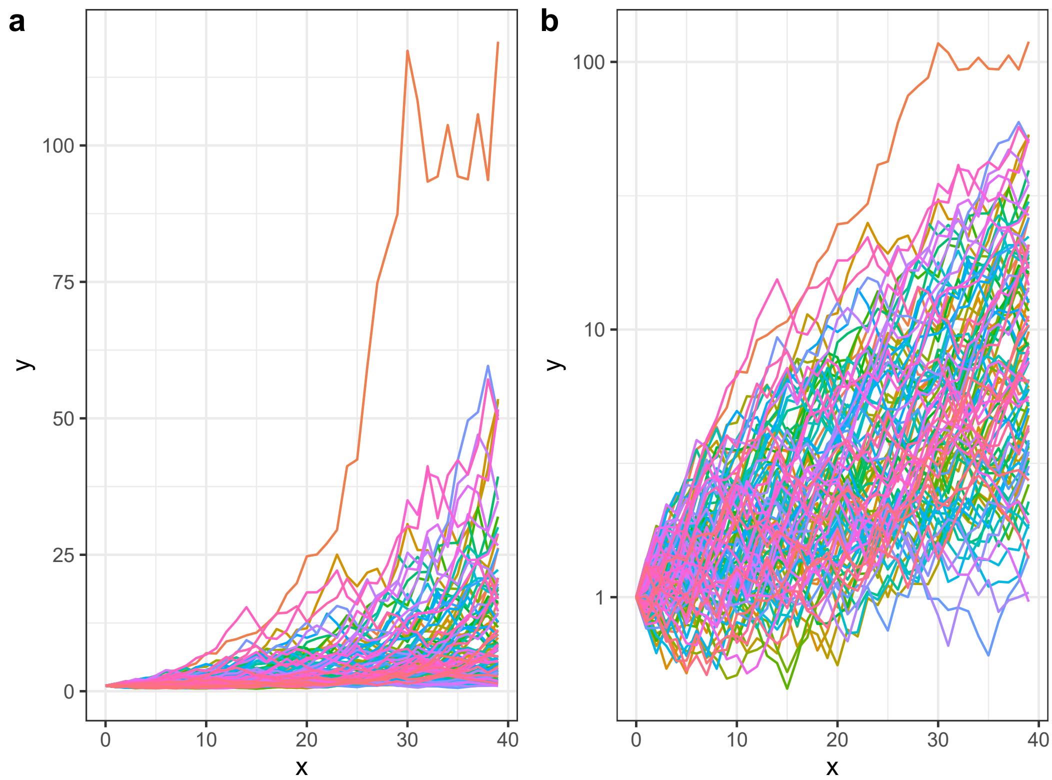

Figure 4.4: *Exponential distribution of* $y = \lambda^x$ *with rate (*λ*) fluctuating randomly and uniformly between 0.75 and 1.4, i.e. with a mean rate of approx. 1.08, over 40 periods (*x*) from an initial value of 1. The plots figure 100 individual runs of the distribution, with linear (a) and logarithmic (b) y-axis. Over time, y-values at any given x are expected to be log-normally distributed by the Central Limit Theorem*

This process is known as *Gibrat's law*, after the French engineer Robert Gibrat who was the first to demonstrate its wide applicability, offering a mathematical explanation of the skewed size distributions that had frequently been observed by economists (Gibrat 1930). Gibrat argued that the log-normal distribution was better fit for modelling firm sizes and salaries than the one already proposed by Pareto (i.e. the power law, see below), since it could account for the entire size range and not only the tail, and since it was theoretically better founded, as

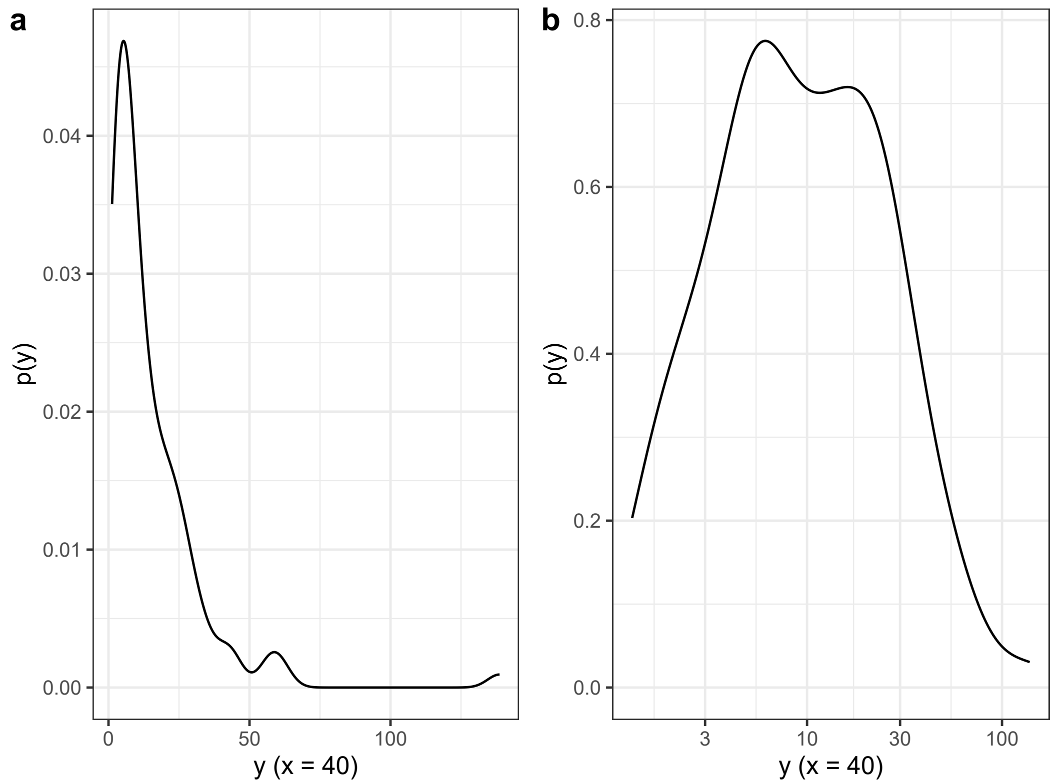

Figure 4.5: *Density function of y-values from Figure 4.4 at* $x = 40$*, with linear (a) and logarithmic (b) x-axis, following a typical log-normal distribution*

Pareto's original model was purely empirical. He termed the model the *law of proportionate effect*, since it emerges – as shown above – when proportionate growth rates are independent of absolute size. This essentially means that growth is exponential (size at any given time is multiplied with a rate, so proportional) rather than linear (additive), and that the range of possible rates is not determined by absolute size. Note however that there must be some randomness in the rate for the growth to result in a log-normal distribution. If the rate is *exactly* the same for all samples, or if it follows the exact same sequence of random rates, the initial distribution will remain the same over time, even though any initial spread will be scaled up or down according to the rates (Figure 4.6). In most economic settings, though there are overall trends that may affect everyone in a population (of firms, employees, cities etc.) at large scales, there are also many smaller factors that will affect individuals differently, causing random variation.

The modern financial market is of course not directly applicable to the Neolithic, but a number of multiplicative processes involving random fluctuations may also be relevant to Neolithic social structure or economy. One obvious example would be that of crop yields over time. Assuming that crop cultivation in a village is organised at a household level, and that house-

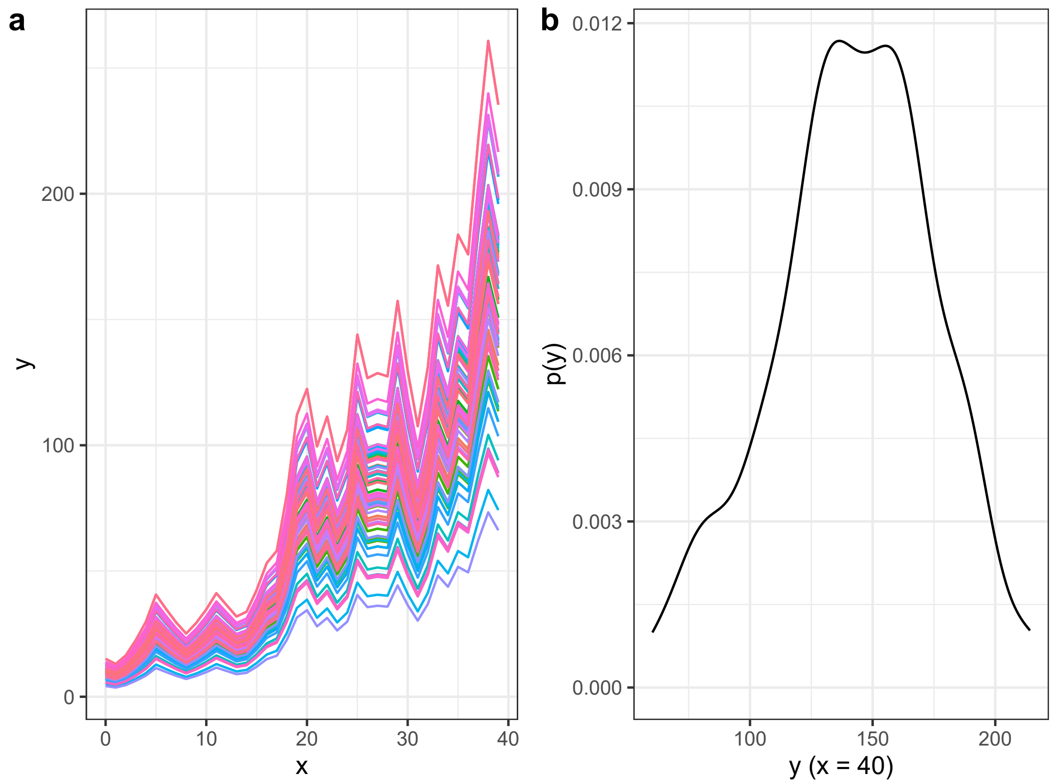

Figure 4.6: *a: Eponential growth of 100 samples drawn from a normally distributed initial population ($\mu = 10$, $\sigma = 2$), all following the same sequence of uniformly distributed rates ($0.75 < \lambda < 1.4$). b: Over time, μ and σ values change, but the distribution remains normal. Scales are linear*

holds grow their crops at separate locations in the vicinity, random differences in soil quality, sunlight, water, exposure to disease and so on, would arguably generate normally distributed yields in one year (the yield volume being the sum of many random effects). But given that the yield the year after also depends on the current yield through the size of the surplus that will be available for sowing, a randomly large yield one year will have better chances of producing an even larger yield the next year, and so on, in the same way a large financial return of a lucky investor one year is more likely to reach an even higher return later if reinvested. Supposing again that house size reflects household size linearly, and that larger yields can sustain larger households, this process could explain the emergence of a log-normal house-size distribution in a village over time.

For Neolithic crop cultivation it is in many cases more reasonable to assume that cultivation took place very close to or even within the village, in which case the conditions for yield volume would be very similar between households. Cultivation could also be organised collectively rather than at the single household level, or as a combination depending on the crop.

In such cases, a good or a bad harvest would affect all households equally, and skewed distributions would not emerge easily. However, even in such scenarios, small random differences in the initial sowing volumes of individual households could grow exponentially over time and produce a log-normal distribution of household yields, though with smaller spread between the highest and lowest values.

A somewhat different mechanism for explaining skewed house-size distributions within Linear Pottery settlements in particular, was proposed by Sara Schiesberg (2010). Assuming a stable population over time, with number of children per woman surviving to reproductive age being Poisson distributed with $\lambda = 2$ (with ever decreasing probability of larger numbers of surviving siblings), and the probabilities of having given ratios of male and female children following a binomial distribution, the combined probability of having x male children surviving to reproductive age would follow a skewed log-normal-like (though discrete) distribution. Schiesberg observed this theoretical distribution of male siblings to be analogous to the empirical house-size distribution of excavated Linear Pottery settlements on the Aldenhoven plateau in North Rhine-Westphalia, Germany, arguing that there were gaps in the continuous size distribution that fitted with the discrete limits between numbers of male siblings. The correlation was furthermore explained as a result of a mainly patrilocal residence pattern, where house size would be a function of number of sons in an extended family (patrilocality being the most widely accepted residence pattern for the Linear Pottery culture, see Section 3.2). The same pattern could have emerged as a function of number of daughters in the case of matrilocality. Though Schiesberg did not model the observed house-size distribution explicitly as a log-normal, her model and the underlying process closely resemble the Gibrat's law described above. A (discrete) Poisson distribution with low rate can well approximate a (continuous) exponential, and a (discrete) binomial can approximate a (continuous) normal – so a combination of the two (by multiplication of exponential and normal probabilities) will equally approximate a log-normal. Thus, simply by the social practice of patrilocal post-marital residence, a skewed house-size distribution would emerge spontaneously, without the presence of any additional structural inequality between community members. In this case, a log-normal distribution would be present in a settlement from its onset, and the skewness could or could not become more pronounced over time, depending on other factors like whether crop surplus would be distributed within the community or kept within households.

A last mechanism that is sometimes referred to in statistical literature as causing log-normal distributions, is that of random additive processes involving variables that by their nature cannot take on negative values, such as weights, heights or densities of physical entities (e.g. N. L. Johnson 1994, Vol. 1:239). While (two-parameter) log-normal distributions only can have positive values (N. L. Johnson 1994, Vol. 1:208), normal distributions will often also have positive probabilities below zero, depending on μ and σ values, and are then poor models of such quantities. A house, as an example, cannot have negative size, but will always be larger than some lower threshold above zero, and should thus also be more adequately modelled as log-normal than normal, even if the distribution looks symmetrical. This will in turn allow for more accurate estimates of other derived parameters, like the confidence limits for the coefficient of variation. However, in cases of symmetrical distributions where μ is much higher than σ, there is little practical reason to prefer a log-normal model over a normal one, as the probabilities of values below zero will be infinitesimally low.

4.2.4 Power-law distributions, preferential attachment and hierarchy

A variable x is power-law distributed when its probability follows the power of itself with a fixed exponent α, so that $p(x) \propto Cx^{-\alpha}$ (Clauset, Shalizi, and Newman 2009, 662). Such distributions decrease very quickly as x increases, and are thus highly skewed, but the probability never reaches 0 – it is said to be *asymptotic* – meaning that they are also very heavy-tailed (Figures 4.2a and 4.3a). Furthermore, a power law can only take positive values, and there is always a minimal threshold $x_{min} > 0$ above which the function holds. The exponent α, often termed *scaling exponent* or *scaling parameter*, will usually lie in the range $0 < \alpha < 3$, though values below 1 are considered rare special cases, when considering size or frequency distributions (Newman 2005, 331–32). Low exponent values give more heavy-tailed distributions and *vice versa*, so power laws with a high scaling exponent (around 3 or above) are those that in practice will be more easily mistaken for other less skewed distributions like exponential or log-normal distributions (Figure 4.7. As with the previously discussed distribution models, the power law is also usually associated with a normalising constant (here denoted C), a factor that ensures that the area under the curve of the PDF sums to 1, and which here is defined as $(\alpha - 1)x_{min}^{\alpha-1}$ (Clauset, Shalizi, and Newman 2009, 664–65). Applying the product rule of exponents, the power-law PDF or density function can be expressed as

$$p(x) = \frac{\alpha - 1}{x_{min}} (\frac{x}{x_{min}})^{-\alpha}. \tag{4.7}$$

In the cCDF or survival function, again the normalising constant is replaced with 1, but rather than left out it is written in the exponential form $(\frac{x}{x_{min}})^1$ which equals 1 when $x = x_{min}$, so that

$$P(x) = (\frac{x}{x_{min}})^{-\alpha+1} \tag{4.8}$$

in the notation of Clauset, Shalizi, and Newman (2009), Eq. 2.6, equivalent to the more complex notation in Newman (2005), Eq. 4.

As shown in Figures 4.2b and 4.3b above, power-law PDFs and cCDFs hold the special property of appearing linear when plotted with logarithmic x and y axes (or equivalently when x and y values are log-transformed). This can be shown by log-transforming the simple functional form above, so that $log(p(x)) = (-\alpha)log(x) + log(C)$, which is a linear model with y-intercept $log(C)$ and slope $-\alpha$. Power functions are in this way similar to exponential functions, with the difference that the variable x is here in the base rather than in the exponent, causing the graph to be linear only when *both* axes are logarithmic (as opposed to one axis for exponential functions). Because of this property, the most common method for estimating α since the first formulation of the model and until recently (Clauset, Shalizi, and Newman 2009; Stumpf and Porter 2012) was to plot the data on logarithmic axes, perform a least squares linear regression and estimate the slope (see Chapter 5).

The main reason for the great attention that has been given to power-law distributions over the years (see below), is its property of *scale invariance*, meaning that the distribution will appear the same no matter the scale in which it is being observed (see chapters 1 and 2 of G. B. West 2017, 1–78 for a general non-technical introduction to scaling in science). Figure 4.7 shows the relation between change in the variable being modelled (x) and its function value (here the PDF function) as a constant proportion of exponents (i.e. a linear relation between two vectors of logarithms). This means that for a power-law distributed variable, a change of scale (zooming in or out on the x-axis, changing order of magnitude no matter the base number) will always result in a proportional change of order of magnitude of the probability or frequency modelled on the y-axis. If x represents a size related variable, and y a frequency (probability in the PDF, rank in the CDF, or absolute frequency), this proportional relation in logarithms generates a self-similar hierarchy, where the same structure of sizes and frequencies is repeated across different scales (Figure 4.8). Thus, even though fractal structures are most often associated with geometric shapes in physical space – like the shapes of plants, rivers or clouds – they can be equally present in non-geometric variables, like income or wealth, magnitude of events, sizes of cities or nodes in networks. Such structures are, as with physical fractals, recognised by their abrupt skewness or high level of inequality, by their hierarchically distributive mode of functioning, and their statistical signature which is the power law. This connection between power-law distributions in general and fractals was first recognised and expanded upon by Mandelbrot (e.g. 1997; short summary in 1982, 341–48).

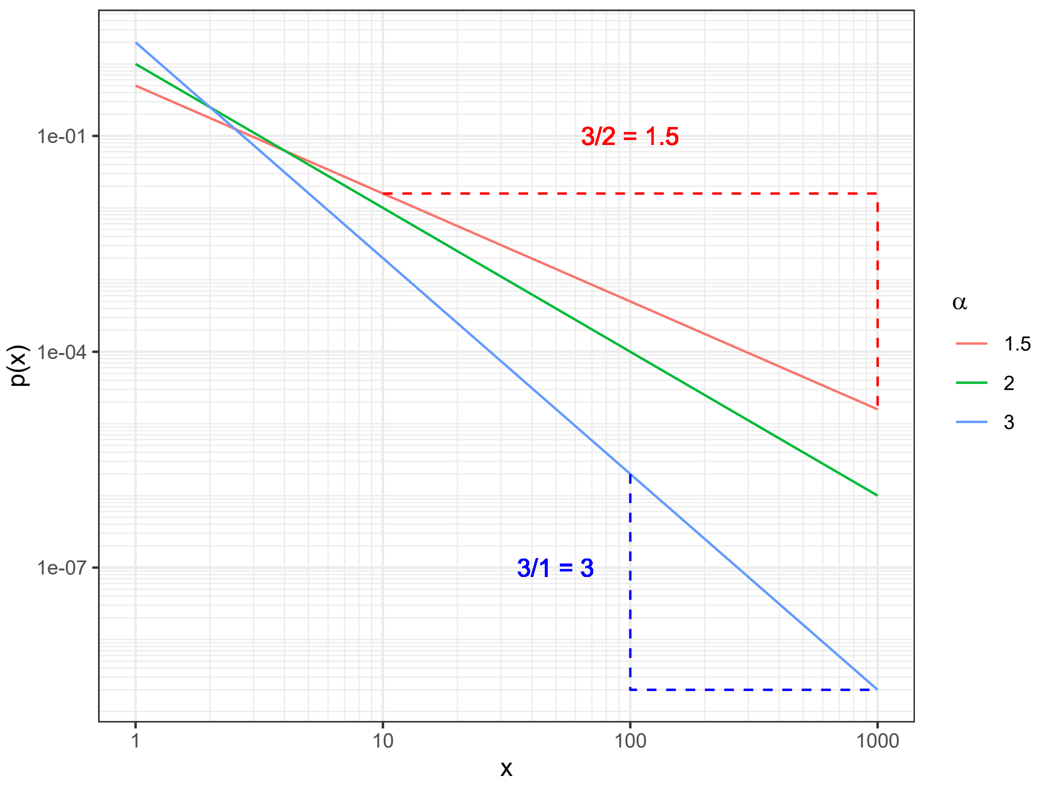

Figure 4.7: *Examples of power-law distributions with different scaling exponent (α) values on logarithmic axes, showing how this parameter reflects change over orders of magnitude. For a model with $\alpha = 3$ (blue), a decrease in probability p(x) of 3 orders of magnitude (powers of 10), e.g. from 0.1 to 0.0001 corresponds to an increase in the size of x of 1 order. For a model with $\alpha = 1.5$, the same decrease in probability corresponds to an increase in x of 2 orders of magnitude. The models appear linear in logarithmic space, but are in reality highly non-linear, as illustrated by the grid*

As already mentioned, the power-law distribution was first formulated by Italian economist Vilfredo Pareto as a model of wealth inequality (1896). Over the 20th century it saw an in-

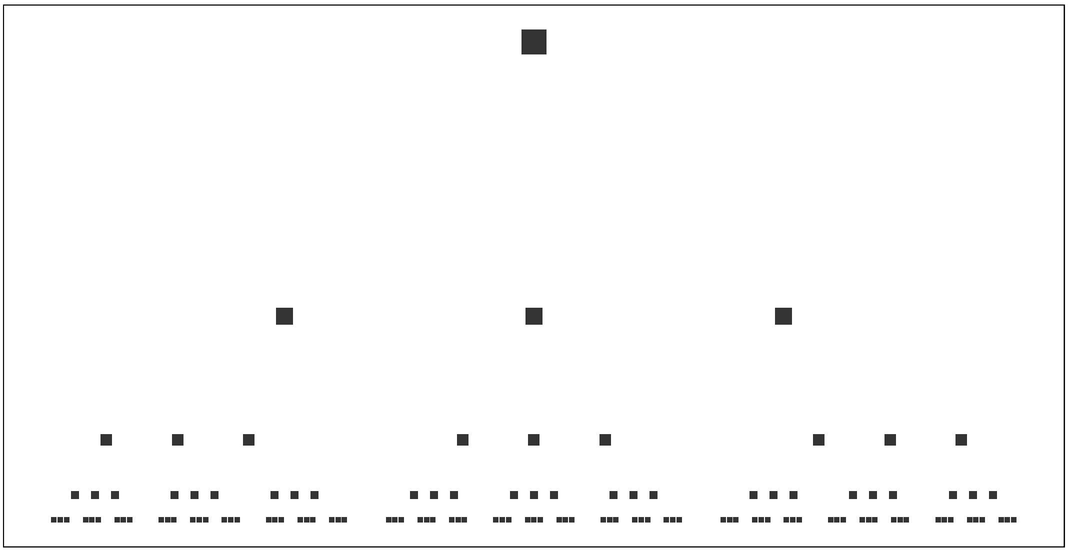

Figure 4.8: *A power-law distribution of sizes arranged in discrete levels, illustrating its characteristic scale invariance. From the largest element on top and downwards, sizes decrease while numbers (frequencies) increase, both exponentially but in opposite direction, generating a hierarchical fractal structure where the same shapes are recognised at different scales*

creasing number of applications in a wide variety of fields in natural and social sciences, modelling phenomena from magnitudes of earthquakes and sizes of moon craters to the intensity of wars, frequency of family names and citations of scientific papers (see Mitzenmacher 2004; Newman 2005 for detailed overviews). The idea that the power law could be a suitable model for city sizes was seemingly first proposed in a short paper by Auerbach (1913), though it is often attributed to Zipf (1949), through which it has become known to and sometimes applied by archaeologists (see Section 4.3). The first influential attempt at an explanatory model of power-law distributions – which at first were primarily descriptive, one of the main critiques against Pareto – was made by Udny Yule (1925), who seeked to explain the observed size distribution of genera by number of species. Yule's model (here following the notation in Newman 2005, 340–42) involves a set of n genera consisting of a variable k number of species each. During a discrete time step interval, a constant $m < n$ number of new species are added to the existing genera by speciation, so that some but not all genera will grow to $k + 1$ in each time step. The probability for each genus of receiving a new species in a given time step is proportional to k or the number of species it already includes, since a speciation is more likely to happen in an already large group of species than in a small group. Finally, for each time step one new species is sufficiently different to be considered a new genus on its own, so that n increases linearly by 1 for each step. Under these conditions, over time k will follow a heavy-tailed distribution with a power-law tail – or strictly speaking a discrete version of it known as the Yule distribution (see Simon 1955).

The precise mathematics involved in this model are quite complex for non-specialists, especially in the version presented by Yule before modern stochastic theory was developed, but even more recent and concise formulations will involve some level of calculus (Newman 2005; Mitzenmacher 2004, 230–33). The essential is however to note that the mechanism it describes – now known as *the Yule process* – is relatively simple and transferable to many natural as well as social settings. The heavy-tailed distribution of species will emerge even in the simplest scenario starting with one genus n_1 consisting of a single species ($k = 1$), and one added species to existing genera per time step ($m = 1$). Then the single genus will have probability $p(n_1) = 1$ of receiving the new species m so that it gets $k = 2$, while the new genus n_2 starts at $k = 1$. In the next time step a new species will be given to one of these two genera, but n_1 has twice as high probability of receiving it as n_2 – that is, probabilities are $2/3$ and $1/3$ respectively. In other words, the probability for any genus n_i with k_i species of winning the round (so to speak) and being attributed with the extra species, is given by $k_i/\sum k$, or the fraction of the total amount of species (in the given time step) that the genus already has. The main difference here with the above described Gibrat's law – which also involves proportional growth – is that in the Yule process the growth is not distributed evenly across the system. There is an additional selecting process that over time gives more to those that already have, thus according a disproportionate advantage to anyone who gets even the slightest advantage by chance from the offset – which is why this is often referred to as a *rich-get-richer* process when applied to economics.

In fact, the Yule process having been recognised more or less independently within a number of disciplines over the years, it has come to be known by a plethora of different appellations, sometimes hiding the fact that they describe similar underlying mechanisms. The term *feedback loop* is derived from acoustics, describing the bothering situation when the sound from an on-stage monitor feeds into the microphones and back to the monitors, and so on, thus very quickly generating a sound so strong and high that it overturns the system. The sound going into the microphones is proportionally amplified to the volume of the sound coming out of the monitor, so that the stronger the input, the stronger the output and by consequence the next input, and so on. Another term, derived from attempts in sociology to explain the power-law distribution of citation frequencies of academic papers, and more generally the rewarding systems in academia, is the *Matthew effect*, alluding to a passage in the Gospel of Matthew (25:29): "*For unto every one that hath shall be given, and he shall have abundance; but from him that hath not shall be taken away even that which he hath*" (Merton 1968). A typical manifestation of such a process is when a prestigious research grant is attributed to a researcher based on academic merit, whereupon this in turn opens up numerous doors allowing for even higher career achievements, while the second best candidate, altough having arbitrarily close merits to the first before the grant, afterwards will have disproportional difficulties of following their pace in career development. For paper citations, one analogous explanation is that a paper that already has many citations, is more likely to be found in literature searches and be cited again than a similar paper with less citations (Newman 2005, 341). In other words, the probability of a new citation is proportional to the citations the paper (or its author) already has, but disproportional to the quality of the paper when compared to other papers of similar quality. The equivalent of the Yule process that has possibly received the most attention within social sciences since the turn of the millennium, is that of *preferential attachment*, first defined in a widely discussed study in network analysis by Barabási and Albert (1999). They were the first to identify scale-free networks of links between websites in the then young World Wide Web, where a few sites were reported to have very high numbers of links to them, while the vast majority of sites only have very few. Their explanation of how such networks emerge – strikingly similar to Yule's explanation of speciation in biological genera – was that as new websites are made and the internet grows, these will tend to link (or preferentially attach) to existing websites that already have many links to them. It should be noted however that around 2010 there was a significant shift towards a more rigorous methodology that would be expected to accompany claims of power-law distributions in empirical data, leading to many previous claims being either weakened or fully rejected [Clauset, Shalizi, and Newman (2009); Stumpf and Porter (2012); Chapter 5].

A range of other more or less related power-law generating mechanisms and processes have been proposed (Newman 2005; Mitzenmacher 2004). Those that specifically relate to the magnitudes of events distributed in time may be of particular use in archaeology, though they have seemingly received little attention so far. Among these are approaches focussed on phase transitions and critical phenomena, like *self-organised criticality* or SOC (Bak, Tang, and Wiesenfeld 1987; Bak 2013). This process models dynamical systems that continuously grow up until a certain critical point, at which it self-regulates downwards to stability through a collapse that is power-law distributed in size and frequency, before resuming the growth process again. The classic illustration of this model is that of a sand pile with a continuous addition of grains on top of it. Once the pile grows to its critical point where its slope becomes too steep, a sand avalanche is triggered and the pile stabilises again. The vast majorities of these avalanches are expected to be small in scale, but from time to time much larger avalanches appear. As the pile grows bigger, the critical point is also gradually raised, since a larger system can generally tolerate larger stresses before it needs to self-regulate. According to the theory behind such systems, the location of the critical point can be predicted with some accuracy, but the magnitude of the stabilising event that occurs when the system reaches this point, is impossible to predict beyond the power-law probability distribution it follows, i.e. that most events will be small but there is also a chance they will be several orders of magnitude bigger. The magnitude of the event is determined by the exact grain of sand in the pile that is the first to yield and thus triggering the chain reaction, but the scale of the potential consequences for the different grains varies greatly. The mechanism is said to be highly sensitive to initial conditions, or *chaotic*, similarly to how negligible initial differences in paper citations or website links over time can lead to disproportionally large differences, while it being virtually impossible to predict at the onset exactly *which* paper or website will come out on top Gleick (1987).

There are of course many ways in which the phenomena described here can be transposed to prehistoric social settings (see Section 4.3 below for examples of how this has been done). One important aspect of social processes that are known to generate power-law distributions, is that they often involve some sort of repeated competition, where actors who win once are more likely to win again later, thus cumulating their advantage. When the variable being modelled is house size, with the underlying assumption that this is a material reflection of wealth and/or power, and the distribution is recognised as a power law, the perhaps most straightforward interpretation is that of the emergence of a social hierarchy. The temporal process of this emergence could look something like the following. Let there be an initial population of households living within a social system that for this purpose can be described as egalitarian (that is, ignoring factors like gender inequality, random differences between households, heterarchical differentiations between kin groups etc.). If, for any reason, a competitive dominance process is triggered, a household that initially holds a random slight advance over others may gain a dominant position over them, as clan leader or chief household. The leader or leading house may then gain certain privileges and duties related to tribute and redistribution of goods, thus strengthening the ties of dominance between them and the group of dominated households. There is however an upper limit to how many households that can be effectively dominated by a single household, so this relation would be unlikely to define the entire population of households. On the other hand, the same process could play out throughout the population, generating a set of leaders or leading houses each dominating a similar group of households – a situation that could be compared to the *house societies* described by Lévi-Strauss [-Lévi-Strauss (1982a); Section 2.1]. If the social, and by extension political, competition continues, it will play out between these house or clan leaders, so that the one that by chance has an arbitrarily small advantage over the others may gain dominance over all of them and thus also their dominated households, and so on up to state societies (e.g. Earle 1997; A. Johnson and Earle 1987). As with the examples of the Yule process and preferential attachment discussed above, at any level of competition – e.g. between chiefs to become king – actors who find themselves further down the hierarchy, like clan leaders, have disproportionally lower chances of reaching that level, though in theory it is not impossible. Furthermore, the power-law relationship between leaders at different levels is recognised in that from any level to the next the number of leaders decreases exponentially while their dominance (the sum of households they each dominate) increases exponentially. The fact that most households remain at a bottom level with more random (*sensu* normal or log-normal) differences between them, does not change the fact that the leadership structure is hierarchical and power-law distributed – meaning that the entire distribution of dominance in the population has a power-law *tail*, as expected for most empirical settings.

There are some critical issues with such a model of the emergence of social hierarchy. Firstly, if house sizes are only *symbolic* materialisations of dominance – meaning that they do not in practice need to be large enough to actually fit all the people they dominate hierarchically – the distribution of these house sizes would have a much shorter spread than what is expected in systems where there is a more concrete physical flow between elements, like oxygen in blood vessels. There is no natural proportion between decision-making power and square metres of floor space, and it seems likely that absolute house sizes in practice would be more constrained by building materials and techniques, which are also subject to change over time. Furthermore, several authors mention as a rule of thumb that a power-law tail should span at least two orders of magnitude in order to be accepted (e.g. Stumpf and Porter 2012, 666; Brown and Liebovitch 2010, 53). For a settlement where the bulk of houses are around 50 m^2 on average this would imply that the largest house (in a hierarchy of largest houses) would need to be in the order of 5.000 m^2 – comparable to the Pantheon in Rome or the Hagia Sophia in Istanbul – which, in addition to effectively excluding any prehistoric context, also seems as an unnecessary strict requirement for recognising hierarchy through house sizes. It is easily conceivable that smaller size differences could equally well be perceived as expressions of large social difference, e.g. if houses that are double the size of the average house (so 100 m^2) belong to clan leaders and double that again (200 m^2) to the chief. In that case, the size distribution would be more difficult to distinguish from a log-normal one, and if still interpreted as a power law, the scaling exponent (α) should be expected to take on a value considerably higher that what is usually expected for natural systems.

Another issue in need of empirical investigation, is how large a social hierarchy needs to be before it can be recognised as a power law, or over how many levels it needs to span. The rule of thumb of an interval of two orders of magnitude could be interpreted as two levels of scale (the base number of 10 is of course entirely arbitrary), which in social terms could translate to at least a complex chiefdom (i.e. a system of commons, intermediate chiefs and a chief of chiefs, A. Johnson and Earle 1987). A system with only a single leader or leading house per settlement as described above, could potentially not be recognised as hierarchical within a distribution fitting framework, even though it may have been lived and felt very much as a hierarchy for the people involved. A large social hierarchy will furthermore often span all settlements across a geographical area forming a settlement hierarchy, in which case studying house-size distributions within single settlements may be misleading. The overall scale of the hierarchy – whether it concerns houses in a settlement or settlements in a regional polity – is best understood from the largest element, and if this is missing from the data for whatever reason, interpretations in terms of social system will tend to underestimate the scale at hand. Luckily for archaeologists however, the largest settlements, as well as the largest houses, are usually the ones that are both best preserved and the easiest to discover, so that more often it is rather the lower end of the distribution that suffers from missing data, which has less importance to the interpretation of hierarchy. It should also be kept in mind that in many – maybe most – hierarchical social contexts, the hierarchy may be highly organic and volatile, and there does not always need to be any discernible discrete levels. I would argue that even in social systems where discrete levels of hierarchy are clearly defined – like in the feudal system of medieval Europe – the hierarchy expressed through the material culture of the involved actors does not always need to tell the same story as the social hierarchy expressed through their titles of nobility.

It should also be noted that there are social situations that do not involve competition of power but that still can generate power-law size distributions. Most importantly is what can in a sense be seen as the opposite of the top-down hierarchy model described above, namely a bottom-up hierarchy, which structurally speaking can be very similar, or even exist as part of the former in a continuous dialectic tension (Furholt, Grier, et al. 2020). Anarchic or democratic societies can form complex (i.e. multi-scale) community structures based on clan or kinship structures (Hamilton et al. 2007; Haude and Wagner 2019, 89–100), and if these structures are materialised in communal houses devoted to political assemblies or rituals, there is little reason to assume that the house-size distributions of such societies should be distinguishable from those of more top-down hierarchical contexts. An archetypical (pre-modern) example of such hierarchically structured democracies is the Iroquois League in the Great Lakes region during early European colonisation (e.g. Haude and Wagner 2019, 127–31; Graeber and Wengrow 2021, 481–92), but contemporary Western bureaucratic democracies are also highly hierarchical (De Landa 2006, 67–91). In any such cases, distribution fitting can allow for identifying the hierarchical structure, but not the actual type of government and to which extent its authority is based on bottom-up or top-down legitimacy. This qualitative aspect must still be investigated through other strands of evidence, like find inventories and symbolic representation within the large buildings. Building size itself will play a different role in democratic systems compared to autocratic ones, though the outcome will often be similar. In the former, communal buildings are by their very nature meant to be accessible and used by large parts of the community, and will therefore often need to be larger than common houses, while in the latter, leaders and high ranking people will be motivated to build houses for their private use that are larger than what they actually need, simply as a means to express their authority. In both cases the hierarchy is materialised in the house-size distribution, even though the absolute relationships between size and social importance may be culturally contingent. Exploring such specificities with hierarchical scaling in material culture has been one of the main motivations for this thesis.

In sum, the important characteristics of power-law generating systems, are that they are *complex*, meaning that they function or operate over several *scales* or orders of magnitude, which again implies that they exhibit *self-similarity* and can be described as *fractals* or *hierarchies*. Furthermore, they are rarely consciously planned, but rather tend to *emerge* spontaneously with growth, as a result of self-regulation and iterated responses to growth, like splitting or feedback that are proportional to size. Finally, even though such systems can be described as fully deterministic, they are also highly sensitive to initial conditions, i.e. they exhibit *chaos*, so that the magnitudes of single outcomes or trajectories can in practice be impossible to predict or forecast into the future, while the system as a whole can be well described, as well as explained backwards in time. Evidence of power-law distributed house sizes should be considered as a clear sign of a hierarchical social structure, though the exact nature of the structure – and specifically whether it is autocratic or democratic, or something in-between – needs to be argued from complementary evidence.

4.2.5 Some variants of power-law distributions

A note must be made regarding terminology on power laws and some closely related distributions. Contrarily to normal, log-normal and exponential distributions, power-law distributions are not defined in a uniform way across all the disciplines where they are applied. Despite all claims of ubiquity, they remain special cases in many contexts, are not systematically taught in basic introductions to mathematics or statistics, and their broadened understanding seems to have suffered from long-standing discipline-specific traditions of defining them. Archaeologists who borrow theory and method from different fields therefore run the risk of talking past each other and not seeing the bigger picture of common phenomena described in different ways. Two alternative and more or less parallel ways of describing power-law distributions run under the names *Pareto* and *Zipf distributions*, after the first researchers who became known to define them under their specific parametrisations.

The so-called Pareto distributions form a group of distribution types with varying numbers of parameters, and are most often associated with applications in economics and finance (Pareto was an economist and his major works were on wealth and income distributions, see Chapter 20 in N. L. Johnson 1994; Pareto 1896). Comparing with the power-law definitions given above (Equations (4.7) and (4.8)), it is important to note that Pareto distributions are defined as survival functions, i.e. cCDFs (complementary cumulative distribution functions) or the proportion of the distribution that is *higher* than a sample X. The scaling exponent for a Pareto distribution (N. L. Johnson 1994 confusingly note this as a, p. 573, but here I prefer to use β for clarity) thus has the value of the power-law exponent $\alpha - 1$, or in negative values $-\beta = -\alpha + 1$, meaning that Pareto plots have a less steep slope than their power-law counterparts. Accompanying their use in applied economics is the appellation of the "80-20 rule", a rule of thumb stating e.g. that in a company 80% of sales will typically go to 20% of clients as a result of the very skewed distribution. This specific rule corresponds to a power-law distribution where $\alpha \approx 2.1$ or $\beta \approx 1.1$ (Newman 2005, 334).

Zipf law distributions, often referred to as "rank-frequency" or "rank-size" plots or rules, are most associated with linguistics, due to Zipf's work showing that word counts (as well as city sizes) tend to follow power laws (Zipf 1949; Arshad, Hu, and Ashraf 2018). Only some minor plotting conventions differ Zipf plots from Pareto plots, but again this influences systematically the value of the calculated scaling exponent. While Zipf distributions are also cumulative, they are not normalised to 1, meaning that absolute size ranks are plotted instead of probabilities. Furthermore, these are plotted on the x axis rather than the y axis, with the implication that the distribution is *discrete* rather than continuous. The y axis (the actual variable being measured) can in principle be discrete or continuous, but the most commonly cited cases are discrete, i.e. count data, like the number of inhabitants in a city or the number of times (frequency) a word appears in a text. The scaling exponent of a Zipf law (or a Zipf exponent, denoted q e.g. in Arshad, Hu, and Ashraf 2018, 78) can thus be expressed as

$$q = 1/\beta = 1/(\alpha - 1). \tag{4.9}$$

The fact that there does not seem to be any consensus on which character to use for the different parameters is an additional difficulty to handle when comparing studies that apply these different approaches. In his extensive review of these three separate research traditions, Mark Newman (a physicist; the power law is the most common variant within the natural sciences) admits that this inconsistent nomenclature "causes much confusion in the literature" (2005, 327), which I will claim is a polite understatement. For clarity, throughout this thesis I only refer to the scaling parameter of the power law as defined in Equation (4.7), which I denote

α, no matter the graphical representation of the distribution (most of the plots here are cCDF plots, and could thus be qualified as Pareto plots). The value of α is furthermore only calculated directly on the data using the maximum likelihood method described in Chapter 5, and is thus not dependent on the type of plot chosen for graphical illustration.

As already mentioned, regular power laws are asymptotic so that as x approaches infinity, y goes arbitrarily close to 0 without ever reaching it. This in itself acts as a limit to its usefulness for modelling most real-world phenomena which are of finite size, even though the power law may be a good approximation over some range of the system. In other words, in a power-law model of house sizes, no matter the value of α, the probability of a house measuring a trillion square metres is certainly negligible, but still technically above 0. Several solutions have been proposed to this problem, allowing the model to end somewhere at $x < \infty$. Among the main candidates are the *power law with exponential cutoff*, the *stretched exponential* and the *parabolic fractal* distributions. The first one is – as the name indicates – a power law with an additional exponential factor, which turns the function increasingly (exponentially) downwards as x gets higher (see Clauset, Shalizi, and Newman 2009, 664 for precise definition). The second is a type of exponential distribution, similar to the one described above (Equation (4.5)), but where the x exponent is raised to a power constant $0 < c < 1$, giving a survival function $P(x) = e^{-(x/x_{min})^c}$, resulting in a tail which is heavier than that of a regular exponential distribution but thinner than that of a regular power law (J. Laherrère and Sornette 1998; Clauset, Shalizi, and Newman 2009, 664). This survival function or cCDF of a stretched expoential is furthermore equivalent to that of the *Weibull distribution*, a well studied model type that is widely implemented in statistical software, making it easier to include in analyses compared to the more piecewise power law with exponential cutoff (comp. N. L. Johnson 1994, Vol. 1:629 ff.).

Finally, the so-called parabolic fractal distribution has been proposed as a quadratic function model on log-transformed values of size to rank (Zipf plot), to account for cases where the distribution follows a parabola of constant curvature on a log-log plot (Jean Laherrère 2000). Jean Laherrère – working for a large oil company modelling the size distribution of oil reserves – noted how studies that claim to show power-law distributions in empirical data often tend to explain away the curvature of the distribution in log-log plots, e.g. by focussing on shorter ranges of the tail, or by constructing composite distribution models. While the log-normal model forms a parabola on log-log plots of its PDF (Figure 4.2), none of the above discussed models have this characteristic for their survival functions. A quadratic function is a simple polynomial including a squared term of the variable, of the type $f(x) = ax^2 + bx + c$. The use of the term *fractal* here relates to the shape of the distribution being evaluated with log-log transformations, so that the function is interpreted as a power law with an additional squared factor, a and b both representing scaling exponents. With quadratic functions, the coefficient to the squared term (here a) determines the curvature of the model, while c has the role of y-intercept as with linear models. The model can furthermore be interpreted as a power law where the scaling exponent α is continuously increased with higher x, in other words where α corresponds to the derivative of the parabola at any given x, forming a continuous spectrum characteristic of multifractals (see e.g. Harte 2001). Though this model seems to have much potential both for modelling and explaining self-affine phenomena within finite systems, it is relatively recent and not easily applicable without specialist knowledge in statistics and programming, and is not further included in this thesis. Future studies would be warranted, e.g. applying the analyses proposed in Jean Laherrère (2000) and J. Laherrère and Sornette (1998) for modelling sizes and frequencies of undiscovered and/or lost archaeological sites within given regions.

4.3 Fitting heavy-tailed distributions in archaeology

The distribution fitting approach adopted in this thesis is inspired by a relatively small number of previous studies in archaeology. Though the relationship between households and house sizes had already been thoroughly studied more generally by archaeologists and anthropologists since at least the 1970s [see Section 4.1], the earliest quantitative study of prehistoric house-size distributions accompanied with theoretical arguments for interpreting power laws as signatures of multi-level social complexity, was perhaps a study by Herbert Maschner and Alexander Bentley published 20 years ago (Maschner and Bentley 2003). They presented a well-argued case for hierarchical scaling between households in a study area on the Alaska Peninsula, apparently discernible in several periods of the region's prehistory, and explained this as the result of an elite emerging from various (though unidentified) competitive socio-economic practices. The study could today be criticised for methodological shortcomings – the authors relied on least squares fitting on log-transformed binned data, they did not systematically propose more than one model for the data, nor propose any quantitative way of selecting the best fits, and their data sets seem to have been severely time-averaged, rendering any claims of hierarchical scaling between households potentially meaningless. All of these issues have been thoroughly addressed in subsequent studies (see Chapter 5). The general analytical procedure and rationale however, remains highly innovative and would merit far more attention than it has received. In my view, this study represented the first turn beyond the study of size *averages* towards a theoretically informed study of size *distributions* in household archaeology.

