Fractal Analysis as Method for studying Social Hierarchy in Prehistoric Settlement Plans

with case studies from the Linear Pottery and Trypillia cultures (5.500 - 3.500 cal. BCE)

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Contents

Preface

Thanks to everyone.

Remember to add abstracts

Furholt, Grier, et al. (2020), Furholt, Müller, et al. (2020), Furholt, Müller-Scheeßel, et al. (2020) are three different publications. Furholt, Grier, et al. (2020), Furholt, Müller, et al. (2020), Furholt, Müller-Scheeßel, et al. (2020) are the same three publications in the same order. gfdsdfgs , Get on with it!

Part I

Frameworks

Chapter 1

Introduction

1.1 Background of the study

1.2 Research question and objectives

What are my goals for this project?

Two approaches: house-size distributions and settlement layouts

1.3 Some definitions

Just the minimum required to present the thesis in general terms.

- Hierarchy Pumain (2006)
- Inequality Kohler and Smith (2018), Midlarsky (1999)
- Social complexity Daems (2021)
- Scale West (2017)

1.4 Main findings here?

1.5 Research ethics

- Open science and open-source scripts
- Terminology and spelling (British English for text. For geographical place names, Slovak special characters are kept as far as possible, even though it can be a pain in the xxx to render in Rmarkdown on Windows OS, and the 2010 Ukranian National transliteration system with only ASCII characters and no soft sign)
- Abstracts in Slovak and Ukranian (and not only in Norwegian)

1.6 Structure of the thesis

This thesis is structured as a monograph in four parts. In the first part the overall framework of the study is exposed, with the general introduction above, the overarching theoretical framework in Chapter 2, and the background of the study material in Chapter 3. Parts two and three are devoted to each their methodological approach to the material: Part II to the study of hierarchy in size distributions, and Part III to the quantification of image textures. Each of these parts consists of three chapters, the firsts of which – Chapters 4 and 7 – expose the theoretical and interpretative background of the applied methods and their relevance to Archaeology. The second chapters – Chapters 5 and 8 – detail the technical specifics of the two approaches and their implementation in this study, and the third chapters – Chapters 6 and 9 – provide the actual analyses and summaries of results. In Part IV, the findings are summarised and further discussed. Chapter 10 gives an attempt of interpreting the results in the context of the culture-historical setting of the European Neolithic, while Chapter 11 reviews the possibilities and limitations of the fractal analysis framework in Archaeology. Concluding remarks and suggestions for further study are given in Chapter 12.

Some readers might react to an apparent deviation from the academic tradition of devoting a separate chapter to research history. This is a deliberate choice, not to suggest that historiography

is unimportant, but rather as a result of the fundamentally interdisciplinary scope of the study. In fact, there is very little extant history of applying fractal analysis in Archaeology – the few studies that, to my knowledge, have been done in this direction are discussed primarily in Sections 4.5 and 7.3. Fractal analysis itself holds a research history of its own (see Section 2.2), and so does the study of Linear Pottery (Section 3.2) and Trypillia societies (Section 3.3), not to mention the general study of social complexity in Prehistory (mainly Section 3.1). In short, instead of trying to shoehorn these parallel histories into a clearly delimited but rather hybrid chapter, I have opted for what I believe to be a more useful approach, namely to fit them in more seamlessly where they belong, in the various associated theory and methods chapters.

An additional note should be made here regarding the writing style of the different parts and chapters. It is my belief that a major obstacle for fractal analysis methods to become more integrated into the standard tool kits of archaeological research, is the excessively technical nature of much of the associated literature. Archaeology as a discipline remains profoundly rooted in the Humanities, as seen in the inbuilt structure of teaching, research and funding institutions in most (at least European) countries. Fractal analysis is derived from pure mathematics, and most applications so far have been developed within the natural sciences (**ref necessary?**). Archaeologists who are trained within a humanistic scholarly tradition cannot be expected to hold a skill level of mathematics more advanced than what is achieved in high school, and code programming is hardly taught at all within the walls of Humanities faculties. Technical details regarding the methods and analyses applied in this thesis are therefore – as much as possible – limited to the devoted Chapters 5 and 8, and readers who are interested in these may also refer to the online code repository for more details and reproducibility **cite repository**. In the rest of the thesis I have opted for a more narrative approach, in an attempt to invite a somewhat larger audience of archaeologists into the fascinating complexities of fractals.

