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Homework 1

ML-NEU
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Exercise:

1. **Exercise 1:**

   To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

2. **Exercise 2:**

   Proof the following distributions are normalized then calculate the mean and standard deviation of these distribution:

   1.Univariate normal distribution.

   2.(Optional) Multivariate normal distribution.

Answer:

1. **Exercise 1:**

   H = Had Test

   T = Testing Positive

   NH = Had not test

   $$
   \begin{aligned}
   P(\mathrm{H}|\mathrm{T}) &= \frac{P(\mathrm{T}|\mathrm{H}).P(\mathrm{H})}{P(\mathrm{T})} \\
   &= \frac{P(\mathrm{T}|\mathrm{H}).P(\mathrm{H})}{P(\mathrm{T}|\mathrm{H}).P(\mathrm{H})+P(\mathrm{T}|\mathrm{NH}).P(\mathrm{NH})} \\
   &= \frac{5\%.98\%}{(5\%.98\%)+(95\%.3\%)} \\
   &= 0.632
   \end{aligned}
   $$

2. **Exercise 2:**

   2.1. Univariate normal distribution:

   2.1.1 Normalize proving:

   The Area under normal distribution curve should be equal to 1

   So that we have :

   $$\varphi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{1}{2\sigma^2}x^2)$$

$\Rightarrow \int_{-\infty}^{\infty} \varphi(x)dx = 1$

$\Rightarrow \int_{-\infty}^{\infty} exp(-\frac{1}{2\sigma^2}x^2)dx = \sqrt{2\pi\sigma^2}$

Let $I = \int_{-\infty}^{\infty} exp(-\frac{1}{2\sigma^2}x^2)dx$

$\Leftrightarrow I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} exp(-\frac{1}{2\sigma^2}x^2 - \frac{1}{2\sigma^2}y^2)dxdy$

$x = r\cos\theta$ and $y = r\sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

$x^2 + y^2 = r^2 \Rightarrow \frac{\delta(x,y)}{\delta(r,\theta)} = \begin{vmatrix} \frac{\delta(x)}{\delta(r)} & \frac{\delta(x)}{\delta(\theta)} \\ \frac{\delta(y)}{\delta(r)} & \frac{\delta(y)}{\delta(\theta)} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & -r\cos\theta \end{vmatrix}$

$\Leftrightarrow r\sin^2\theta + r\cos^2\theta = r$

$\Rightarrow I^2 = \int_0^{2\pi}\int_0^{\infty} \exp\left(-\frac{r^2}{2\sigma^2}\right) rdrd\theta$

$= 2\pi\int_0^{\infty} \exp\left(-\frac{r^2}{2\sigma^2}\right) rdr$

$= 2\pi\int_0^{\infty} \exp\left(-\frac{u}{2\sigma^2}\right) \frac{1}{2}du$

$= \pi\left[\exp\left(-\frac{u}{2\sigma^2}\right)(-2\sigma^2)\right]_0^{\infty} = 2\pi\sigma^2$

$\Rightarrow I^2 = (2\pi\sigma^2)^{\frac{1}{2}}$

2.1.2 Mean:

We have :$Z = \frac{(X-\mu)}{\sigma}$

$\Rightarrow E[Z] = \int_{-\infty}^{+\infty} xf_Z(x)dx = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} xe^{\frac{-x^2}{2}}dx = \frac{-1}{\sqrt{2\pi}}e^{-x^2}2\Big|_{-\infty}^{+\infty} = 0$

$X = \mu + \sigma Z \Rightarrow E(X) = E(\mu) + E(\sigma Z) \Longleftrightarrow E(X) = \mu + E(Z)E(\sigma)$

$E(Z) = 0 \rightarrow E(X) = \mu$