This approach was further explored by Brown et al. (2012) and expanded upon by Strawinska-Zanko et al. (2018), who identified a shift from less heavy-tailed distributions (exponential) to power-law distributions (Pareto) of house sizes in the Maya region approximately coinciding with the pre-Classic/Classic transition, which is traditionally considered the onset of state-level organisation. Following political scientist Manus Midlarsky (1999), Strawinska-Zanko et al. (2018) argued that this transition to a power-law distribution of wealth (proxied through house sizes) could be explained as resulting from increasing competition for agricultural land ownership following population growth. Furthermore, they identified a trend towards more pronounced inequality, both through lower α values and higher Gini indices, until the end of the Classic period, with an abrupt shift to more equal distribution in the post-Classic (higher α and lower Gini). Despite the inclusion of only four settlements in this case study, it remains highly interesting since the overall economic and demographic development of the Classic Maya is very well documented from a range of other approaches and intensive research. The study also contributed with detailed discussion of methodological issues, comparing the performance of different procedures. In a study by Crabtree et al. (2017) a similar analysis was performed on data from the Mesa Verde region in the American South-West over the 7th to 13th centuries CE. This cultural context is also very extensively documented, and holds the additional advantage of a fine-grained temporal sequence supported by dendrochronology, allowing for detailed analyses of near-coeval features. With the same motivation of using distribution models as proxies for underlying generative mechanisms, the authors systematically compared best fit log-normal and power-law models of settlement sizes as well as the sizes of *kivas* – a special category of communal ritual structures – across the study area. Though the results were not entirely unanimous, both indicators pointed towards a settlement hierarchy in the Pueblo II phase from ca. 1030-1140, centred around Chaco Canyon receiving tribute from surrounding areas, which again is coherent with the current understanding of the period based on other strands of evidence. Furthermore, they implemented preferential attachment mechanisms into an agent-based model of the regional socio-demographic development accordingly, largely reproducing the observed temporal patterns. This also seems to have been the first archaeological study consistently performing distribution fitting by maximum likelihood estimation rather than least squares, and comparing the fit of different models quantitatively. For this they used the same R package *poweRlaw* that is used here in the following chapters (Gillespie 2015).

The analysis of the size distribution of special communal structures across a region, as was done by Crabtree et al. (2017), is in some sense a bridge between analyses of house-size and settlement-size distributions. The study of settlement hierarchies have a long tradition in archaeology, either qualitatively (or semi-quantitatively) applying central place theory (Christaller 1966; Niels Müller-Scheeßel 2007; Chen 2011), or quantitatively with reference to the Zipf law – in archaeology usually termed the "rank-size rule" (Zipf 1949). However, such applications in archaeology have suffered gravely from theoretical confusion, a certain level of dogmatism, and generally poor groundings (see Grove 2011 for a more extensive discussion and review). Studies routinely reveal a misunderstanding of what is meant by *law* in statistics, as well as what appears as an obsession with what Zipf termed as the "ideal" case of a slope of 1 (see Hodder 1979 to mention only one example). Investigators who find that their observations do not fit these assumptions have routinely felt the need to explain away deviations rather than see them for the actual results that they are – namely settlement samples *without* hierarchical scaling, which were unknown to Zipf since he studied modern capitalistic societies, not prehistoric ones.

As Grove (2011) points out, there is no reason to assume that archaeological settlement systems should follow a Zipf law at all – not all cultural settings exhibit functional hierarchies between settlements at a regional scale – and when they do, the reasons for why the slope should "ideally" equal 1 remain unsubstantiated. It should be kept in mind that Zipf's primary results were on word count distributions, which have since become widely recognised as one of very few domains that systematically follow power-law distributions over large scale ranges (Clauset, Shalizi, and Newman 2009; Stumpf and Porter 2012; Arshad, Hu, and Ashraf 2018). This is not directly transferable to archaeology. Rather, as with the distribution-fitting methodology presented further on, one should aim for first test whether the observed settlement system at all does scale hierarchically, and preferentially also test the assumption against competing models (Clauset, Shalizi, and Newman 2009). If a power-law (or Zipf law) is retained, variations in slope values should be celebrated as indications of cultural variation, rather than lamented as deviant results.

Fortunately, Zipf law modelling on settlement systems has recently been entirely re-branded as "settlement scaling theory", primarily by urban geographers (and some archaeologists) at the Santa Fe Institute Duffy (2015). There, they have considerably strengthened the theoretical and methodological framework, laying the ground for far more interesting results than have been provided through rank-size analyses. While this framework has seemingly for the time being only been applied to North-American contexts, it would seem natural to extend them to European prehistoric contexts like the Linear Pottery and the Trypillia. However, this has not been done as part of this thesis.

In any case, it seems clear that the least optimal option when studying size distributions in archaeology, is to not perform any distribution fitting, but rather assume they are heavy-tailed, or avoiding the question altogether, but still conclude with hierarchical scaling Wilk (1983).

Chapter 5

Methods: Distribution fitting

In this chapter I go into some more detail around the methods used in Chapter 6, and the reasoning underlying my choices of methods. As mentioned earlier (Sections 1.2 and 4.2.4), I consider power-law distributions as a statistical signature of hierarchical structures, and wish to test whether such structures may be reasonably shown to exist among houses in European Neolithic villages, or if house sizes in these contexts are better explained by other non-hierarchical models. Results from these analyses add to current debates surrounding the development of social and political organisation in the Neolithic, and to the question of the emergence of stratified societies more broadly. Furthermore, and as also mentioned in Section 4.2.4, the methodological procedure leading to claims of power-law distributed data is not entirely straightforward, and is an issue that has undergone important developments in recent years, often leading to refutations of earlier claims. It is therefore critical to be explicit as to the methods being applied in studies like this one, and not simply report results obtained in some unspecified way through obscurely documented software.

In the following I will present the methodological procedure applied in the distribution fitting on house sizes done in Chapter 6, followed by a series of tests of this procedure on synthetically generated data, with the goal of obtaining a more detailed view of the accuracy and limits of the method. Due to limited space and for simplicity, I will concentrate on the choices of methods and procedure, and not on the under-the-hood functioning of different statistical tools like maximum likelihood estimation and calculation of the Akaike information criterion. For more details on these there is a number of good introductory volumes, some of which – like Shennan (2008b) and Baxter (2003) – are also specifically aimed at archaeologists.

5.1 Modelling heavy-tailed distributions

The standard method for fitting power-law models to empirical data throughout the 20$^{\text{th}}$ century was through least squares linear regression on log-transformed x and y values (e.g. Harrison 1981; Mitzenmacher 2004), the same way exponential and log-normal models could be fit more easily to data by log-transforming x values. Conscious about the still frequent lack of statistical training among archaeologists, Brown, Witschey, and Liebovitch (2005) and Brown and Liebovitch (2010) presented the log-linear regression method as sufficient because of its simplicity of application compared to more sophisticated methods. Brown and Liebovitch (2010) furthermore provided a detailed discussion around how to plot the data in order to obtain the most accurate parameter estimates. The central problem with fitting power-law models to data, is that power-law distributions characteristically have an overwhelmingly large proportion of the data at lower values, while the scarce high values are typically several orders of magnitude higher. Density plots of empirical data (with the PDF on the y axis) require binning, so that the plotted data points in reality correspond to bar heights in a histogram. The applied bin width will furthermore have a heavy influence on the appearance of the plot, where small bin widths are best to represent the many low values and large bin widths are best for the few high values. Brown and Liebovitch proposed multi-scale PDFs, combining histograms of different bin widths before performing regression, which they showed give better results on synthetic data (2010, chap. 2). Logarithmic binning – increasing bin width exponentially so that points appear to be spaced as constant increments on a log-transformed x axis is also a possibility that has been proposed (Newman 2005, 325–26). Plotting the cCDF instead of the PDF has the advantage of avoiding the bin-width issue altogether, since y values then are a function of the rank of each data point. This also allows for using all the data and not reducing it into bins, and is shown on synthetic data to give more accurate α estimates (i.e. absolute slope of cCDF $+1$, see Eq. (4.9)). However, the inconvenience with fitting the power-law model to the cCDF, as pointed out by Brown and Liebovitch (2010), is that it does not necessarily form a straight line, but will in particular be curved when $\alpha \leq 1$, making it more difficult to distinguish visually from other heavy-tailed distributions.

From the early 2000s, physicists and mathematicians started to criticise the frequent use of log-linear regression methods for modelling power laws, since they were shown to introduce systematic biases to the parameter estimates no matter the adopted plotting method, and calls for the use of more robust methods like maximum likelihood estimation (MLE) were put forth (e.g. Newman 2005, 325–27; Stumpf and Porter 2012). A new methodological tool kit was proposed by Clauset, Shalizi, and Newman (2009), which has since seemingly become the new gold standard for fitting heavy-tailed distributions. One of their main critiques of earlier practices, came from the recognition that in nearly all real-world contexts where power laws are claimed to exist, this behaviour only kicks in from some lower threshold or x_{min} in the terminology of Clauset, Shalizi, and Newman (2009). In earlier studies the value of this threshold was simply set by guessing from the looks of the plot and trying to fit a line covering as much as possible of the data. More formally, this also impeded proper normalising of the distribution. The method proposed by Clauset, Shalizi, and Newman (2009) consisted in testing a range of different x_{min} values and picking the one that gave the best MLE fit to the data by minimising the KS or Kolmogorov-Smirnov statistic (the largest observed distance between the model and the data), and was reported to perform very well on synthetic data. Next they tested the plausibility of the power-law model through bootstrapping, i.e. generating a large number of synthetic random data sets with the same estimated parameter values, each time measuring the KS statistic compared to the ideal model. The fraction of runs giving a KS statistic higher than that of the empirical data gives the p-value, which they argued should lead to a rejection of the power-law hypothesis when $p < 0.1$. They estimated the number of bootstrapping runs necessary for robust p-values being between 1.000 and 10.000, which for a few data series is not dramatic, but for larger numbers of data series quickly becomes computationally intensive. But most importantly, they argued that the bulk of previous studies claiming to find power laws in empirical data never actually tested and compared their model with alternative models, which they argued should be done even in convincing cases where a power-law model could not be excluded as a good fit through bootstrapping. The method they proposed for comparison and selection between competing models, was Vuong's log-likelihood pairwise comparison test, though pointing out that any good statistical model selection method could serve this purpose (see Clauset, Shalizi, and Newman 2009, 663 for an overview of their "recipe for analysing power-law distributed data"). These methods were later implemented with functions and documentation in the R package *poweRlaw* (Gillespie 2015), which has been used and cited in at least some archaeological studies since (Crabtree et al. 2017; Haas et al. 2015).

For the present study I have largely chosen to follow the instructions advocated by Clauset, Shalizi, and Newman (2009), but with a few modifications, for reasons that are discussed in more detail below. Early experiments with the *poweRlaw* package indicated that the proposed bootstrapping procedure possibly represented a slight overkill in the present context, requiring much computing time with relatively limited gains. Furthermore, the pairwise model comparison using the Vuong's log-likelihood test appeared good but somewhat tedious, requiring a nested algorithm eliminating competing models one by one. Therefore I decided instead to opt for testing all candidate model simultaneously using the Akaike Information Criterion (AIC), or rather the version of it designed to correct for small sample sizes – the AICc. The AIC score is also calculated from the log-likelihood of each model, and indicates which of the candidates accounts for the given data with the most weight. While not implemented in the *poweRlaw* package, it is a frequently used statistical tool implemented in a number of accessible R packages. Here I used the *AICcmodavg* package (Mazerolle 2023) because of its useful functions for manually tackling the differences in how base R and *poweRlaw* models are coded. I also tested using the Bayesian Information Criterion (BIC) on the data sets, but quickly came to the conclusion that the AICc was sufficient for the types of models being used here, with maximum two parameters. The tested model types for the distribution tails were power law, log-normal, exponential and stretched exponential/Weibull, all of which were fitted using the *poweRlaw* package.

Having defined the best possible power-law model to the data series and selected the best candidate model for the distribution tail from the same x_{min} value applying the above procedure, I went on to test for the best model of the whole data series, without setting a lower threshold. This excludes per definition the power-law model, and I also excluded the Weibull distribution since it is more general and can mimic other models without providing very useful theoretical explanations. The Weibull distribution – much like the gamma distribution – can be a very handy tool for modelling empirical data when the goal is to use a single model type for data series with multiple types of shape, or for prediction in many practical settings, but it has much more limited explanatory power since it lacks a broad theoretical generative mechanism like the Central Limit Theorem or preferential attachment. The reason for including it when comparing tail models is that it provides a good power-law-like approximation with finite variance, i.e. it allows for upper bounds. The tested models for the whole data range were therefore the normal, log-normal and exponential distributions, all fitted with MLE using the broad and well established *MASS* package (Venables and Ripley 2002), and selected with AICc as with the distribution tails.

5.2 Testing for false positive power-law tails

The difficulty of comparing power-law models with other common candidate models (like log-normal or exponential), is that they, unlike the others, by definition need a specified lower bound above 0, denoted x_{min}. Comparison of multiple models with AIC scores is only meaningful when done over the same range of data (this also applies to the Vuong's log-likelihood test for pairs of models proposed by Clauset, Shalizi, and Newman 2009). However, comparing multiple models over the range in a data set which has already been recognised as providing the best possible fit for a power-law model, gives this latter model a potential advantage over the other ones. Log-normal models, for instance, can explain the entire range of a data distribution, where a power law can in most cases only explain the highest values in the distribution tail. The fact that these two model types have frequently and for a long time represented competing explanations for the same empirical data sets, may reflect this apparent incomparability between them (e.g. Bee, Riccaboni, and Schiavo 2011; Gibrat 1930; Harrison 1981; Mitzenmacher 2004; Sheridan and Onodera 2018). One can suspect then that this procedure of distribution fitting and model selection would favour power-law models unreasonably. At the same time, one of the main findings of the Clauset, Shalizi, and Newman (2009) study, was that power-law behaviour was only confirmed beyond reasonable doubt in one out of 24 empirical data sets which had been reported as power-law distributed in earlier studies, leaving the impression that the methodology would be conservative rather than lenient. In many cases however, the study remained inconclusive, especially regarding comparisons between power-law and log-normal models to empirical data sets (comparisons between power-law and other models were generally more conclusive). The authors admitted "*In general, we find that it is extremely difficult to tell the difference between log-normal and power-law behaviour. Indeed, over realistic ranges of* x *the two distributions are very close, so it appears unlikely that any test would be able to tell them apart unless we had an extremely large data set*" (Clauset, Shalizi, and Newman 2009, 689). Extremely large data sets are of course a luxury that is rarely afforded in archaeology, and if these two models are that close in many situations, one can ask whether picking one over the other really matters in the end. This question is further developed in Chapter 11.

The reliability of this methodology can to some extent be assessed using synthetic data. A first question to address is whether sample size affects the selected distribution model for the tail, and if so what size should be considered a minimum for the results to be reliable. In Figure 5.1, using random number generator functions in base R (R Core Team 2023) and with the *poweRlaw* package (Gillespie 2015), I reproduced Fig. 5a in Clauset, Shalizi, and Newman (2009), namely examples of a power-law, a log-normal and an exponential distribution, with the addition here of a stretched exponential, illustrating how they all can look roughly linear on log-log plots with their survival functions/cCDFs. Using the same parameter values, but with four different sample sizes (10, 100, 1.000 and 10.000) on each model, these test distributions were run through the distribution fitting algorithm described above (Figs. 5.2 and 5.3). The power law was correctly identified no matter the sample size, but for the smallest sample size ($n = 10$) all other distribution types also gave power-law tails. For $n \geq 1000$ all distribution types were correctly identified also in their tails, while for $n = 100$ this was only the case for the stretched exponential and the power law. These results are in agreement with the analysis based on p-values obtained from bootstrapping presented by Clauset et al. (2009, 676 ff.), but the method opted for here is far less computationally intensive. Selecting the best model alternative directly based on AICc is also a less complex operation compared to the sequence of first bootstrapping and then performing pairwise comparisons of log-likelihood as proposed by Clauset, Shalizi, and Newman (2009). The inconvenience with the method proposed here, is of course that there is no guarantee that any of the models tested for are in reality appropriate – we only find out *which* one of them is the *most* appropriate. The p-value approach in (Clauset, Shalizi, and Newman 2009) does allow for positively rejecting hypotheses that clearly do not fit the data. However, early experiences (not presented in further detail here) gave the impression that this made little practical difference, at least on the data sets analysed in this thesis. Most sample sizes are in the order of 100 or lower, in which cases bootstrapping remained inconclusive, while it was still interesting to have an indication of which model that gave the best fit. The results shown in Figure 5.2 indicate that for sample sizes below ca. 100 power-law interpretations should be treated with care, and should not be trusted as n approaches ca. 10.

However, it must be noted that in order to generate the non-power-law distributions with a defined lower threshold as done here (and in Clauset, Shalizi, and Newman 2009), a much larger number of data points is in reality needed if we also consider those falling below that same threshold. For example, to get 1000 data points with values above $x = 15$ following the log-normal distribution shown in Figure 5.1, they need to be filtered out from a total distribution almost ten times larger. When considering entire data distributions without lower bounds, as is usually the case e.g. when analysing archaeological house-size distributions, the sample size will potentially also need to be much larger for correct model selection, although exactly *how* much larger should depend on the type of distribution and parameter values of the data. Similarly, when Clauset et al. argued that with the MLE method for estimating α in a power-law model, sample sizes around $n \geq 50$ would usually be enough for the estimates to be within 1% accurate (2009, 669), sample size is here referring to the number of data points actually being considered when fitting, which is $n > x_{min}$ only. For this number to be 50 or higher, the total size of the distribution could often need to be 500 or higher, which is far more than most of the house counts per village in this study. This matter of sample size is a question that is perhaps less relevant to physicists and mathematicians, but that may be of crucial importance to archaeologists who regularly suffer from limited amounts of data.

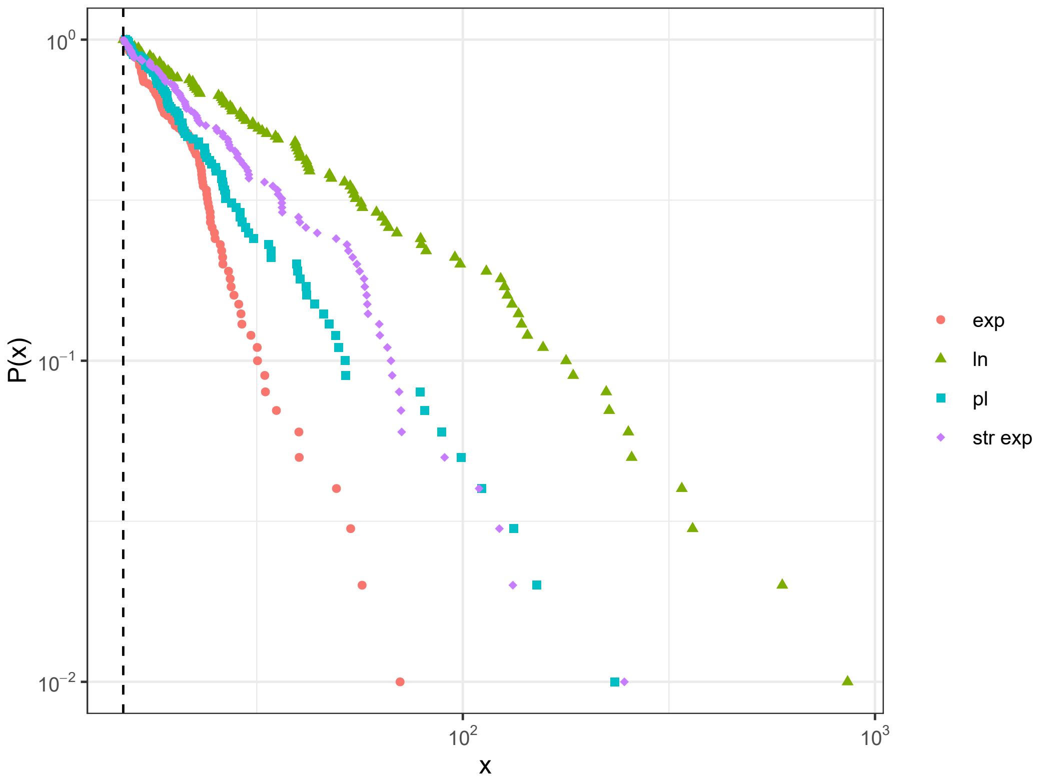

Figure 5.1: *Synthetic data series drawn from four different distribution types: exponential ($\lambda = 0.125$), log-normal ($\mu = 0.3$, $\sigma = 2$), power-law ($\alpha = 2.5$) and stretched exponential/Weibull (shape $= 0.5$ and scale $= 3$), all with $n = 100$ data points and $x_{min} = 15$. Plot equivalent to Fig.5a in Clauset, Shalizi, and Newman (2009), with deviations due to random fluctuations only. Scales are logarithmic, and all four series appear as roughly straight lines, though only one is a true power law*

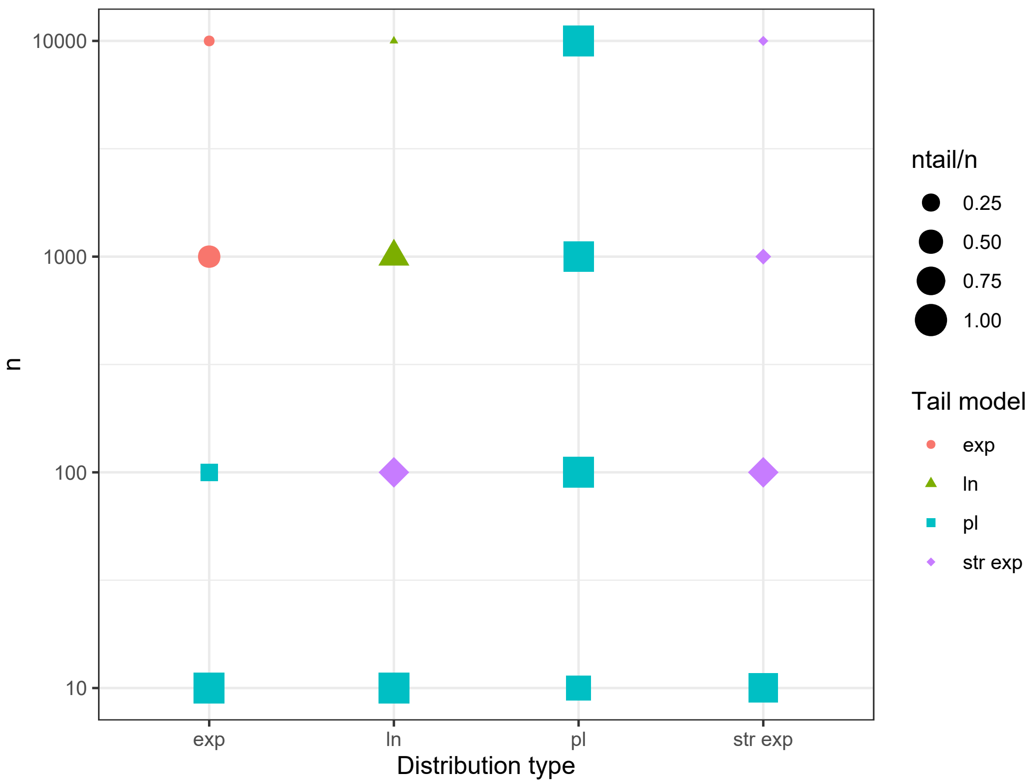

Figure 5.2: *Selected tail models for the same synthetic data sets, each with four sample sizes ($n = 10^1, 10^2, 10^3, 10^4$). For each tail model, x_{min} is set at the value which gives the best power-law fit. Point size indicates fraction of data points thus included in the tail model. For power-law distributions, all samples are correctly identified, while this is the case only for large samples ($n > 10^2$) of log-normal and exponential samples, smaller samples being interpreted as having power-law or stretched exponential tails*

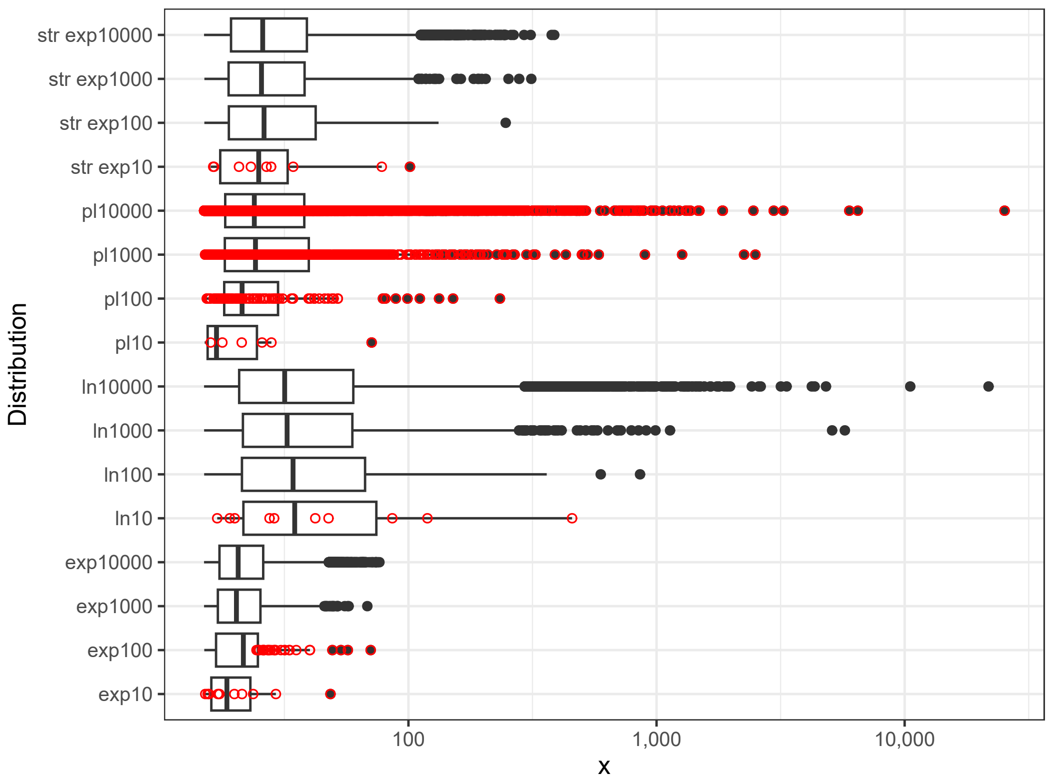

Figure 5.3: *Box plot of all the synthetic data sets, overlaid (in red) with the data points interpreted as power-law tails. X axis is logarithmic – however the log-normal distributions do not appear symmetric since they are truncated with a lower threshold. Note that especially for log-normals and power laws, larger samples give longer tails. If the model predicts a probability of having a value of 10.000 or more as only 1 in 10.000 or 0.01%, a sample size of 10.000 will probably allow for one such value*

A second question more specifically related to the selection between log-normal and power-law models, is whether certain parameter combinations increase the likelihood of log-normal distributions being incorrectly interpreted as power laws. In his extensive review of power-law generating mechanisms, Newman (2005, 347–48) showed algebraically how log-normal distributions can be mistaken for power laws especially when the range of the data that is being analysed is short, and when the value of σ is high. More specifically, since the PDF of a log-normal on log scales is a quadratic function – i.e. a parabola – sufficiently smaller sections of this will be nearly indistinguishable from straight lines, and can thus be well modelled as a power law (Figure 4.2b). The curvature of the function is characterised by its quadratic term, which is a fraction with x in the numerator and σ in the denominator, written $-\frac{(\ln x)^2}{2\sigma^2}$, essentially causing a flatter curvature with higher values of σ since this term then will vary more slowly with x (see Eq. 84 in Newman 2005 for more details). The difficulty of distinguishing the two model types empirically undoubtedly lies in the close relationship between them, both being defined as some enhanced exponential distribution (Mitzenmacher 2004). Figure 5.4 shows the cCDF plot of 36 synthetically constructed log-normal distributions, with μ values ranging from 1 to 6 in integer increments, and σ values from 0.5 to 3 in increments of 0.5, each with sample size $n = 1000$ and no truncation (i.e. $x > 0$). When plotted this way (with the survival function of the variable), low σ values generate angular curves, while high σ values generate more parabolic curves.

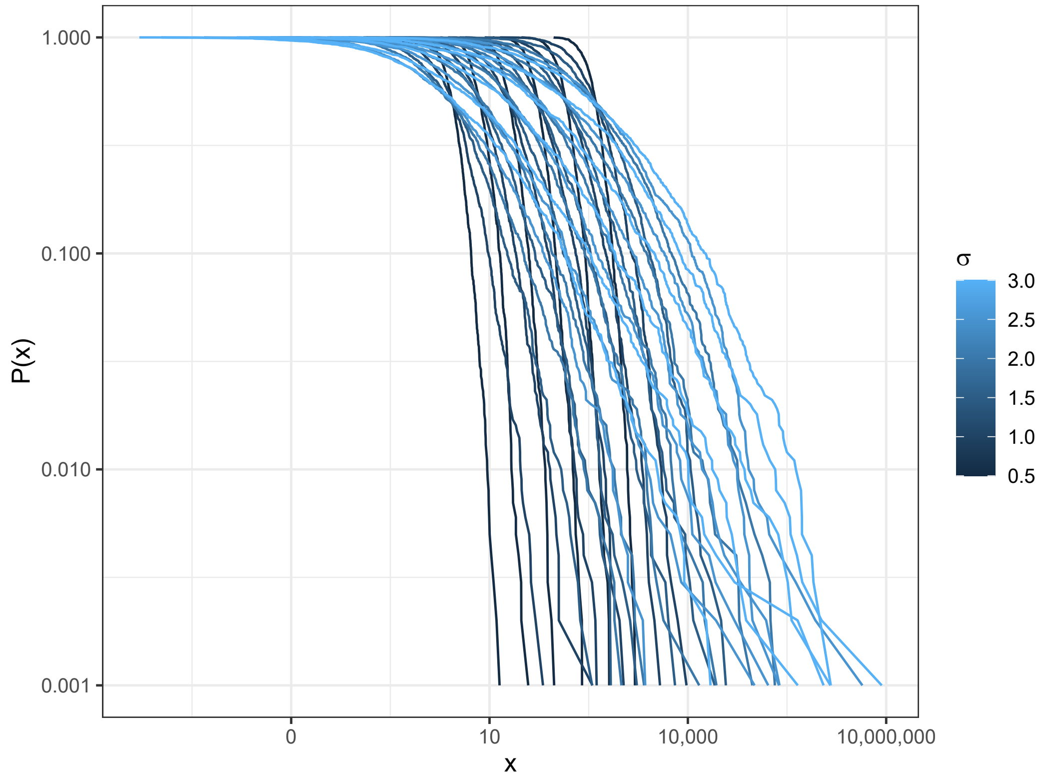

Figure 5.4: *cCDF plot of 36 synthetic log-normal distributions with parameter values* $1 \leq \mu \leq 6$ *and* $0.5 \leq \sigma \leq 3$. *Each distribution is generated with* $n = 1000$ *data points, but rendered here as lines for clarity. Scales are logarithmic*

Running these distributions through the distribution-fitting and model-selecting algorithm, more than half of them are interpreted with a power-law tail (19 of 36, Figures 5.5 and (**ref?**)(fig:05-ln-pl)). These are seemingly spread throughout the parameter space, with the only clear pattern being that for the highest σ values, the power-law tails cover only smaller fractions of the data.

What are we then to conclude from these preliminary tests? Clearly, it is a challenge to confidently distinguish between log-normal and power-law distributed data in the high ends of distributions, as noted in the beginning of this section. If the proposed methodological procedure seemingly serves well to identify power-law distributions when that is what they really

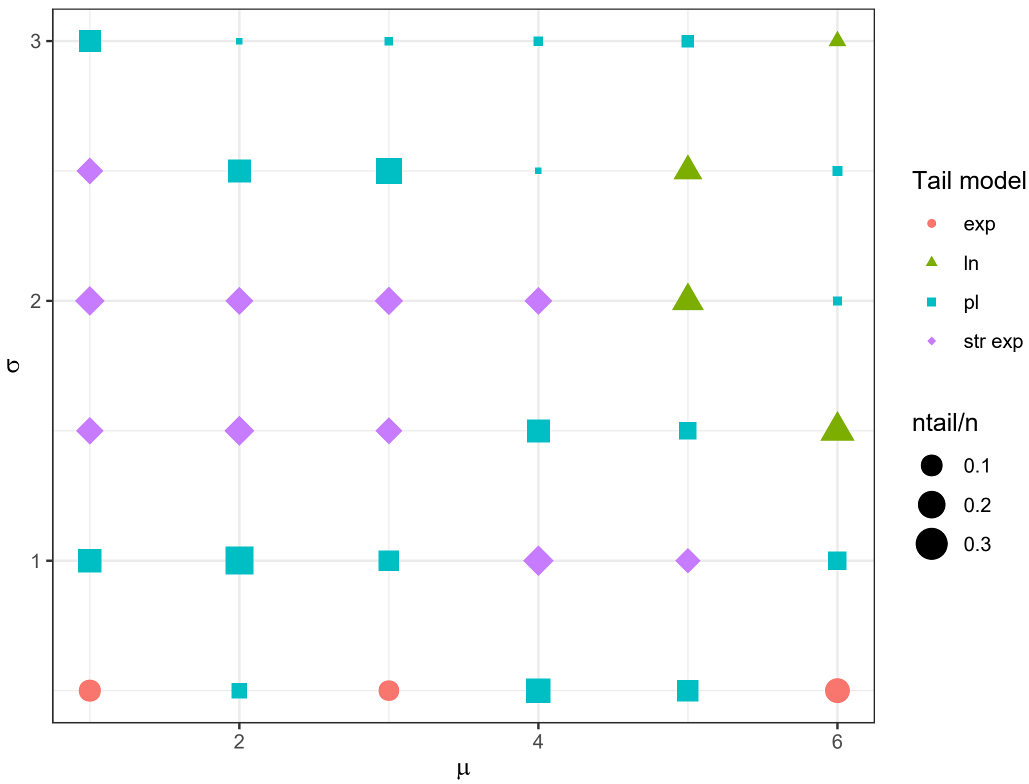

Figure 5.5: *Interpreted tail models of the same log-normal distributions. 19 of 36 distributions have tails that are best modelled as power laws. Symbol size indicates fraction of the data included in the tail, with x_{min} parameter set for best possible power law fit. See text for details*

are (i.e. there are no false negatives), it also seemingly identifies these erroneously in the tails of log-normal distributions half of the time, irrespectively of the log-normal parameter values, and with (for archaeologists) optimistic sample sizes. A more positive way to look at this issue, is to acknowledge that in truly log-normally distributed data, some definable portion of the upper tail is in many cases indistinguishable from a power law, and actually best modelled as such. There may well be precise mathematical reasons behind this, but further insight to whether such power-law tails are confidently indicative of social hierarchies when observed on material culture proxies such as house sizes, would perhaps require large-scale systematic testing on ethnographically documented cases, which would clearly go beyond the scope of this thesis.

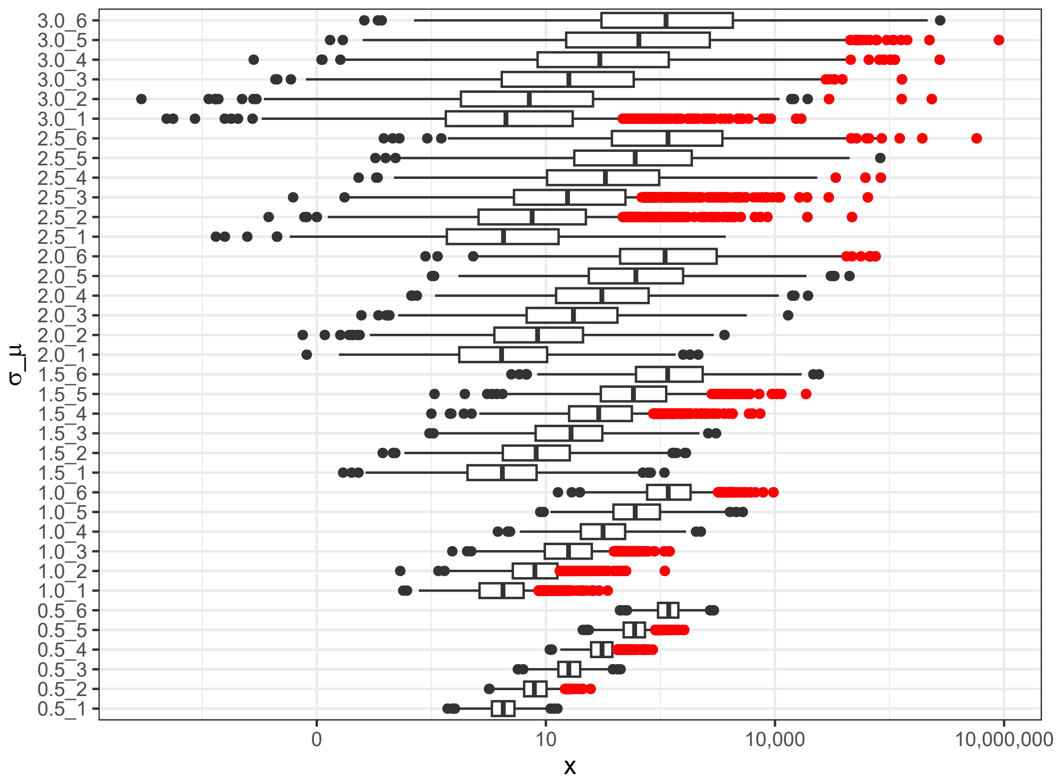

Figure 5.6: *Box plot of the same 36 synthetic log-normal distributions, overlaid (in red) with data points included in tails interpreted as power laws. The power-law tails stretch across the log-normal data in a range from 0.3% (3 data points out of 1000) to 23.5%*

5.3 False positives from data aggregation

Data aggregation or lumping of samples may be done for two main reasons in archaeology. Firstly, when the sample size is too small lumping together several contemporary samples can raise the sample size to an acceptable level (spatial lumping). And secondly, in cases when it is impossible to temporally disentangle elements within a settlement, i.e. to reduce temporal resolution to coeval elements, we are forced to proceed with temporal lumping (accepting a low temporal resolution, Perreault 2019, 56–61 ff.). The problem can be further broken down to two case types: a) all lumped samples are really drawn from the same underlying distribution (referred to as the i.i.d. condition in the previous chapter), and b) they are not similarly distributed. For spatial lumping this degree of similarity can be assessed, but not necessarily for temporal lumping. But even when it can be assessed, the lumping needs to be justified in social terms. As an example, several spatial samples (e.g. villages in a region) can have house-size distributions in which no significant differences are observed using statistical tests like ANOVA, but at the same time be functionally entirely independent, in which case it is arguably more logical to augment the size of a single sample through simulation and evaluate plausibility through bootstrapping, rather than by lumping of all available samples. On the other hand, house sizes can be significantly different between quarters or suburbs within a city or metropolitan area, or even between cities in a region or country, but if they all function together in a coherent system, they may reflect a spatial segregation between different strata in the society, in which case it can make much sense to lump and analyse them together. When it comes to temporal lumping, given that a settlement does not undergo substantial cultural changes during its timespan (change in archaeological culture), a workaround to evaluate whether the house-size distribution evolves significantly over time may be to target a number of size categories for ^{14}C dating, and to check that they all stretch over the entire range of the settlement's duration, and if yes, accept to analyse the whole distribution as one.

In order to build an appreciation of the possible effects of data aggregation on the identification of power laws, I constructed a set of 20 synthetic data series in successive steps. The first series consisted of 100 log-normally distributed random numbers with $\mu = 4.5$ and $\sigma = 0.4$, corresponding to sizes of ca. 90 for the mean and 1.5 for the standard deviation when exponentiated. These values were set to be close to realistic values of house-size distributions in Neolithic settlement as presented in the next chapter. The second data series consisted of the previous plus an additional 100 random numbers with the same parameter values, and so on for 20 iterations, so that the last series consisted of 2000 points. These series were then run through the distribution fitting algorithm for best power-law fits and model selection with AICc (Figures 5.7a and 5.8a). Five of the 20 series were interpreted with power-law tails (1/4), and these were clustered in two groups. The example is only illustrative, and would need larger and more systematic analyses to be considered general, but these results seem to indicate that under these conditions (increasing sample size with identically distributed, though not independent samples) power-law tails appear somewhat randomly, not necessarily as a result of larger or smaller sample sizes. However, once a power-law tail has appeared, it lingers for one or two iterations since the following series are only copies of the previous with some additional data.

It is important to note here that this additive process is not equal to simply constructing random log-normal sequences with gradually increasing sample size, as was done above (Figures 5.2 and 5.3). There it was shown that for log-normal and power-law distributions, data range increases with sample size since larger samples allow for data points with values that have lower probability of occurring. The data series presented here have internally very similar ranges of x values, since for every iteration new data is added *as if* the distribution only had the original sample size of 100, which is closer to what actually happens when we mix together analytically different phases of a settlement. A settlement with a long duration like 20 generations can thus have a house-size distribution that easily appears as a log-normal distribution with an upper truncation when all houses are analysed together. Though this point is not pursued further here, such truncations could then in themselves be seen as indications of temporal depth in a settlement – i.e. if the modelled house-size distribution would be expected, given the sample size, to yield some fraction of data points with markedly higher values than what is observed, it could be a sign of substantial temporal mixing. However, as is shown here, such mixing does not necessarily affect the interpreted model of the data, nor the modelled parameter values, *given that the samples are identically distributed* (see script in supplementary material for further details).

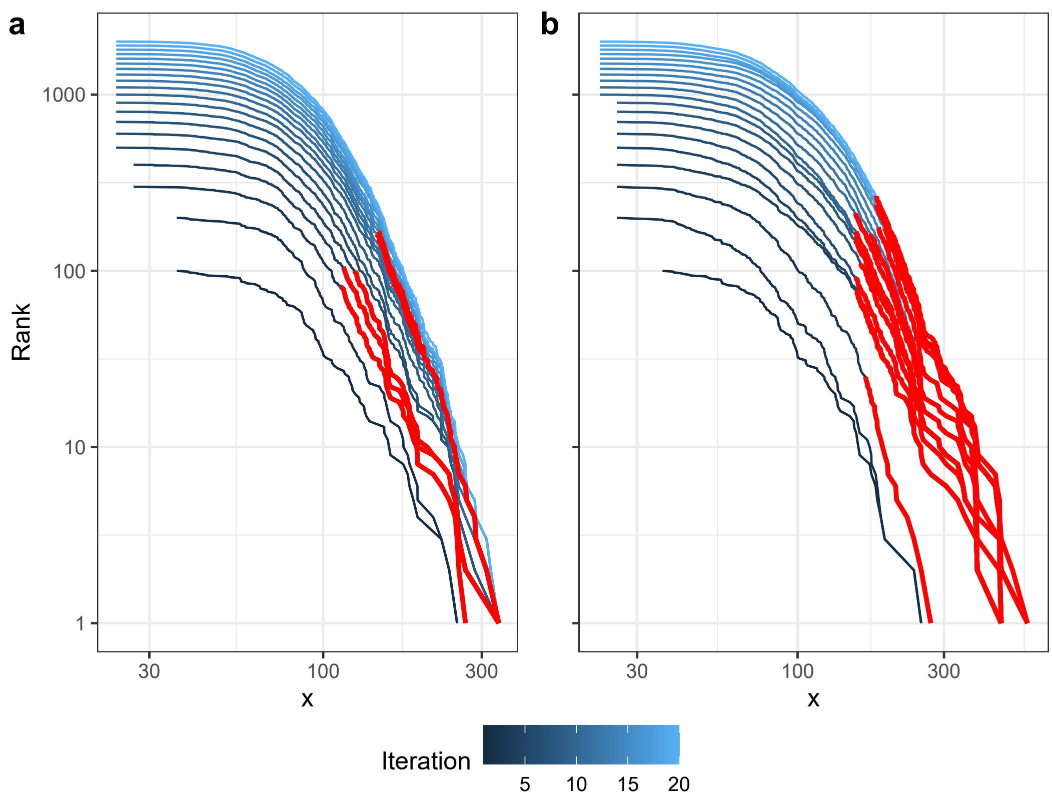

Figure 5.7: *Twenty series of sequentially aggregated log-normal distributions, starting with 100 data points and 100 more added for each iteration. Parameter values for every group of 100 data points are fixed at $\mu = 4.5$ and $\sigma = 0.4$ (a) or uniformly fluctuating between $4 < \mu < 5$ and $0.3 < \sigma < 0.5$ (b). Both settings give distributions that resemble those of Neolithic house-size distributions. Red lines indicate power-law tails. The series overlap to a large extent, so y axis is plotted with rank rather than normalised cCDF to facilitate readability. Scales are logarithmic*

This situation quickly changes when the samples are made up of differently distributed sub-samples. In Figures 5.7b and 5.8b, the same data series are constructed, but allowing for the μ and σ values to fluctuate randomly within very small ranges, between 4 and 5 for the mean and 0.3 and 0.5 for the standard deviation. The resulting distributions then become slightly more skewed over time, and after the first two iterations almost all distributions are modelled to have power-law tails. This is phenomenon of increased variance resulting from analytical lumping is described and further discussed in Perreault (2019, 61–79). In the specific case of house-size distributions, the implication is that if there are significant changes occurring over the time span of the settlement being analysed – e.g. that houses become larger or smaller over time, or that there is growing or reduced inequality over time – this will affect the overall distribution with increased variance, potentially leading to false positive power laws. If individual dating of houses is difficult to achieve, changes in the house-size distribution can to some extent be seen using temporal trends in construction techniques or raw material use as proxies. But if these material factors are stable over time and the change in house-size distribution is induced solely by social factors that are more difficult to observe directly, like post-marital residence patterns or kinship structures, there may be no way of distinguishing trends over time without dating houses individually. The issue of temporal resolution is a major concern in any social archaeology, and a pragmatic attempt at dealing with it is given in the following chapter.