END Chapter

Here I skipped a line.
Here I added a backslash.

fdsasdfasdf You can label chapter and section titles using `{#label}` after them, e.g., we can reference Chapter 1. If you do not manually label them, there will be automatic labels anyway,

e.g., Chapter **??**.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

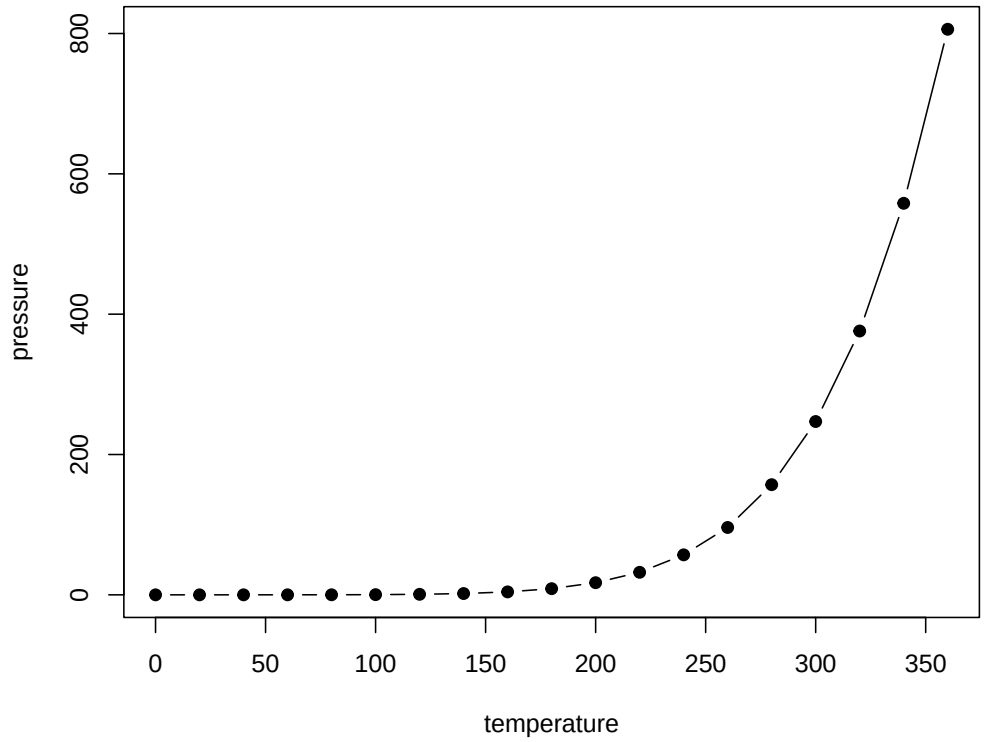

Figure 1.1: Here is a nice figure!

