New Trends in Condensed Matter Physics (Dec 10, 2024, ISSP), Poster #10

dedicated to the memory of Ian Affleck (1952-2024)

The Ground State of the $S=1$ Antiferromagnetic Heisenberg Chain is Topologically Nontrivial if gapped

Hal Tasaki

A 10-minute presentation video for the poster – check it out if I'm not around!

preprint arXiv:2407.17041 (with link to the webinar on YouTube)

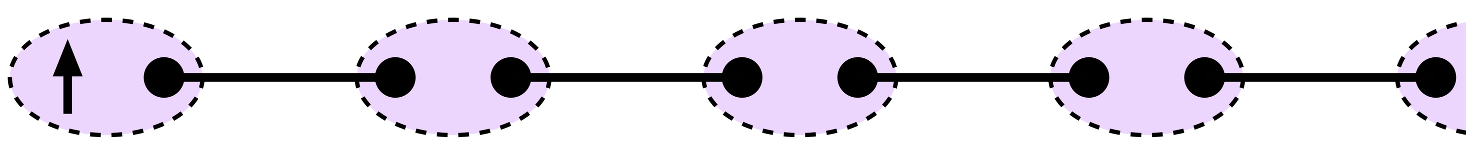
two conjectures on the S=1 Heisenberg AF chain

$$\hat{H}_{\mathrm{HAF}} = \sum_j \hat{\boldsymbol{S}}_j \cdot \hat{\boldsymbol{S}}_{j+1}$$

C1: the ground state is unique and gapped — Haldane 1981

C2: it belongs to a nontrivial symmetry protected topological (SPT) phase — Gu, Wen 2009, Pollmann, Turner, Berg, Oshikawa 2010

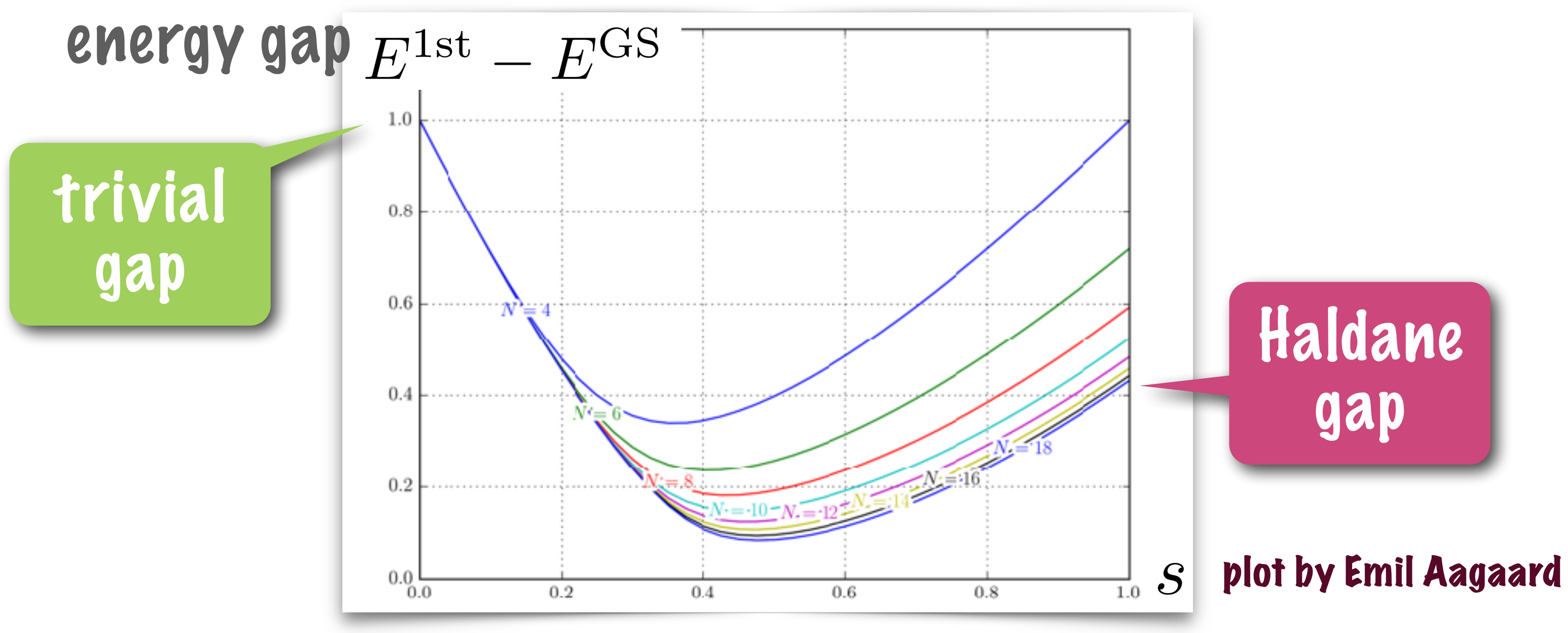
two properties that follow



(1) the presence of gapless $S=\frac{1}{2}$ mode at the edge of the half-infinite chain