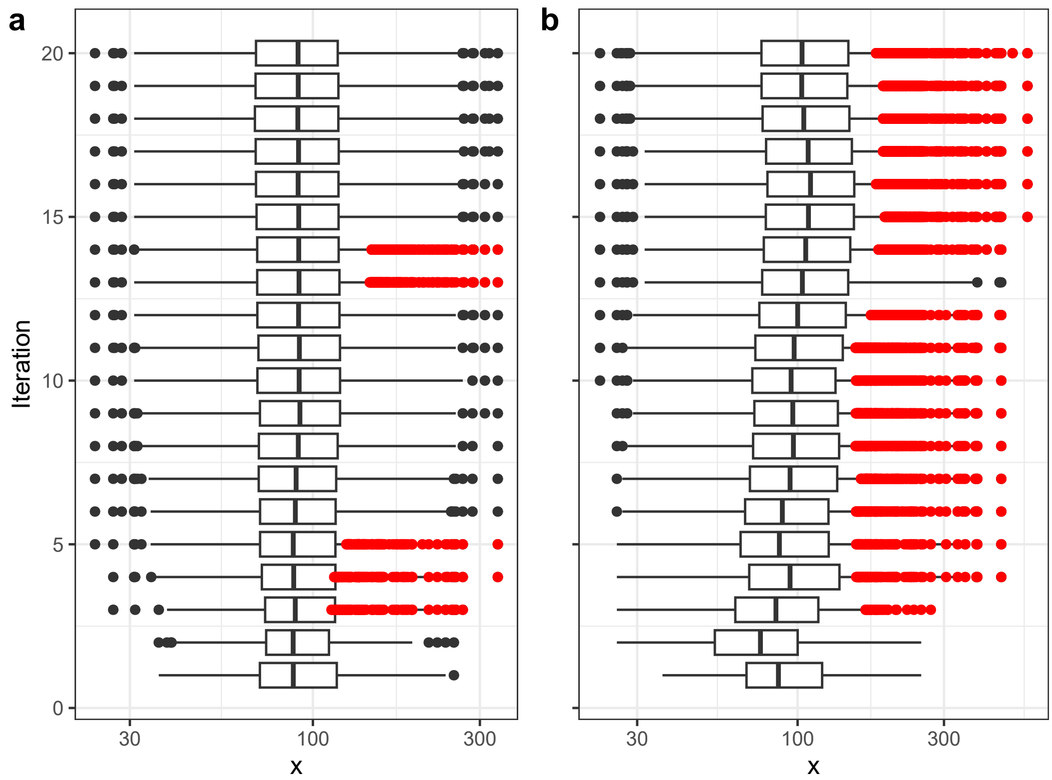

Figure 5.8: *The same aggregated distributions as above in box plots, illustrating how the data ranges increase much more slowly with sample size than expected for log-normal distributions. Sample size is 100 for iteration 1 and increases by 100 to 2000 in iteration 20. Fluctuating parameter values (b) increase variance and the probability of finding power-law tails (red points). X axis is logarithmic*

5.4 Summary of methodological procedure and tests

The overall goal of this part of the thesis is to identify power-law structures in the house-size distributions of the sampled Neolithic settlements of Linear Pottery and Trypillia material culture. As it was shown in the previous chapter, unlike other common size distribution models like the normal, log-normal and exponential, the power law is characteristic of hierarchically scaling structures, and it is assumed here that when such structures are observed in house-size distributions they are indicative of some sort of socially relevant hierarchy, as they are very unlikely to emerge from simple random additive or multiplicative processes like those relating to the Central Limit Theorem or Gibrat's law. However, a number of caveats have been presented so far.

Firstly, a power-law distribution does not suffice to say what *type* of hierarchy is in play, only that there *is* a hierarchy. The idea that hierarchical structure in society equals despotism is a prejudice that should be kept out of the analysis. Rather, the exact political organisation of the society needs to be studied archaeologically through multiple angles. However, distinguishing between cases where there is and where there is not hierarchy remains still very useful.

Secondly, even though I have strived to follow best practice in terms of statistical methodology, some issues remain. One of these is that sample sizes in the following chapter are probably near the lower limit of what is acceptable for the distribution fitting and selection algorithm to be effective. Testing on synthetic data sets indicated that the tail models should ideally include hundreds of data points, and that at $N \approx 10$ the model selection is unreliable with a high risk of false positive power laws. Furthermore, especially log-normal distributions are known to often produce power-law tails, and in the limited parameter scan provided here there is no obvious pattern between the values of mean and standard deviation and the probability of identifying a power-law tail. The observations done here on synthetic data sets seemingly show that random fluctuations in the tail are sufficient to produce power laws more or less independently of the parameter values. It remains unknown how relevant this issue is for interpreting archaeological house-size data, since the intensity of random fluctuations in house size is difficult to model precisely. It is possible that the frequent (but not constant) power-law behaviour in the tail is a mathematically inherent property of log-normal distributions. The question of the extent to which the presence or absence of power laws in otherwise log-normal house-size distributions confidently translates into presence or absence of social hierarchy should be addressed in future ethnographic or ethnoarchaeological studies.

Lastly, tests on aggregated log-normal distributions with synthetic data indicate that lumping or mixing of data series does not affect the risk of obtaining false positive power laws given that the mixed sub-distributions have identical parameter values. However, if the mixed distributions differ, even by small random fluctuations in the parameter settings, the aggregated data set quickly runs a much higher risk of giving false positive power laws. In the present context, this issue is especially relevant for cases when archaeological settlements are documented primarily through remote sensing, and when a majority of houses lack individual dating, impeding any further separation into coevally existing settlement plans. It should be noted that of all the caveats mentioned here, the main problem is the identification of false positive power laws (so-called “type 1 error”), while failing to recognise actual power laws (false negative or “type 2 error”) is seemingly much less of an issue.

The applied distribution fitting algorithm can be summed up by the following (see also script in online supplementary material for further details):

- A lower threshold (x_{min}) is selected by fitting power-law models to the data by maximum likelihood estimation. The fitting is done recursively from different threshold values, and the one that gives the model with the lowest K-S statistic is selected.
- Other candidate models for the tail of the distribution are fitted (log-normal, exponential and stretched exponential/Weibull), also by MLE and with the same lower threshold as the power-law model.
- The best tail model is selected by lowest AICc score. The result shows whether a power-law model gives a better fit to the tail than the other models.
- Models are calculated for the whole distribution, without lower bound, excluding the power law and stretched exponential, but including the normal distribution. These are also fitted by MLE and selected based on AICc score.
- Reported values are the selected models for the whole distribution and the tail, as well as model parameter values, sample size (N), size of the tail (N_tail) and proportion of the data included in the tail (Tail_P), as well as the Gini index calculated on the whole distribution

In order to test if observed power laws are only resulting from data aggregation, the same analysis is also performed on separate quarters or neighbourhoods for two settlements where this information is available (Nebelivka and Vráble), and on modelled coeval settlement plans for Vráble. In all cases, samples of size 10 or lower are excluded from the analysis, and results for distribution tails that include 10 or less houses are disregarded.

Chapter 6

Results: Distribution fitting

6.1 Settlements

Running the distribution-fitting algorithm presented in Chapter 5 on the house-size distributions of the 13 settlements in the current sample (settlements with 10 or less houses were excluded), resulted in five settlements being interpreted as having power-law tails. These were the Trypillia sites at Maidanetske, Nebelivka and Moshuriv, as well as the Linear Pottery sites at Vráble and Horný Oháj (Figure 6.1 and Table 6.1). The remaining eight settlements were interpreted as having exponential tails. It is important to note however, that this simply means that the exponential model was the best out of the tested models, on the tail length that gave the best possible power-law fit. In other words, for these eight settlements, the power-law model is effectively excluded, since other models better explain whatever tail could be interpreted as a power law. Analysing the entire distributions the same way without setting a lower threshold value (x_{min}) resulted in all settlements but one being interpreted as having log-normally distributed house sizes. Only Úľany nad Žitavou (Linear Pottery) was interpreted as normally distributed (Table 6.1). This result can also be suspected from the logarithmic box-plot, where most distributions are near symmetric (Figure 6.1).

As shown in Figure 6.2a, with the exception of Horný Oháj (Linear Pottery), the power-law tails are much longer than those of the other settlements (b). Even though they do not span two orders of magnitude in size – as predicted earlier due to material and physical limitations of houses as well as the symbolic nature of house size as expression of hierarchy (Section 4.2.4) – they do so on the cCDF which is based on house rank. Two orders of magnitude

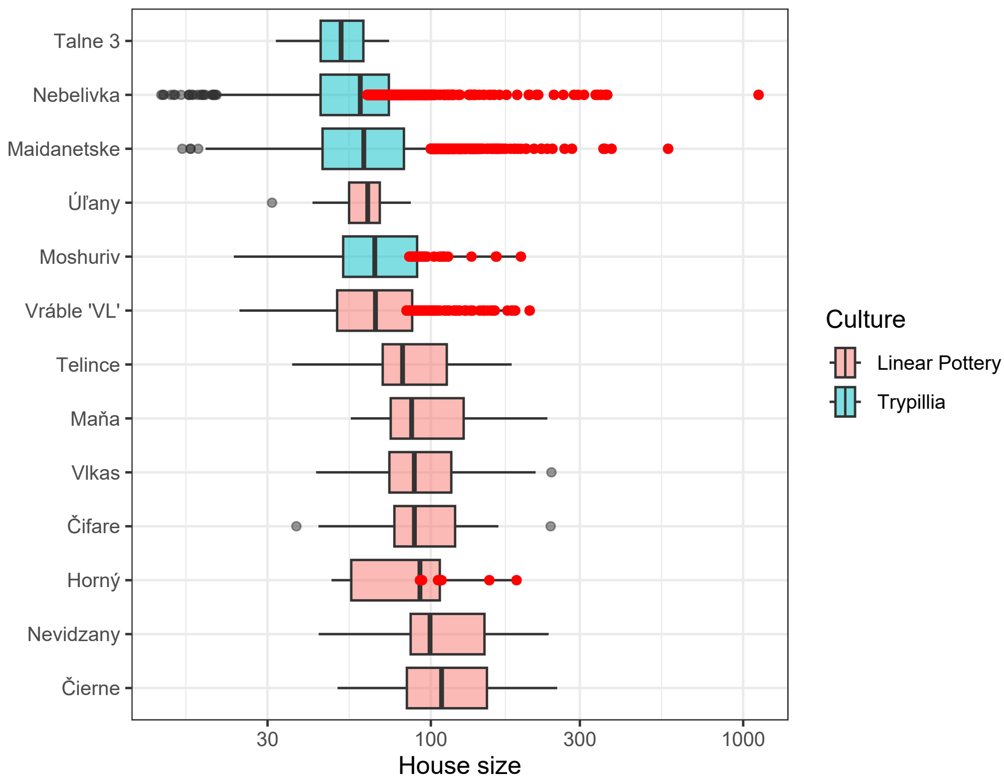

Figure 6.1: *House sizes of the 13 analysed settlements, arranged according to median house size. Red dots represent houses with size* $\geq x_{min}$ *within distribution tails interpreted as power laws. X axis is logarithmic*

in rank represents a decreasing proportion of data points as total number increases, meaning that such power-law tails are "easier" to achieve for larger settlements, which is already an indication that this behaviour is inherently connected to settlement size. Table 6.1 shows that the power-law settlements are indeed the largest ones in the sample, again with the exception of Horný Oháj. While these results are not at all surprising when it comes to the two Trypillia mega-sites – indeed the hierarchical scaling of buildings there is already recognised without quantitative analysis – there are two results that may come as more surprising.

Firstly there is a clearly recognised power law in the house-size distribution of Vráble, indicating the probable presence of a hierarchical scaling relationship in its buildings. This is not obvious simply from looking at the site plan, and relates furthermore to the question of social organisation and hierarchy which is much more open and unsettled in Linear Pottery than in Trypillia research. The power-law tail of Horný Oháj should be treated with more caution, as it seems more likely to result erroneously from the very small sample size. Secondly – though the total sample of settlements is admittedly small – the single most relevant predictor for the presence of scaling in house sizes seems not to be cultural attribution but rather settlement size, since the largest Linear Pottery settlement shows scaling while the smallest Trypillia settlement (Talne 3) does not. This is an indication that social organisation should not be considered as uniform within even well defined archaeological cultures and regions, despite common practice in archaeology. Here, in the case of near contemporary settlements with shared material culture located only some kilometres apart, there are two examples of settlements with deviating size that also deviate in how their houses scale internally, possibly indicating very different structures of social organisation between them and the other settlements in their respective regions (hierarchy seen in Vráble and not the rest of the Žitava valley, and equality seen in Talne 3 and not at other sites in the Bug-Dnieper interfluve). It should be worthwhile to test whether this relationship – between scaling in house sizes and settlement size – also cross-cut archaeological cultures on larger samples.

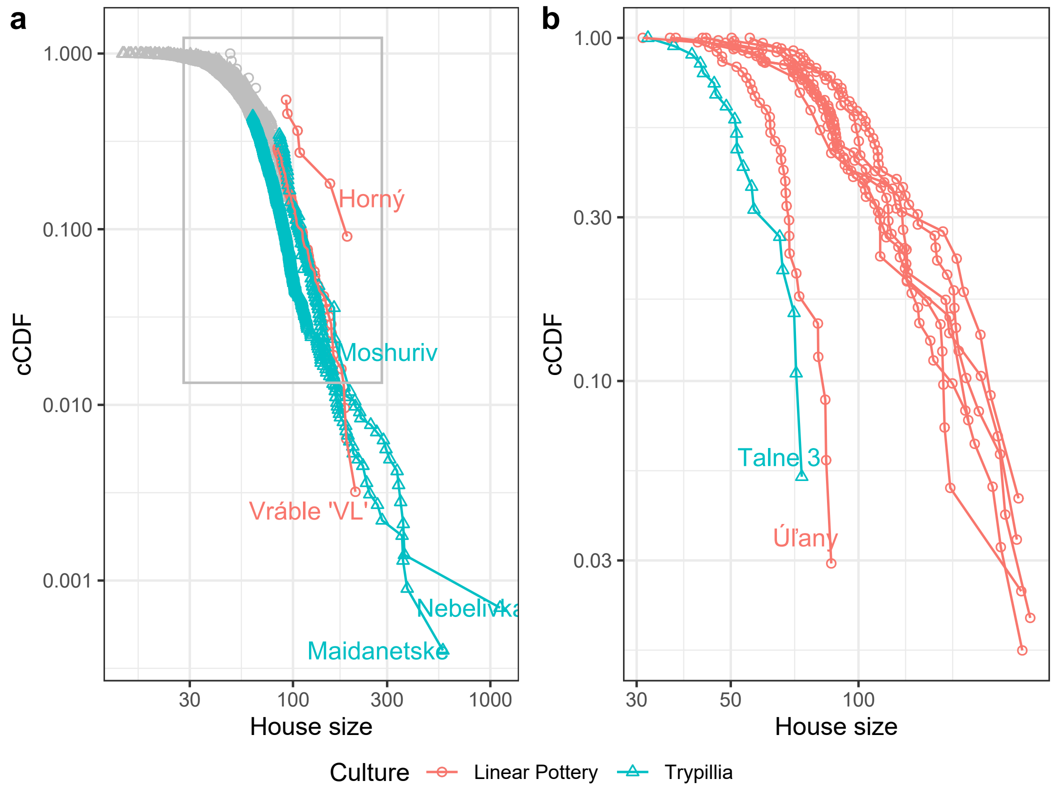

Figure 6.2: *Analysed house-size distributions for whole settlements. a) Distributions with identified power-law tails. For clarity, only the tails (house sizes $\geq x_{min}$) are coloured and connecting lines are added within each settlement. The grey frame represents the extent of panel b. b) House-size distributions without power-law tails. Two settlements are atypical with their absence of large houses. Scales are logarithmic*

Figure 6.2b shows that the house-size distributions of both Talne 3 (Trypillia) and Úľany nad Žitavou (Linear Pottery) are clearly different from those of the other settlements without power-law tails. At the same time, only Úľany nad Žitavou was interpreted as normally distributed – i.e. not skewed but symmetric, with only random differences between houses – when analysed formally. Talne 3 houses were interpreted as a log-normal distribution, but with a very low standard deviation. A Shapiro-Wilk normality test of the house sizes of Talne 3 gives a test statistic $W = 0.958$ and $p = 0.541$, which is far from enough to exclude the null-hypothesis of a normal distribution. In other words, for the practical purpose here of interpreting probable underlying mechanisms of house-size differences, there is no significant difference between houses at Talne 3 and the distribution can be considered normal.

Table 6.1: *Main results of distribution fitting analysis on settlements, ordered according to retained tail model (Tail) and parameter values (T_Par1), from steepest to shallowest (see Figure 6.2). Model and Gini are evaluated on the entire distribution for each settlement. Par1 and Par2 represent μ (mean) and σ (standard deviation) for normal and log-normal distributions, and T_Par1 represents λ (rate) for exponential and α (scaling exponent) for power-law distributions. x_{min} is the house size from which the tail parameters are estimated. N is the number of houses per settlement, N_tail is the number of houses in the distribution tail, and Tail_Pr is the proportion of N being part of the tail, or N_tail/N*

Settlement	Model	Par1	Par2	Tail	T_Par1	xmin	N	N_tail	Tail_P	Gini	Culture
Úľany	norm	62.349	12.542	exp	0.109	63.6	34	17	0.50	0.113	Linear Pottery
Talne 3	ln	3.941	0.226	exp	0.077	45.7	19	14	0.74	0.126	Trypillia
Vlkas	ln	4.534	0.387	exp	0.025	81.2	61	40	0.66	0.221	Linear Pottery
Čifare	ln	4.535	0.398	exp	0.024	70.2	41	34	0.83	0.217	Linear Pottery
Telince	ln	4.452	0.417	exp	0.024	59.3	13	12	0.92	0.228	Linear Pottery
Maňa	ln	4.597	0.381	exp	0.021	65.8	29	27	0.93	0.224	Linear Pottery
Čierne	ln	4.703	0.376	exp	0.021	89.2	49	36	0.73	0.211	Linear Pottery
Nevidzany	ln	4.659	0.434	exp	0.020	86.0	22	17	0.77	0.240	Linear Pottery
Moshuriv	ln	4.211	0.395	pl	6.306	85.3	84	29	0.35	0.219	Trypillia
Maidanetske	ln	4.112	0.443	pl	5.508	100.0	2243	301	0.13	0.249	Trypillia
Vráble ’VL’	ln	4.198	0.389	pl	5.235	83.6	313	91	0.29	0.221	Linear Pottery
Horný	ln	4.423	0.445	pl	4.916	92.3	11	6	0.55	0.253	Linear Pottery
Nebelivka	ln	4.045	0.432	pl	4.764	62.6	1435	629	0.44	0.239	Trypillia

From the table it is clear that the Gini index follows the model type and parameter values closely but not exactly. The advantage of using the Gini index for the purpose of quantifying inequality is that it is more straight-forward to calculate, and it allows for a unified single measure facilitating comparison between different distribution types (e.g. Kohler and Smith 2018; Kohler et al. 2017). The disadvantage is that there is no clear and reliable way of identifying underlying mechanisms from this index alone – in the present case it would be impossible to single out which settlements show signs of hierarchical scaling in their houses, let alone determining how many of the houses this would concern. My conclusion from this is that the Gini index is useful when the question to be answered relates specifically to inequality within populations, and when cross-cultural comparisons or long temporal trends are more emphasised than the social organisation of specific cultural contexts.

Looking at how the power-law distributed houses are spread spatially within the various settlements allows for further interesting patterns to emerge. At Vráble, the whole range of hierarchically scaling houses is present at each of the neighbourhoods, though apparently somewhat less in the south-west (Figure 6.3). There seems to be a weak tendency for larger houses to cluster throughout, i.e. that large houses tend to be close to other large houses, and with interstices between clusters being filled by regular small houses of size below x_{min}. However, observing the entire settlement plan in a single block does not allow for further evaluation of whether these clusters actually represent coeval groups of larger houses or if they rather form temporal sequences, following the much discussed "yard model" of Linear Pottery settlement development [Zimmermann (2012a); 3.2]. Either option speaks against an interpretation of house-size differences being only random.

In the much larger Trypillia settlements of Maidanetske (Figure 6.4) and Nebelivka (Figure 6.5) the hierarchical scaling of houses is seen throughout the settlements, indicating that there is no general sector that has obviously more of smaller or larger houses than others. As is widely recognised by the researchers who have studied these settlements in detail (e.g. Hale 2020, 127–37; Ohlrau 2020, 61–64), the single and by far largest building is located at around 3 o'clock on the inner house ring (at least for Nebelivka, but most probably also Maidanetske), while other smaller mega-structures or "Assembly Houses" are regularly dispersed throughout the main street and, to a lesser extent and mostly for Nebelivka, outside the outer house ring. The most interesting result from the distribution fitting analysis provided here, is that the scaling behaviour also goes far beyond the typological distinction between mega-structures

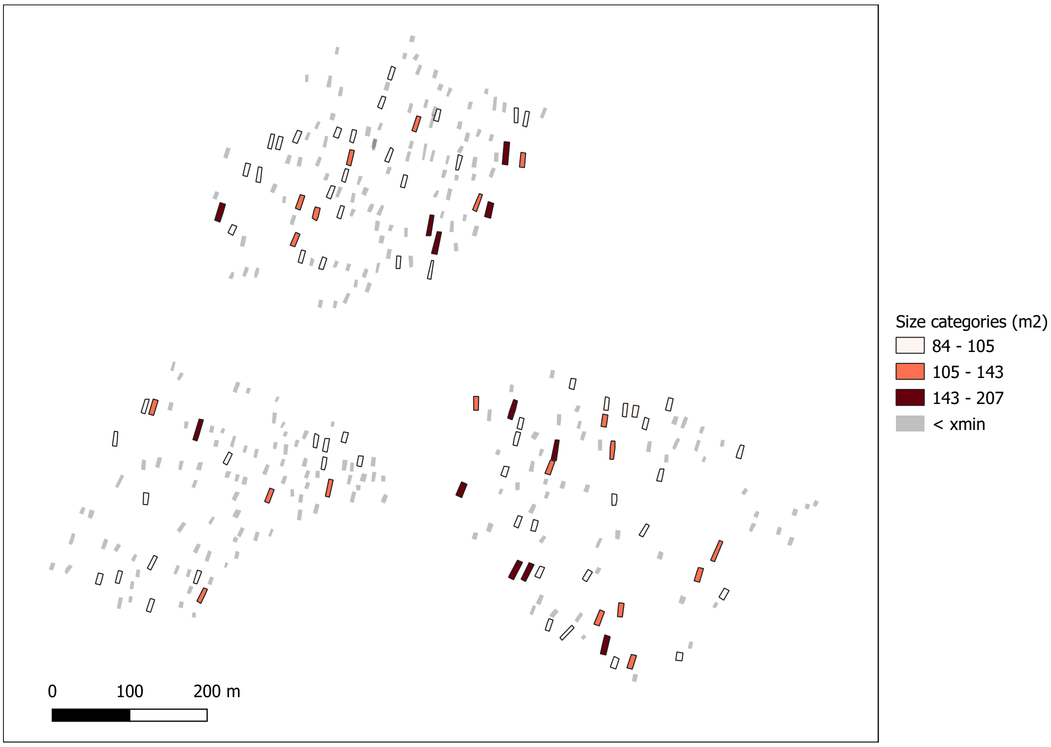

Figure 6.3: *Power-law distributed houses at Vráble (Linear Pottery), arbitrarily grouped to three levels using the Jenks optimisation method integrated in QGIS for readability. Figure made by author after Nils Müller-Scheeßel et al. (2020)*

and domestic houses. For both settlements, extra large domestic houses are more concentrated around the main street than in the inner streets and plazas, and while these are quite regularly spaced in Maidanetske, in Nebelivka they seem to cluster near the Assembly Houses, further underlining the size hierarchy. But again it is difficult to know at the present stage to which extent these clusters represent coeval groups of larger houses or temporal sequences, given the duration of the settlement. It does however suggest in any case, that large domestic houses had privileged access to the Assembly Houses by close proximity, while the smaller houses were mostly confined to the inner streets and the borders between Quarters, i.e. the furthest away from the Assembly Houses.

In the small Trypillia settlement of Moshuriv, power-law distributed houses are also spread across the settlement plan, with the largest house situated in the east end of the inner house ring, following the same scheme as the mega-sites, as noted by Ohlrau (2020, 241–43). Here, much like at Maidanetske in miniature, the hierarchically scaling houses are evenly spread without obvious clustering, the only pattern being that the three houses just below the largest one in size also are located on the inner house ring (Figure 6.6).

Since the main goal in this thesis is not only to measure and interpret archaeological data, but

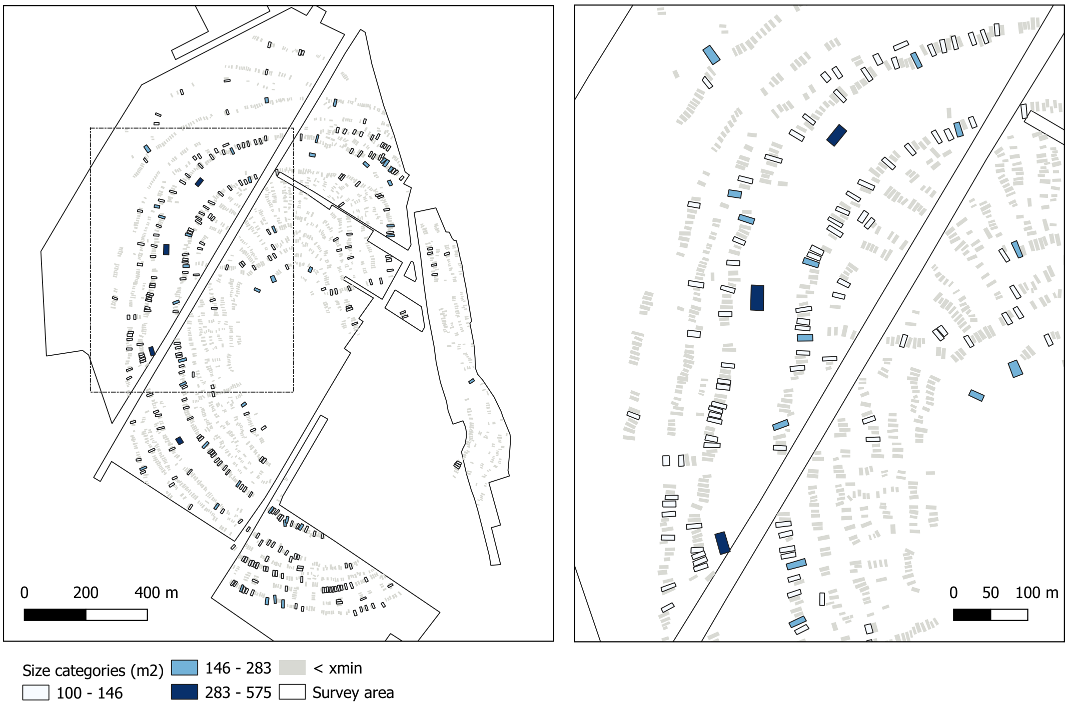

Figure 6.4: *Power-law distributed houses at Maidanetske (Trypillia), grouped to three levels with Jenks optimisation. The levels are arbitrary but overlap well with the typological distinction between mega-structures (dark blue) and other houses. Hierarchical scaling includes far more houses than the mega-structures, and is distributed across the settlement. Figure made by author after Ohlrau (2020)*

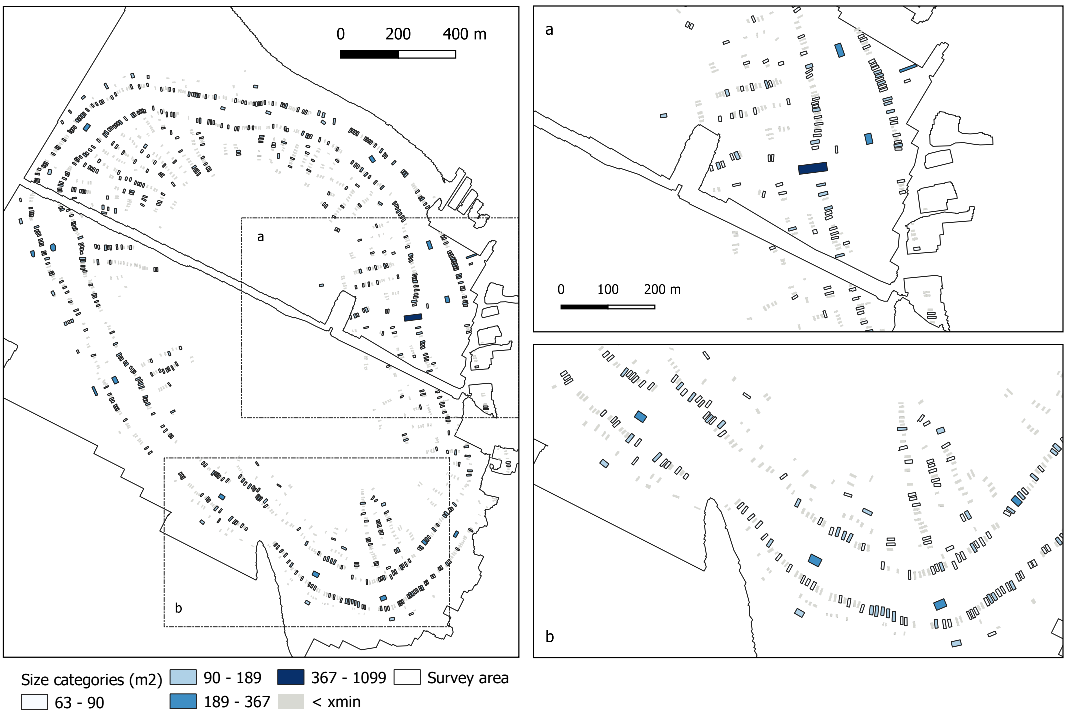

Figure 6.5: *Power-law distributed houses at Nebelivka (Trypillia), grouped into four arbitrary levels with Jenks optimisation, the two largest of which are overlapping with the typological levels of the “Mega-structure” and the “Assembly Houses”. Figure made by author after Hale (2020)*

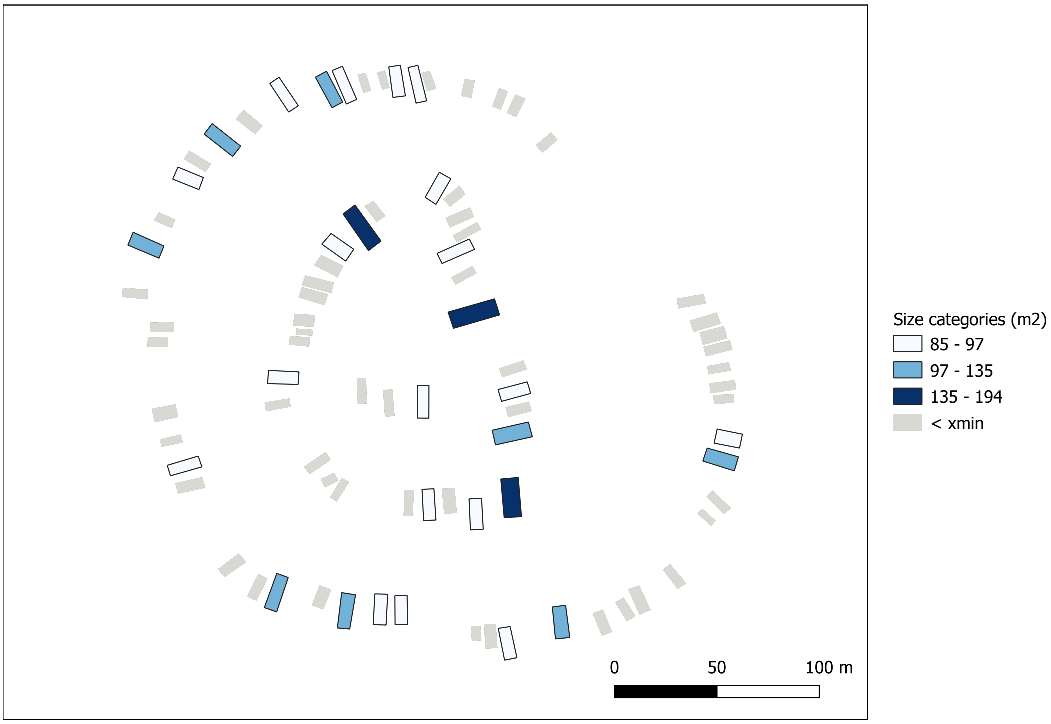

Figure 6.6: *Power-law distributed houses at Moshuriv (Trypillia), grouped arbitrarily into three size categories using Jenks optimisation. Made by author after Ohlrau (2020)*

also to assess the reliability of the methods and the robustness of the results, in the following sections I propose further iterations of this analysis on gradually more filtered sub-sets of the data, to see whether the power-law interpretations continue to hold. Various factors specifically related to archaeological data could be thought to influence size distributions in ways that would preclude any meaningful interpretations in terms of social organisation. More specifically, if power-law distributed house sizes are to be considered as statistical signatures of social hierarchy, the possibility of them being no more than artefacts resulting from methodological choices like spatial or temporal lumping needs to be excluded.

6.2 Quarters/neighbourhoods

Two of the large settlements with power-law house-size distribution tails – Nebelivka and Vráble – were selected for further distribution fitting analysis within separate quarters or neighbourhoods. The Linear Pottery site of Vráble is clearly organised into three distinct neighbourhoods, termed North (N), South-West (SW) and South-East (SE) by the research team in the dedicated publications (Furholt, Müller, et al. 2020; Winkelmann et al. 2020). The Trypillia mega-site of Nebelivka has also been subdivided into quarters by its research team (following their terminology here), but the limits between them were set somewhat more arbitrarily based on a series of criteria, including natural topography, placement and orientation of streets and entrance ways, as well as the locations of the large so-called Assembly Houses (Hale 2020, 123). This resulted in a series of 14 defined quarters arbitrarily labelled A–N, which is reused as is here. It should be noted that the single largest building in Nebelivka – the "Mega-structure" of approximately 1.200 m^2 – is not included in any of these quarters, but lies in the interstice between quarters A and B along with a few other houses. Though this delimitation of quarters at Nebelivka could have been done differently, which is clearly recognised by the researchers themselves, I do not pretend that I could make any better judgement. The number of houses per quarter is included in Table 6.2 below.

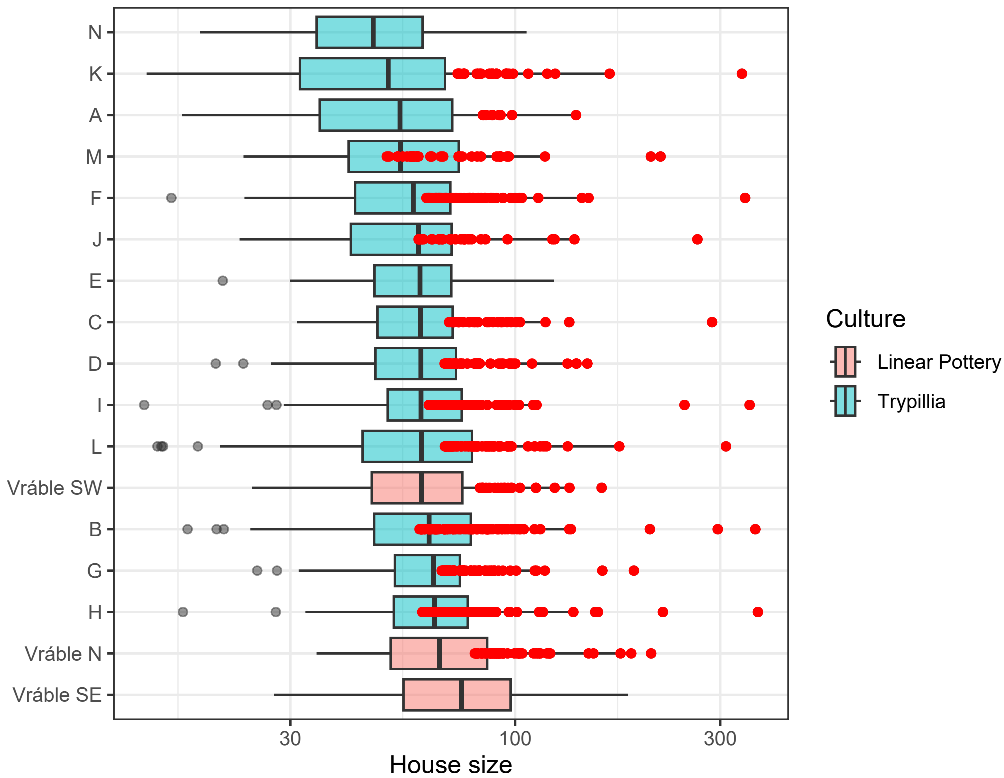

Figure 6.7: *House-size distributions of individual quarters for Nebelivka and neighbourhoods for Vráble, arranged according to median house size. Red dots indicate houses of size* $\geq x_{min}$ *in cases where the distribution tail was interpreted as a power law. X axis is logarithmic*

When analysed separately following the same distribution-fitting algorithm as with the entire settlements, 14 of the 17 data series were recognised as having power-law tails (Figures 6.7 and 6.8). The only deviations were Nebelivka quarters E and N, and the SE neighbourhood of Vráble. This clearly shows that the power laws observed at the total settlement level do not merely result from analytically stacking together separate more lightly skewed distributions like log-normals, but rather from genuinely different scaling patterns in house sizes for these settlements. The α estimates, between 3.6 and 6.8 (with one outlier for Neb. A), were largely similar to the ones observed for the entire settlements (5.2 for Vráble and 4.8 for Nebelivka; Tables 6.2 and 6.1). Each of the quarters and neighbourhoods were furthermore identified as log-normal when analysed over their entire size range, with μ and σ estimates very similar to those observed for the total settlements.

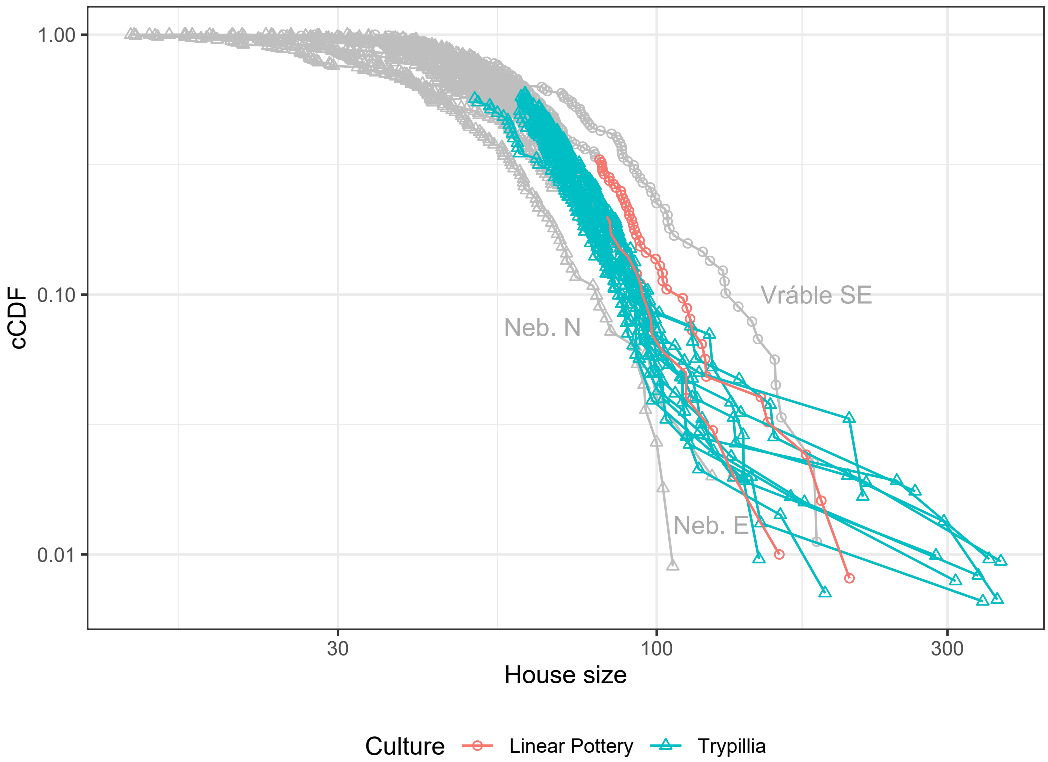

Figure 6.8: *Survival function of the same house-size distributions of quarters/neighbourhoods. Coloured dots represent houses within power-law tails. Three series were not interpreted as power laws. Scales are logarithmic*

Table 6.2: *Results of distribution fitting on separate quarters at Nebelivka and neighbourhoods at Vráble, arranged by tail model and parameter. Par1 and Par2 indicate μ and σ for log-normal distributions, and T_Par1 is α for power-law and λ for exponential tail distributions. Gini index is calculated on the entire sample*

Quarter	Model	Par1	Par2	Tail	T_Par1	xmin	N	N_tail	Tail_P	Gini	Culture
Neb. N	ln	3.830	0.408	exp	0.063	55.7	111	41	0.37	0.224	Trypillia
Neb. E	ln	4.057	0.333	exp	0.061	59.3	50	27	0.54	0.179	Trypillia
Vráble SE	ln	4.296	0.424	exp	0.034	84.4	89	37	0.42	0.236	Linear Pottery
Neb. A	ln	3.940	0.441	pl	10.490	84.1	51	10	0.20	0.240	Trypillia
Neb. G	ln	4.137	0.295	pl	6.763	67.5	141	60	0.43	0.160	Trypillia
Vráble SW	ln	4.090	0.367	pl	6.704	82.8	100	20	0.20	0.202	Linear Pottery
Neb. D	ln	4.085	0.361	pl	5.684	68.5	104	38	0.37	0.196	Trypillia
Neb. F	ln	4.035	0.385	pl	5.359	62.2	151	69	0.46	0.217	Trypillia
Vráble N	ln	4.215	0.358	pl	5.350	80.6	124	41	0.33	0.208	Linear Pottery
Neb. C	ln	4.097	0.327	pl	5.295	70.2	101	28	0.28	0.191	Trypillia
Neb. L	ln	4.040	0.501	pl	5.285	68.6	126	54	0.43	0.255	Trypillia
Neb. K	ln	3.866	0.548	pl	4.974	73.5	120	27	0.22	0.300	Trypillia
Neb. I	ln	4.113	0.393	pl	4.832	63.0	104	49	0.47	0.218	Trypillia
Neb. J	ln	4.043	0.416	pl	4.620	59.6	57	29	0.51	0.244	Trypillia
Neb. H	ln	4.175	0.390	pl	4.451	60.8	106	63	0.59	0.221	Trypillia
Neb. B	ln	4.112	0.433	pl	4.411	59.9	149	86	0.58	0.241	Trypillia
Neb. M	ln	3.985	0.466	pl	3.614	50.3	60	34	0.57	0.271	Trypillia

In addition to testing whether the power-law interpretations hold when the analysis is done at the quarter rather than settlement level, performing distribution fitting on separate quarters allows for observing intra-site differences between quarters. The immediate interpretation of the two non-power law quarters at Nebelivka would be that their houses are more similar in size and that this could indicate a flatter hierarchical structure, or an absence of hierarchy altogether. The association between settlement size and power-law distributions noted above could also be valid here. With only 50 houses, quarter E is the smallest of the Nebelivka quarters (Table 6.2, Figure 6.9). With a smaller population the inhabitants could possibly do well without a long-range hierarchy beyond the (comparatively small) assembly house located within its section of the central street. However, this interpretation does not fit for quarter N which is in the mid-range of house count with 111 houses. Here the explanation could be taphonomical, as the quarter is lacking an Assembly House which could be located in an unpreserved part of the site (Hale 2020, 123). Interestingly, quarter A also has a small house count (51) *and* is lacking an assembly house, but its house-size distribution is all the same interpreted as a power law, albeit with atypical parameter values – x_{min} is the highest among the Nebelivka quarters, and α is by far the highest – indicating that these factors may influence model selection without determining it entirely. For quarter E in particular, another possibility which is linked at least to house count, is that its quarter borders are drawn incorrectly so that the quarter should either be larger or included into one of the neighbouring quarters, which would then probably still be power-law distributed. The explanations based on taphonomy and quarter border definition work under the assumption that since most of the quarters have power-law distributed house sizes, all of them should originally have had this, and that the data is somehow distorted. The social explanation of differing degrees of hierarchical organisation between quarters remains a possibility, but at this stage any of these explanations are difficult to exclude. Given that the Nebelivka mega-site is exceptionally well preserved for its size – the site plan is very nearly complete – this illustrates the difficulty archaeologists face when trying to interpret only partially preserved settlements. I personally see the Nebelivka plan as extremely regular between quarters, and the mentioned deviations in quarters N and A as resulting from taphonomy (missing data) and in quarter E as incorrectly drawn quarter borders. I also question the border drawn between quarters J and K, though this seemingly has not influenced their internal house-size distributions noticeably.