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 1.1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 1.1.

```
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package (**R-bookdown?**) in this sample book, which was built on top of R Markdown and **knitr** (Xie 2015).

Table 1.1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

Test: Morgan (1965).

Test: Mandelbrot (2021). jløkj fdsasdfadfasdfadf new text

Chapter 2

Theoretical framework: Complexity and Fractals

2.1 Very short introduction to Complexity Theory / Dynamical Systems Theory

Lit. use Daems (2021) and Bentley and Maschner (2007).

For Dynamical Systems, use Awrejcewicz and Grzelczyk (2020) and Devaney (2020), but don't go into detail.

Describe complexity, dynamical systems, chaos, feedback loops, criticality, emergence.

Mention the most common applications in Archaeology: ABMs, **and what**? check in literature.

2.2 Very short introduction to Fractals and Fractal Analysis

Lit. use Mandelbrot (1982), Falconer and Falconer (2013) (general), also Brown and Liebovitch (2010), Brown, Witschey, and Liebovitch (2005) and Diachenko (2018) (for Archaeology)

Two sentences on history of fractals? Mandelbrot. Maybe cut?

Fractals as irregular patterns/structures

Fractals as hierarchy

Self-similarity and scale invariance

Processes/mechanisms that produce fractals:

- Cascading bifurcations and confluences (splitting or merging - tree structure/arborescence/branching, and relation to size. Terminology borrowed from biology and fluid dynamics (including turbulence/turbulent flow)

The role of randomness - tidy and messy fractals (romanesco broccolis are not more fractal than regular broccolis, only more regular.

The relationship with criticality and chaos: deterministic *and* unpredictable

Fractals embedded in

- Space (hence "fractal geometry"): geomorphology, plants, ocean and wind currents, galaxies, also human constructed features (see Chapter xx for details)
- Time series: earthquakes, finance (not applied in this study, though should be done later)
- Networks/abstract: hierarchical organisations, income distributions, word counts, 1/f or pink noise, www. etc. (see Chapter xxx for details)
- Pure mathematics: Julia and Mandelbrot sets, strange attractors (don't go into details!)

No, not everything is fractal: e.g. Central Limit Theorem

Fractal analysis for studying irregular phenomena (methods described in more detail Chapters xxx), and thus as a tool for quantitative empirical research.

2.3 Very short introduction to micro-macro approaches in social theory

- Lévi-Strauss and Structuralism
- Giddens and Structuration Theory
- Delanda and Assemblage Theory
- Latour and Actor-Network Theory

What these approaches all have in common, is that they are entirely qualitative (**check**).

- That's not a problem in itself.
- Quant approach is both possible for the stated purpose, and desirable for reasons of comparative analysis.
- Data deluge (refer to chap. on geomagn data). The goal here is to establish a quantitative framework for studying social complexity and hierarchy in archaeological/prehistoric settings. Further articulating fractal analysis with existing social theoretical approaches is not the primary goal here, as it could constitute a separate research project. In the present thesis, bla bla.

END chapter.

Math can be added in body using usual syntax like this

p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt{(\frac{p(1-p)}{n})} \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

You can also use math in footnotes like this[1].

We will approximate standard error to 0.027[2]

[1] where we mention $p = \frac{a}{b}$

[2] p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt{(\frac{p(1-p)}{n})} \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

Chapter 3

Material and data: social complexity in the European Neolithic

3.1 Studying social complexity in Archaeology and Prehistory.

- Grave goods
- Burial monuments
- The denominator problem
- The use of ethnography
- Other approaches (osteological, isotopes **refs**)
- This project: house-size distributions and settlement layouts (details in subsequent chapters), just very short argumentation

3.2 The Linear Pottery culture complex

- General intro to the culture
- The Žitava valley and research project

- Organisation of Linear Pottery society: egalitarian or hierarchic?

3.3 The Cucutení-Trypillia culture complex

- General intro to the culture
- The B2/C1 and the mega-sites of the Southern Bugh – Dnipro interfluve

3.4 Reading site plans from geomagnetic imagery

- Caveats: fill in here.

3.5 Synthetic data

- And why I'm not (this time) relying on ethnographic data.
- Don't go into technicalities here, just the reasoning.

Part II

Size distributions

Chapter 4

House sizes and social meaning

4.1 What is social hierarchy?

- Political assumptions – all hierarchical social structures are not despotic top-down rule. Democracies can also be very hierarchical. Matter of scale rather than political system Graeber and Wengrow (2021). But, tendencies? Use Pumain (2006), also furholt2020b.
- Nested and non-nested social hierarchies, hierarchical hunter-gatherers? Hamilton et al. (2007), Whitridge (2016)
- Biologically defined thresholds to group size? Dunbar's number and controversies, Dunbar (2022), West et al. (2023) (add published papers). Scalar stress Johnson (1982), Alberti (2014), Zhou et al. (2005)

4.2 Social typologies and their critique

- Tools for classifying societies, or evolutionary model? Discussion of Johnson and Earle (1987), Testart (2005), Service (1971)
- Anti-evolution critique in Yoffee (1993), Yoffee (2005), Graeber and Wengrow (2021). Anarchistic critique: Crumley (1995), Haude and Wagner (2019),

4.3 Possible reasons for house-size difference

Often underlying assumptions: household wealth or size? Or both?

- Kinship and households: who lives in a house? Sahlins (2013), Ensor (2013), Ensor, Irish, and Keegan (2017)
- Do clan leaders have bigger houses? check Haude and Wagner (2019)

Discuss some archaeo references

Functional difference:

- Ethnography of initiation houses, communal/assembly houses, ritual houses, including Barley (2011), Godelier (1986), Wilk (1983), Fraser (1968), Haude and Wagner (2019)
- Caveats: building materials and constraints

4.4 Interpreting distribution types and their underlying mechanisms

- Power-law distributions, hierarchy and scale invariance.
