

# DAT200

Sammendrag av klassifikasjonsmodeller.

## Perceptron

$\eta$ : learning rate. How big the weight updates should be.

Classes:

$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$

Threshold value:

$$z = w_0x_0 + w_1x_1 + \dots + w_mx_m \quad w_0 = -\theta, x_0 = 1$$

Threshold function:

$$\phi(z) = \begin{cases} 1 & z \geq 0 \\ -1 & \text{otherwise.} \end{cases}$$

$\phi(z)$  was originally 1 for  $z \geq \theta$  (therefore  $w_0 = -\theta$ )

Weight update:

$$w_j = w_j + \Delta w_j$$

Weight change:

$$\Delta w_j = \eta(y^i - \hat{y}^i)x_j^i$$

## Adaline

$\eta$ : learning rate. How big the weight updates should be.

Classes:

$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$

Threshold value:

$$z = w_0x_0 + w_1x_1 + \dots + w_mx_m \quad w_0 = -\theta, x_0 = 1$$

Activation function:

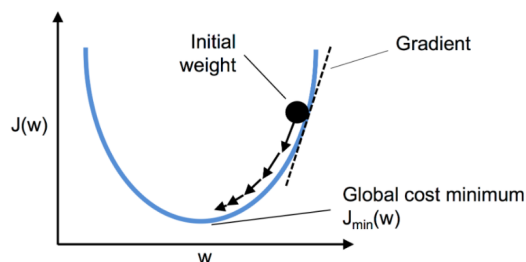
$$\phi(z) = \phi(w^T x) = w^T x \text{ (linear activation function)}$$

Threshold function:

$$\varphi(z) = \begin{cases} 1 & \phi(z) \geq 0 \\ -1 & \text{otherwise.} \end{cases}$$

Cost function:

$$J(w) = \frac{1}{2} \sum_i (y^i - \phi(z^i))^2$$



Weight change:

$$\Delta w = -\eta \nabla J(w)$$

$$\Delta w_j = \eta \sum_i (y^i - \phi(z^i)) x_j^i$$

(where  $\nabla J = -\sum_i (y^i - \phi(z^i)) x_j^i$ )

## Perceptron vs. Adaline

Similarities:

- Binary classification.
- Linear decision boundary.
- Threshold function ( $\phi(z)$ ).

Differences:

- Perceptron uses a step function ( $\phi(z)$ ), Adaline uses a linear activation function ( $\phi(z)$ ).
- Perceptron compares true class labels to predicted labels, Adaline compares true class labels to continuous output from  $\phi(z)$ .
- Perceptron updates weights immediately after misclassification, Adaline updates all weights at the end of each iteration.

## Logistic regression

$\eta$ : learning rate. How big the weight updates should be.

Classes:

0 1

Threshold value:

$$z = w_0 x_0 + w_1 x_1 + \dots + w_m x_m \quad w_0 = -\theta, x_0 = 1$$

Activation function:

$$\phi(z) = \frac{1}{1+e^{-z}} \text{ (sigmoid activation function)}$$

Threshold function:

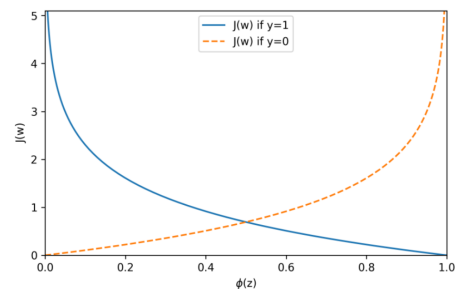
$$\varphi(z) = \begin{cases} 1 & \phi(z) \geq 0.5 \\ 0 & \text{otherwise.} \end{cases}$$

Cost function:

$$J(w) = \sum_{i=1}^n [-y^i \log(\phi(z^i)) - (1 - y^i) \log(1 - \phi(z^i))]$$

For  $y^i = 0$ :

$$J(w) = -\log(1 - y_{pred}) = -\log(1 - \phi(z^i))$$



For  $y^i = 1$ :

$$J(w) = -\log(y_{pred}) = -\log(\phi(z^i))$$

Weight change:

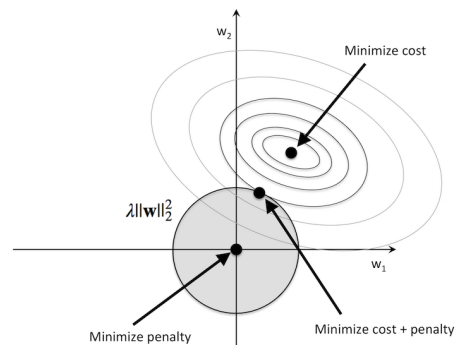
$$\Delta w = -\eta \nabla J(w)$$

## Regularisation

C: regularisation strength. How greatly to punish large weights.

Cost function:

$$J(w) + \frac{\lambda}{2} \|w\|^2 = J(w) + \frac{1}{C} \|w\|^2 = J(w) + \frac{1}{C} \sum w_j^2$$



## Support vector machines (SVM)

C: error penalisation. How greatly to punish misclassifications.

Margin:

$$\frac{w^T(x_{pos} - x_{neg})}{\|w\|} = \frac{2}{\|w\|}$$

Goal: minimise  $\frac{2}{\|w\|}$

## Radial Basis Function Kernel SVM (RBF Kernel-SVM)

$\gamma$ : penalise misclassifications. ( $\|x^i - x^j\|^2$  is the (Euclidean) distance between two points.)

Kernel function:

$$\kappa(x^i, x^j) = \exp\left[-\frac{\|x^i - x^j\|^2}{2\sigma^2}\right] = \exp[-\gamma \|x^i - x^j\|^2]$$

## Generally

Hyperparameter: large values = overfitting. (Goal: Penalises error)

Cost function: find (global) minimum. (Goal: Minimise cost)