Regarding Vráble SE, in addition to not having a power-law tail, the house-size distribution

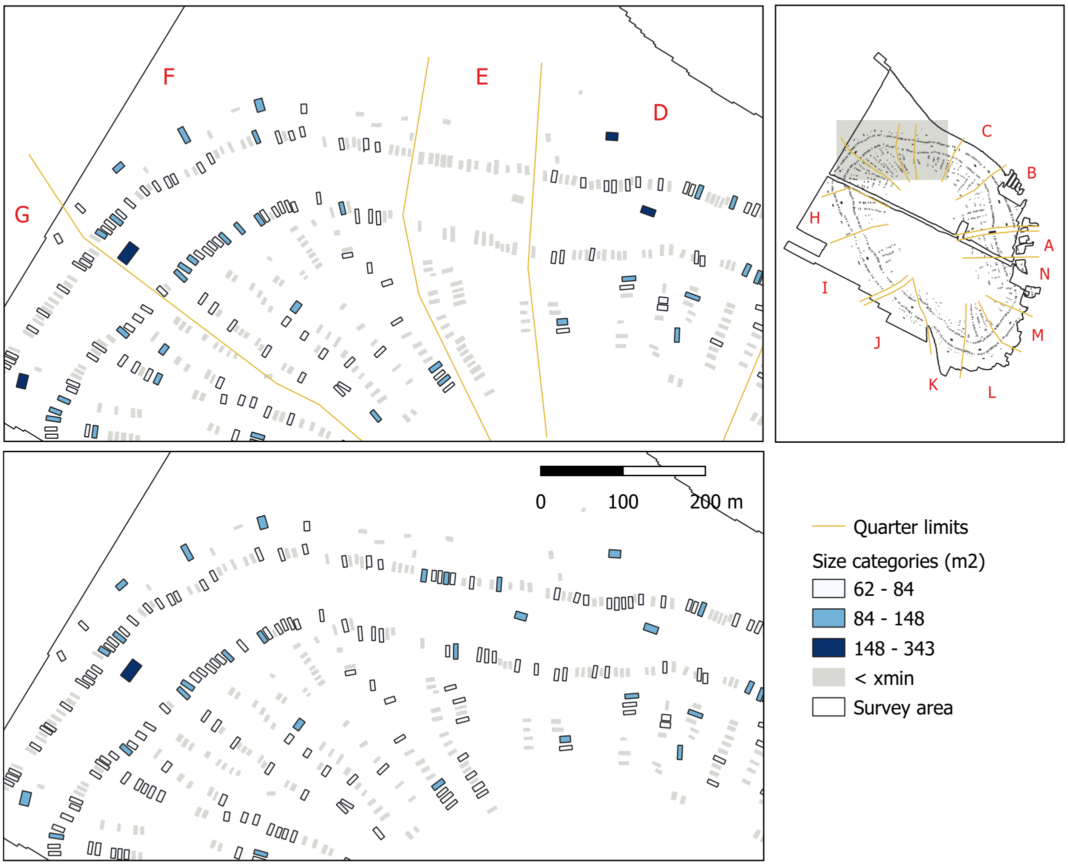

Figure 6.9: *Power-law distributed houses in quarters D to G in Nebelivka, fitted by quarter (top) and for the settlement as a whole (bottom). Quarter-wise distribution fitting does not identify hierarchical scaling in quarter E, though many of the houses there are included in the power-law model for the whole settlement. Size categories are arbitrary (three levels with Jenks optimisation) and values differ between quarters. Legend values correspond to quarter F. Figure by author after Hale (2020)*

is also distinguished from the two others with a markedly higher median, i.e. generally larger houses, as well as higher mean and standard deviation (Table 6.2). On the survival function plot on Figure 6.8 this translates to a wider and more regular parabolic curve of house sizes, more typically log-normal, as opposed to the more abruptly descending straight lines of the two other neighbourhoods (in red on the plot). For the N and SW neighbourhoods, the combination of a less skewed log-normal for the main body of the distribution (i.e. regular houses) and an actual power law for the largest 41 and 20 houses respectively (1/3 and 1/5 of the houses), could indicate a more marked difference between these, which could be interpreted either as an emerging social elite or the presence of some large houses with special social functions. However, any further interpretation in social terms depends on how many of these houses were actually coeval, i.e. in use at the same time. If most of these large and hierarchically scaling houses were in reality spread out through time, so that only a very few existed at any single one moment, it would not make much sense to speak of a social hierarchy, and the distribution should not be interpreted as a power law. For interpreting social relationships between different households and how they function in daily life, it is essential that the studied households be contemporary – a requirement that is often hard to meet in archaeology, but an attempt is done in the following for Vráble only.

6.3 Temporal samples (Vráble)

Following the procedure for defining data series with houses in Vráble considered to be coeval at given time samples, as presented in Section 3.2, these series were analysed through the same distribution fitting algorithms as before. Ten of the 16 time samples had more then 10 houses, and these were situated consecutively in time between 5230 and 5050 cal. BCE, the remaining samples being at the start and end of the settlement's duration. Of these 10 samples, seven had tails in their house-size distributions that were interpreted as power laws (Figures 6.10 and 6.11).

A last attempt to test if these power laws were simply statistical artefacts of lumped data sets, was done parting from the following hypothesis: At any given time during Vráble's duration, the three neighbourhoods were largely independent from each other and there was no hierarchical relationship between them. Within each neighbourhood, household sizes were log-normally distributed, or lightly skewed, as a result of their generalised post-marital residence pattern of patrilocality, and in addition there was always one single building markedly larger than the others, intended for communal ceremonial use. Analysing all houses in Vráble together, even by considering only temporally coeval houses, would be enough to generate a false impression of a hierarchy between the largest houses.

In order to investigate if this was the case, only the houses from the south-west neighbourhood of Vráble were selected out (being the ones for which the construction date model was the most secure), and the single largest house of each time sample was excluded. It then remained only five time samples with more than 10 houses, namely the ones situated between 5190 and 5110. Out of these, three were still interpreted with power-law distributed house-size distributions (Figures 6.10b and 6.11b). These power laws could thus not be explained as resulting from neither spatial nor temporal lumping, nor from the eventuality of a special

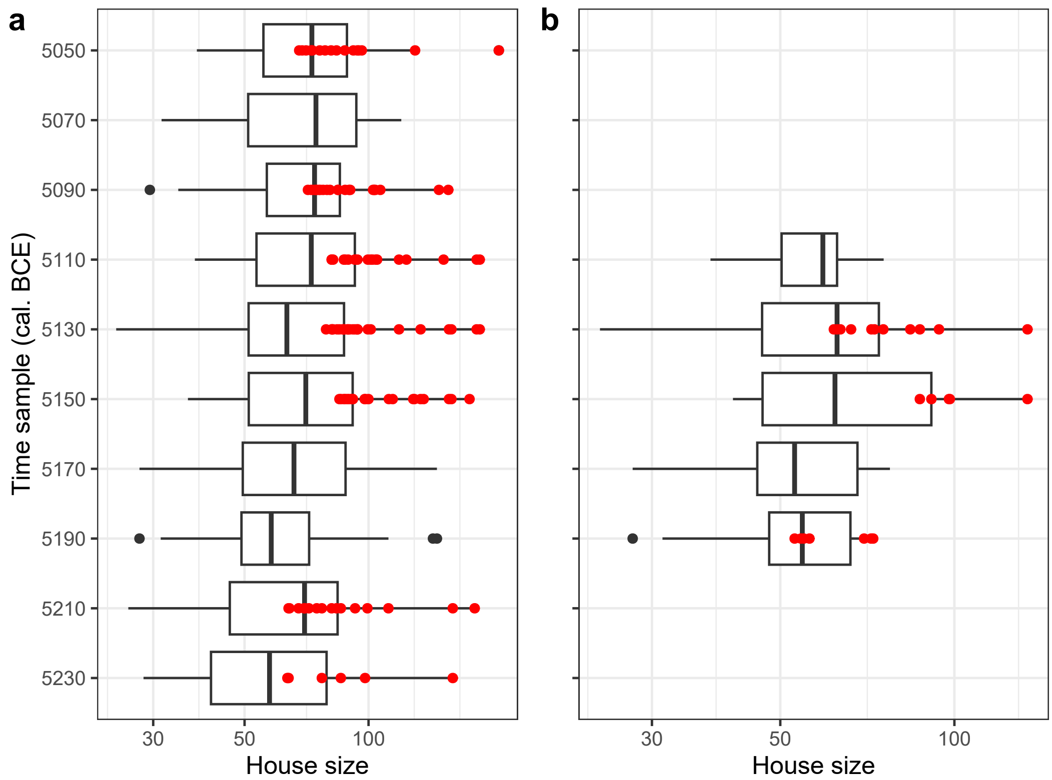

Figure 6.10: *House-size distributions at single time samples for the entire settlement at Vráble (a) and the South-West neighbourhood only (b). Red dots indicate houses with power-law distributed size. Only distributions including more than 10 houses are represented. In plot b, the single largest house of each sample is also excluded. X axis is logarithmic*

function of the single largest house, and it seems therefore reasonable to conclude that they represent actual hierarchical scaling which can be further interpreted in terms of social organisation. It also seems plausible that analysing the three neighbourhoods together may be justified, which allows for identifying this scaling behaviour over a larger range of the settlement's duration. Though conclusions should not be taken too far given the many uncertainties related to the temporal modelling applied here, it is interesting to note that power-law tails are identified throughout the duration of the settlement, and are seemingly not a feature that is specifically related to the later phases only. This seemingly contradicts the current understanding of the temporal trajectory of inequality in Linear Pottery society, which is generally seen as increasing towards the later phases, leading to the rising tensions seen in the widely discussed massacre deposits, including in the very same Vráble settlement [Furholt, Müller-Scheeßel, et al. (2020); Nils Müller-Scheeßel et al. (2021); Section 3.2]. Not surprisingly, the more conventional inequality measure of the Gini index draws a similar picture of the temporal development of house sizes at Vráble (6.3). Throughout the phases with 10 or more coeval houses, the index remains rather stable at values between 0.28 and 0.20, and if anything it shows a more decreasing than increasing trend. In other words, the dynamics of changing house sizes at Vráble do not give support to an interpretation of the eventual decline of the settlement as a result of rising inequalities (though this does not exclude the possibility such inequalities being expressed through other material proxies).

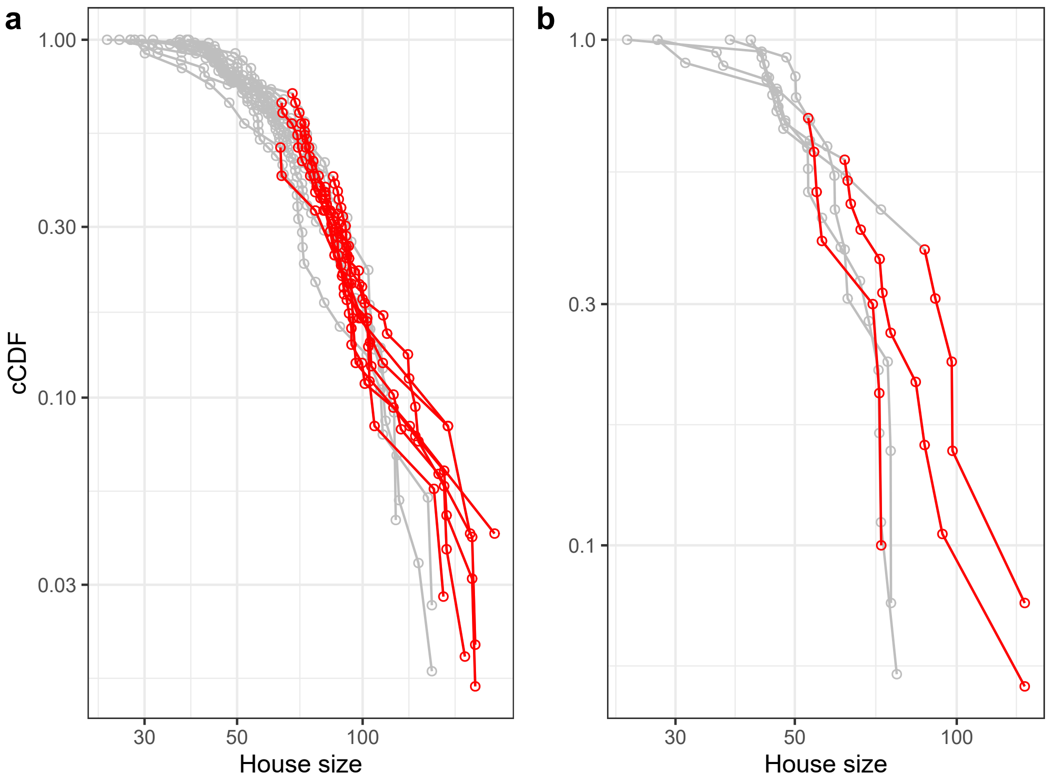

Figure 6.11: *Survival function of the same house-size distributions of time samples for Vráble (a) and Vráble SW only excluding the largest house of each sample (b). Coloured dots represent houses within power-law tails. Power-law tails persist despite the gradual breaking down of the data set. Scales are logarithmic*

Table 6.3: *Distribution analysis results for Vráble, subdivided into time samples with coeval houses, arranged chronologically. The analysis was also done on the South-West neighbourhood separately, where the single largest house for each sample was excluded. In both series, only samples consisting of 10 houses or more were analysed. Par1 and Par2 are μ and σ for log-normal and normal distribution models. T_Par1 is α for power-law and λ for exponential tail models. Tail_P is the proportion of data points (N) in the tail model (N_Tail), or N_Tail/N. Gini index is calculated on the entire sample distribution*

BCE	Model	Par1	Par2	Tail	T_Par1	xmin	N	N_tail	Tail_P	Gini
Vráble										
5230	ln	4.059	0.487	pl	4.232	63.5	12	6	0.50	0.279
5210	ln	4.188	0.477	pl	4.314	63.9	24	16	0.67	0.258
5190	ln	4.114	0.372	exp	0.041	51.4	38	28	0.74	0.210
5170	ln	4.199	0.360	exp	0.038	60.9	58	35	0.60	0.204
5150	ln	4.288	0.398	pl	5.235	85.0	53	22	0.42	0.228
5130	ln	4.224	0.395	pl	5.041	78.9	64	25	0.39	0.225
5110	ln	4.281	0.368	pl	4.907	81.3	49	19	0.39	0.215
5090	ln	4.224	0.378	pl	5.542	71.3	36	21	0.58	0.206
5070	norm	75.086	25.864	exp	0.044	70.0	22	14	0.64	0.197
5050	ln	4.297	0.357	pl	5.010	67.9	24	17	0.71	0.201
Vráble SW*										
5190	norm	53.755	14.740	pl	7.911	52.9	10	7	0.70	0.151
5170	norm	54.851	13.923	exp	0.063	44.3	18	15	0.83	0.145
5150	ln	4.186	0.375	pl	8.045	87.1	13	5	0.38	0.213
5130	ln	4.100	0.365	pl	5.489	61.9	19	11	0.58	0.196
5110	norm	58.895	10.918	exp	0.074	48.2	13	12	0.92	0.103

Three examples of the spatial distribution of coeval power-law houses is seen in Figure 6.12. Through most of the analysed time samples there is seemingly only one or two very large houses per neighbourhood, and in the first and last of the samples that are large enough for distribution fitting (5230 and 5050 cal. BCE) there is only one for the entire settlement (Figure 6.10a). This further indicates that it would be in the middle phases of Vráble in particular – roughly between 5150 and 5090 following the model – that the three neighbourhoods developed into independently functioning settlements, possibly with increased competition between them, as is also suggested by the construction of the enclosure and palisade around the south-west neighbourhood shortly after and by 5070 BCE (Furholt, Müller, et al. 2020, 493–98). A single house per phase which is clearly larger than the rest is a regular trend in Linear Pottery settlements, though absolute size categories vary between contexts (Coudart 1998, 49). This largest house does not necessarily represent any obvious architectural particularities indicating special function like the Modderman 1a type (1970), though they do tend to have double posts (possible elevated granaries) in the frontal section [Coudart (1998), pp. 41-2; 3.2]. Though this is not yet confirmed through excavation in Vráble (#CHECK), the largest houses are generally upscaled versions of smaller common types, in particular the type 1b, or *Großbau* with ditch in the posterior section only. And though it is difficult to prove with house sizes alone, it seems fully plausible that such largest houses be inhabited by clan leader households. Once the settlement grew large enough for more than one clan to take on a leading role – i.e. too large for one clan to control alone – the settlement would split and crystallise into three more or less independent factions, each with a clan-leading household during the middle phases. Following this model, it would then be the competition between three clan households or Houses that lead to the increased tensions and violence seen in the late phases of the settlement, rather than generally increasing levels of inequality. Given that the above described power-law distributions stretch well beyond these largest houses in the Vráble neighbourhoods, it is furthermore tempting to interpret the placement of subsequent houses in the size hierarchy as a function of their kin proximity to the leading household, with the smallest houses (those outside the power law) representing recently established or otherwise poorly integrated households (Hachem and Hamon 2014, see also Section 3.2).

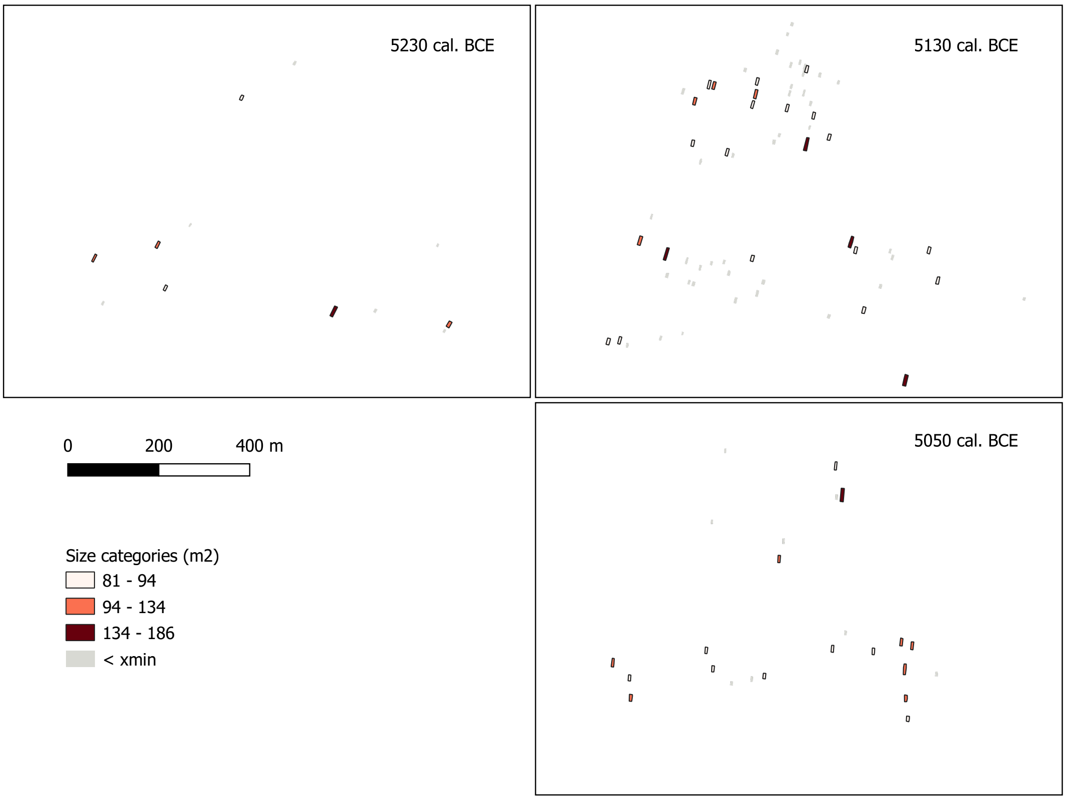

Figure 6.12: *Vráble at three temporal samples, with house sizes above x_{min} grouped into three arbitrary classes by Jenks optimisation. Parameter values differ slightly between time samples (see table), and the legend categories refer to the 5130 sample. Counter-clockwise shift in house orientation is used as proxy for construction date*

6.4 Summary of findings

The distribution fitting analysis on the house sizes of entire settlements resulted in power-law tails being clearly identified in four out of 13 cases. These were the Trypillia mega-sites of Maidanetske and Nebelivka, as well as the smaller Trypillia site of Moshuriv and the Linear Pottery settlement of Vráble. The much smaller Linear Pottery settlement of Horný Oháj was also identified with a power-law tail, but the result was disregarded due to the small sample size. These four settlements were also the largest of the sample of settlements. Furthermore, one Linear Pottery settlement – Úľany nad Žitavou – had normally distributed house sizes, and one Trypillia settlement – Talne 3 – had a log-normal house-size distribution with very little skew which in practice was indistinguishable from a normal distribution. While these two settlements were not the smallest in the sample, they were clearly within the lower end of settlement sizes. While the normal distribution is by definition symmetric, the power-law distribution is the most asymmetric or unequal of the models compared here. Though the settlement sample used here is small, I see these results as indications that absolute settlement size is more determinant for the shape of the house-size distribution than cultural belonging. Given that settlements within archaeological cultures like the Trypillia and the Linear Pottery vary greatly in size, this has potentially great implications for how we conceptualise their social organisation. In other words, the daily life in large Neolithic settlements may have been more similar across cultures than to small settlements within the same culture, much like urban life of large contemporary cities may have more in common more across nations than it has between a large city and its surrounding rural area. More specifically, the house-size distributions of these large Neolithic settlements show hierarchical scaling which – at least in theory – can be related to preferential attachment processes where the already large have advantages for growing larger. How exactly this may translate to the specific cultural settings of the Trypillia and the Linear Pottery contexts is however open for debate, and other strands of evidence from their material culture indicate that these hierarchies may have been socially quite different. These issues are further discussed in Chapter 10.

Following the methodological tests given in the previous chapter, the distribution fitting was also performed on separate quarters for Nebelivka and neighbourhoods for Vráble. The results were largely similar, with a few exceptions, notably the south-east neighbourhood of Vráble (one of three) and the N and E quarters of Nebelivka (two of 14). For Nebelivka, the result of two non-power-law quarters could largely be questioned with missing data and unclear quarter boundaries, and the hierarchically scaling houses were wildly spread across the site both when analysed as a whole and by quarters. Compared to Maidanetske, for which this spread was seemingly very even and unstructured, in Nebelivka the scaling houses were at the same time spread throughout the quarters as well as clustered, so that large houses tended to be found near other large houses, and small houses near other small houses, seemingly reflecting some level of segregation between privileged and unprivileged households. For Vráble, the picture was somewhat more confusing since the interpretation of house scaling would differ more between the perspective of the whole settlement and that of the separate neighbourhoods. In particular Vráble SE has a scaling of house sizes which is very similar to that of Vráble N when all is analysed together, while it has no scaling when analysed separately. This difference should perhaps not be overstated however, since other parameter values showed that the south-east neighbourhood did not have a markedly more symmetrical house-size distribution than the other two sections – both its standard deviation and Gini index were actually higher than those of the other two (Table 6.2). Judging from the cCDF plot in Figure 6.8, Vráble SE was seemingly not recognised as having a power-law tail since its two or three largest houses were smaller than what should be expected for a power law given the size of the other large houses.

The houses of the Vráble settlement were furthermore attributed to one or more temporal samples based on modelled construction date and house durations. Sixteen samples of 20 year intervals were defined, 10 of which had more than 10 houses attributed to them and were thus analysed further. Of these, seven were interpreted as having power-law distributed house sizes. These were spread in time, and neither the distribution types nor the more standard Gini index give support to interpretations of increasing inequalities toward the later occupation phases of Vráble. Rather, it was shown that in the early and late phases there was only a single very large house dominating the hierarchy of houses at the whole settlement, while in the middle phases there were usually two or three, and when mapped to the settlement plan, these were situated in the three different neighbourhoods. From this it could be interpreted that it was increasing competition and tension between leading Houses or clan leading households in the three neighbourhoods – rather than increased inequalities in general – that lead to the violence observed in the skeletal material, as well as the construction of the enclosure surrounding the south-west neighbourhood, in the late phases of the settlement shortly before its decline. An additional distribution-fitting and model-selection test was done on coeval houses of the south-west neighbourhood only, and excluding the single largest house for each phase, to ensure that the power-law signal was not a statistical artefact resulting from lumping the three neighbourhoods and from a scenario where a single communal building would dominate otherwise moderately skewed distributions. In this sub-set, only five time samples were large enough for analysis, three of which still gave power-law distributed tails. It was concluded that the observed hierarchies in house sizes swere real, at least in the case of Vráble, and that they most probably represent some sort of hierarchical social organisation, which is discussed further in Chapter 10.

Part III

Settlement Plans

Chapter 7

Village planning in prehistory

Systematic relationship between settlement layout and social structure in small-scale societies has been the topic of some ethnographic studies. In a comparative study of ethnographic examples from all over the world, Fraser (1968) showed that some tendencies existed, but that there was also large variation. One trait was that the more mobile the society, the less strict where the overall planning principles, while planning tended to be more organised in more settled, as well as hierarchic societies. Religious, cosmologic and defensive concerns were shown to influence the relative placement of buildings, but not in any uniform way cross-culturally. More recent kinship studies have attempted to link different kinship structures with settlement layouts Souvatzi (2017), but methods for effectively quantifying these layouts and thus allowing for cross-cultural regularities to be apprehended have been largely lacking. Factors that may be hypothesised to affect village layouts in systematic ways include:

- Political structure (but, as with hierarchy, an organised layout does not necessarily equate top-down despotic decision making, as other strong social institutions may be anarchic or heterarchic)
- Kinship, matrimonial and locality structures
- Cosmology (e.g. Linear Pottery house orientations)
- Economic and ritual functions of village elements (constructed and non-constructed)
- Local landscape setting (to be factored out)

The maybe most promising results in this direction in prehistoric archaeology have been presented by Furholt (2016), who identified two distinct village types in the early Neolithic of the Aegean region, based on layouts observed and coded into a range of quantitative and qualitative variables. Furholt applied correspondence analysis to reduce the dimensionality of his data set, resulting in a rather clear distinction between a compact bee-hive-like “Anatolian village” and a more dispersed “Balkan village”, which would become representative of later developments in south-east Europe. He interpreted these layout differences in terms of social organisation, in that the Anatolian type would have tighter and stronger overarching community structures, while the Balkan village showed stronger independence between households.

While intriguing, the assumptions underlying interpretation are difficult to test, from the lack of comparable material, in particular from ethnography. Furthermore, studies like that of Fraser (1968) provide settlement plans which by quality and precision generally do not meet the standards and requirements of archaeology, and it is difficult to perform any meaningful quantitative analyses on them.

In this part of the thesis, I wish to contribute to this topic, with the application of fractal image analysis. In particular, I believe that the theory underlying fractal geometry may well translate into social processes involved in the emergence of settlement layouts.

7.1 The geometries of conscious planning vs. emergent behaviour

The rationale of the following analysis is simple: Regular Euclidean patterns – like rectangular grids, circles and straight lines – result from conscious planning, while chaotic, irregular fractal patterns are indicative of independent households. This distinction should not be considered as binary, but along a continuum, and it is hypothesised that the degree of planning versus household autonomy can be effectively quantified through fractal analysis (see Bruvoll, n.d.).

As mentioned earlier, fractal geometry was first developed by Mandelbrot (1982), who defined certain measures to describe patterns that were too irregular to be quantified through traditional geometry. The main measure he proposed was termed the *fractal dimension*. This was meant as an extension of the Euclidean dimensions (what we refer to when saying 2D or 3D), by taking on fractional values in addition to integer values. A pattern with a fractal dimension of 1.5 would be neither a curve (1D) nor a plane (2D), but something in-between, that is, a broken pattern which does not fill its embedding dimension – in this case a two-dimensional plane. Such a pattern could be the Sierpinski triangle or the von Koch curves seen in Chapter 2, or real world features like coastlines, river systems, Moon craters or national borders.

The fractal dimension was defined by Mandelbrot as

$$D = \frac{logN}{log1/r}$$

where N is element count and r is scale (as fraction of the whole). In Mandelbrot (1982), he effectively applied this measures to a range of mathematically constructed fractals, showing its usefulness as quantification of highly irregular shapes.

He furthermore noted that while this dimension was indicative of the degree of space-filling or completeness of patterns, in many cases patterns with very different image textures could be shown to have the same fractal dimension. As *D* is a summary measure (calculation methods are explained in the next chapter), it does not say everything about the data it describes, similarly to how a mean of a data distribution does not in itself say anything about the spread of the data. For describing the texture of fractal patterns, he tentatively defined *lacunarity*, mathematically expressed as

$$L = 1 + (\frac{\sigma}{\mu})^2$$

or the variance to square mean ratio of foreground pixel mass (see Hingee et al. 2019 for an extensive review of the development of lacunarity measures since Mandelbrot). It quantifies the distribution of voids or gaps in the pattern (*lacuna* being Latin for gap or lake), and has since been included in a variety of studies of texture, however mostly within the natural sciences.

While these measures are held to quantify the regularity or irregularity of spatial patterns, applying them to archaeological settlement plans is not straight-forward. First of all, it is not entirely beyond doubt that fractal settlement plans are always unconsciously emergent (Eglash 1999). Also, it is not obvious how exactly fractal dimension and lacunarity estimates on empirical plans differentiate between Euclidean and fractal patterns. I recently showed that random noise in synthetically generated images was correlated to both *D* and *L*, but there are challenges involved in transferring this to archaeological settlement plans (Bruvoll, n.d.). The interaction between other texture-related variables and their relative influence on fractal dimension and lacunarity estimates are largely under-studies, seemingly in any discipline (in archaeology these discussions are largely absent, as reviewed below).

The relationship between image density (built-up area) and fractal dimension was evaluated by Thomas, Frankhauser, and De Keersmaecker (2007), who showed that these two parameters, under certain conditions (constant observation window, prefactor values close to 1), are exponentially correlated. They furthermore showed that observation window size and shape, as well as centroid placement, have little influence on *D*, while they have more influence on density when the pattern is not homogeneous. They do show, however, that images with the same density may have quite much variation in *D*, which is reflected in the layouts. Judging from their examples, more clustered layouts give higher *D* values, while more dispersed or dusty layouts give lower *D*, when density is constant. According to Thomas, Frankhauser, and De Keersmaecker (2007), density is a crude measure of the overall intensity of the pattern, while fractal dimension is characterises the morphological structure, though it is not directly descriptive. Lacunarity measures were tested on urban patterns by Myint and Lam (2005), but have not yet become a standard tool in quantitative urban studies.

7.2 Fractal image analysis in archaeology

Intra-site spatial analysis approaches have in general received much attention in archaeology, and analytical methods and techniques specifically related to architecture and spatial organisation are known as space syntax, point-pattern analysis, Ripley's K, visibility graph analysis, viewshed analysis and access analysis, to name a few (see Gillings, Hacigüzeller, and Lock 2020 for a recent comprehensive review). However, few of these, if any, use complexity theoretical frameworks (the percolation analysis proposed by Maddison 2020 is seemingly an exception), and I believe fractal analysis can become a valuable supplement to the panoply of spatial analysis methods, in particular because of its explicit theoretical framework.

In archaeology, seemingly all previous applications of fractal image analysis has been done in

Mesoamerican contexts. The earliest example of which I am aware, is a study by (Oleschko et al. 2000; commented on by Brown, Witschey, and Liebovitch 2005) on the Ciudadela complex in Teotihuacán, Mexico, where the measured fractal dimension was reported to be close to that of the mathematical fractal known as the Sierpinski carpet. From this the authors argued that the two may have similar underlying generative mechanisms. Brown and Witschey (2003) adapted the method further to archaeological settlement plan drawings, and compared measurements on different quarters and settlements from classic Maya Yucatán.

The study which contributed the most with impetus for the analysis presented here, was done by (Farías-Pelayo 2017; more detailed in Farías-Pelayo 2015), who, apparently as the first in archaeology, added lacunarity measures to better apprehend the particular patterns of lithic artefacts, petroglyphs and site topography (not settlement layout) within the Xajay culture in central Mexico. By combining fractal measures of different material series into a single index, she claimed to better characterise this culture which has proven difficult to characterise through traditional typology.

These limited developments are the reasons for the largely exploratory and experimental nature of this part of the thesis, where the methodological discussions provided are probably of as much, if not more interest than the results from the archaeological analysis.

Chapter 8

Methods: Fractal image analysis

In this chapter I present briefly the techniques for calculating fractal dimension and lacunarity estimates on archaeological and synthetic settlement plans, as applied further on. The chosen procedure for preparing images of these plans for analysis is also detailed. I review some issues related to the interpretation of these measurements, mainly in terms of visual pattern characteristics and textures. In order to evaluate how various parameters influence fractal dimension and lacunarity estimates on images, some tests on synthetically generated images are provided and discussed. Results of analyses on images of archaeological settlement plans are presented in the following chapter.

8.1 Calculating fractal dimension and lacunarity

As noted in the previous chapter, the fractal dimension of spatial patterns was originally estimated by Mandelbrot by deciphering visually the relations of size and frequency of elements between the *initiator* and the *generator* of theoretical (i.e. fully regular) fractal sets like the Koch curve or the Sierpinski triangle (e.g. Mandelbrot 1982, 39 ff., see also the Figures above in Chapter 2). Empirical fractals on the other hand usually also include some stochastic elements, rendering this technique much more difficult to use in practice, as the generator of the pattern is harder or even impossible to discern. A number of more systematic methods have therefore been developed since the 1980s for estimating the fractal dimension of empirical patterns, the by far most popular being the so-called box-counting method (Li, Du, and Sun 2009; Klinkenberg 1994). The principle of the method is quite simple:

- Cover the pattern with a regular square grid of a given mesh ("box") size, and count the number of boxes that intersect with the pattern.
- Do this for a range of box sizes (usually in exponential steps from the pixel size up to about half of the image length), recording the number of boxes for each size.
- Fit a linear model to the logarithms of box counts to box sizes, and the slope of the line corresponds to an approximation of the fractal dimension of the pattern.

Readers of Section 4.2.4 will recognise that the relationship between box sizes and box count is a power law with fractal dimension being defined as its scaling exponent, and while one may wonder why the log-log fitting method is accepted here when it was banned in previous chapters, there are some differences between the two contexts. Firstly, we are not dealing here with univariate distributions and their underlying generative mechanisms, but rather the relation between two variables. Secondly, the fractal dimension (in the following denoted *D*) is *defined* as this scaling exponent, that is the logarithm of frequency (number of elements) divided by the logarithm of their sizes relative to the whole. In theory, the graph of boxes to box sizes should always follow a power law – i.e. a straight line on a log-log plot – but with a fractional slope value when the pattern is fractal and an integer slope value when the pattern is a fully Euclidean shape.

There are some caveats regarding the interpretation of this estimate however. One is that there is nothing preventing a fractal shape of having an integer dimensional value. Its defining characteristic is that unlike Euclidean dimensions it can take on any value from 0 to 3 for spatial patterns and higher for more abstract patterns, and the numbers 0, 1, 2 and 3 are values just like any other in this continuous range. An integer dimension is thus in itself not enough to claim that a pattern is Euclidean and not fractal (though fractals with exactly integer dimensions are rare). But it is more important to consider what exactly is meant by Euclidean shapes in this context, namely a shape that entirely fills its embedding Euclidean dimension like a square or circle in two-dimensional space or a straight line segment in one dimension. An entirely regular grid pattern consisting only of identical squares or circles with some space between them does not fill its embedding space, and will have a fractional dimension when analysed through box counting, even though it can hardly be described as scale-invariant or fractal (examples are given below). For such cases, the total area of the pattern, approximated by the box count times box size, will be unstable down to the image resolution, as if the pattern were truly fractal. The dimension value obtained through box counting is thus in itself not a guarantee for the pattern being either Euclidean or fractal, which is why it is more accurate to refer to it as the *box-counting dimension* or D_{box}. This has led to numerous confusions and erroneous interpretations in earlier studies involving box counting Oleschko et al. (2000), and it illustrates the difficulty involved in interpreting the obtained values. This is also largely why I have opted for a more prudent empirical approach using synthetically constructed images to guide interpretation, testing for the effects of different variables one-by-one as presented below.

Some similar issues concern the estimation of lacunarity in empirical patterns. While Mandelbrot mainly relied on regular constructed fractals for estimating lacunarity, empirical patterns involving stochastic processes require more formal methods for analysis. The main method for estimating lacunarity is the so-called gliding-box method (Allain and Cloitre 1991; Hingee et al. 2019; Cheng 1997; Plotnick et al. 1996). Similarly to box counting, it involves evaluating the pattern over a range of scales (box sizes), but in stead of a grid, a single box is glided incrementally across the image with overlaps (rendering the method computationally heavier). For each increment, the pixel mass (number of foreground or pattern pixels) within the window is recorded, and the average spread of this over all box displacements for a given box size is calculated, giving a lacunarity index. This index – i.e. lacunarity for a given box size, is then plotted according to box size, and unlike the fractal (box-counting) dimension, the shape of the lacunarity curve can effectively give an indication of whether or not, or over which scales, the analysed pattern is fractal. For regularly self-similar mono-fractals, the curve of the lacunarity index forms a straight line in log-log scales, with the slope being equal to *D–E*, i.e. the fractal dimension of the pattern minus its Euclidean dimension (Plotnick et al. 1996, 5463; Allain and Cloitre 1991; Mandelbrot 1982, 315–17). Since real-world patterns are rarely self-similar over very large ranges, and even less so for image renderings of them which depend on pixel resolution, this lacunarity index curve tends to follow a power law less strictly than that of box counts, even when the pattern is fractal-like over some scales.

Mandelbrot never gave any single definition of a summary statistic for lacunarity across scales like the fractal dimension, and several such summary measures have subsequently been proposed. The software plugin *FracLac* for *ImageJ* offers principally three of these, namely the exponent and prefactor values of a power-law approximation of the lacunarity index, as well as its arithmetic mean (Karperien 2013). There is little literature on the relationship between these, and it is not always clear which one of these is applied. Farías-Pelayo seems to be using exponent lacunarity in (Farías-Pelayo 2017) and a mixture of this and mean lacunarity in (Farías-Pelayo 2015). The value range of the power-law exponent should be expected to vary much less than that of the mean or prefactor, and these latter two cannot be below 1, while the exponent can in theory go down to 0 (a full pattern with no gaps and $D = 2$ for spatial patterns, see also Bruvoll, n.d. for further discussion on this issue). As mentioned above, the power-law exponent is directly correlated to D in the case of self-similar mono-fractal patterns.

For practical applications, Hingee et al. (2019) recently showed that the lacunarity index is mathematically equivalent to spatial covariance, which has the further advantage of being more tolerant to irregular image outlines as well as missing data. From this they proposed a series of estimators that they showed to give more reliable results than the standard lacunarity estimate, and implemented these in the *R* package *lacunaritycovariance*. While both irregular outlines and missing data are continuously relevant issues in archaeology, I did not pursue these possibilities further here, though I made use of the gbl()-function (gliding-box lacunarity) from this package, with its default “GBLcc.pickaH” estimator, for calculating the lacunarity index of images. Fractal dimension was in this thesis calculated in *R* using the fract2D()-function from the *fractD* package (Mancuso 2021). For both analyses, box sizes (widths) were set to the default values in *fractD*, namely 1, 2, 4, 8, 16, 32, 64, 128, 256 and 512 pixels.

The analysed images of empirical settlement plans were prepared in QGIS 3.20.1, from shapefiles that were kindly shared by their creators on behalf of the research projects in which the spatial data was collected [Hale (2020); Ohlrau (2020); Nils Müller-Scheeßel et al. (2020); 3]. In order to focus the analysis on architectural features only, all other spatial features were removed, while house polygons were rendered with black fill and no stroke, and rasterised to a Boolean (black-and-white) image with 0.5 m pixel resolution, and cropped to the minimal extent of the corresponding vector layer (Figure 8.1). Differences in image processing between the *lacunaritycovariance* and the *fractD* packages made it necessary to store the images in two versions in separate folders, in .jpeg format with greyscale rendering (0 for black and 255 for white) for fractal dimension and in .tiff format with binary values (1 for black and 0 for white) for lacunarity. This preparation process is obviously not ideal, and is due to the lack, for the time being, of implementation of these methods in a single coherent *R* package.

This also makes the use of ready-made image analysis softwares like the above mentioned *FracLac* plugin even more convenient. However, the advantages of still performing the analysis in *R*, include the full transparency and adaptability of the method implementation, as well the increased possibility of combining the analysis with other methods. The analysed images are reproduced in the Appendix A, and are available in their original sizes and formats in the online repository, along with the code for analysing them.

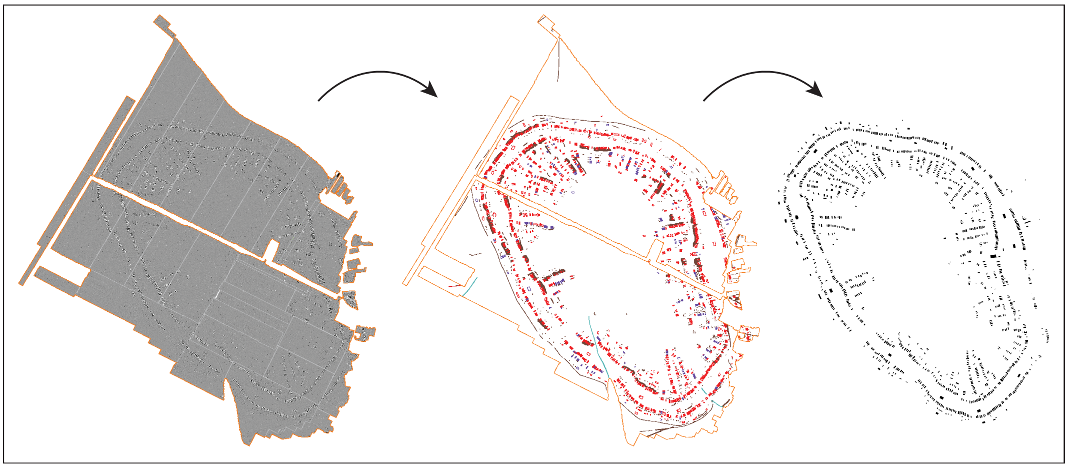

Figure 8.1: *The applied image preparation procedure. Raw geomagnetic images (left) were interpreted and redrawn as shape-files by GIS specialists for publication within their respective research projects (middle). For the analyses presented here, architectural features were selected and rendered as black-and-white images with minimal rectangular frame by author. Example from the Nebelivka settlement plan: left and centre images adapted from J. C. Chapman et al. (2018), © D. Hale under the ADS Terms of Use*

In the following, I attempt to address the difficulty of interpreting fractal dimension and lacunarity measures of spatial patterns by testing the methods on synthetically produced images, designed to isolate relevant variables one by one. The chosen variables are perceived to be of importance specifically in the present context where the images of interest represent architectural features on archaeological settlement plans. While this is not a sufficient analysis for effectively modelling the relative effects that any of these parameters may have on D and L measurements in any image, it allows for an intuitive illustration of the complexity involved in analysing spatial textures. The chosen image variables which are selected here for empirical testing are image size, element count, element size, image density, element size distribution, self-similarity/clustering and random noise. While the first three are important for assessing the requirements of the analysed images for the methods to function properly, the latter two are are of more direct interest for the hypothesis which is proposed in this part of the thesis, namely that fractal image analysis may allow for quantification of the degrees of hierarchy and conscious planning in a society evident through the layout of its settlement plans. The variables density and size distribution are important aspects of any settlement plan, but say little by themselves about the extent to which the layout is planned or not, and their influence on fractal dimension and lacunarity measurements can thus in this context be regarded more as side effects. As is shown below and further discussed in Chapter 11, while spatial clustering, element size distribution and noise do have effects on both fractal dimension and lacunarity measurements, in many practical settings these effects tend to be drowned by those induced by other variables like image size and density. The possibility remains that these methods may prove useful for comparisons of settlements of similar size and density only, or by defining more formal model by which undesired effects of other variables may be factored out. Here I suggest thresholds from which image size and resolution seem to give stable results, and I provide only some very simple models are tentatively constructed to factor out image density. While more formal and complete modelling for these purposes would be warranted for obtaining more fine-grained results, this goes beyond the scope of the present study.

For the following tests, all images were constructed in R as data tables with x and y coordinates for each element ("house"), rendered with the *ggplot2* package (Wickham 2016). More specifically, in each image houses were represented as a geom_tile() layer with black fill and given height and width, and the background was set to white, while the theme_void() function allowed for removing the axes and grids altogether. With this procedure it was possible to generate incremental changes in a single variable on otherwise identical images. However, as some of the variables are inherently connected, full separation was not always possible. Again, all the generated and analysed images are included in Appendix B, and the original files and code for generating them, are available in the online repository.