(2) the existence of a topological phase transition in the model that interpolates between the Heisenberg model and the trivial model

$$\hat{H}_s = (1-s)\sum_j (\hat{S}^z_j)^2 + s\,\hat{H}_{\mathrm{HAF}} \qquad s \in [0,1]$$



everything is rigorous by now for the artificial model $\hat{H}_{\mathrm{AKLT}} = \sum_j \{\hat{\boldsymbol{S}}_j \cdot \hat{\boldsymbol{S}}_{j+1} + \frac{1}{3}(\hat{\boldsymbol{S}}_j \cdot \hat{\boldsymbol{S}}_{j+1})^2\}$

C1: Affleck, Kennedy, Lieb, Tasaki 1987, C2: Pollmann, Turner, Berg, Oshikawa 2010, Tasaki 2018, Ogata 2018

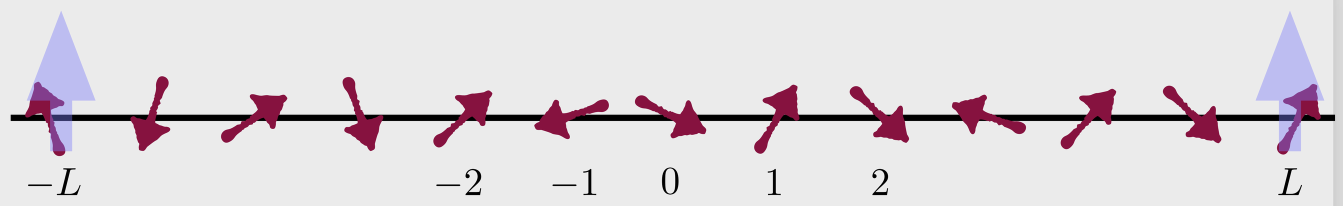
but we should not give up justifying the conjectures for the original Heisenberg model!

we prove C2 for the $S=1$ Heisenberg AF chain, assuming C1

model on a finite chain with $\hat{H}_L = \sum_{j=-L}^{L-1} \hat{\boldsymbol{S}}_j \cdot \hat{\boldsymbol{S}}_{j+1} - h(\hat{S}^z_{-L} + \hat{S}^z_L)$

(easily proved to have a unique ground state if $h > 0$)

E_L^{GS} the ground state energy, $E_L^{1\mathrm{st}}$ the first excited energy



assumption: there are constants $h>0$, $\gamma>0$, and L_0 s.t. $E_L^{\mathrm{GS}} - E_L^{1\mathrm{st}} \ge \gamma$ for any $L \ge L_0$

theorem: the Heisenberg AF chain has a nontrivial topological index $\mathrm{Ind}(\hat{H}_{\mathrm{HAF}}) = -1$

corollary: the above properties (1) and (2)

remark: in case the assumption is NOT valid, the Heisenberg AF chain $\hat{H}_{\mathrm{HAF}} = \hat{H}^{(1)}$ itself is critical. I have thus shown the property (2) without any unproven assumptions

the main ingredient of the proof

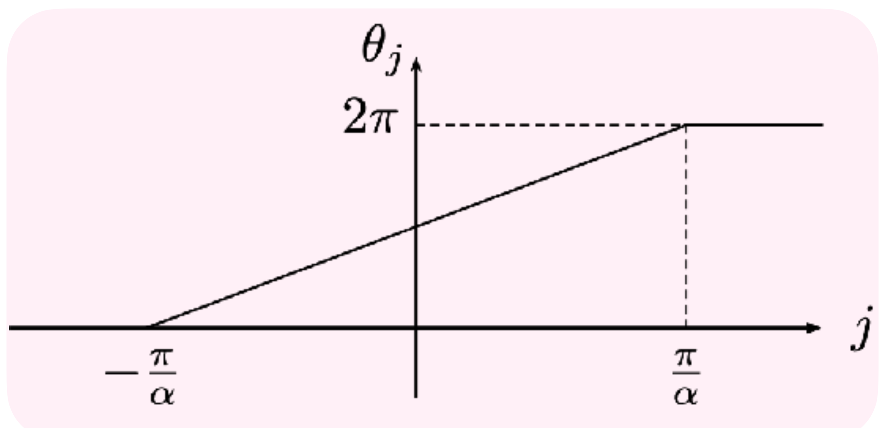
Tasaki's topological index for chains with $\mathrm{U}(1)\times\mathbb{Z}_2$ symmetry — Nakamura, Todo 2002, Tasaki 2018

arbitrary spin-rotation about the z-axis + inversion about the origin — Fuji, Pollmann, Oshikawa 2015

$$\mathrm{Ind} = \lim_{\alpha\downarrow 0}\lim_{L\uparrow\infty} \langle \Phi_L^{\mathrm{GS}} | \hat{U}^{(\alpha)} | \Phi_L^{\mathrm{GS}} \rangle \in \{-1, 1\}$$

twist operator $\hat{U}^{(\alpha)} = \exp[-i\sum_j \theta_j^{(\alpha)} \hat{S}^z_j]$

$$\theta_j^{(\alpha)} = \begin{cases} 0, & j \le -\frac{\pi}{\alpha}; \\ \pi + \alpha j, & -\frac{\pi}{\alpha} \le j \le \frac{\pi}{\alpha}; \\ 2\pi, & j \ge \frac{\pi}{\alpha}, \end{cases}$$



future problems

show that Tasaki's index is identical to the Ogata index (which is more robust)

prove C1, the Haldane conjecture! a super-clever comupter-aided proof??