- Normal distributions and the Central Limit Theorem.
- Exponential distributions and growth rates.
- Combinations: Log-normal, stretched exponential, parabolic fractal
- Notes on terminology: Power law, Pareto and Zipf, Newman (2005)
- A law (distribution) is not a law (of nature), see Grove (2011) for review of the long-lasting confusion in Archaeology (e.g. Hodder (1979)), also “rank-size rule”

4.5 Fitting heavy-tailed distributions in Archaeology

- Lit. use Strawinska-Zanko et al. (2018), Crabtree et al. (2017), Maschner and Bentley (2003), Grove (2011) ++
- Zipf law and Settlement Scaling theory, Bettencourt (2021), Gomez-Lievano, Youn, and Bettencourt (2012), Lobo et al. (2020). Connection with Central Place Theory, e.g. Müller-Scheeßel (2007), Chen (2011). Why I'm not doing settlement scaling in this study.

END chapter

Chapter 5

Methods: Distribution fitting

5.1 Heavy-tailed distributions, testing for power laws

- Technical characteristics of power laws, Newman (2005), do I need more?
- Old style distribution fitting (which is used in Brown and Liebovitch (2010) and Brown et al. (2005) and check. Mitzenmacher (2004), Harrison (1981)
- New style presented in Clauset, Shalizi, and Newman (2009), Stumpf and Porter (2012), implemented in R with the poweRlaw package Gillespie (2015), and used in Strawinska-Zanko et al. (2018) and Crabtree et al. (2017). More recent?

5.2 Methodological procedure

- Reminder of main goal for this part of the study: identify power-law structures in the house-size distributions of the Linear Pottery and Trypillia samples.
- Synthetic data genaration:
  - Why?
    1. Process: how long/much does in take for a normal distribution to become power-law? And reverse sense?

2. Temporal resolution issue: does the temporal palimpsest of several phases with e.g. log-normal distributions produce false power-law signals?

– How?

* Random number generation and iterated multiplicative (1.) or additive (2.) sequences, with K-S testing (Gillespie 2015) at each stage. Report when the distributions become power laws.

- Present data set with categories (settlements, quarters/neighbourhoods, time samples for Vráble)
- Parameter settings: dist. types, xmin, testing the pl hypothesis etc. Minimal house-count cutoff (min. sample size). Isolating top house.

5.3

END chapter

Chapter 6

Results: Distribution fitting

6.1 Synthetic distributions

Fill in analysis for

1. Multiplicative process
2. Additive process

6.2 Settlements

House-size distributions of whole settlements (both Linear Pottery and Trypillia)

6.3 Quarters/neighbourhoods

House-size distributions of separate quarters (Nebelivka) and neighbourhoods (Vráble). Can I include separate Nebelivka neighbourhoods as well? (prob. too small, but check)

6.4 Temporal samples (Vráble)

If time, I can add more Žitava sitesre, but not necessary. I can also analyse temporal samples of Vráble neighbourhoods separately.

6.5 Summary of findings

END Chapter

Part III

Settlement Plans

Chapter 7

Village planning in Prehistory

7.1 Settlement layout and social structure

Or the social organisation of village layout. Research background:

Lit. use Furholt (2016), Fraser (1968), Ensor et al. (2017) (not the correct reference, I should ask him!)

Use the Trypillia volumes. Also Müller-Scheeßel (2019), Trebsche, Müller-Scheeβel, and Reinhold (2010)

Factors affecting village layout:

- Political structure (but, as with hierarchy, an organised layout does not necessarily equate top-down despotic decision making).
- Kinship, matrimonial and locality structures
- Cosmology (e.g. Linear Pottery house orientations)
- Economic and ritual functions of village elements (constructed and non-constructed)
- Local landscape setting (to be factored out)

7.2 The geometries of conscious planning vs. emergent behaviour

- I need to find some references here!
- Euclid: grids, lines, circles – how humans think in shapes. Social settings: architect/planner, strong common institutions/ideals (examples?)
- Mandelbrot: irregular, self-similar, scale independent (i.e. fractal) shapes – emergent, not consciously preconceived. Self-organisation. Does the "no pattern" case exist? Emergence from repetitive sequences of simple choices/mechanisms. Examples.
- Binary or continuum? Needs to be studied empirically.

7.3 Fractal image analysis in archaeology

END Chapter

Chapter 8

Methods: Fractal image analysis

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8.1 Fractal dimension and lacunarity

- Box-counting, lit. Mancuso (2021), Li, Du, and Sun (2009), Klinkenberg (1994)
- Gliding-box algorithmsc, Allain and Cloitre (1991), Hingee et al. (2019), Cheng (1997), Plotnick et al. (1996)
- Caveats:
  - Fractional box-counting dimension does not equal self-similarity in a simple way
  - My summary L is not equal to the one used in *FracLac* and thus by Farías-Pelayo (2017)

8.2 Image preparation

- Procedure for archaeological samples, same as in article
- Procedure for synthetic sample

END Chapter

Chapter 9

Results: Image analysis

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END Chapter

Part IV

Synthesis

Chapter 10

Discussion: Social complexity in Linear Pottery and Trypillia settlements

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END Chapter

Chapter 11

Discussion: Fractal Analysis and Archaeological data

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END Chapter

Chapter 12

Conclusion and Outlook

12.1 Things I would like to have done, but that didn't fit into this study

- Ethnoarchaeology: Measure house sizes and settlement layouts in contemporary settings, and relate to social organisation (largely overlooked by ethnographers)
- Test distributions and settlement layout analysis on other settings: Lake dwellings, later/historic periods, other materials (e.g. megaliths)… Add more complex distribution models, add observation windows on images. Try on remote sensing imagery.
- Settlement Scaling on Neolithic settings
- Time series: Hurst exponent and scale invariance in temporal development of e.g. regional settlement or population
- Integrate – bridge the gap – between opposite theoretical (nat. and soc./hum.) approaches to the same phenomena
- Chaos and strange attractors in Archaeology
- More?

12.2 Concluding remarks

END Thesis

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Appendix A

This is my first appendix

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Appendix B

This is my second one

Bla bla bla