8.2 Effects from image size, element size and element count

To test the effect from image size on fractal dimension and lacunarity measures, it was necessary to proceed in three steps (Figure 8.2). A first series was created in which a single image was simply scaled up from 40^2 to 420^2 pixels in 20 steps. The smallest image thus represented four square houses of size 10*10 pixels, which with 0.5 m per pixel length (the resolution used with the actual settlement plans in the next chapter) gives houses of 25 m^2. Each house filled the lower left quarter of a 4 times larger square, so that image density was kept at 0.25. Density is defined here as fraction of pattern pixels to the whole number of pixels. For the largest version of this pattern, each house thus had a side length of 52,5 m, giving a total surface of ca. 2.756 m^2 per house, which is far above comparable values for houses in prehistoric settlement plans. Since in reality house sizes do not increase linearly with settlement size, but are more constant, a second series was created where the image size was incremented the same way, but with a constant house size of 25 m^2, so that in order to also keep image density constant, house count (N) was increased proportionally with image size. For this series, the largest image then had $N = 21^2 = 441$ houses, thus emulating settlements of different sizes but with similarly sized houses and densities between houses. In other words, different image sizes could not be made keeping both density, house count and house size constant. A third series of 20 images was made where image size was kept constant at 420^2 pixels, but with the same range of house count from 4 to 441 as well as house sizes from 105^2 to 10^2 pixels, or 2.756 to 25 m^2.

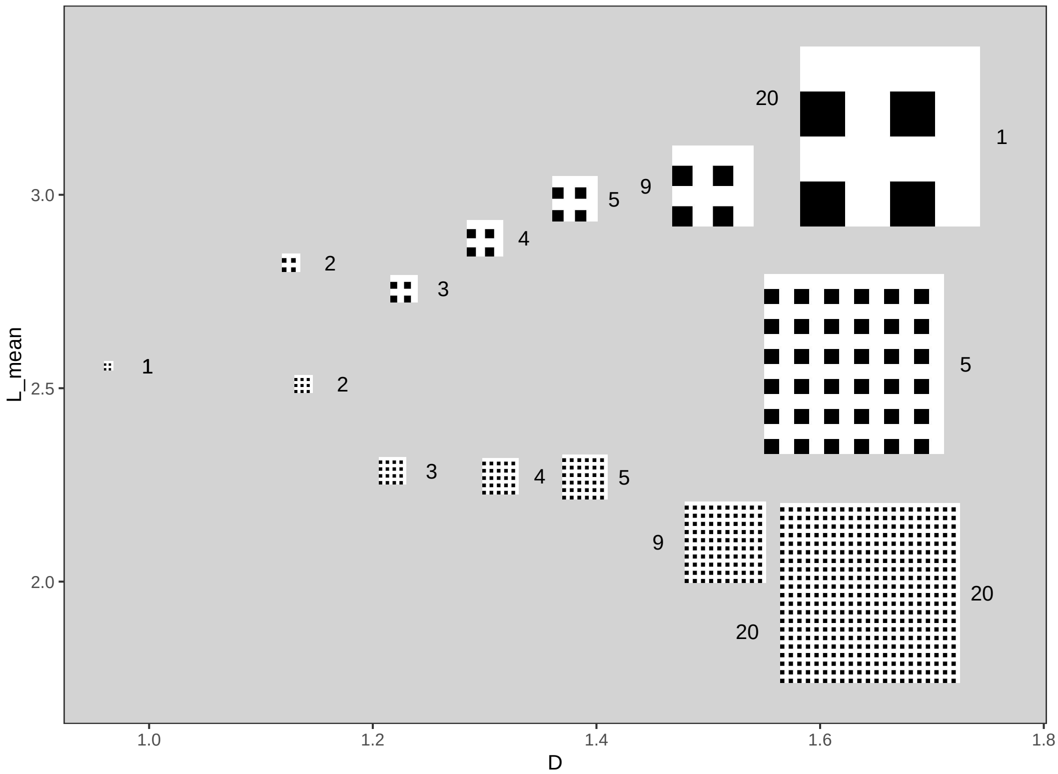

Figure 8.2: *Effects from image size, element count and element size on fractal dimension and lacunarity. Density is fixed at 0.25 for all images. Number labels represent iteration within each series. Image size is variable in the upper and lower series, house size in the upper and the vertical series, and house count in the vertical and the lower series. The three corner images are identical for two series each. Images are selected here to prevent overlaps*

The resulting fractal dimension and lacunarity measures for all images in these three series are shown in Figures 8.3 to 8.5. In all three cases – increased image size and house size, increased image size and house count, and increased house count and decreased house size – there are clear tendencies. In the two first series (upper and lower images in Figure 8.2), fractal dimension increases with image size, seemingly independently from the evolution in the pattern given that density remains constant. In the third case, when image size is kept constant (right side images in Figure 8.2), changes in fractal dimension are much smaller and less systematic, however lacunarity changes with some fluctuation. Regarding the two different summary measures of lacunarity – mean L and exponent L (see above) – though not identical, they respond in very similar ways to changes in all three image series. However, it is important to notice that the direction of change in lacunarity is opposite depending of the summary measure in the first and third series, while identical in the second. This difference is not easy to explain, and underlines the importance of reporting explicitly which lacunarity measure is being used. As seen on these plots, the exponent lacunarity typically has values below or around 1, while the mean lacunarity may have much higher values and cannot be below 1 – an observation that can serve as a rule of thumb for decrypting results where the exact type of measurement used is not explicitly reported.

In the first two series, where image size is variable, it should furthermore be noted that the changes in D and L measures between iterations roughly slow down as image size increases, potentially indicating that they approach the "true" values of the given pattern, and that the measurements of the smallest images are only statistical artefacts. Another parameter which is related to image size is the range of box widths used when analysing the images, and further study could allow for more elaboration on the possible influence this could have on the resulting estimates. In any case, the range of dimension values from 0.96 to 1.66 is considerable given that these images are practically the same besides image size. The range of lacunarity in these series is more moderate compared to the results from other variables presented below. The step-wise decrease in lacunarity seen especially in the image series of increasing N with image size (Figure 8.4), is a further indication that the choice of box widths relative to the resolution of the image has a marked influence on the results.

The series with fixed image size and changes in house count and house size also show some convergence in results at higher iterations, though with seemingly more random volatility (Figure 8.5). This indicates that with a given image size, zooming in and out on the pattern has very little influence on the measured fractal dimension (as would be expected from the

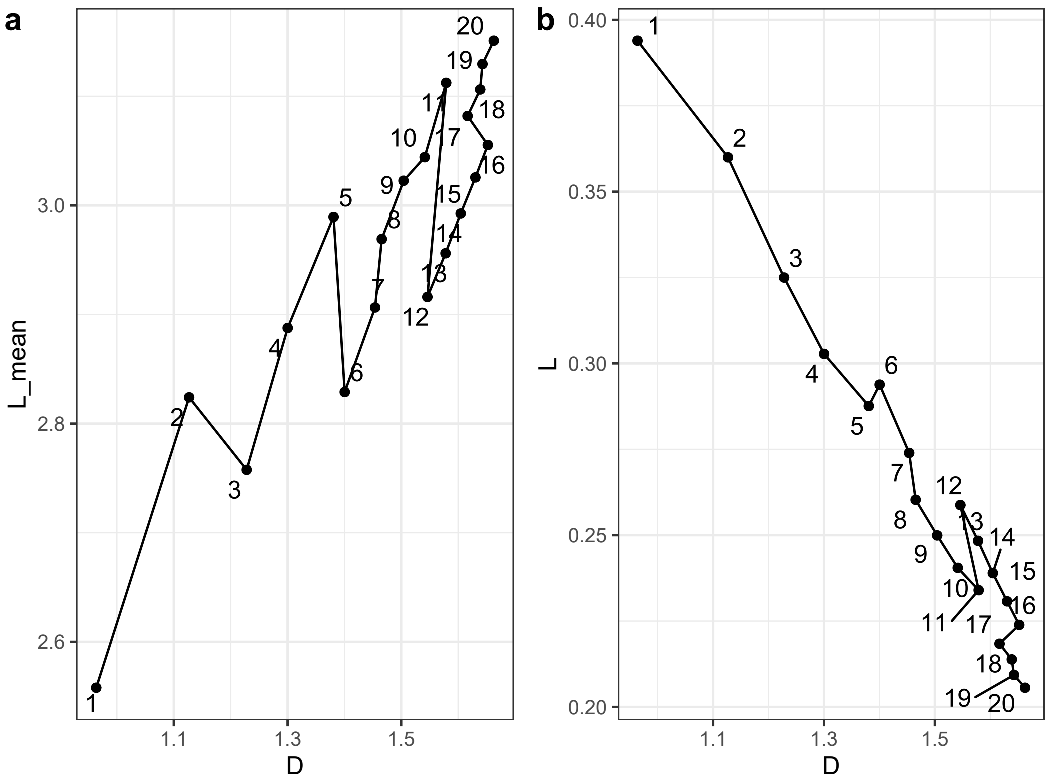

Figure 8.3: *Different image sizes and house sizes, with* N *= 4 and density = 0.25. Resulting fractal dimension (*D*) and mean lacunarity (*L_mean*, plot a) and exponent lacunarity (plot b). Images are numbered by iteration*

theory), as long as the pattern is spatially homogeneous. Lacunarity estimates on the other hand, become influenced when zooming in too much, though it quickly stabilises when more of the pattern is included by zooming out. Judging from the results of all three size related image series, there might be some lower threshold of image size and resolution beyond which the box-counting and gliding-box methods become inaccurate, and that the "real" values for this regular square grid pattern of 0.25 density are close to those in the lower right end of Figure 8.2, namely $D \approx 1.64$ and $L\_mean \approx 2.0$. For image size, the results start to converge at iteration 12 or around 260*260 pixels (Figure 8.4), given the box size range and fitting method used here. Furthermore, the choice of a 0.5 m per pixel resolution used in this thesis was arbitrary but came from the acknowledgement that higher resolution would only induce a false sense of accuracy, while lower resolution could potentially lump together archaeological spatial data. In Figure 8.5, estimates begin to stabilise at iteration 7 for mean lacunarity and 5 for exponent lacunarity. Using the more conservative option of 7 (discarding the most inaccurate data), the length of a house is here covered by $420/8/2 \approx 26$ pixels, which, if this represents 5 metres gives a resolution of ≈ 0.19 metres per pixel, which again is about 0.04

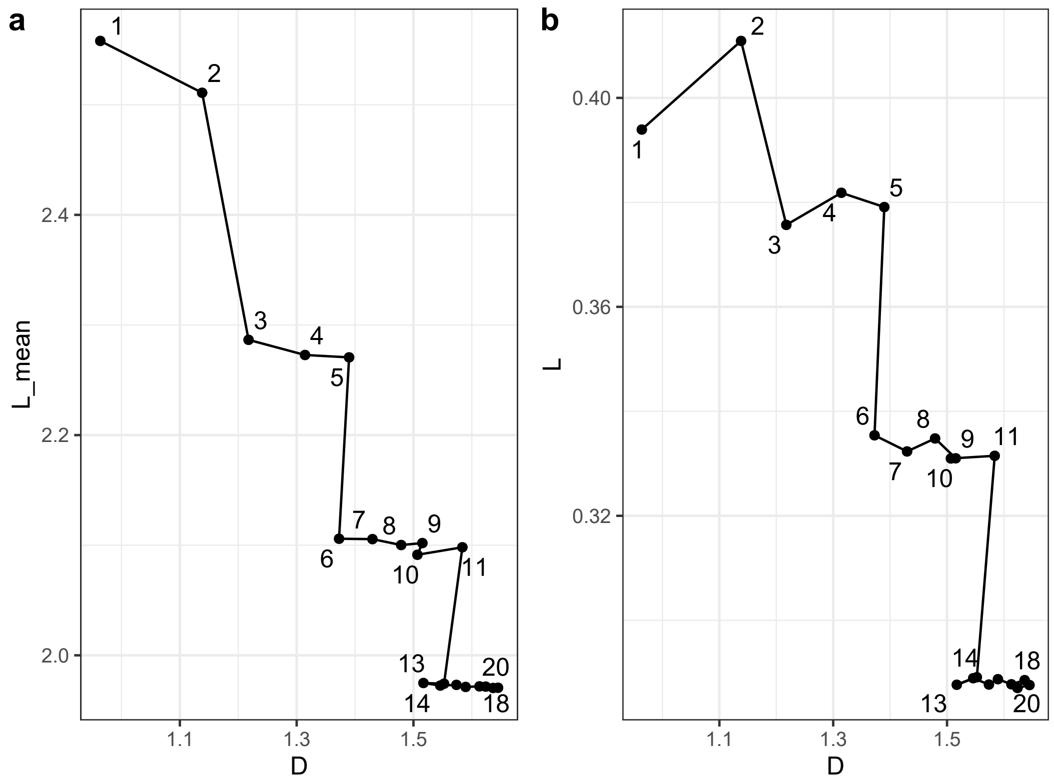

Figure 8.4: *Variable element count (*N*) and image size, with constant house length of 10 pixels and density at 0.25. Resulting fractal dimension (*D*) and mean lacunarity (*L_mean*, plot a) and exponent lacunarity (L, plot b). Images are numbered by iteration*

or 4% of the house length. In other words, the results presented here indicate that when single pixels represent 4% or less of the smallest features that are being analysed, lacunarity estimates become inaccurate, while fractal dimension estimates remain more unaffected, again given the box widths and fitting method applied here. This sets a tentative upper threshold to image resolution, while the lower threshold should be determined by what gives an acceptable spatial representation of the data. The 0.5 metres per pixel resolution applied in Chapter 9 lets one pixel represent 10% of a 5 metres long wall, thus being within the threshold. The other way around, 0.5 metres is 4% of 12.5 metres, which would be the maximum size of the smallest mapped feature before the resolution would be too high for the lacunarity estimates to be accurate. This threshold value should however at this point only be understood as a rule-of-thumb, as its precision is difficult to quantify any further here. The effect of image resolution on D and L estimates of images with other pattern layouts and densities also remains unknown. And again, in these analyses image resolution is only an issue since the analysed pattern is not strictly speaking scale-invariant. Rather it has a clear lower bound, leaving large open spaces in the image when zooming in too much. This is also the case with settlement plans, which, no matter the degree of clustering and self-similarity, will always have a lower bound set by the human scale, since houses can only be so small.

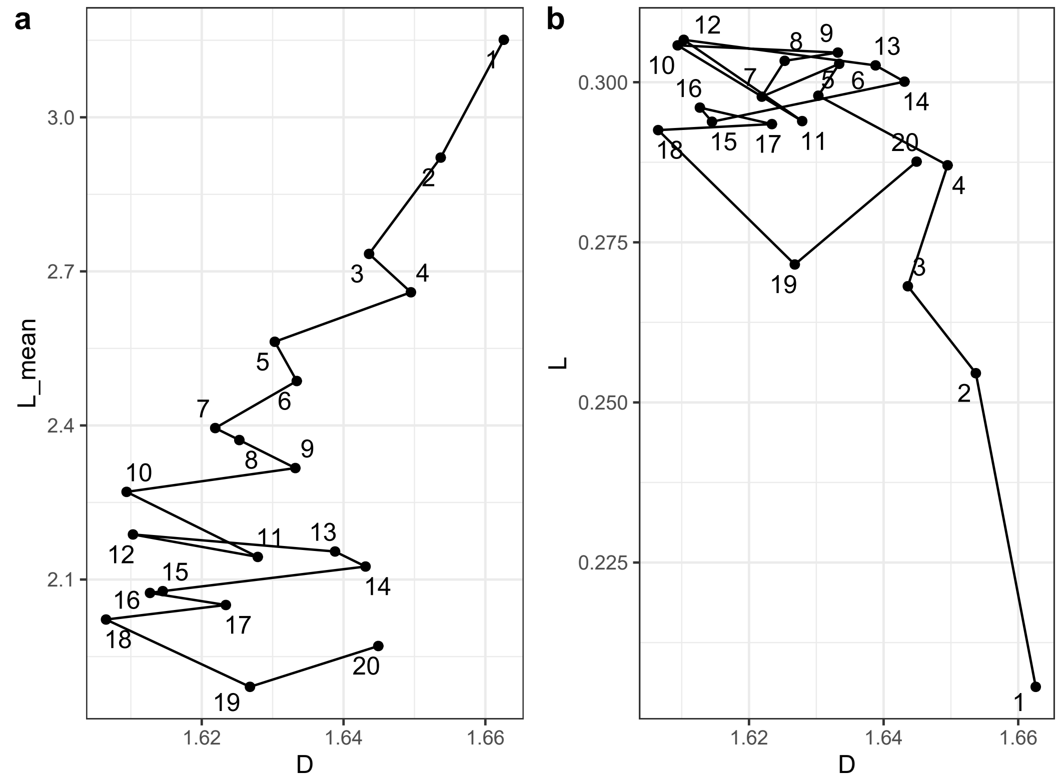

Figure 8.5: *Variable house count (*N*) and house size, with fixed image size of 420^2 pixels and density at 0.25. Resulting fractal dimension (*D*) and mean lacunarity (*L_mean*, plot a) and exponent lacunarity (*L*, plot b). Images are numbered by iteration*

8.3 Effects from density and size distribution

While density and size distribution of elements are not directly connected to layout type and regularity, as different layouts may have varying degrees of both, they are nonetheless important aspects of settlement plans. Different chrono-cultural contexts give settlement plans of different densities, and this aspect was one of the main differences between the “Anatolian village” and “Balkan village” types of the early Neolithic as shown by Furholt (2016). The density between houses may be highly reflective of the organisation of village life as a whole, as well as of differences between areas of larger settlements. The importance of size distribution of houses for understanding the underlying social system and potential hierarchy between houses was shown in the previous chapters of this thesis.

For both of these variables, a series of 20 images was constructed with internal incremental change between images, keeping other variables constant. In order to avoid the issues related to image size and resolution presented above, image size was set here to 540*540 pixels, with a regular grid of $N = 27^2 = 729$ houses (i.e. corresponding to rather large settlements from an archaeological point of view). Over the 20 images, density was set to vary incrementally from 0.05 (one pixel per house) to the upper limit where all houses percolate into a single filled square (density = 1, Figure 8.6). Fractal dimension and lacunarity estimates of the images with variable density followed clear and regular trends, with *D* and *L* being seemingly exponentially correlated to density, as seen in how the step lengths between iteration points in Figure 8.7b decrease at a constant rate. *L_mean* being already exponentially correlated to *D* (see further discussion below), it seems to follow a power-law relationship to density, meaning that low densities get very high values of *L_mean* and inversely. As should be expected the range in *D* values in this series is very wide, from approximately 1.0 (the dimension of a straight line segment with no surface) to above 1.8. In theory the single black box of which the last iteration image consists should have a dimension of 2, illustrating the limits of the box-counting method. This wide range also shows clearly that a fractional dimension value obtained from an image through box counting does not in itself mean that the analysed pattern is actually fractal (i.e. exhibiting self-similarity at a range of different scales). The observed lacunarity values of the last image of this series (no space between points) were, however, arbitrarily close to the theoretical values of 0 for *L* and 1 for *L_mean*.

The exponential relationship between fractal dimension and density of spatial patterns was noted by Thomas, Frankhauser, and De Keersmaecker (2007), who – based on the layouts of different suburbs of modern Brussels – argued that *D* gave additional information on texture and clustering that density could not give. The constant relationship between *D* and density here shows that the two measures are largely equivalent, *given a constant layout*. To further investigate whether *D* and *L* measurements actually capture anything more than density, in the remaining image series density was kept constant at 0.25.

A natural follow-up from the preceding chapters is to test to which extent different size distributions of houses are reflected in fractal dimension and lacunarity of settlement plans, all other things being equal. A series of 20 images were generated with gradually increased inequality between house sizes, with image size kept at 540^2 pixels and N = 729 (27 rows and columns in a square grid). The size distribution was defined as a log-normal with an arbitrary mean at $\mu = 3.5$ and standard deviations varying in linear increments between $0 < \sigma < 0.9$.

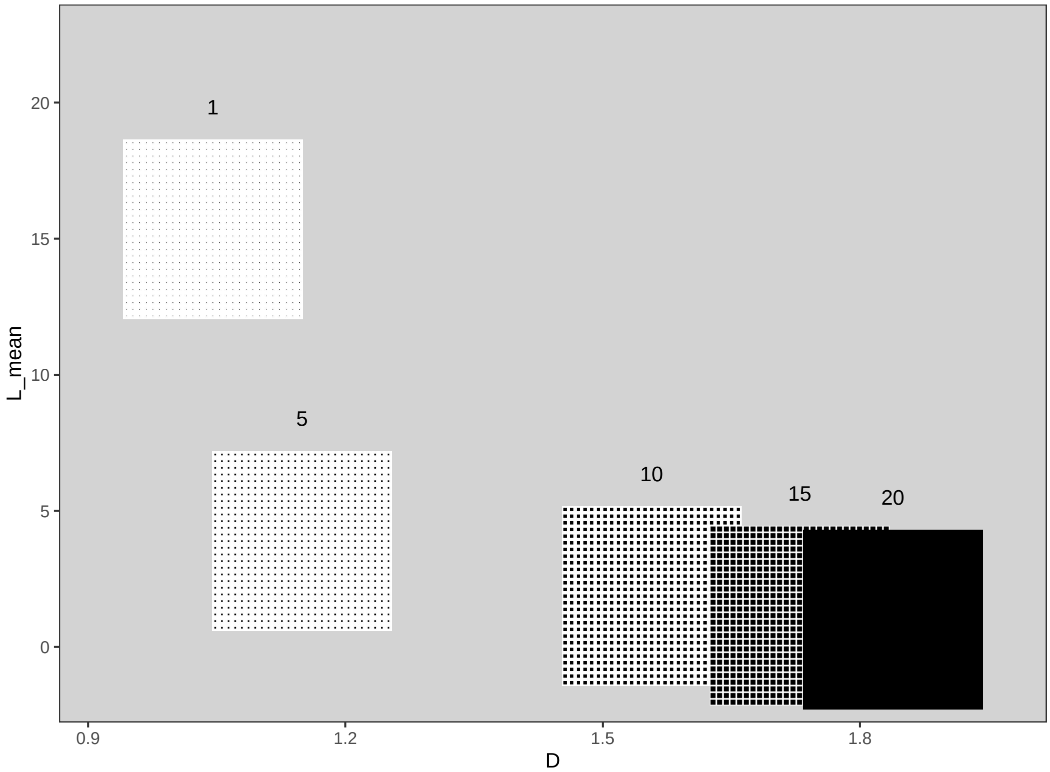

Figure 8.6: *Fractal dimension and mean lacunarity of images with identical size and layout, but with densities varying with linear increments from 0.05 to 1. Number labels represent iteration, and images are selected to prevent overlaps*

The first image was thus uniform with identical house sizes, while the last image had some sizes that were much larger than most of the others. For each image, the size distribution was normalised to 1 and multiplied with the desired fraction of the total image area so that density was kept at 0.25 (with small deviations due to occasional overlaps between neighbouring houses). This series thus illustrates settlements of identical size and layout, but where the size distribution of houses range from (nearly) identical to very unequal (Figure 8.8).

The fractal dimension of the images with variable house-size distributions dropped from the value of the first iteration ($D \approx 1.56$) to around 1.52 where it fluctuated with no clear pattern for the remaining iterations (Figure 8.9). Lacunarity – both the mean and exponent summary measures – increased gradually for almost every iteration, showing how this measure quantifies the increasing irregularity of the gaps between elements, and not only the sum of the gap sizes (i.e. density). Though this trend seems clear enough, the range of lacunarity values on this series remains moderate compared to those obtained from the series with variable density.

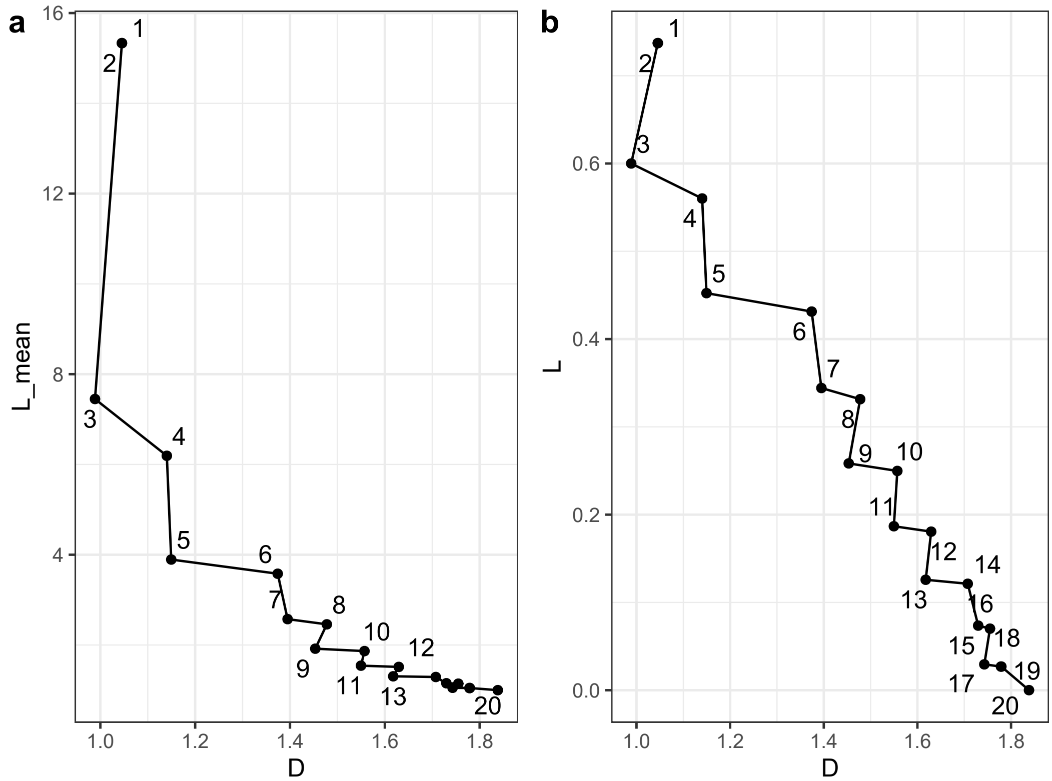

Figure 8.7: *Fractal dimension and lacunarity measures on the same 20 images with varying density. Because of pixelation the first image was rounded up to be identical to the second, with density = 0.01. Number labels represent iterations*

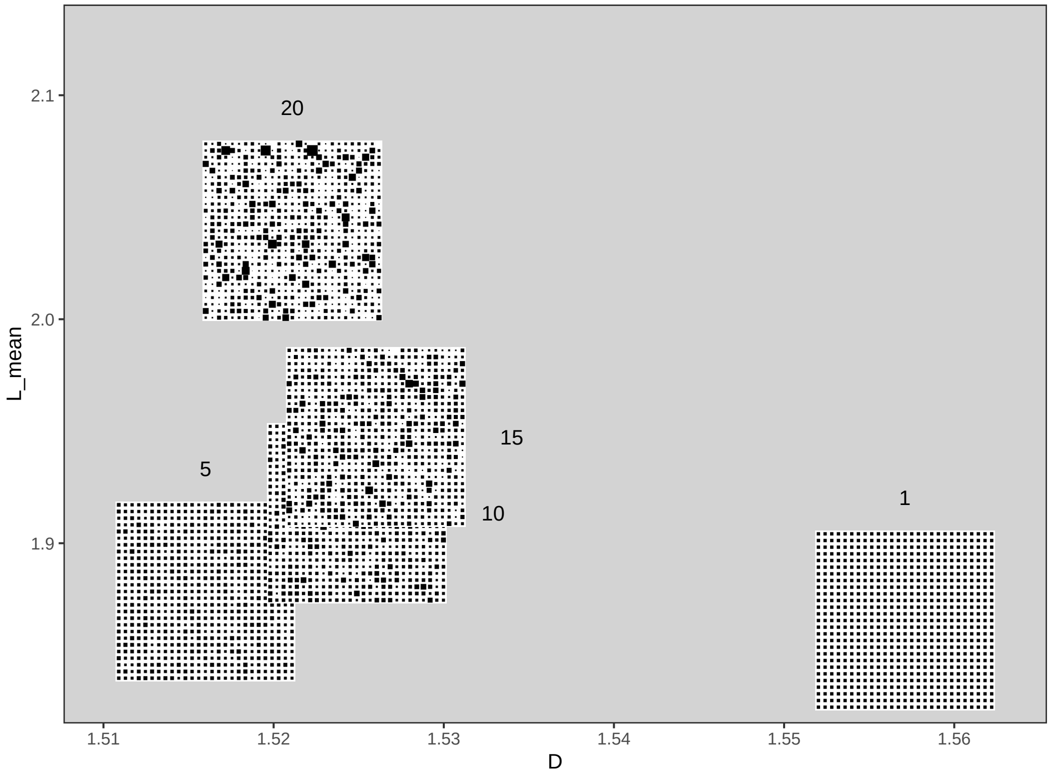

Figure 8.8: *Fractal dimension and mean lacunarity of images with identical size, layout and density but with varying size distributions of single elements, from uniform (image 1) to log-normal with $\sigma = 0.9$ (image 20), in linear increments of σ from 0. Number labels represent iteration, and images are selected here to prevent overlaps*

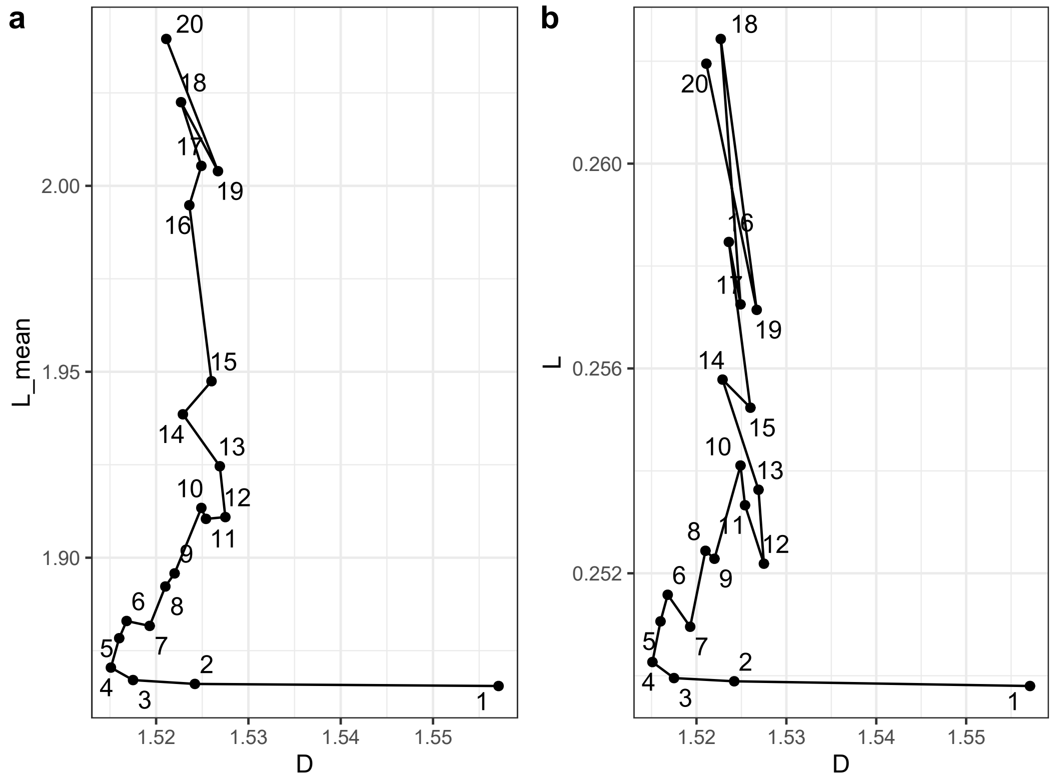

Figure 8.9: *Fractal dimension and lacunarity estimates of the whole image series, with 729 square points varying in size distribution from uniform (log-normal with $\sigma = 0$) to log-normal with $\sigma = 0.9$. For each image, sizes were normalised so that they together covered 25% of the total image area, notwithstanding some overlaps causing image density to descend to a minimum of 0.242. Label numbers indicate iteration*

8.4 Quantification of self-similarity and random noise

To assess the effect of spatial self-similarity or hierarchical clustering, an image series was generated where, as before, image size, N and density were kept constant, and the size distribution was kept uniform, but where the spaces between houses were gradually increased or reduced so that houses increasingly would form hierarchical clusters. This can be done in a number of ways – in this case two levels of clusters were generated (a third being the settlement as a whole), the lower consisting of 3*3 houses and the upper of 3*3 lower clusters (Figure 8.10). This configuration was the reason for choosing 27 rows and columns in the first place for all these image series, since it can conveniently allow for such hierarchical clustering without having non-integer numbers of houses (or houses of different sizes). The first image in this series was thus an entirely regular grid identical to the first image in the size distribution series described above, but the last image here represents a rather different layout, illustrating settlements where hierarchical organisational levels result in self-similar clustering of the overall plan, as described by Brown and Witschey for classical and post-classical Maya urban settlements (2003, 1625–28). As they argue, the underlying hierarchical levels of social organisation (e.g. family, lineage, clan, state) have been widely observed and described by ethnographers and historians, while the materialisation of such hierarchies in settlement plans is far more rarely recognised, and even less so in terms of fractal geometry.

The resulting fractal dimension and lacunarity estimates on these images show a very similar trend as the one observed on the size distributions image series (Figure 8.11). However, in this case D continues to drop from the first until the last iteration, albeit with some seemingly random fluctuations. Furthermore, after the first five iterations (which are in reality quite close to the entirely regular grid, see Figure 8.10), both lacunarity measures given here start to increase and do so regularly until the end of the series. Though again the ranges in both dimension and lacunarity are rather moderate here compared to those seen for density and image size above, the trend seen here is clear enough to conclude that hierarchical clustering is reflected in both these measures, in good accordance with the theory.

As a last test series of images to facilitate interpretation of fractal dimension and lacunarity estimates of settlement plans, 20 images were generated from the same point of departure as before – i.e. image size of 540*540 pixels, 27*27 houses each of 10*10 pixel size, giving 0.25 image density – but where an increasing amount of spatial noise was added for each iteration.

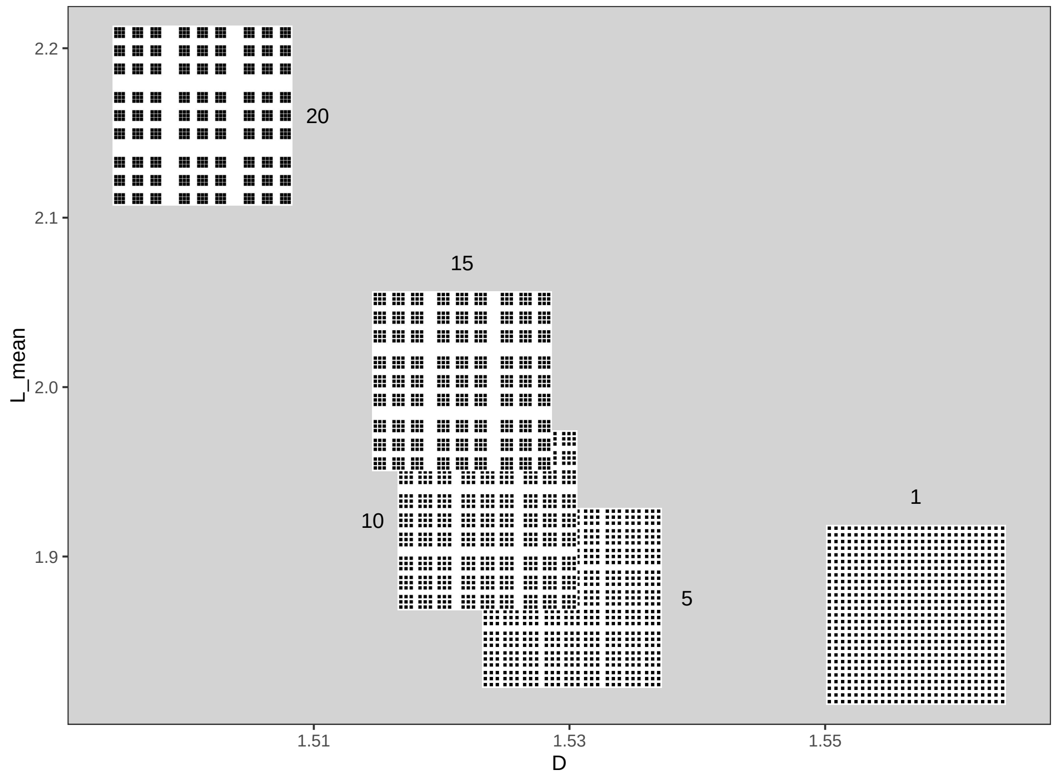

Figure 8.10: *Fractal dimension and mean lacunarity of images of identical size, layout and density, but with degrees of hierarchical clustering, from no clustering (image 1) to high clustering in two levels, where the space between clusters represent 8% of the superior level's total length and the points start to percolate (image 20). Number labels represent iteration, and images are selected to prevent overlaps*

Again, this could be done in a number of ways. In this case, each house was allowed to do an independent random walk with a step length of 2.4 pixel equivalents in one out of 12 circular directions (this caused some greyscaling in the resulting images, but the applied box-counting and gliding-box functions round such values to 0 and 1). Each random-walk step represented one iteration in the series (see Bruvoll, n.d. for a similar approach). An advantage of using square synthetic images for this test rather than simulations of actual settlement plans, is that the effect from such spatial noise can be more easily isolated from density. When elements walk in random directions, some will inevitably walk out and away from the initial pattern, thus decreasing image density no wether the image is extended to include them or not. To solve this issue and making sure that density remained as constant as possible given how it was shown above to influence the resulting estimates, the image size was here kept constant and houses that walked beyond the boundaries were moved across the image to the opposite side. Some decrease in image density was however difficult to avoid given the frequent overlaps of the areas of houses that walked very close to each other. As a result, the lowest density in

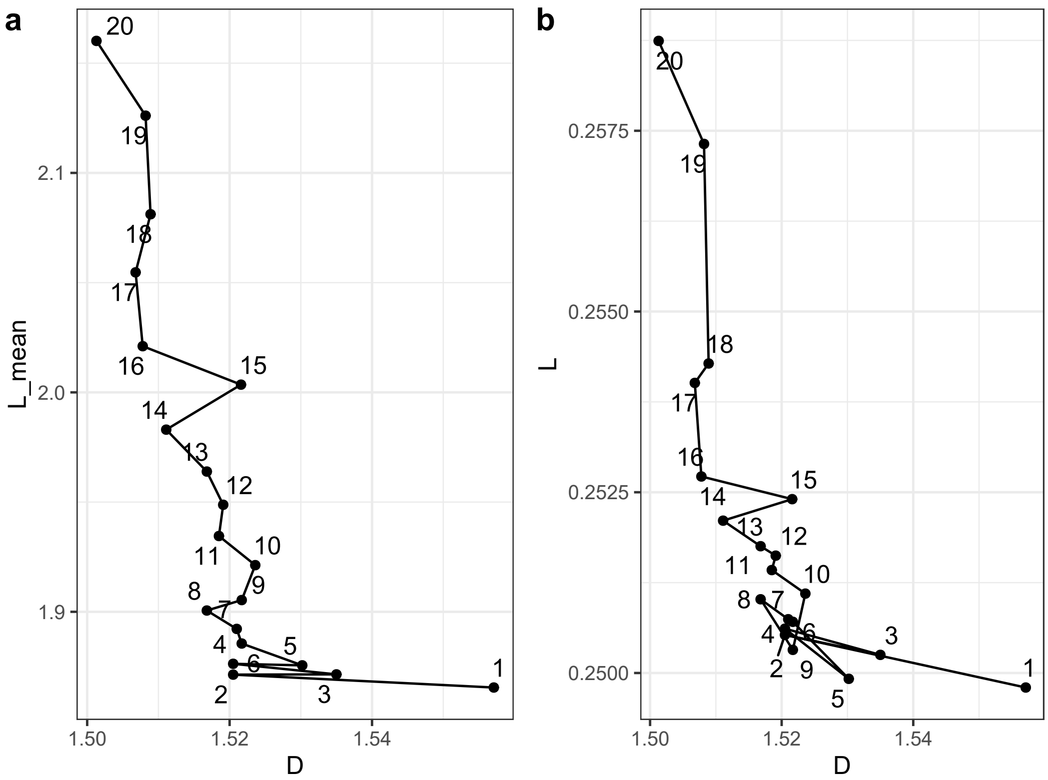

Figure 8.11: *Fractal dimension and lacunarity measures on the same images with varying degrees of spatial clustering, and fixed image size, density, element count and size distributions. Number labels represent iteration*

this series extended to 0.232, which is still fairly close to the goal of 0.25. The resulting more or less noisy images in this series thus illustrate the range of possible settlement layouts from one where every house is strictly allotted to a predefined place following a simple geometric grid, to one where houses are constructed completely without any regard to the placement of surrounding houses (Figure 8.12). Neither of these extremes are of course very likely for any real archaeological setting, but it seems obvious that the former case will be more characteristic of societies with strong overarching institutions that regulate everyday life while the latter case is more representative of very loose social ties between independent households and an absence of any overarching decision-making authority (Fraser 1968).

The resulting D and L measures from these images follow the same general trajectory as the images with variable clustering and size distribution, with lower fractal dimension and higher lacunarity for each step of added noise (Figure 8.13). The fractal dimension estimates oscillate in a few cases, possibly because of sudden random correspondence between the initial pattern grid and the box sizes used in the calculations. The lacunarity estimates – both the mean and the exponent lacunarity – followed very regular increase after the second iteration.

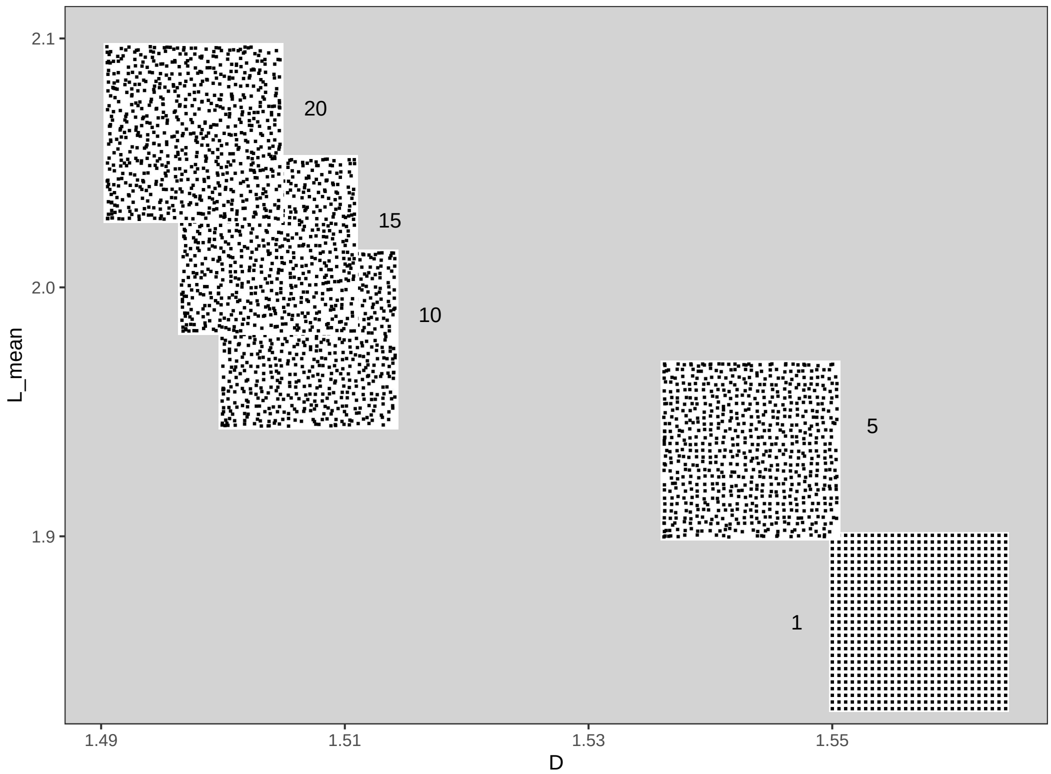

Figure 8.12: *Fractal dimension and mean lacunarity of images of identical size, layout and density, with varying degrees of added spatial noise, generated by letting each house independently perform a random walk of constant step length in random directions, from no steps (image 1) to 19 steps (image 20). Number labels represent iteration, and images are selected to prevent overlaps*

Here again, the trend seems sufficiently regular to conclude that random noise is effectively quantified by both fractal dimension and lacunarity, in a similar way to the other changes in pattern presented above.

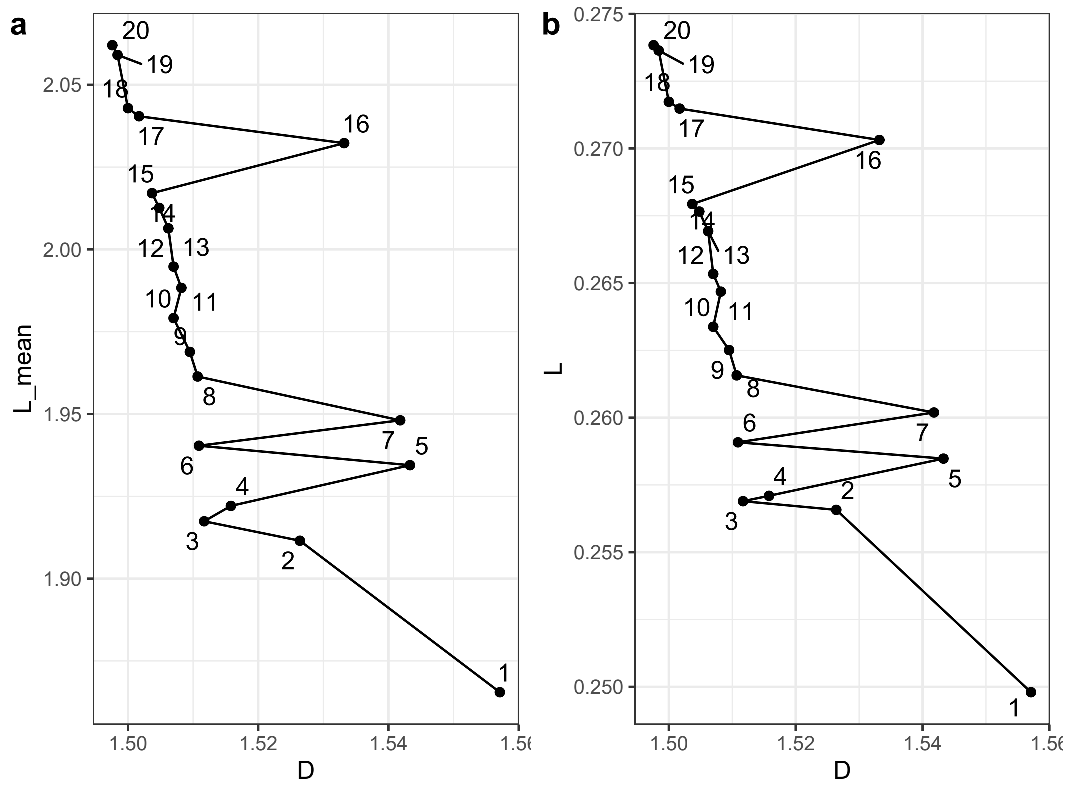

Figure 8.13: *Estimates of fractal dimension (*D*) and mean lacunarity (plot a) and exponent lacunarity (plot b), of 20 images with increasing degrees of added random spatial noise. Number labels represent iteration*

8.5 Summary of procedure and tests

Calculations of fractal dimension and lacunarity on synthetically generated test images provide useful clarifications on how these measures respond to changes in some crucial variables. The tests on image series with varying image sizes and zooms on a regular square grid pattern indicate that both D and L measures become increasingly inaccurate when estimated on images smaller than about 260*260 pixels, given the box sizes used here in the analyses. The values were calculated from bi-logarithmic linear fits to results for box sizes ranging from 1 to 512 pixel lengths, where null values were excluded from the fit whenever the analysed images were smaller than the box size. Reducing the maximum box sizes remains a possibility that could allow for more accurate estimates from small images, which again could prove useful whenever run time becomes an issue, since larger images are computationally much heavier to analyse. With the test images analysed here, too small images gave too low fractal dimension estimates, while lacunarity was seemingly less affected (Figure 8.2).

Another requirement for reaching acceptable accuracy in fractal dimension and lacunarity results, is that the image resolution should seemingly not exceed pixel sizes smaller than about 4% of the side length of the smallest mapped features. Letting pixels be smaller than this (i.e. *higher* image resolution) only adds a false sense of accuracy given that the analysed patterns are not strictly speaking scale independent but rather bound to the scales that are relevant for human agency. A too high resolution thus generates too large void spaces between elements, leading to too high lacunarity values, while fractal dimension in this case remains less affected.

Other variables, which were more related to the visual appearance of the pattern, also gave characteristic results. Size distribution of pattern elements, level of hierarchical clustering and random spatial noise all affected D and L measures in a similar way, in that larger deviations from homogeneity drew the former values down and the latter up in a consistent way. Density on the other hand, had a different effect on D and L estimates in two ways. Firstly, increased image density gave consistently *higher* fractal dimension and *lower* lacunarity values, and secondly – though the intensity of the different variables are difficult to compare – the effect from density appeared here as much stronger than that of the others. Figures 8.14 and 8.15 illustrate how a minor adjustment in image density made more difference in both fractal dimension and lacunarity than the whole range of variation in noise, clustering and size distribution. One caveat here though, is that these variables are often in practice combined in empirical settlement plans – there is usually both some level of clustering, noise and inequality in sizes – and it remains unknown whether these factors combined give stronger signals compared to that of density. In any case, it seems clear that image density must be taken into consideration when interpreting the layout of settlement plans from their fractal dimension and/or lacunarity.

From the results presented above, another trait seemed characteristic of the link between density, fractal dimension and lacunarity. All other things being equal, density was exponentially correlated to both D and L, and power-law correlated to L_mean (Figure 8.7). In fact, for a regular square grid pattern with given density, these fractal analysis measures can seemingly be very well predicted with simple linear models on log-transformed density (Table 8.1). These models can be used tentatively to compare D and L values of images with varying density, by first subtracting the values expected from density alone. However, it must again be stressed such an analysis probably is premature at this stage, since several factors remain unaccounted for. For example, regarding the Neolithic settlement plans compared in the next chapter, the

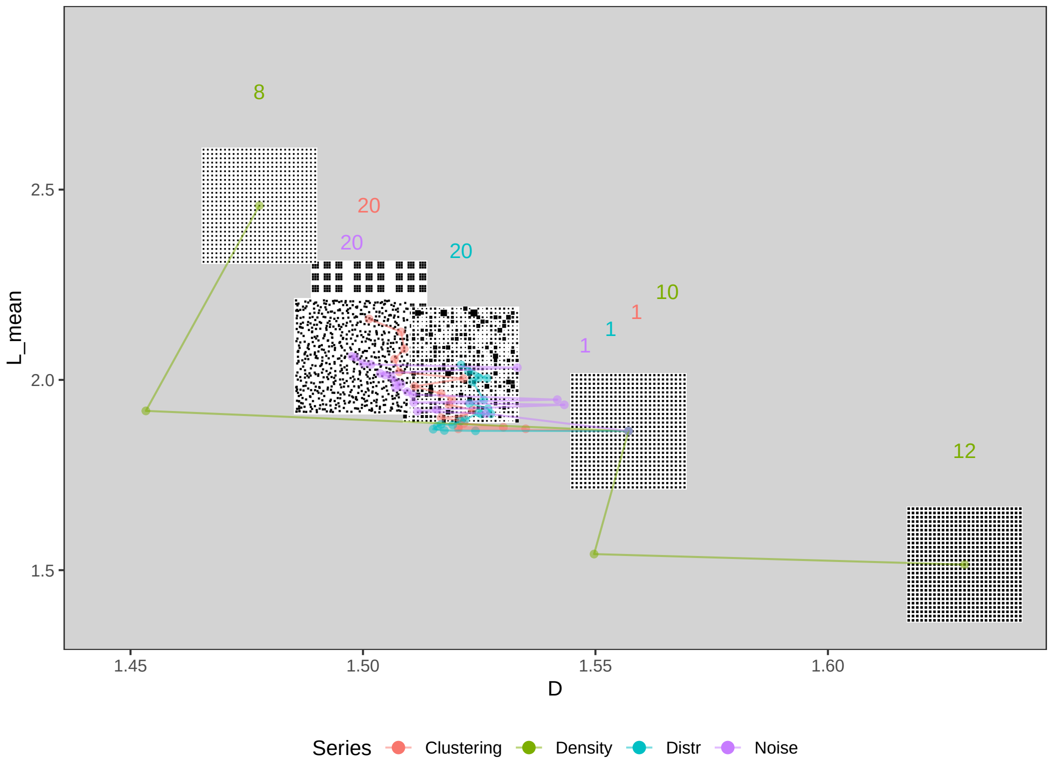

Figure 8.14: *The first and last images of the clustering, size distribution and noise series, as well as images 8, 10 and 12 of the density series, showing how only small increments in density – here from 0.16 (im. 8) to 0.36 (im. 12) – generate changes in fractal dimension and lacunarity that are larger than those induced by the whole range of the other variables. Density image 10 is identical to image 1 of the three other series, and has density value 0.25*

effects on D and L from *different* settlement layouts remain untested (the standard Trypillia layout is radial and not a grid). Also, effects from size distribution, clustering and noise have only been evaluated here at a single given density value, and these may behave very differently at other density levels. As mentioned, these factors are usually mixed to some extent and may have non-linear effects that for now remain unknown. In empirical settings grid orientation and edge effects quickly become issues of their own that are also not further investigated here.

Table 8.1: *Linear models of fractal dimension (*D*), exponent lacunarity (*L*) and mean lacunarity (*L_mean*) with the log-transformations that give the best fit, evaluated by the coefficient of determination (R^2). The third model can be written in power-law form as* $L_{mean} = 0.862 * density^{-0.61}$

Model	coeff.
D = 0.188*log(density)+1.792	0.931
L = -0.166*log(density)+0.008	0.994
log(L_mean) = -0.610*log(density)-0.148	0.995

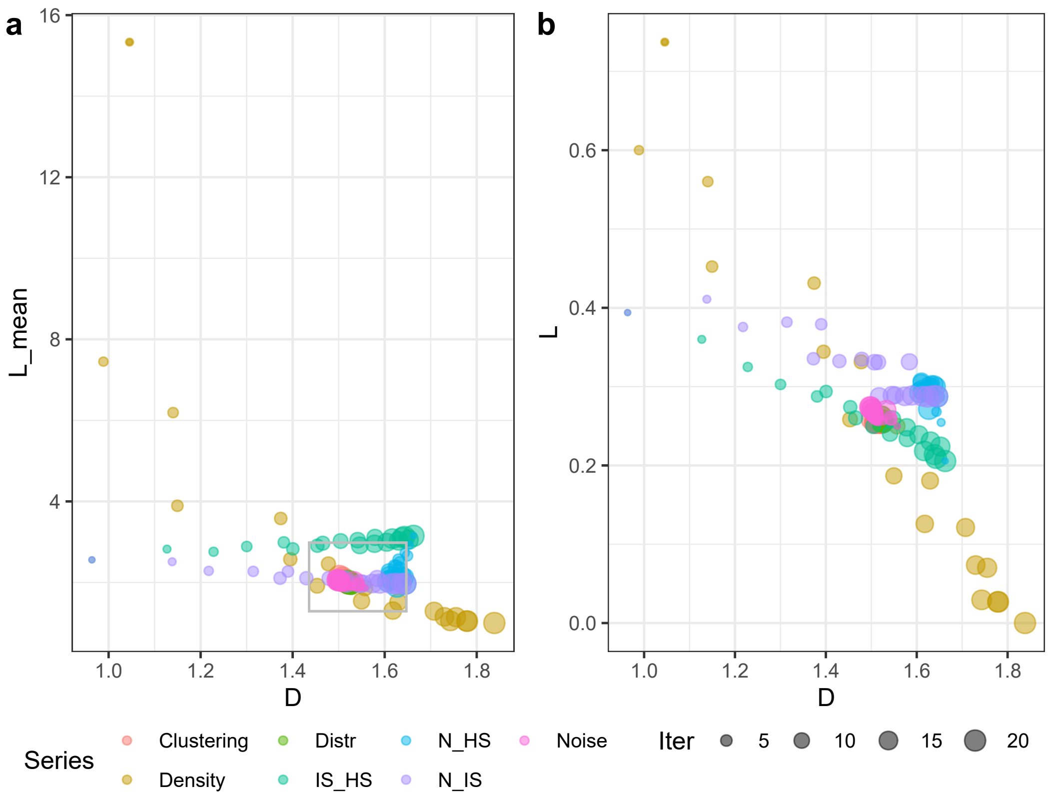

Figure 8.15: *Fractal dimension and lacunarity estimates of all synthetic images analysed in this chapter.* L_mean *and* L *show largely similar distributions but with some marked differences. See text for details. The grey frame in plot a shows the extent of Figure 8.14*

Lastly, in this chapter I have largely opted for presenting results for both exponent lacunarity (here denoted simply *L*) and mean lacunarity (*L_mean*), since both are found in the literature as summary measures of lacunarity, but not always with explicit mention of what they represent mathematically (as in Farías-Pelayo 2017, 2015). The results here show that they are largely equivalent, a part from the differing value ranges. For two image series – varying image size and house size (Figure 8.3), and varying house size and house count (Figure 8.5) – the results followed similar but reverse trajectories, without any obvious reason. And for density, as mentioned, *L_mean* was power-law correlated while *L* was only exponentially correlated (Figure 8.7). I have elsewhere shown how prefactor lacunarity is very closely correlated to mean lacunarity, arguing that these two are practically linearly equivalent (Bruvoll, n.d.; see also Karperien 2013). While studies in various fields refrain from using summary measures of lacunarity at all, preferring to rather show the full distribution of lacunarity to box sizes (e.g. in ecology, see Hingee et al. 2019), my impression here is that any of these summary measures may be used, as long as it remains clear which one it is. This relative equivalence being demonstrated, and for ease of presentation, in the following chapter I will largely focus on mean lacunarity values, while exponent and prefactor lacunarity values are given in the full data table in the data repository.

In the following chapter, the same methods of analysis are applied to images of archaeological settlement plans, in series consisting of total settlement plans, single quarters/neighbourhoods and temporally coeval sub-samples. The goal is then primarily to investigate how they perform with real archaeological spatial data, and to which extent they quantify different spatial layouts and textures that can be interpreted in terms of social organisation.

Chapter 9

Results: Image analysis

The same Neolithic settlements that were analysed in Chapter 6 were here analysed with image renderings of their plans following the procedure presented in Section 8.1. Resulting estimates of fractal dimension and lacunarity (summarised by power-law exponent and mean) are shown for all images in Figure 9.1. The spread of results in the scatter plot is very similar to that of the synthetic images analysed in the previous chapter (Figure 8.15), with a strong linear correlation between D and L, and exponential correlation between D and L_mean. Fractal dimension estimates are however much lower and lacunarity estimates are higher for the empirical settlement plans than for the synthetic ones, possibly resulting from the generally lower image densities (see Appendix ref(add ref) for the complete results). Settlements with higher house counts are also consistently situated towards the lower right of the plot, while the temporal samples of Vráble, with fewer houses and large voids between them, fall towards the upper left. The image of Vráble 1, showing only a single house, is a clear outlier. It also violates the minimal image size prerequisite suggested previously, and will be excluded from the further analysis. The clear separation between the temporally coeval and the cumulative settlement plans illustrates how these are not easily comparable, which is a very common problem in archaeology. It shows how crucial it is to take into consideration the temporal resolution of the data both when formulating research questions and when interpreting results (Perreault 2019). In the following, the results are further discussed by image series, starting with whole settlements, followed by quarters and neighbourhoods for Nebelivka and Vráble, and lastly by temporal samples for Vráble. The results presented in this chapter are also partially presented in Bruvoll (n.d.).

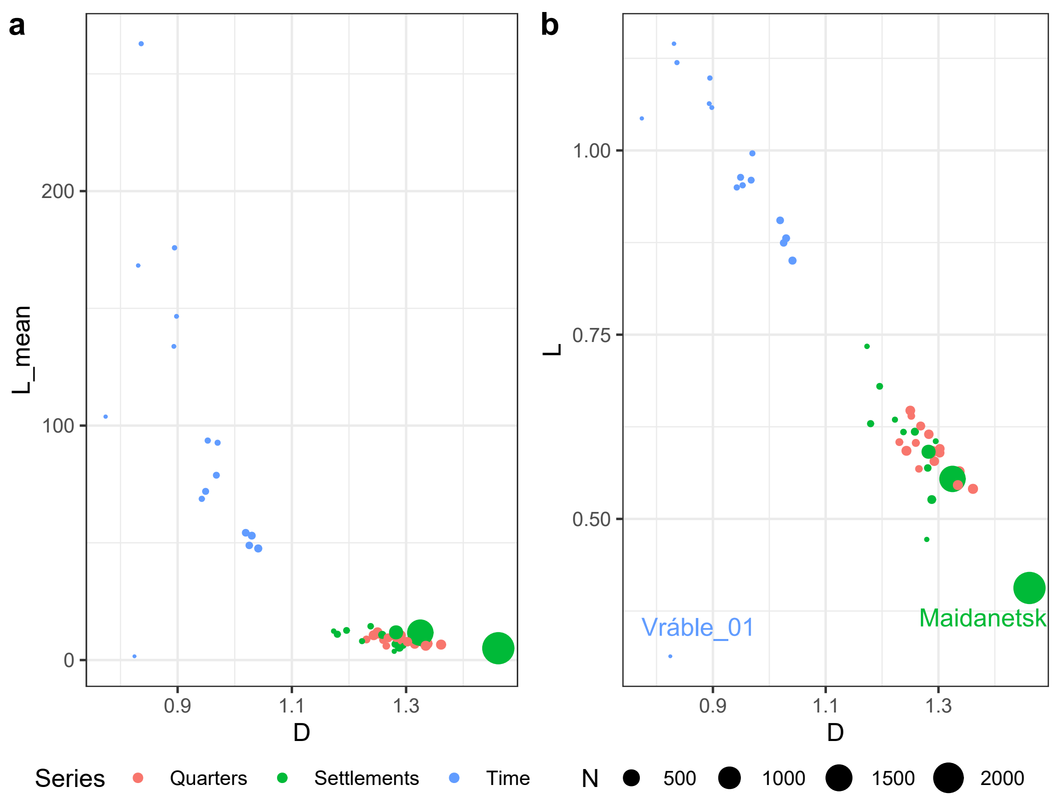

Figure 9.1: *Fractal dimension and mean lacunarity (plot a) and exponent lacunarity (plot b) for all 46 images analysed in this chapter. See online repository for full data table*

9.1 Settlements

Fractal dimension and mean lacunarity of the plans of whole settlements are shown in Figure 9.2a. Lacunarity values divide the images into two distinct groups, cross-cutting cultural adherence, while dimension values seem closely correlated to settlement size (proxied through house count *N*). The plot can to some extent be reproduced directly by replacing *D* with *N* and *L_mean* with density, also setting both axes in logarithmic scales and reversing the y axis (Figure 9.2b). In this image series, the small Linear Pottery settlement of Horný Oháj and the Trypillia settlement of Talne 3 had image sizes that were below the threshold proposed in the previous chapter, meaning that their fractal dimensions are possibly underestimated. When comparing these values to the visual appearance of the settlement plans, at least for the Linear Pottery settlements it seems clear that the upper group of settlements (those with high lacunarity values) are subdivided into more or less separate neighbourhoods with open spaces between them, i.e. more clustered, while the lower group – specifically Nevidzany and Čierne – only consist of one more dense and regular grid-like layout (Figure 9.3). For the two Trypillia mega-sites, Nebelivka and Maidanetske, this separation is somewhat less obvious, since they both follow the characteristic radial layout. Nebelivka does arguably have more open space between quarters than Maidanetske, which in turn is more homogeneously "filled", and the higher overall density of the latter (0.084 to 0.031 of Nebelivka) may contribute to its higher fractal dimension. The smaller circular settlement of Moshuriv is also highly regular in its distribution of open spaces, leading to lower lacunarity.

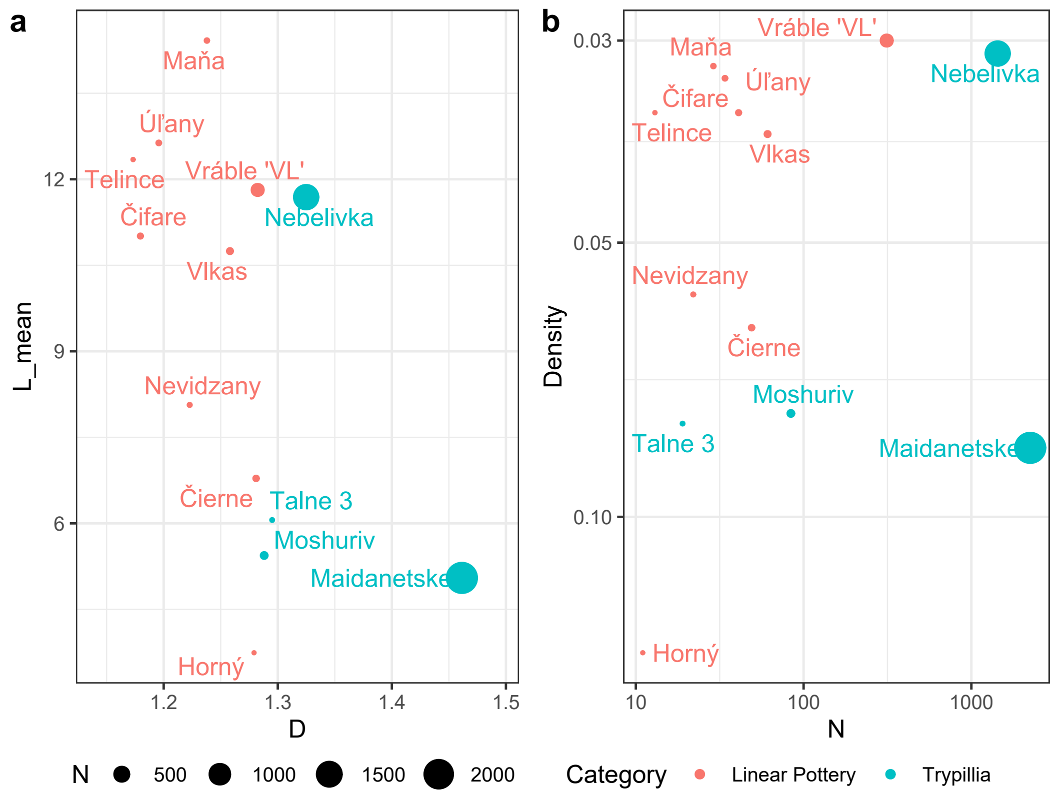

Figure 9.2: *Settlements plans quantified through their fractal dimension (*D*) and mean lacunarity (*L_mean*, plot a) and house count (*N*) and density (plot b). In plot b, scales are logarithmic, and the y axis is reversed, giving the most similar results to those shown in plot a. The settlements of Talne 3 and Horný Oháj had images that were smaller than the lower threshold proposed in the previous chapter*

The models formulated in the preceding chapter for D, L and L_mean as functions of density on regular grids (Table 8.1) were tentatively applied to these results, and the residuals – i.e. the differences between the modelled and the empirical values for each image – are shown on Figure 9.4. The residual values do not show actual values of D and L, (D values are below 1 and close to 0) but value points of how much the empirical values differ from the modelled ones. The plot thus gives an idea of fractal dimension and lacunarity estimates that could be obtained for these settlement plans if they all had the same density. However, as already mentioned, these results are only illustrative, since the analysis does not take into account a number of relevant factors, like the different layout concepts differentiating Trypillia and Linear Pottery

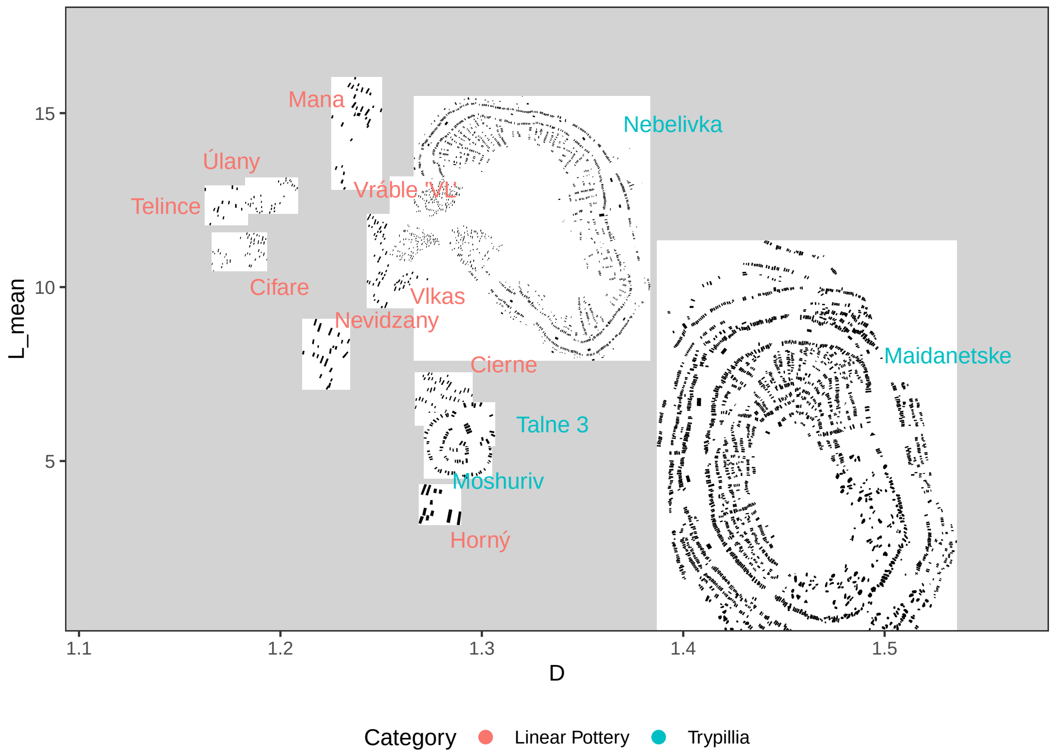

Figure 9.3: *Plans of the same settlements, plotted by* D *and* L_mean*. Image sizes are not internally to scale – size differences are reduced to facilitate readability, as the mega-sites are in reality orders of magnitude larger than the smallest ones*

settlement plans. The resulting scatter plot shows similarities to both plots in Figure 9.2. The differentiation by lacunarity between clustered and less clustered settlements is maintained, while fractal dimension separates the three largest settlements – the two Trypillia mega-sites as well as Vráble – more clearly from the remainder. These were also the ones showing the clearest power-law distributions of house sizes in Chapter 6. In the synthetic images analysed earlier, clustering, noise and unequal size distributions resulted in lower fractal dimension for images with the same density, while here almost all settlements have *higher* dimension than what would be expected from a regular grid with the same density. Comparing the values in relative rather than absolute terms, and following the conclusions from the analysis on synthetic images, the three largest settlements here could be interpreted as being overall more regular, less clustered and noisy (though with more unequal size distributions) than the other settlements. Since fractal dimension here is seemingly correlated to settlement size, it could also be that the effects from noise, clustering and size distribution be weaker relative to the overall plan, though this effect does not appear on the results for lacunarity.

Any attempt of interpreting these results in terms of social organisation is not self-evident.

However, some points can be made. Firstly, it is clear that fractal dimension and lacunarity are not sufficient for distinguishing Trypillia and Linear Pottery settlement plans quantitatively. Neolithic specialists may find this result disappointing, pointing to the visually very obvious difference between grid and radial layouts. On the other hand, it could be objected that these cultures do share concepts in spatial organisation that are quite close – houses are largely free-standing, often organised in rows, and overall densities are similar – and it remains quite possible that these methods could distinguish more easily between more differentiated plans like early Neolithic Anatolian villages, Alpine wetland sites or Bronze or Iron Age semi-urban or urban settlements. Secondly, the results do seem to differentiate effectively between clustered and non- (or at least less) clustered settlements, which again points to the social coherence within the settlement. Villages that are clearly clustered into separate neighbourhoods may show signs of higher levels of inter-group competition and potentially violent tensions, as seen in the skeletal material at Vráble. Contrarily to the results obtained from synthetic images, household inequality proxied through house sizes seems not to be effectively reflected in fractal dimension nor lacunarity, since the settlements that previously have been shown to exhibit the highest levels of inequality also have the highest fractal dimension values, especially when controlled for effects from density. Expected effects from size distribution as well as spatial noise probably drowned from the size of the largest settlements. More sophisticated modelling could possibly also control for a larger range of disturbing factors such as image size, house count, grid orientations etc. Lastly, even though the quantitative distinction between clustered and non-clustered settlements is interesting, the results obtained here (Figures 9.2a and 9.4) are not substantially better than those that could be obtained more directly through house count and density (Figure 9.2b), which are far easier to calculate.

9.2 Quarters/neighbourhoods

The quarters of Nebelivka and the neighbourhoods of Vráble were analysed in the same way, and their fractal dimension and lacunarity estimates are shown in Figure 9.5a, while for comparison, house count and image density are shown on Figure 9.5b. For this image series, the spread of estimates was rather moderate, and again no clear distinction was observed between Linear Pottery and Trypillia layouts, even though they are visually organised in strikingly different ways (Figure 9.6). For both settlements, there was no clear partition of sections into

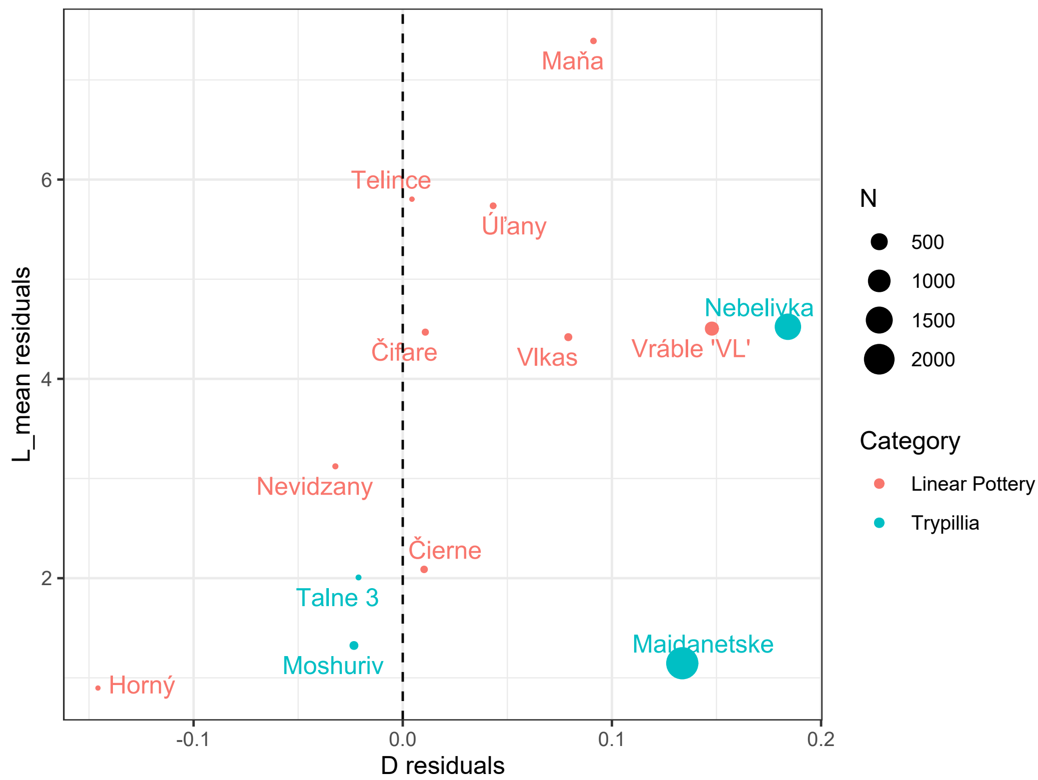

Figure 9.4: D *and* L_mean *residuals for the same settlements after subtracting expected values due to image density alone, modelled on the synthetic images with variable density presented in Chapter 8. See Table 8.1 for details*

separate groups – rather, the whole series formed a continuous spread across the plot. It is clear that density was correlated to lacunarity for these images, since the relative order of images on the y-axis is nearly the same in plots a and b of Figure 9.5. However, fractal dimension was in this case less obviously correlated to house count than for the total settlement plans – or, this correlation had a lesser effect since the various quarters and neighbourhoods were of more uniform sizes (both by image size and house count).

If some distinction is still to be made between images on opposite ends of the scatter plot, for both settlements the most compact sections were plotted towards the lower right end of the plot, while the most patchy or stretched-out images were towards the upper left. For example, quarters L, F, D and G at Nebelivka are all dominated by inner more or less parallel streets, and have relatively high *D* and low *L_mean* values, while quarters N, A and K are more patchy or dusty – one could even say lacunar – and are evaluated to correspondingly low *D* and high *L_mean* values. Interestingly, the results here do seem to reflect those obtained from the distribution fitting analysis in Chapter 6, where quarters N and A were judged as suffering from missing data (notably lacking Assembly Houses). Quarters K and J, which

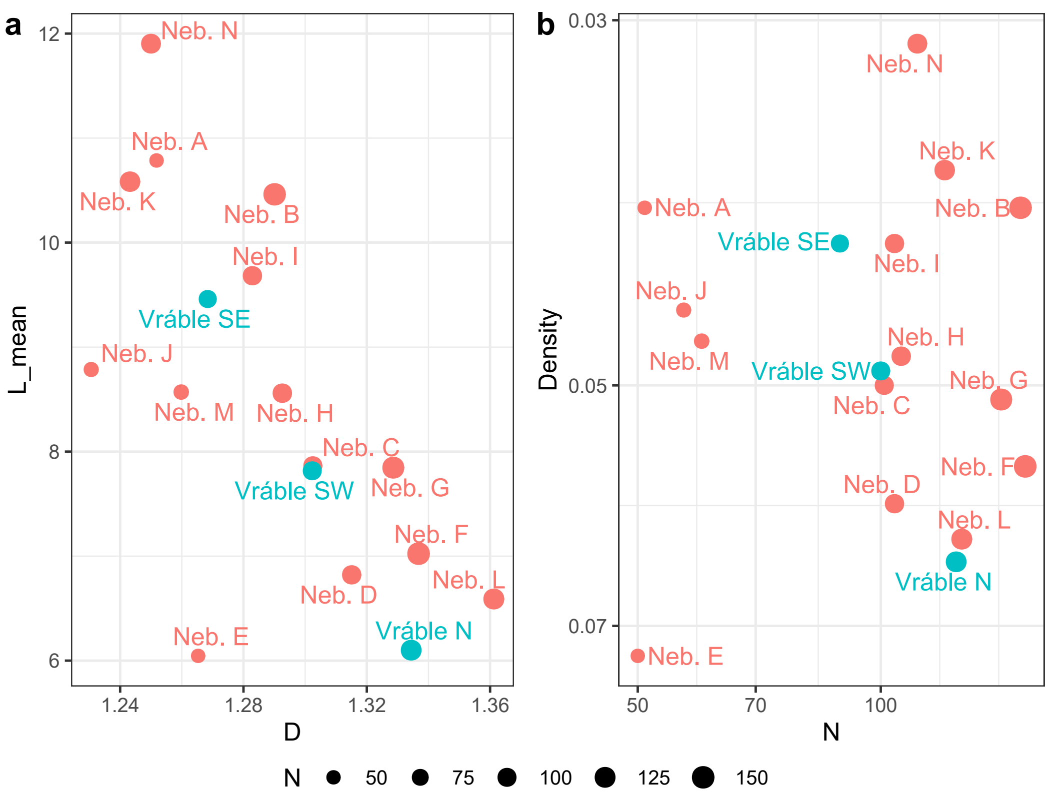

Figure 9.5: *Fractal dimension (D) and mean lacunarity (L_mean, plot a) and house count (N) and density (plot b) of the plans of separate Nebelivka quarters and Vráble neighbourhoods. The image size of Nebelivka E was below the lower threshold of 260*260 pixels. Axes in plot b are logarithmic, with the y-axis reversed, in order to reproduce the spread in plot a*

here have the lowest fractal dimension values, were there interpreted as having quarter borders erroneously drawn by the researchers – in any case their outlines are irregular compared to the other quarters. Quarter E was also speculated to be wrongly interpreted as a separate quarter, causing it to "lack" the power-law distribution of house sizes which characterised the other quarters. Here, quarter E is an outlier regarding both fractal dimension and lacunarity, but the analysed image is also the only one falling below the size threshold of 260*260 pixels previously proposed. Taking these methodological caveats into account, it would seem the Nebelivka quarters had an even smaller spread in fractal dimension and lacunarity, since all the quarters with the most atypical values can seemingly be explained away as non-representative of their original layouts. Despite this, the observation still holds, that quarters dominated by dense inner street grids are placed to the lower right of the plot, while those dominated by the open main street are to the upper left.

For Vráble, the three neighbourhoods also show similar results, with differences being apparently gradual rather than categorical. Also here, the image with the most densely packed

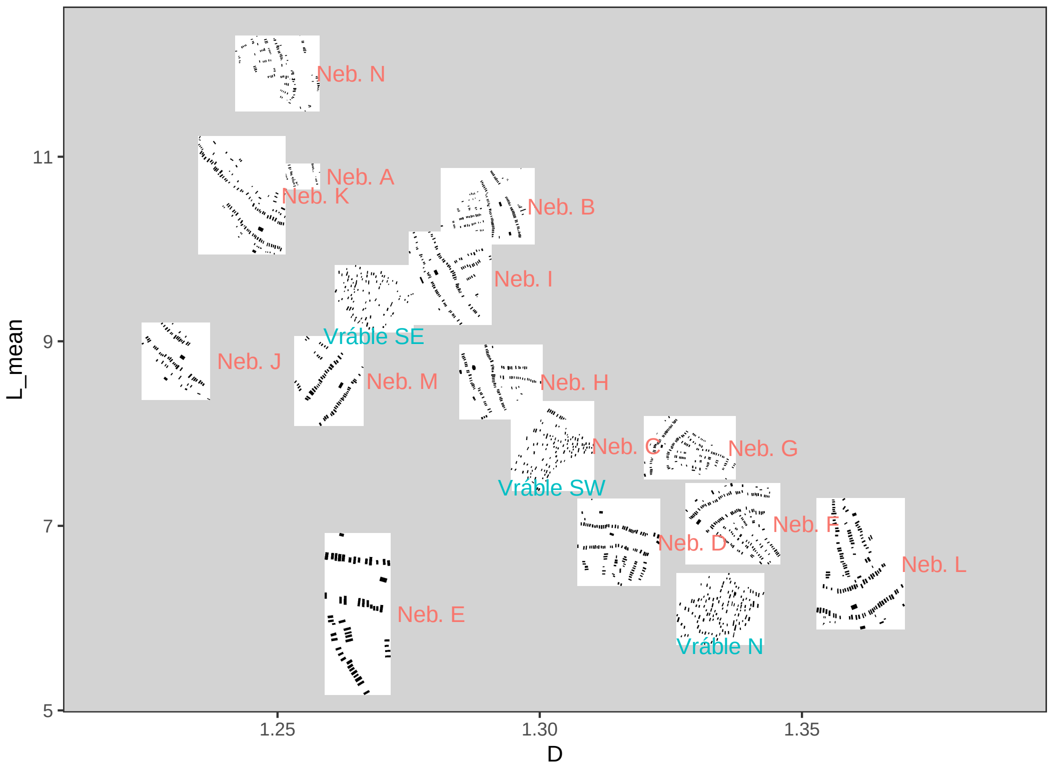

Figure 9.6: *Plans of Nebelivka quarters and Vráble neighbourhoods, placed according to fractal dimension (*D*) and mean lacunarity (*L_mean*). Image sizes are transformed to allow for better visibility of smaller images. The two layout types representative of Trypillia (Nebelivka) and Linear Pottery (Vráble) settlements largely overlap*

and grid-like layout – the northern neighbourhood – also had the highest *D* and the lowest *L_mean*, and inversely for the most patchy image of Vráble South-East. This latter neighbourhood was also the one of the three that was shown to not have a power-law tail to its house-size distribution in Chapter 6, similarly to Nebelivka quarter N. However, the perhaps most surprising result from analysing this series, is that these rather subtle differences in plan regularity had a greater effect on the fractal dimension and lacunarity estimates than the more obvious differences in layout between Linear Pottery and Trypillia settlements. Again, it should be worthwhile to test these analyses on other and more different data sets, to see whether they capture other types of layout differences more accurately.

Lastly, when *D* and *L_mean* estimates are controlled for effects from density, as was done with whole settlements above, residual *D* values become somewhat more randomly distributed, while lacunarity remains more unaffected (Figure 9.7). Even though the caveats already mentioned for this modelling must be repeated here, it would thus seem that there is very little to no significant difference between quarters and neighbourhoods in these two large settlements, further suggesting that there is no noticeable intra-site socio-economic differentiation either, which is in agreement with current understanding of Linear Pottery and Trypillia social organisation (see Chapter 3). Further analysis on more recent settlements (e.g. Bronze Age tell settlements or Iron Age *oppida*) could potentially contribute to our understanding of when such intra-site differentiation first became important factors for urban or semi-urban life.

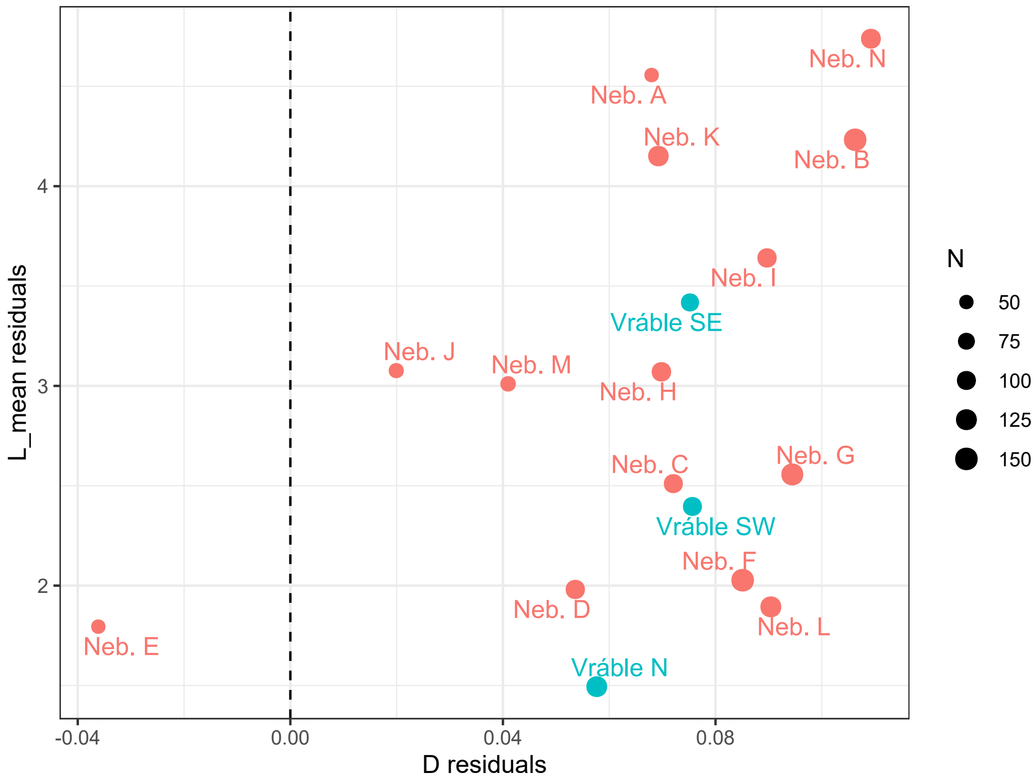

Figure 9.7: *Fractal dimension and mean lacunarity estimate residuals after controlling for effects from density, on the same quarter and neighbourhood images. Values expected from density are modelled on the density series of synthetic images in the previous chapter, see Table 8.1*

9.3 Temporal samples (Vráble)

The partitioning of the Vráble settlement plan into 16 coeval plans separated in time by 20 year intervals was done following the same procedure as in Chapter 6, and an image for each plan was generated, setting the image size to the minimal x and y extent, as with the total settlement and quarter images analysed above. As before, the first time sample consisted of a single house, which by any standards would not be representative of a settlement plan, and for the purpose of this analysis was excluded for having a too small image size (its deviating results are shown on Figure 9.1). Samples two and 16, representing Vráble at 5270 and 4990 BCE according to the model, were different in that they only included houses from two of the three neighbourhoods, resulting in relatively smaller image sizes, while all the remaining images included houses from all three neighbourhoods and thus had similar sizes. Thus, as the village grew over time and subsequently declined, it follows that it effectively also densified until its peak around 5110 before thinng out again until its abandonment. Given how image density was already shown to be determinant of an image's fractal dimension and lacunarity estimated by box-counting and gliding-box algorithms, and since in this case image density would be furthermore highly dependent on house count, it is not surprising that *D* and *L* results of these images can be largely predicted by density and N (Figure 9.8).

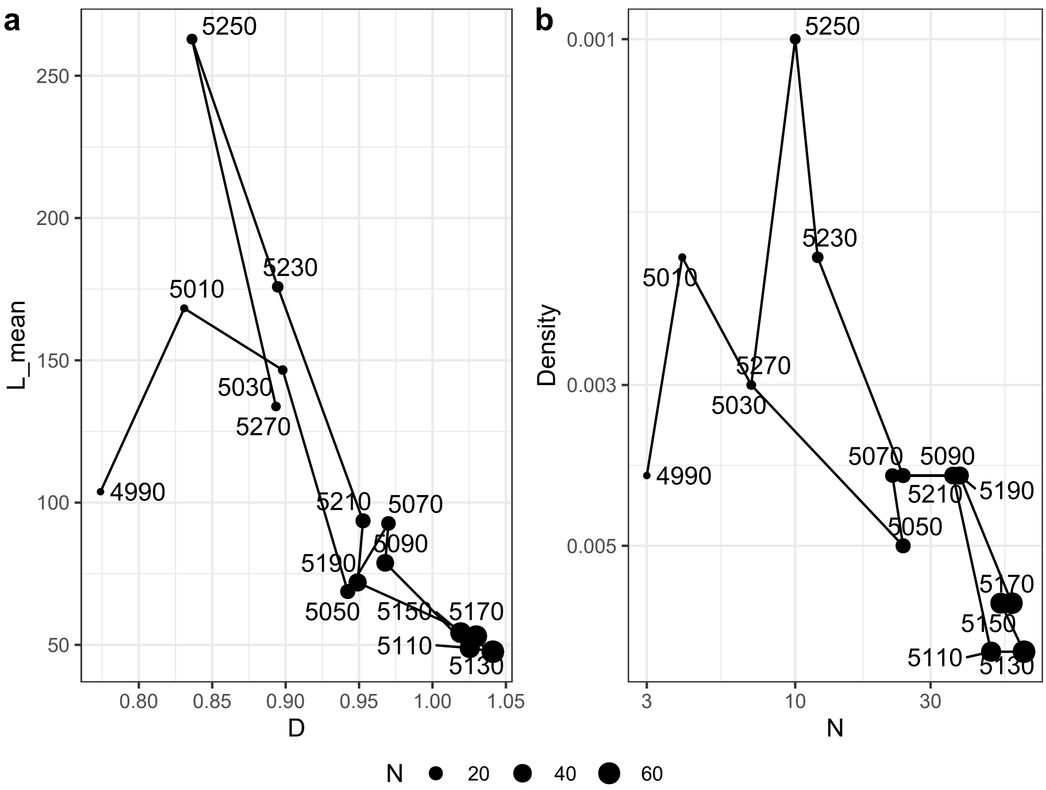

Figure 9.8: *Fractal dimension (*D*) and mean lacunarity (*L_mean*, plot a) and house count (*N*) and image density (plot b) of the site plan of the Linear Pottery settlement of Vráble, subset into 15 coeval time samples with 20 year intervals. Axes in plot b are logarithmic and with reversed y axis in order to emulate plot a*

While the fractal dimension and lacunarity results of these images are inversely proportional, in accordance with the results of the previously analysed series, it is then not clear if they simply reflect this temporal trend of density or add any other information on grid regularity or clustering (the house-size distribution was shown in Chapter 6 to be stable over time in Vráble). However, when looking at the visual aspects of the different images, the first and last few images of the series among those that include houses from all neighbourhoods – i.e. samples 3, 4, 14 and 15 (representing model years 5250, 5230, 5030 and 5010 BCE) –

do not exhibit the clear clustering into three neighbourhoods seen in the middle phases of the settlement (Figure 9.9, all images are included in Appendix A). These are also the images that get exceedingly high lacunarity values

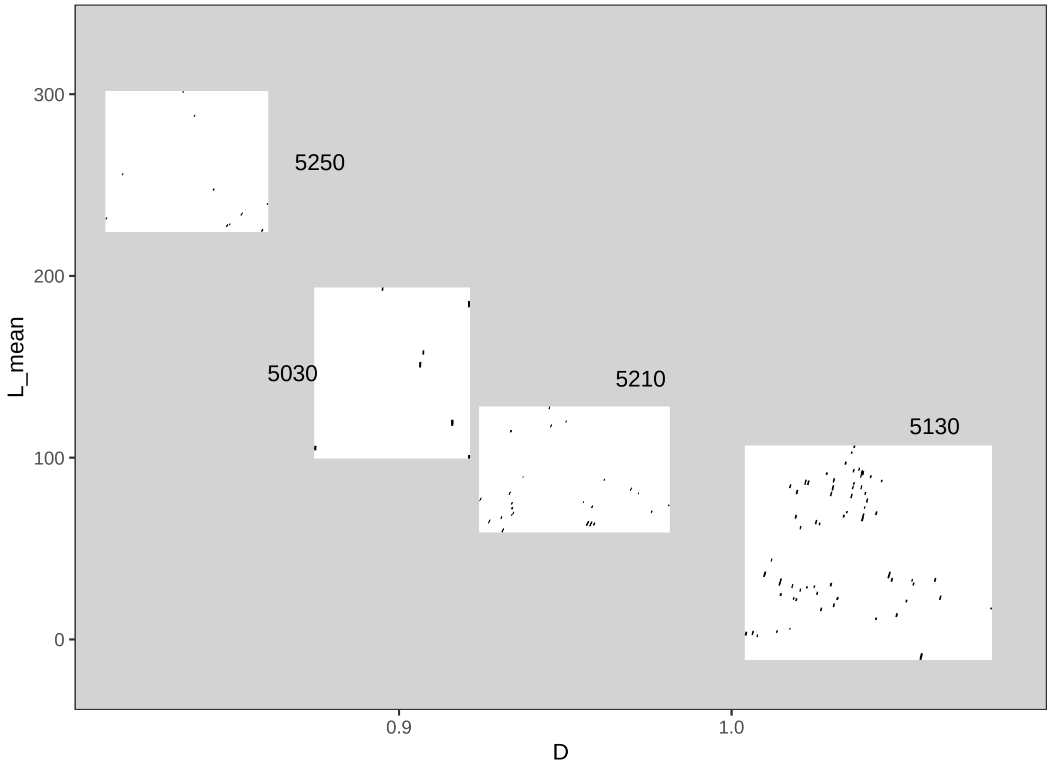

Figure 9.9: *The temporal development of Vráble (Linear Pottery), as seen through the fractal dimension (*D*) and mean lacunarity (*L_mean*) of coeval samples of its settlement plan. Images are selected here to prevent overlaps*

When subtracting *D* and *L_mean* values following the models that were made on synthetic images with varying density in Table 8.1, *D* residuals show no clear pattern besides a seemingly random (normal) spread around a mean of 0.19, i.e. a slightly higher value than would be expected from a perfectly regular grid (Figure 9.10). Residual values of *L_mean* on the other hand do still show some spread for the same deviating time samples, possibly indicating that lacunarity in this case captures the crystallisation of three distinct neighbourhoods in the settlement between approximately 5210 and 5050 BCE. If this were to be interpreted as a higher level of clustering in the middle phases however, from the experimental results in the previous chapter we should expect higher rather than lower lacunarity in these phases. Another way of looking at it is to remark that the distribution of gap sizes becomes more equal with increased clustering in the middle phases of Vráble because of the overall densification, while in the synthetic images clustering was generated with increased gap differences (Figure

8.10). Such apparently trivial differences in how clustering is generated may thus seemingly determine the direction of change in lacunarity values, illustrating once again the difficulty of interpreting this variable directly. But again, these results are to be taken with a particular pinch of salt, considering the many uncertainties (house orientation as proxy for construction date, modelled house duration, *D* and *L* values modelled from density of regular grid images) that had to be accepted in order to generate them.

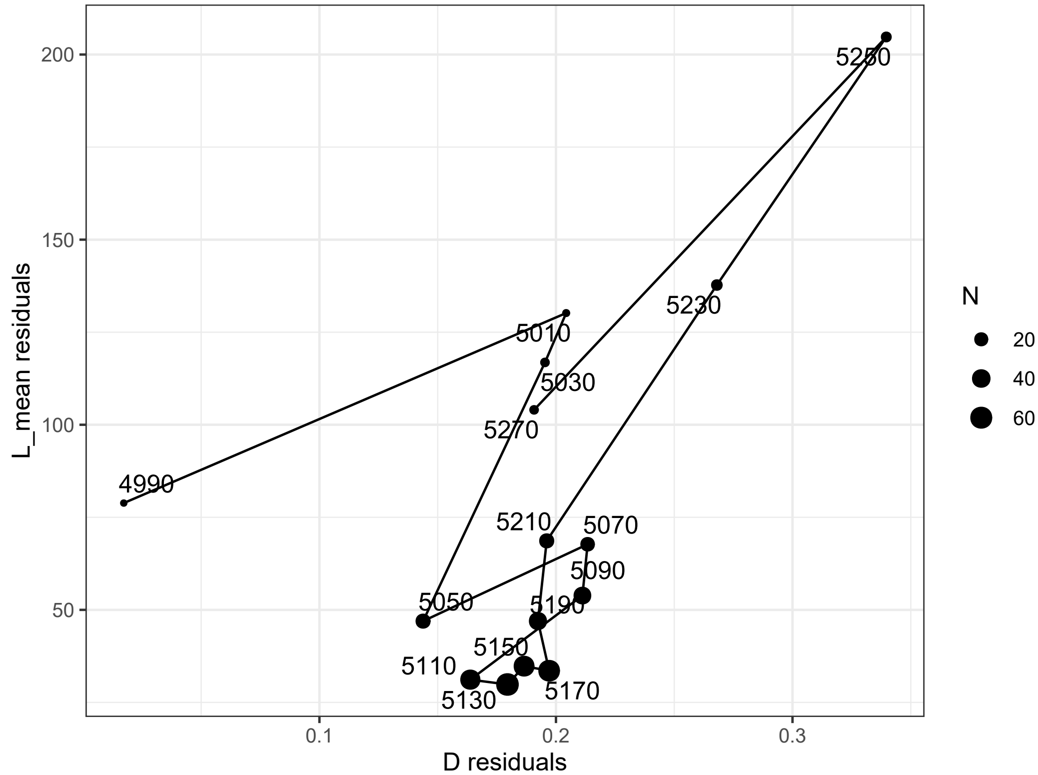

Figure 9.10: *Fractal dimension and mean lacunarity residuals after controlling for effects from image density, following the models presented in Table 8.1 in the previous chapter.*

9.4 Summary of findings

In this chapter, fractal dimension and mean lacunarity estimates were calculated by the box-counting and gliding-box methods on binary images of the 13 settlements, 17 single quarters/neighbourhoods and 16 time samples analysed in Chapter 6, in total 46 images. The results for all three series were shown to be strongly correlated to other known variables like image size, house count and density, as expected from the tests on synthetic images, and the scatter plots of *D* and *L_mean* could in all three cases be largely emulated by plotting *N* (house count, as proxy of settlement and image size) and density on logarithmic scales. The goal of this analysis being to quantify and compare different levels of spatial irregularity and clustering between these plans, the question must then be asked whether fractal analysis in the end brought any more insights than what could be obtained much more easily – both with far less code and far shorter computing time – through other and more direct variables. The patterns that were seen in fractal dimension and, especially, lacunarity results also persisted when modelling away the effects that would be expected from the variation in density between images, and it may seem that while lacunarity and density do not measure the exact same things, they are in practice also correlated, so that the plans that show higher lacunarity also consistently show lower density. These variables are thus difficult to separate analytically in empirical data.

The results for the image series of cumulative settlement plans gave an apparent partitioning by lacunarity into two groups of settlements which cross-cut size categories as well as cultural adherence (Figures 9.2 and 9.3). Settlements with low lacunarity like Maidanetske, Moshuriv and Čierne were more homogeneously compact, while those with higher lacunarity like Nebelivka, Vráble and Úľany nad Žitavou were more clearly clustered into separate neighbourhoods. However, this clustering also gave them lower density since the space between clusters was also included in the images, which would explain the similarity between these results, and why this partition remained after subtracting the effects from density alone. Fractal dimension was also more sensitive to either house count or image size (which were closely connected in these images), and this tendency was strengthened in the D residuals after removing effects from density (Figure 9.4). Considering the results from the analysis of synthetic images, it is possible that any spatial irregularities (noise) would be similar in the small and large settlements, but that the effects of these on fractal dimension estimates become relatively smaller in larger images/settlements with the same resolution (see Bruvoll, n.d. for further discussion on similar observations with synthetic images of variable sizes).

Results on separate quarters did not yield a partition between different types, but rather a continuum from regular, compact and grid-like to loose, dusty or irregular spatial patterns. The most interesting observation here was perhaps that images of typical Linear Pottery and Trypillia layouts – i.e. grid-like and radial with perpendicular streets – that are visually very easy to distinguish, were not differentiated by the fractal analysis, but rather overlapped with larger spread within each group than between them (Figures 9.5 and 9.6). However, the analysed images also did overlap in image size, house count and density, and the gradient in results did seem to capture another difference between the images, more difficult to spot, in compactness. But again the results are difficult to separate analytically from the differences in density and size.

The fractal dimension and lacunarity results of temporally coeval plans at the Linear Pottery settlement of Vráble showed very close resemblance to those of house count and density. In this image series, image size was much more constant than in the quarters and settlements series, but density and house count followed a clear trajectory of growth by concentration followed by dilution after the peak of the settlement, as seen in Chapter 6. Here, fractal dimension and lacunarity estimates thus followed the same trajectory, but when effects from density were modelled out, some of the variability in lacunarity remained (Figure 9.10). This seemingly contradicted the previous observation that increased clustering gives higher lacunarity, since the three neighbourhoods at Vráble were most visible in the images with the *lowest* lacunarity. However, lacunarity did in this case probably capture the subtlety that the *distribution* of gap sizes after all became more equal in the middle phases of the settlement when density was highest. In the early and late phases, the few present houses were already located in what would become or had been the separate neighbourhoods, and these were then, relatively speaking, much more separated in space than when the village was more fully settled.

Overall these results were perhaps less satisfying than those obtained through distribution fitting, and little more insight is gained regarding Trypillia and Linear Pottery social organisation. The fractal dimension and lacunarity estimates obtained here could be largely reproduced by log-transformed house count and density, meaning that the utility of these methods is not clearly demonstrated. More careful modelling on larger synthetic data sets, accompanied with more in-depth theoretical exploration as well as tackling of methodological issues like edge effects, image orientation and missing data, all seem like necessary requirements for luring out the positive correlation seen on the synthetic images between fractal dimension and lacunarity on the one hand, and clustering, size inequality and spatial noise.

Part IV

Synthesis

Chapter 10

Social complexity in Neolithic settlements

While the primary goal of this thesis has been to study fractal analysis methods themselves and how they can become better integrated into archaeological research, this assessment could hardly be done without reference to how well they account for patterns in real-world data. For the case studies included here – the sampled Linear Pottery settlements in the Žitava valley, Slovakia, and Trypillia settlements in the Sinyukha basin in Ukraine, which have largely been analysed together – a number of observations of more culture-historical nature can be made, which can furthermore be interpreted in light of recent research on these culture groups and settlements.

As seen in Chapter 3, for both the Linear Pottery and Trypillia culture groups there has been much discussion among specialists regarding their social organisation and possible levels of hierarchy. Both archaeological cultures are well defined by a number of characteristics of material culture, and they have been extensively documented for decades. However, recognising specific traits of social organisation in archaeology is a complicated matter, and it is often difficult to evaluate different hypotheses against one another. Also, despite obvious similarities in material culture within large geographical zones – as is seen in house architecture within the Linear Pottery area – specific traits of social organisation that are seen in one study area are not necessarily valid for the whole archaeological culture. Broad generalisations may be useful for apprehending complex phenomena, but they also have a tendency to make us blind to evidence of variation. This is a frequently recurring discussion in archaeology, and indeed in most academic disciplines (Graeber and Wengrow 2021; Furholt 2021).

Expert opinions on the social organisation, and specifically the degree of hierarchy, have evolved with somewhat different dynamics in Linear Pottery and Trypillia research. For the former, there has been a pattern of oscillation over time between interpretations tending towards strong egalitarianism (e.g. Pavúk 1972; Gimbutas 1991; Coudart 2015) *contra* lineage- or clan-based society dominated by elites with inherited status (Velde 1990; Jeunesse 1996; Bogaard 2012; Bentley et al. 2012; Augereau 2021), with perhaps the majority of researchers seeking some middle ground (e.g. Last 2015; Whittle and Bickle 2014; Hamon and Gomart 2021; D. Hofmann and Lenneis 2017). The most striking feature of this debate is how the different points of view are forwarded largely from the same material. There seems to be at least a week tendency of science-oriented research concluding with more hierarchy, and humanities-oriented research arguing for less hierarchy or at least more variation. Characteristically, the middle-ground advocates tend to be those who manage to incorporate both approaches in their analyses. In Trypillia research on the other hand, interpretations of social organisation have largely followed national schools of thought. Researchers in British-led projects have stressed the communal nature of large buildings and the lacking material evidence for social elites, arguing for a largely democratic-egalitarian society (Gaydarska 2020; J. Chapman, Gaydarska, and Hale 2016). In German-led projects, interpretations have focussed on temporal dynamics and indications of a gradual shift from communal to elite-led governance, which is claimed to have contributed to the ultimate decline of the mega-site phenomenon (R. Hofmann et al. 2019; Müller et al. 2016). It should be noted that these two interpretations are not mutually exclusive, as the British-led research has been focussed on Nebelivka, which is one of the earlier mega-sites (Müller et al. 2022). Ukrainian researchers have followed the more culture-historical tradition, constructing detailed typo-chronological sequences with the goal of tracing migratory movements of coherent social groups, largely avoiding hypothesising on their social organisation (Diachenko 2012; Kruts 2012; Ryzhov 2012).

10.1 Hierarchy in Neolithic house sizes

The distribution-fitting analysis presented in Chapter 6 allowed for the identification of power-law distributions in both Trypillia and Linear Pottery house sizes. While house sizes are regularly reported in archaeological settlement studies, this approach has to the best of my knowledge not been applied to settlements in Neolithic Europe before. As seen in the theoretical overview in Section 4.2, power-law distributions are clear indications of hierarchically scaling structures. The power-law distributions observed in these house-size samples therefore constitute a strong argument for some sort of hierarchy in their interrelations. What exactly this hierarchy consisted of however, is a different matter.

For the Linear Pottery settlements, and apart from the deviating results for Horný Ohaj which was probably due to the small sample size, only the largest settlement of Vráble was interpreted to have power-law distributed house sizes. While the distributions of the smaller settlements were almost always skewed (with the exception of normally distributed Úľany nad Žitavou), they were markedly less so than that of Vráble, and were interpreted as log-normals. The Vráble settlement, with its total of 313 houses is also markedly larger than the other settlements, of which Vlkas is the largest with 61 houses (Table 6.1). It was hypothesised that the result for Vráble was a statistical artefact reflecting arbitrary data aggregation rather than an actual hierarchy. However, further analysis on separate neighbourhoods and time samples revealed hierarchical scaling also there, including for sub-samples with less than 20 houses, indicating that the observed difference in house-size distributions between Vráble and the other settlements was a robust result.

Temporal modelling was not done here for the smaller sites, but following (Nils Müller-Scheeßel et al. 2020; see also Furholt, Müller, et al. 2020) it can be assumed that not all houses were coeval there either, but rather that they also developed dynamically over time. If there was a threshold of village population size above which Linear Pottery households would begin to organise hierarchically (Chapter 2), it is likely that it would be found already at a low number of coeval houses – possibly around a dozen. There is currently not enough evidence here to estimate such a threshold with any precision, both because of the very limited geographical scope of this study, but also since this number is also where the distribution-fitting algorithm applied here becomes unreliable. It is also very difficult to attach such a threshold in number of coeval houses to any absolute population estimate, as these vary widely between authors for the Linear Pottery culture. As mentioned earlier, Coudart (1998) estimated the typical Liner Pottery hamlet to consist of about five coeval longhouses with a total population of 150-200 (amusingly close to Dunbar's infamous number). Coeval house numbers for the temporal samples of Vráble are much higher, roughly in the range of 20-60 houses, and even when considered separately, temporally coeval social groups within single neighbourhoods could in many phases reach twice or three times as high populations, i.e. totals of 300-600 people, if we accept her estimation methods (Table 6.3). It could be argued that whenever Linear Pottery social groups grew beyond about 200 inhabitants – which was evidently rare given that the vast majority of known settlements were indeed smaller – a possibility emerged for clan leaders to take on a distributive role and actively seek to increase their political and economical dominance over other households. In a settlement like Vráble, where the whole was clearly sub-divided in three distinct neighbourhoods, competing interests between such clan leaders could over time lead to significant tensions. This can again be discerned in the construction of the enclosure and the associated human remains surrounding the south-west neighbourhood dated to its later stage of development (Furholt, Müller-Scheeßel, et al. 2020; Nils Müller-Scheeßel et al. 2021).

While admittedly speculative, this interpretation is corroborated by evidence from Linear Pottery cemeteries indicative of higher social status for local-men-with-adzes (Bentley et al. 2012; Whittle and Bickle 2013) and their probable link to the largest houses (Velde 1990). Furthermore, the argument presented by Jeunesse (2022) regarding differences between generally "rich" and "poor" cemeteries, could be seen as reflective of the difference between villages that have and have not passed the critical point of emerging social hierarchy. This point does not depend on the accuracy or even the relevance of Dunbar's number, which has been much criticised (Chapter 2), but on the observation that larger settlements have power-law distributed house sizes while smaller settlements do not. Testing this hypothesis on wider samples should be a priority for future research.

The link between settlement size and hierarchically scaling house sizes was also seen in the analysed Trypillia settlements. The clearly hierarchical relationship between building sizes at the mega-sites is hard to miss, and has already been recognised by the researchers who have studied them in recent years (R. Hofmann et al. 2019; J. Chapman, Gaydarska, and Hale 2016). However, the distribution-fitting analysis allowed for a more detailed appreciation of just how far down the scale this hierarchy appears to have gone. While R. Hofmann et al. (2019) acknowledge that some large houses located at entrance points and street corners may have had special functions even though they have not been previously classified as typical "mega-structures", the results presented in Chapter 6 indicate that as much as 44% of houses at Nebelivka have sizes distributed in a way that is best described as a power law. Furthermore, mapping where these houses are located in the settlement reveals a strong tendency of concentrations of larger houses near mega-structures and in the main streets, while smaller houses are located more often in side streets and near the borders between quarters (Figure 6.5). Though interpretations of the largest structure at Nebelivka have been somewhat inconsistent (Burdo and Videiko 2016; J. Chapman, Gaydarska, and Hale 2016), the public rather than domestic function of at least the medium-sized mega-structures/assembly houses seems well established (R. Hofmann et al. 2019). As it has been repeatedly stated throughout this thesis, there is no inherent contradiction between hierarchical structures and communal social organisation, and fully egalitarian democracies can very well solve a number of organisational issues through hierarchical scaling of institutions. In contemporary democratic states, this sort of hierarchy often translates to law-regulated bureaucracies (De Landa 2006), and can be recognised materially through a number of communally organised networks of public and institutional buildings, as well as infrastructure, which will – at least in theory – tend to follow power-law size distributions on various levels (Batty 2005; D'Acci 2019). There is thus no need to explain away this hierarchy simply because Trypillia settlements are thought to be egalitarian. However, these large houses which are centrally placed within the neighbourhoods at Nebelivka have received far less systematic attention in later years than the mega-structures, and there has been an overall consensus that Trypillia domestic houses were constructed with extreme conformity (e.g. Chernovol 2012). At the same time, as reviewed by R. Hofmann et al. (2019), in most cases the supposed public buildings could not possibly fit but a fraction of the use-groups they would represent, meaning that some sort of delegation of tasks must have taken place. It would seem Trypillia household research would draw benefit from the active search for variability that has characterised Linear Pottery research for a long time, as such an approach could potentially nuance the currently dominant view of Trypillia households as strictly egalitarian.

The results for Maidanetske were somewhat less conclusive. The observed power-law distribution of house sizes covered "only" 13% of the total house count (which still represents 300 houses). Several differences both in data quality and settlement characteristics could explain this: The largest building structure is missing from the documentation, but is thought to have been there originally judging from the overall layout. There is more missing data than in Nebelivka – as the south-east part of the settlement was not included in the distribution analysis because of poorer data resolution. But probably most importantly, it is increasingly being recognised that Nebelivka had a much shorter occupation span than Maidanetske, both being situated in the extremes of the more standard duration of mega-sites (Müller et al. 2022). At the same time, it remains at this stage impossible to reconstruct coeval settlement plans of Maidanetske at single time samples, because of the enormous size of the settlement and the colossal effort that would be needed to date enough houses independently. Given the distortions that was shown to result from arbitrary data aggregation using the method applied here (Chapter 5), it is likely that the results from Nebelivka are the most reliable in this context.

Interestingly, a power-law distribution of house sizes was also recognised at the much smaller settlement of Moshuriv (84 houses) but not at the smallest Talne 3 (19 houses). As with the results for the Linear Pottery settlements, this could again be an indication that hierarchical scaling between households somehow kicks off above a certain threshold. The fact that this was seen for both the Linear Pottery and the Trypillia settlements, and given the diverging views on their modes of social organisation, would lend support to the interpretation of a mechanism which is not primarily culturally contingent. What would need further cultural explanation, is how some settlements came to surpass the threshold to structural hierarchy while others did not. It should be kept in mind that the sizes of the mega-sites as well as Vráble are highly unusual within their cultural contexts.

10.2 Settlement layouts and scaling

The results from the image analysis approach were overall less conclusive, and more work remains before it can be effectively implemented in archaeological research. However, also there the resulting fractal dimension and lacunarity values outlined a grouping of the settlements which cross-cut cultural attribution. The obtained values seemingly quantified the level of clustering, hierarchical or not, so that the settlements with high D and low L values were homogeneously compact in contrast to heterogeneously clustered settlements (Figure 9.3). Interestingly, this partition also cross-cut size categories and distribution models, generating the impression of a highly complex relation to clustering which is seemingly unrelated to these other variables. The method did not deliver as expected regarding the quantification of the level of overall planning (spatial regularity vs. noise). However, it is possible that the sampled settlements were in reality too similar for this effect to become discernible, and the procedure would merit to be tested on much larger and more varied samples. There are highly diverging opinions as to just how regular both Linear Pottery and Trypillia settlement plans really are, and a tool for quantifying this aspect reproducibly would be welcome. The approach with the best performance for this purpose is for now the multivariate correspondence analysis approach, as proposed by Furholt (2016).

Though it was not part of this project, a settlement scaling approach seems to be both possible and warranted for Linear Pottery and Trypillia regional analyses. In the current framework as proposed by Lobo et al. (2020), Bettencourt (2021) and others, it does not appear to have been applied yet to prehistoric Europe (the closest to my knowledge being the analysis of Irish Bronze Age stone circles proposed by Grove 2011). Traditional rank-size distributions for the Trypillia sites in the Sinyukha basin have recently been provided by Shatilo (2021, 218–22), showing rather clearly that they do not follow power laws in any phases (though she words it differently). This indicates that there was no settlement hierarchy for which the mega-sites functioned as central places, but rather that they appeared through an agglomeration process, draining populations from the wider area of smaller communities as well as with influx from migrations from the south-west, as has also been suggested by others (Diachenko and Menotti 2012; Diachenko and Zubrow 2015). While the rank-size analysis in this case seems to work well, a more formal and theoretically sound settlement scaling approach would provide more convincing results. Settlement hierarchies have for some time and increasingly been suggested for Linear Pottery micro-regions (see Section 3.2), but usually with qualitative assessment only. Applying settlement scaling analysis on Linear Pottery contexts at different geographical scales could shed more light on the processes involved in the initial spread of agriculture in these regions and the subsequent possible emergence of social elites.

Chapter 11

Fractal Analysis and Archaeological data

11.1 The distribution fitting approach

A number of observations can be made regarding the results obtained from the analyses on synthetic data distributions presented in Chapter 5 and their implications to empirical analyses.

It is widely recognised that confidently differentiating between log-normal and power-law distributions is a complicated matter (Clauset, Shalizi, and Newman 2009; Stumpf and Porter 2012), and confusions between the two have a deep history in statistical research (Sheridan and Onodera 2018; Mitzenmacher 2004; Harrison 1981; Gibrat 1930). The question raised by Stumpf and Porter (2012), of whether it really matters in the end, is intriguing. The generative mechanisms underlying these two heavy-tailed distributions are in many ways similar, but not identical. And only the power law is considered to quantify actual hierarchical/fractal structures, which is the focus of this thesis. However, it is also recognised that strict power laws are extremely rare in nature and society (if they even exist at all) since they involve infinite variance, while most of the phenomena that we are interested in are known to exist within clearly limited bounds. As presented in Chapter 4.8, several statistical workarounds have been proposed as solutions to this problem, with composite distributions like the power law with exponential cut-off, or the parabolic fractal distribution. In that case – and given that the log-normal and the power law are reportedly so difficult to distinguish – why cannot the log-normal distribution also function as a step-in to account for finite effects?

I believe it is a matter of convenience: the log-normal distribution is so malleable that it can effectively mimic any level of tailedness and asymmetry (kurtosis and skewness in statistical terms) from a perfectly symmetrical normal distribution to an extremely unequally distributed power law. This property makes the log-normal very useful for many purposes, but it also makes it more difficult to distinguish between qualitatively different underlying mechanisms. If the goal is – like in this case – to identify a critical point where a system changes its behaviour, like when an undifferentiated social group begins to organise hierarchically, it is not very helpful to model its expressions as a log-normal. Identifying or excluding a power law is more indicative, since it says more clearly if there is or is not a hierarchical structure at hand.

In the results of the empirical analyses presented in Chapter 6, the seemingly most characteristic difference between power-law and non-power-law tails (log-normal distributions) for settlements, was the proportion of the distribution included in the tail (Table 6.1). Tails that were interpreted as power laws consisted of a maximum of 44% of the data (at Nebelivka, and excluding Horný which was too small for confident results). Log-normal distributions without power-law tails on the other hand (interpreted as exponentials) had a minimum of 66% and up to 93% of the data in them. All tail x_{min} values were set at the point where they gave the best possible power-law fit. This may indicate that the distinction *is* meaningful after all.

Furthermore, the power laws identified for whole settlements were largely persistent when the distributions were subdivided into separate quarters, neighbourhoods and time samples, as already discussed in the previous chapter. Tests on synthetic data series indicated that aggregated data series (settlement data with low temporal resolution) would not generate false positive power-law distributions, as long as the data aggregation did not involve stacking of essentially differently distributed sub-sets, i.e. different settlements or phases with marked shifts in material culture. Also, small sample sizes were shown to increase the risk of false positive power laws, while large sample sizes reduced the risk. Only the small settlement of Horný Ohaj seems to have given a false positive power-law tail, the other ones being the largest settlements in the sample. Together, these arguments further indicate that the settlements with identified power-law tails were indeed different from the rest, and that this difference was linked to settlement size.

The results from the distribution fitting approach were overall very convincing – the underlying theory is relatively sound and has great explanatory power, and the methods seem to be reliable also when applied to archaeological data. Interesting culture-historical conclusions were presented in the previous chapter. Future research could focus on extending this application to data sets of poorer quality. In this study, the analysed settlements were sampled primarily based on their extensive documentation and completeness. In most archaeological settings, settlements are not documented in their entire extent, but are "cropped" by a limited observation window, or subject to erosion, or with more complex stratigraphy. Also, as briefly discussed already, the closely related approach labelled Settlement Scaling should be more broadly applied to prehistoric settlement systems.

11.2 The image analysis approach

Fractal analysis of synthetically generated image series with incremental changes in single variables, presented in Chapter 8, resulted in some critical observations. The three tested variables which were considered to capture essential features of settlement layouts, namely hierarchical clustering, size distribution of pattern elements and random noise, all showed clear correlations both with fractal dimension and lacunarity, even though these tests were not large enough for formal statistical modelling. The first image in each series (which was identical for all three), represented an entirely regular grid, emulating a highly rigid, strictly planned settlement layout. As clustering, noise or size variability were added, fractal dimension decreased while lacunarity increased, at more or less the same rate for all three series.

I believe this to be the most important finding of this thesis: fractal image analysis does quantify deviations from geometrically regular spatial patterns, and can – for now in theory only – be used as a proxy for degree of settlement planning versus household autonomy.

However, a number of caveats were also made apparent, which together impeded the production of useful empirical results in this occasion. Firstly, variations in image density was shown to have a much greater effect both on fractal dimension and lacunarity than any of the targeted variables (Figure 8.14), meaning that a slight change in density could potentially preclude any meaningful insights related to the other variables, if results were to be taken at face value (Thomas, Frankhauser, and De Keersmaecker 2007). The settlement images analysed in Chapter 9 admittedly all had densities within comparable ranges, but it should be expected that if this analysis also was to include settlement types of much higher densities – like the early Neolithic "Anatolian village" type discussed by Furholt (2016) – these would show markedly higher fractal dimension and lower lacunarity estimates. Any interpretation in terms of regularity of spatial layout of such results would be flawed, since they would rather simply reflect the differences in settlement density which would drown the signal from the targeted effects.

The strong effect from image density was seen in the diverging results between the time sample images of the Vráble plan and the remainder of empirical settlement plans (Figure 9.1). The temporal samples constituted the only one of the image series where density was markedly different from the other (much lower density values), and this was reflected in the results with a clear partition between this series and the others in the scatter plot of fractal dimension and lacunarity. A rudimentary attempt was made to model away the effects from density on D and L results from the empirical images, which left apparently random D residuals, while L (the mean lacunarity was used) residuals were largely unaffected. This was not a formal analysis for many reasons, but the results still point in some directions:

- For fractal dimension, no clear pattern could be discerned that could not be reduced to image density. This could be explained either as the analysed images being too similar for any meaningful patterns to be detected, or that other factors (image size, resolution, edge effects) could prevent these patterns from being detected.
- For lacunarity, the observed patterns were minimally affected by modelling out effects from density. Since it was shown that the two variables strongly correlated, the fact that lacunarity still could not be reduced to density indicates that the two are also inherently connected beyond a simple correlation. Lacunarity quantifies the regularity (the size distribution) of gaps in a pattern – it depends on the presence and quantity of gaps and thus on density. On the other hand, lacunarity did prove to quantify textures independently of density. Only larger and more systematic studies could allow for further understanding of how these effects can be made more clear.

From this, it could be tentatively concluded that lacunarity would be more useful for the purpose intended here, than fractal dimension. Another result of this analysis was that the even though the absolute value ranges differ between the two summary statistics of lacunarity reported here – exponent and mean lacunarity – the distribution of results were to a great extent equivalent, and any one of them can be used, as long as it is clear which one it is. I have elsewhere shown that prefactor lacunarity also is very strongly correlated to mean lacunarity, to the extent that their measured effects are nearly indistinguishable (Bruvoll, n.d.). However, since the absolute value ranges between these summary statistics differ, it is important to report which one is being used to allow for comparisons between studies (Farías-Pelayo 2017, 2015).

Another problematic side of using fractal dimension and/or lacunarity as proxies for village planning, is that the different texture-related variables which were shown to have similar effects do not translate into social processes in a uniform way. Random noise – here obtained by letting houses perform random walks – clearly illustrates non-adherence to overall rules and planning. A settlement plan where every household constructs wherever they like without any concern to the placement of other houses, would resemble white noise. Unequal size distributions on the other hand, are illustrative of hierarchy, as seen in the chapters devoted to distribution fitting. Higher levels of hierarchy would be expected to lead to more detailed settlement planning, maybe even more so in democratic political systems than in autocratic ones. Hierarchical clustering is also, clearly, hierarchical. However, its social implications are not necessarily straightforward.

In their early paper pioneering the use of these methods in the study of archaeological settlements, Brown and Witschey (2003) made the case that hierarchical clustering – "clusters of clusters of clusters" – were indicative of nested levels of social cohesion in classical Maya society, from family units, lineages and clans, to larger sodalities and ultimately the city state at the highest level. However, they interpreted this simply from obtaining a fractional dimension value (rather than an integer) from box-counting of the settlement plan, which at this point should be easily recognised as a hasty conclusion. Similarly, Oleschko et al. (2000) compared the Ciudadela complex in Teotihuacán with the mathematical fractal known as the Sierpinski carpet, simply from the close fractal dimension result they obtained (at $D \approx 1.89$). The input images they used for the analysis were seemingly black-and-white aerial and satellite photographs, dominated by shadows, vegetation, tracks and other unrelated surface features. While using remote sensing imagery (including geomagnetic) directly in fractal analysis of archaeological features would be an intriguing and potentially rewarding exercise, it seems at the current stage that the method is far from ready to be used uncritically. One possible avenue that seems to become ever more realistic, is using machine learning and artificial intelligence techniques for automating the recognition of architectural features from remote sensing imagery, to then be subject to analysis (see e.g. Guyot et al. 2021; Olivier and Vaart 2021). Though we should not expect machines to do the critical thinking for us any time soon, they could probably take over the laborious task of finding and redrawing features of interest (which would of course need control checks), thus speeding up such studies considerably, and opening up the possibility of performing analyses on drastically larger data sets.

Chapter 12

Conclusion and Outlook

This thesis was devoted to the study of social hierarchies in prehistory, and how they could be approached through the application of fractal analysis methods. While the subject might seem highly specialised, I have argued that this framework is underexploited in archaeology, and that the methods which are explored throughout these chapters could also be applied to a range of other contexts and research questions. Fractal analysis is inherently linked to dynamical systems and complexity – concepts that archaeologists could benefit from to a much larger extent than we do today.

One of the main conclusions from the analyses that have been presented here, is that the studied social groups seem to have started to organise hierarchically at some point as a response to population growth in larger settlements, irrespectively of cultural attribution and purported political system, indicating – despite the small sample size, but in accordance with associated theory – that hierarchical structuring emerges spontanously with growth under given conditions and from simple rules, not as a result of conscious planning.

In archaeology, such explanations are often unpopular, since we like to believe that social organisation is culture-specific and largely determined by human agency. However, this view can paradoxically lead to seeing archaeological cultures as monolithic in how their societies are organised, while there might well be more intra-culture variation than what is often recognised. From this I argue that we must be open to the possibility that social life could have been as different between small and large settlements within single archaeological cultures as it would be between settlements of different cultures within the same period. While exceptionally large settlements in Neolithic and Chalcolithic Europe may lack certain traits that would qualify them as urban, there might well have been distinctions at play between their inhabitants and those of smaller surrounding hamlets, somewhat analogous of how city and countryside dwellers today perceive each other as different in so many ways.

With the preceding chapters I have attempted to bring the world of fractals and complexity closer to archaeology. Undoubtedly, this framework will seem remote to the preoccupations of many archaeologists. The mathematical nature of the methods is an obvious obstacle to their more widespread use. However, fractals and related concepts are already being used as metaphors, with little to no mathematics involved, illustrating how these fundamental types of structuring can be recognised also in qualitative ways (e.g. J. Chapman et al. 2006; Sherratt 2004; Sindbæk 2022; Whitridge 2016).

A plethora of possible directions for future research could be listed here. I will mention only a few, which I find particularly promising, given the current state of research:

- Ethnoarchaeology! Measuring house sizes and settlement layouts in live settings and relating them to social organisation is a possibility largely overlooked by ethnographers. Research in urban science and human geography is little concerned with the questions asked in this study. Archaeologist, and prehistorians in particular, are concerned with small scale societies and how they relate to their material culture. Perhaps we should bring a total station and a drone next time we go for participant observation?
- The distribution fitting and image analysis approaches should be more widely tested in other archaeological settings: Alpine lake dwellings, Anatolian villages, later historic periods, other materials, burials, megaliths… The real question is where to start.
- The methods themselves can be further improved. The performance of more complex distribution models, like the power law with cut-off and the parabolic fractal need to be systematically tested on archaeological material. Image analysis can be improved by adding observation windows. Multifractal spectral analysis should be tested, and the direct use of remote sensing imagery for analysis should be explored.
- Settlement Scaling Theory, with its formal framework and methodological and theoretical advantages over its outdated rank-size predecessor, needs to be applied to prehistoric European settings, with the potential of improving our understanding of regional settlement systems.
- Time series: Hurst exponent and scale invariance in temporal development of e.g. re-

gional settlement or population dynamics remain practically unexplored in archaeology.

- Integrate theory: Bridge the gap between distant theoretical traditions (natural sciences and humanities) and their respective approaches to the same phenomena
- Take the full step into the world of dynamical systems, and apply concepts like chaos, strange attractors and self-organised criticality to archaeological material.

That is a rough sketch to a roadmap of infinite possibilities for archaeologists who wish to take a step or two off the beaten track. It is the list I wish I had read before starting to work on this thesis. As such, it is perhaps the best contribution I can bring to the archaeological table.

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Appendix A

Archaeological plan images

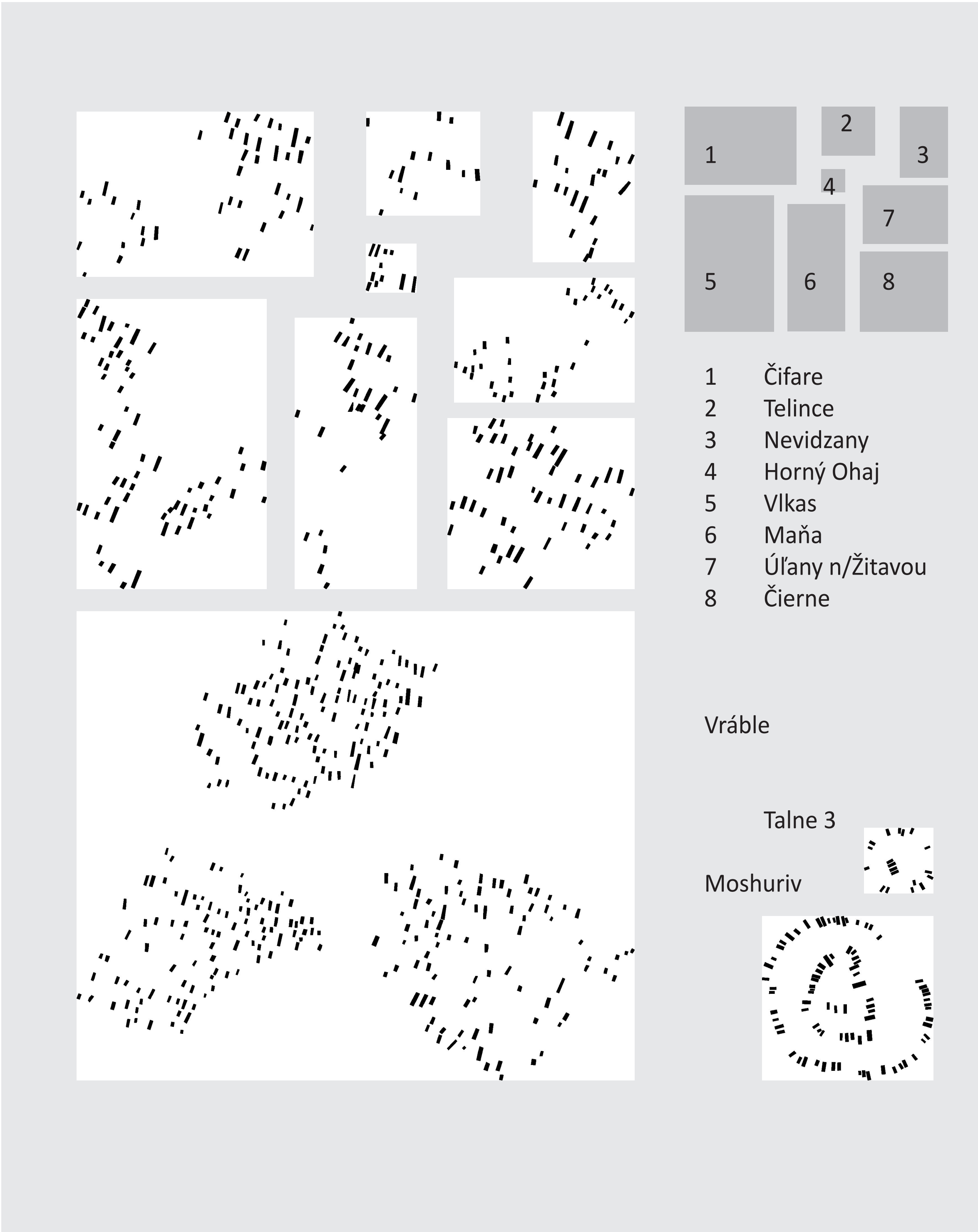
The following plates include image series of archaeological settlement plans analysed in Chapter 9. The different image series are not to scale between them.

- Settlements – images are to scale within series
- Quarters – each plate is to scale internally, but not between each other
- Time samples – images are to scale within series
  - Table A.1 – time samples, modelled sample dates and house orientations

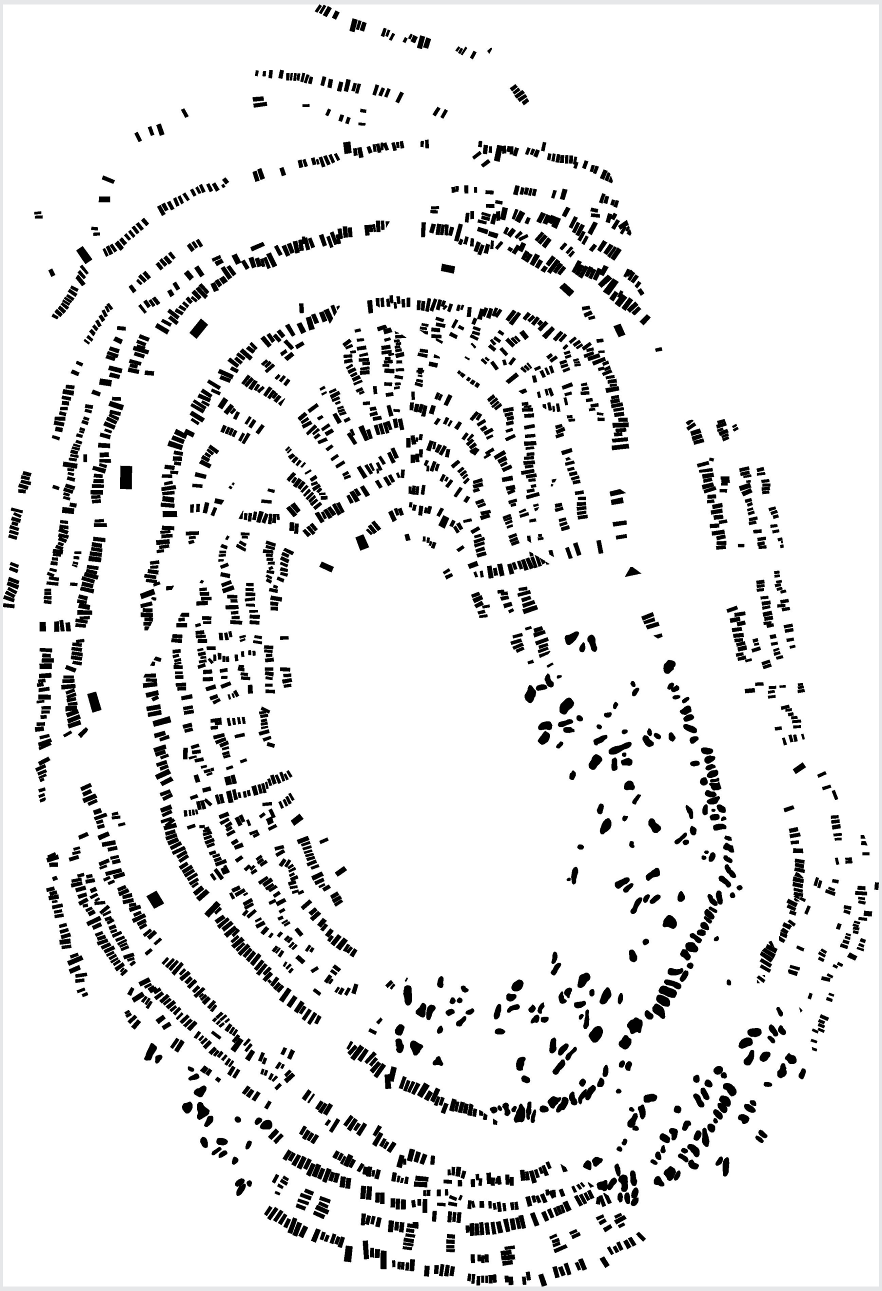
See overview figures in Chapter 3 for absolute scales, and details on image preparation.

Original image files can be downloaded from project repository.

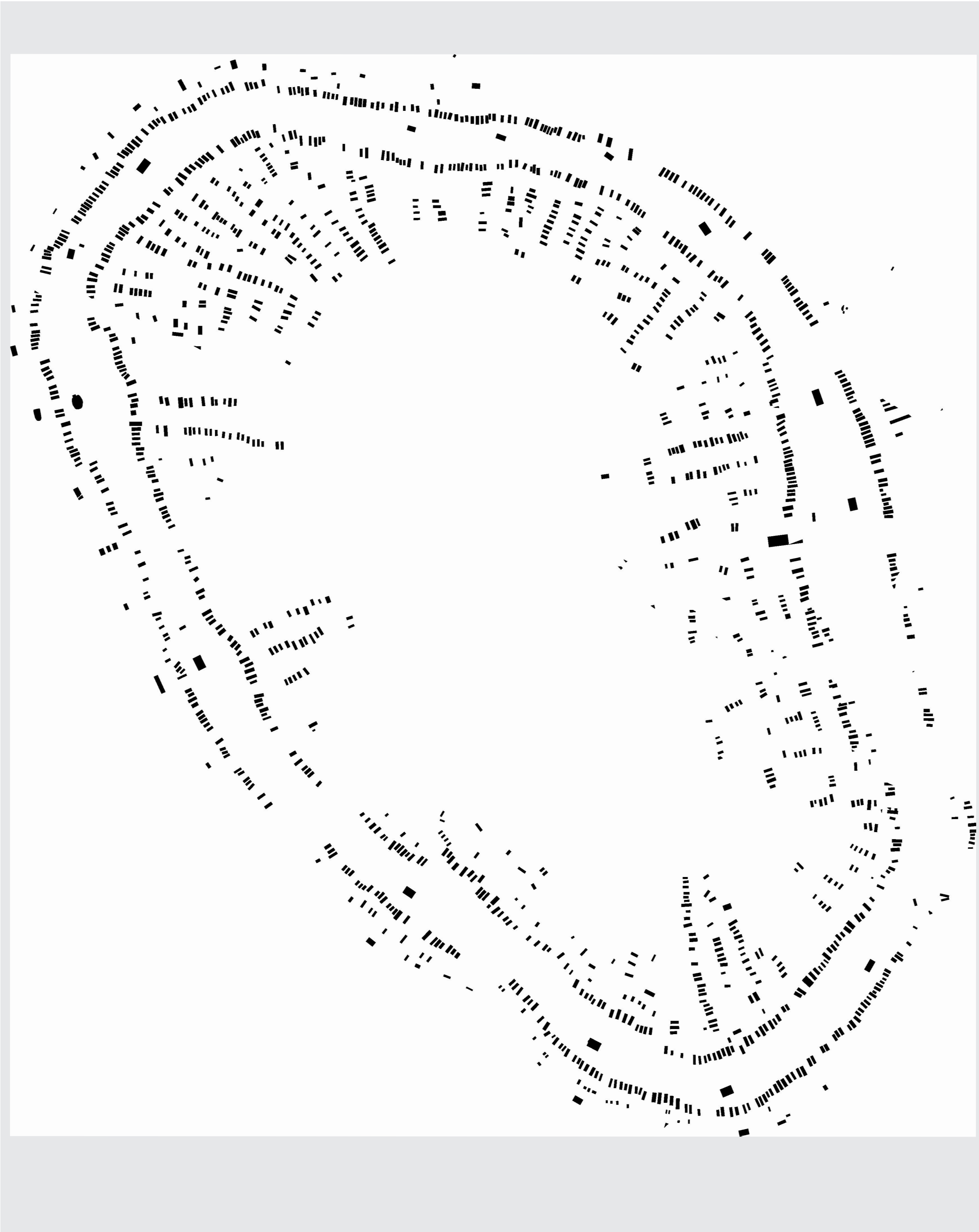
Settlements:
Linear Pottery and Trypillia



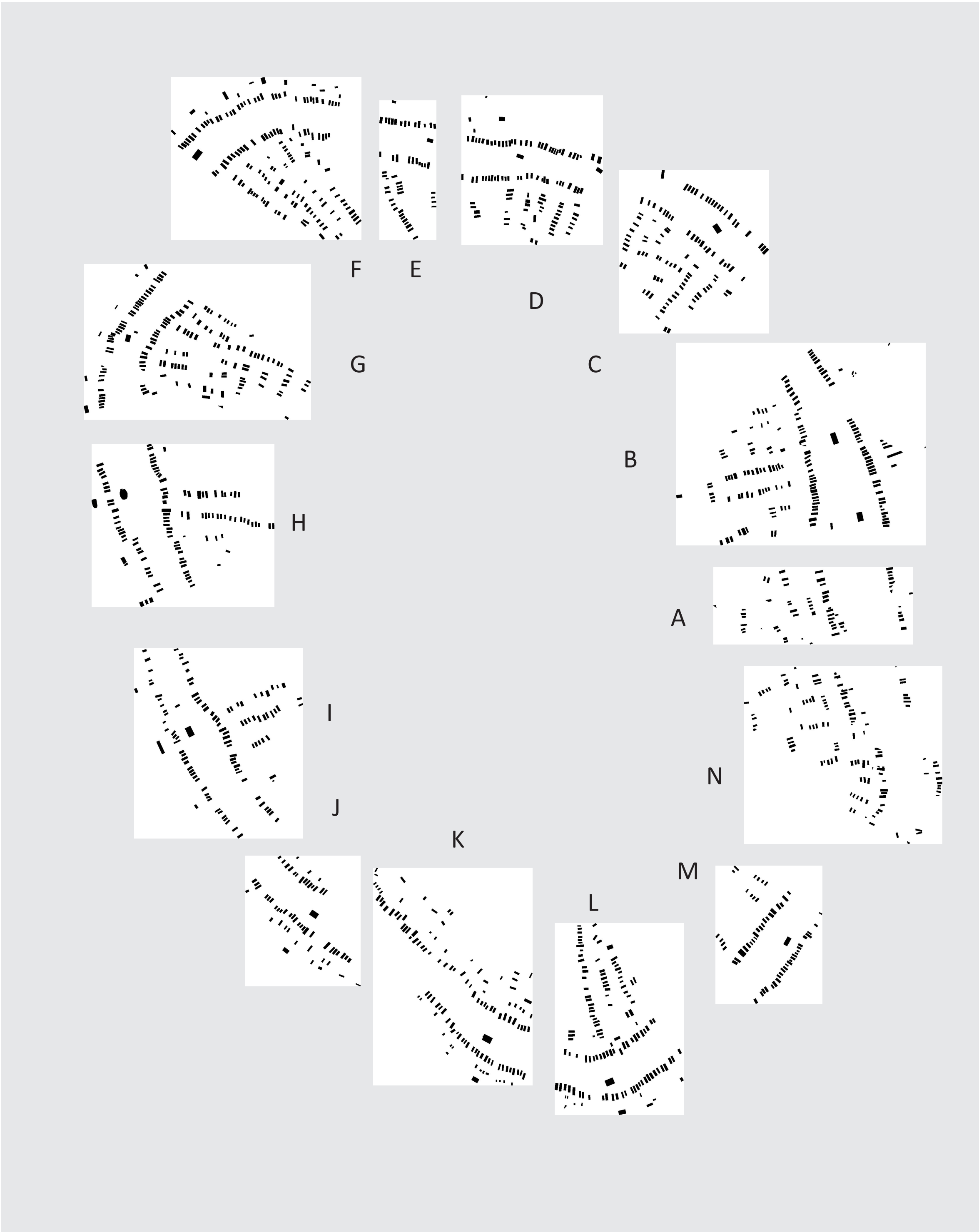
Settlements:
Maidanetske



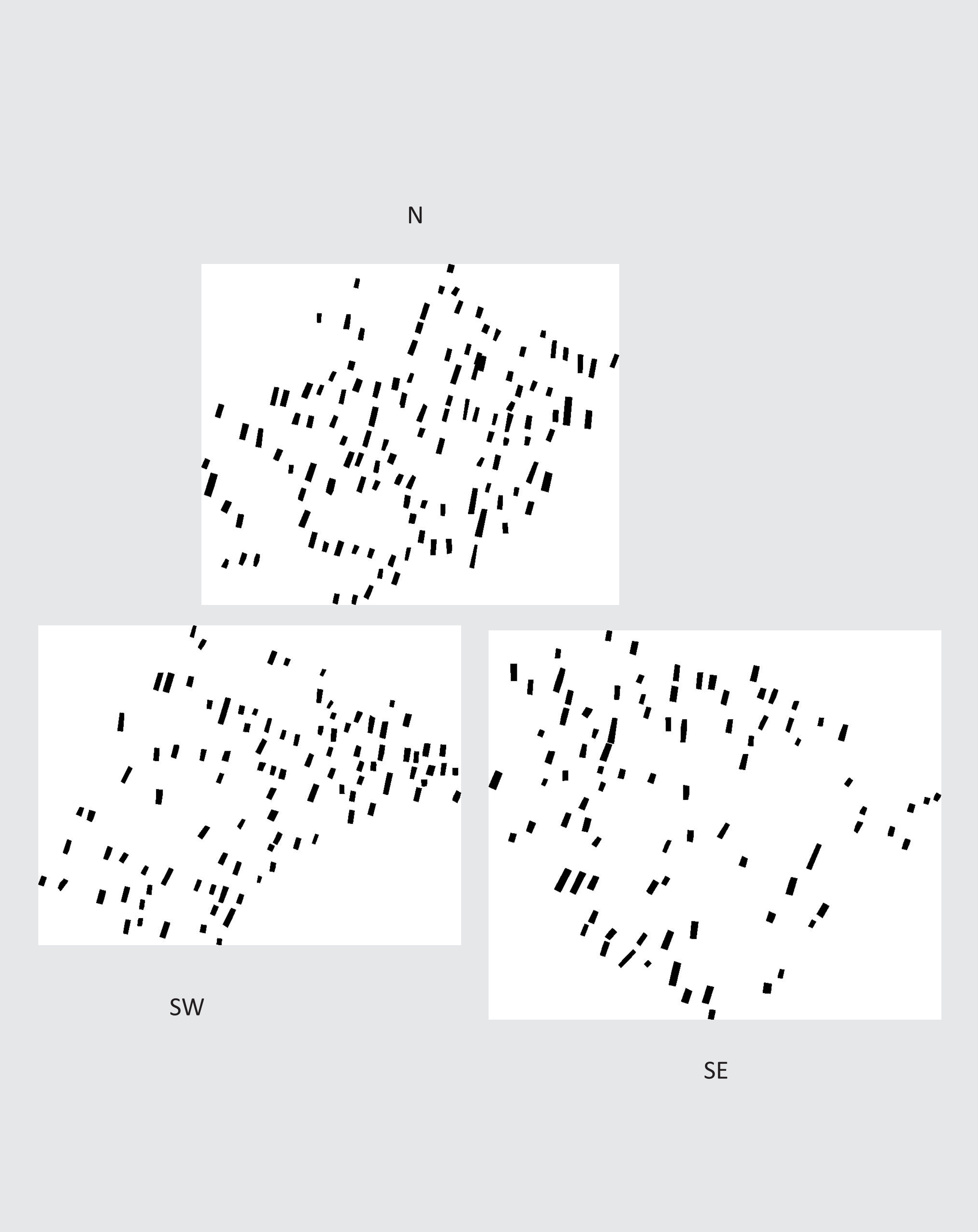
Settlements:
Nebelivka



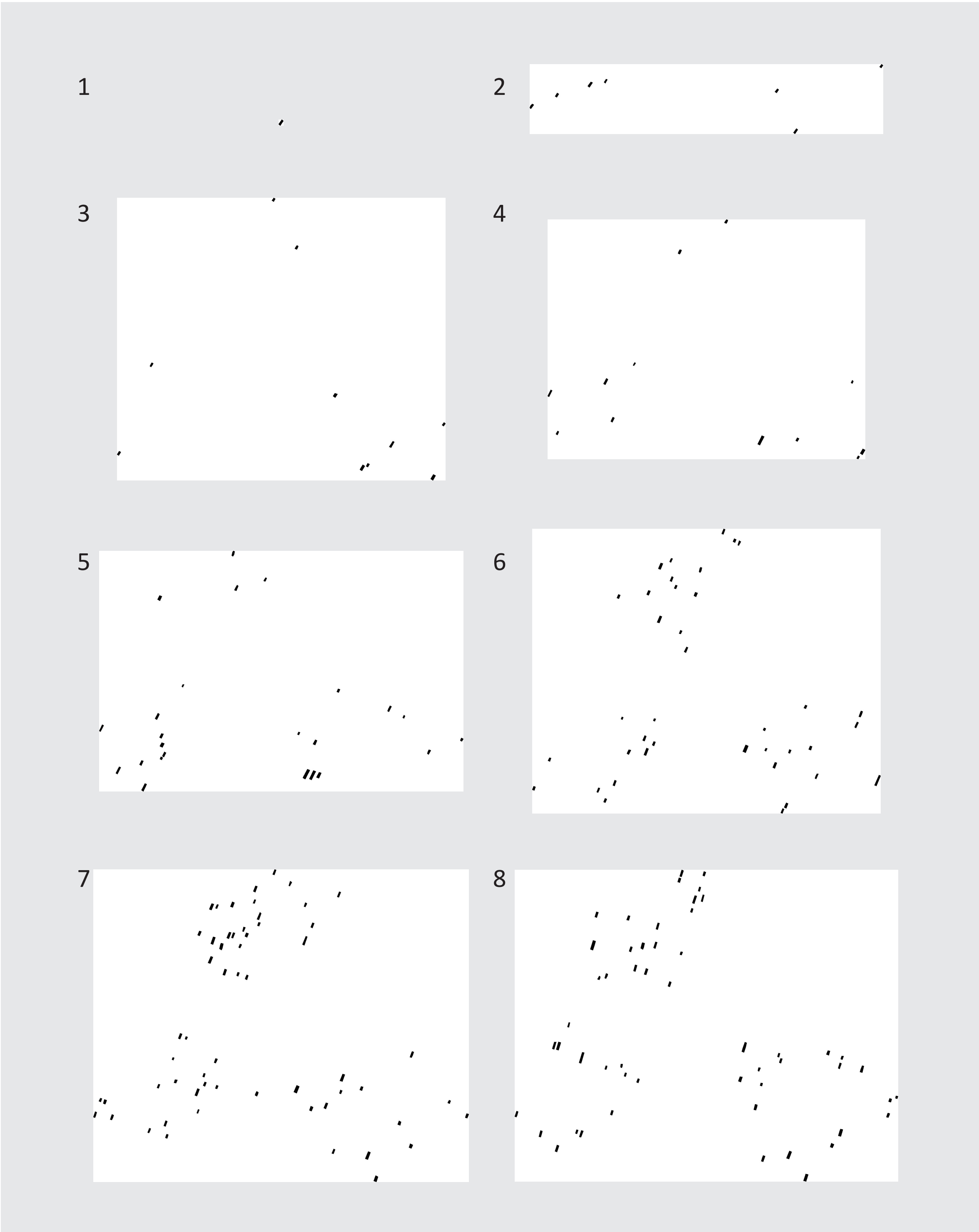
Quarters:
Nebelivka



Quarters:
Vráble



Time samples:
Vráble 1 - 8



Time samples:
Vráble 9 - 16

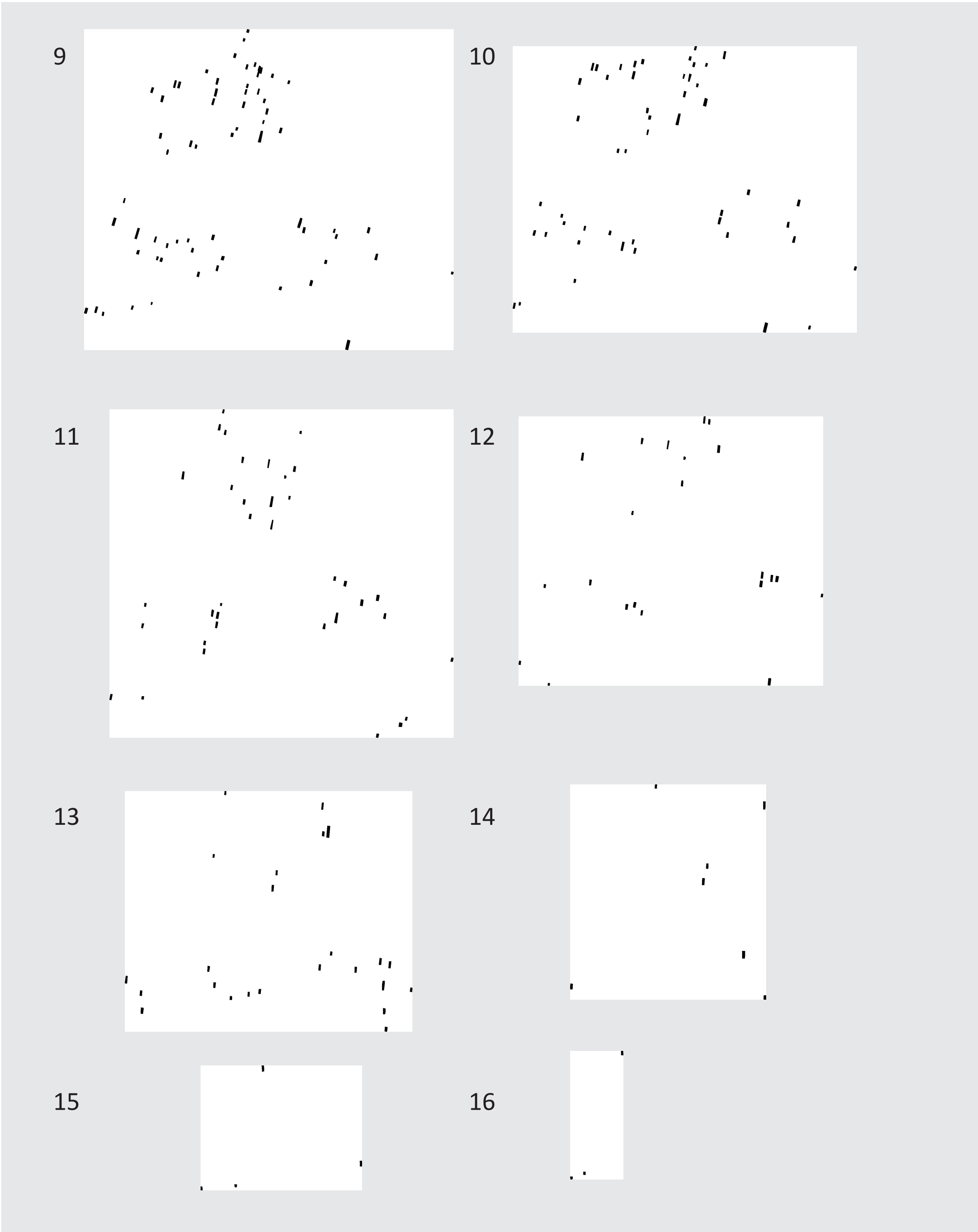


Table A.1: *Time samples, modelled sample dates, maximum and minimum house orientations of included houses. Details in Section 3.5*

dates	sample	degrees	abs_degrees	abs_degrees_max
5290	1	33.96494	33.96494	37.525805
5270	2	31.37522	31.37522	34.936085
5250	3	28.78550	28.78550	32.346365
5230	4	26.19578	26.19578	29.756645
5210	5	23.60606	23.60606	27.166925
5190	6	21.01634	21.01634	24.577205
5170	7	18.42662	18.42662	21.987485
5150	8	15.83690	15.83690	19.397765
5130	9	13.24718	13.24718	16.808045
5110	10	10.65746	10.65746	14.218325
5090	11	8.06774	8.06774	11.628605
5070	12	5.47802	5.47802	9.038885
5050	13	2.88830	2.88830	6.449165
5030	14	0.29858	0.29858	3.859445
5010	15	-2.29114	357.70886	361.269725
4990	16	-4.88086	355.11914	358.680005

Appendix B

Synthetic images

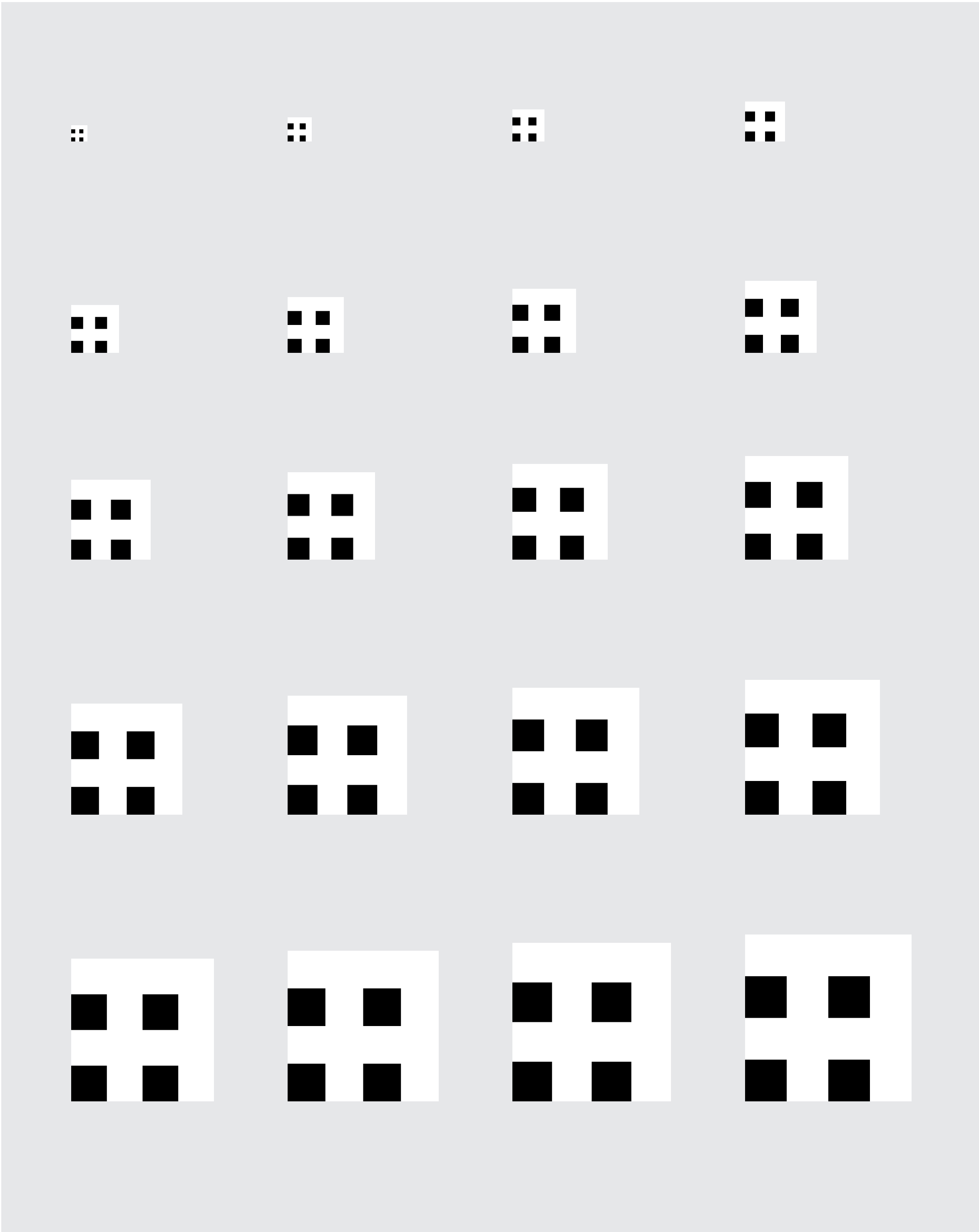
The following plates contain the synthetic image series used for analysis in Chapter 8, in the following order (plates are read row-wise from top left):

- Variable image size and resolution – pattern is zoomed in as the image gets larger
- Variable image size and observation window – as image gets larger, a larger extent of the pattern becomes visible; resolution stays the same
- Variable resolution and observation window – pattern is zoomed out as larger extent is included; image size is fixed
- Variable image density (from here, image size, resolution and observation window are constant)
- Variable size distribution of elements
- Variable level of hierarchical clustering
- Variable level of added random noise

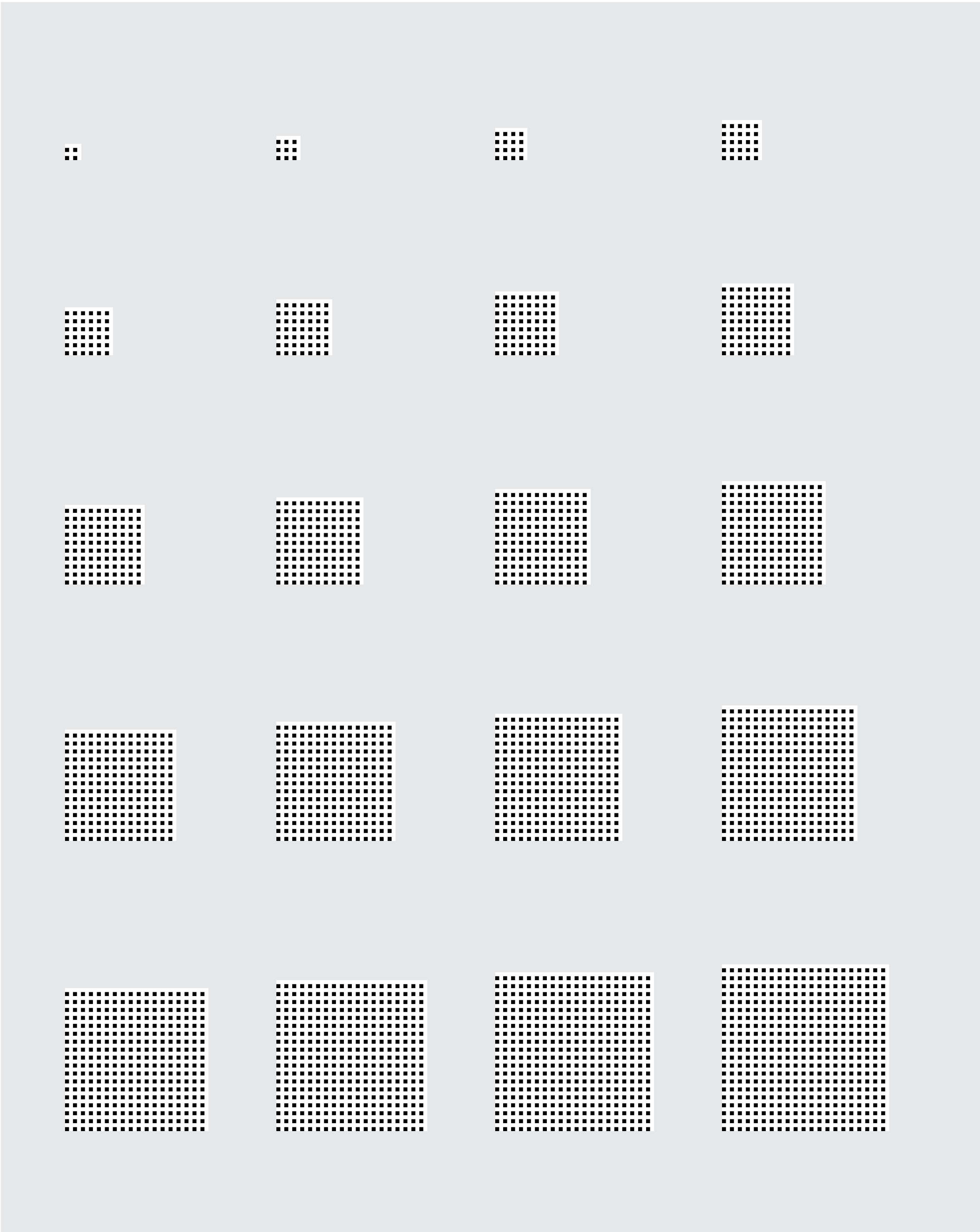
See chapter text for details.

Original image files and R code for generating them can be downloaded from project repository.

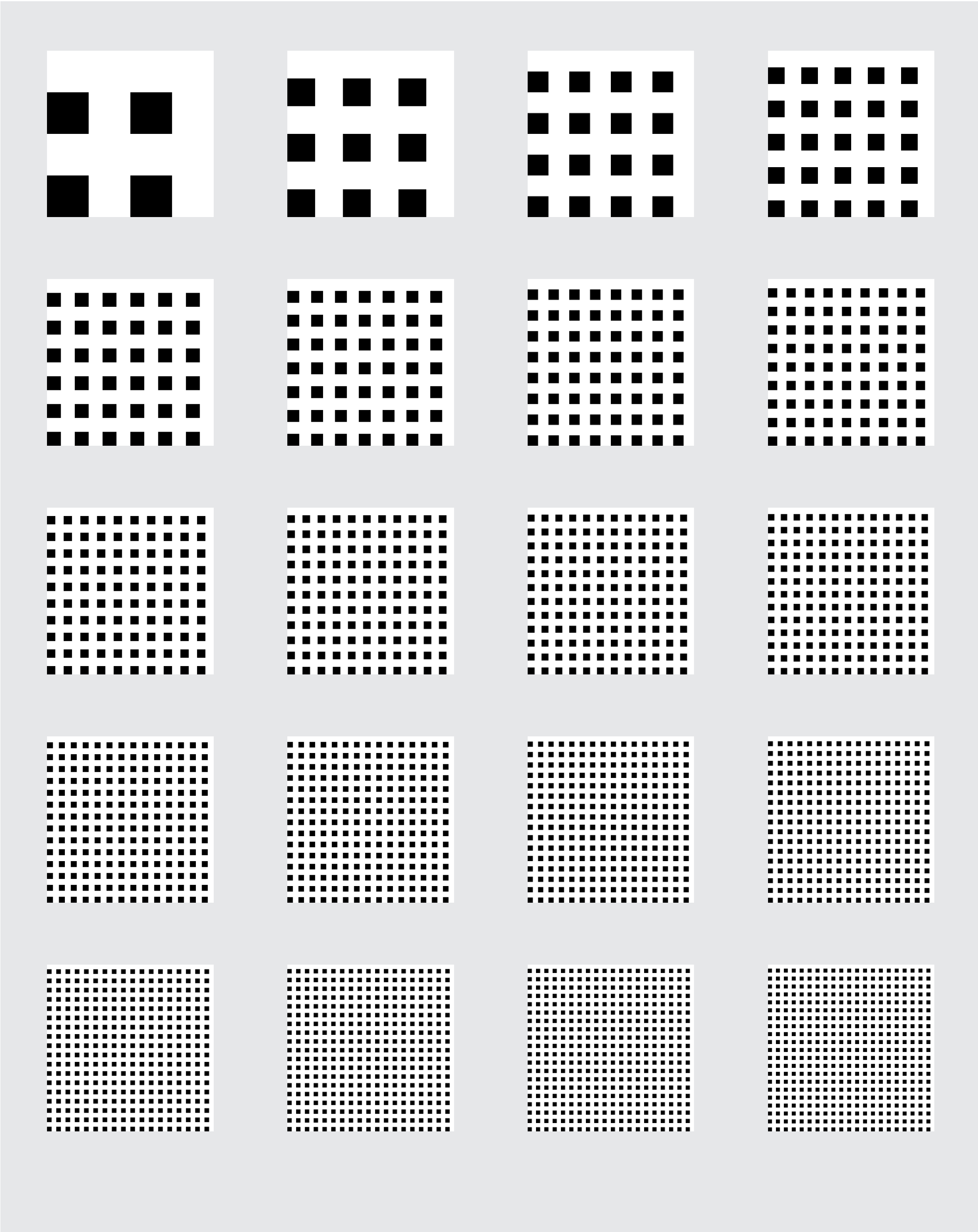
Variable Image Size
(resolution)



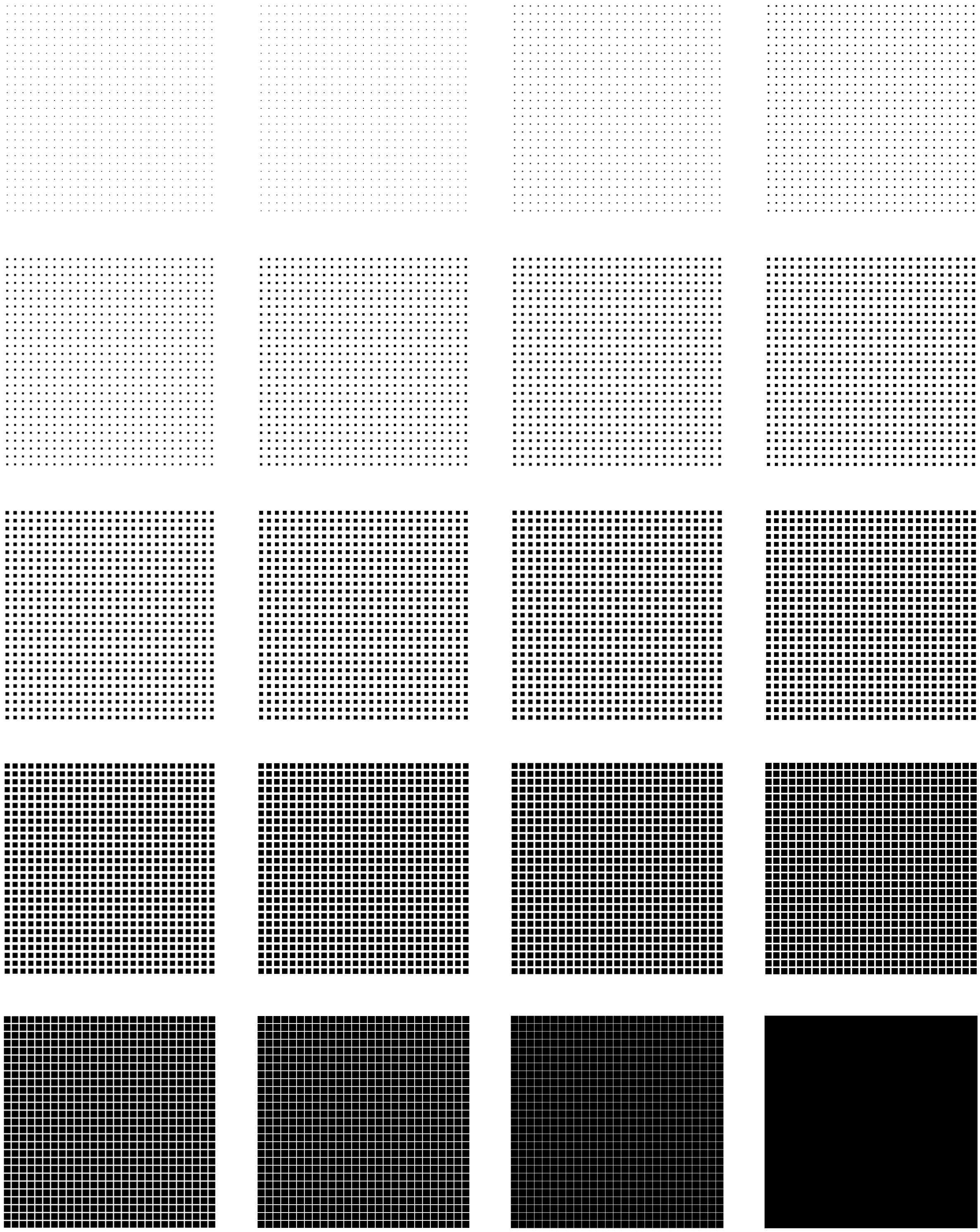
Variable Image Size
(observation window)



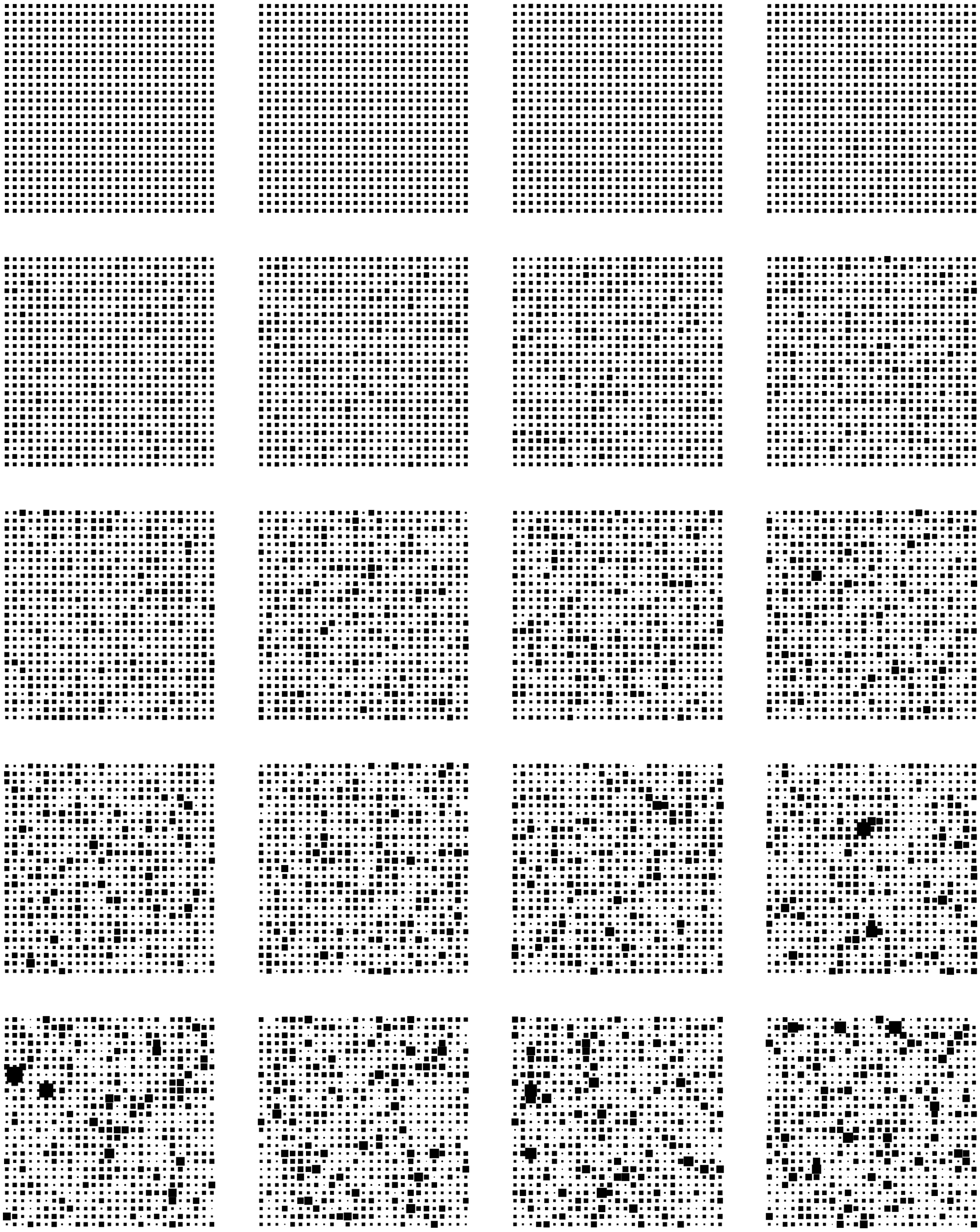
Variable Resolution
(observation window)



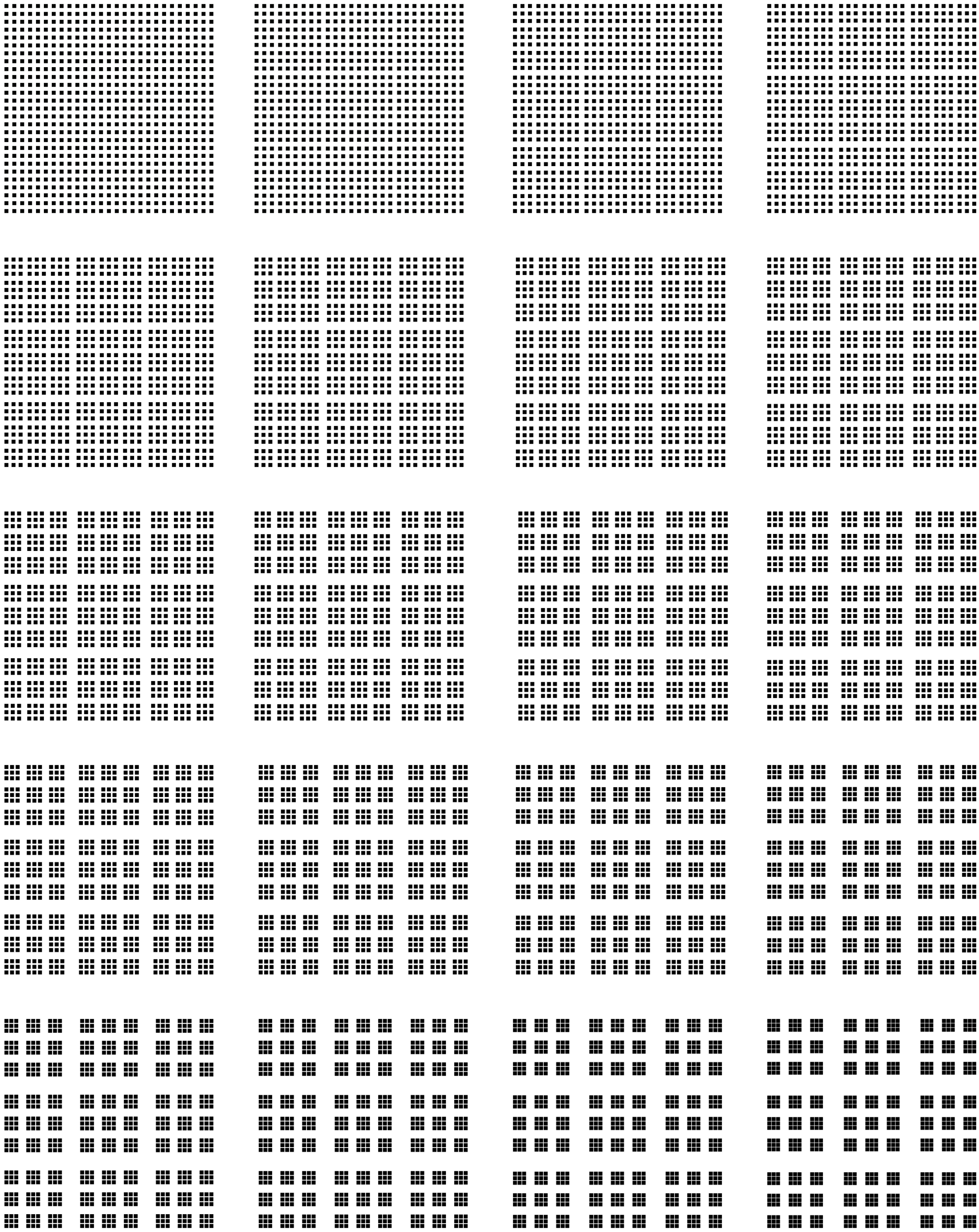
Variable Density



Variable Size Distribution



Variable Clustering



Variable Spatial Noise