FIZ447E - Homework 2

Posting date: March 2, 2015
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Problem - Computer simulation of repeated coin tossing

Imagine you are tossing a coin with an equal probability $P_{\text{heads}} = P_{\text{tails}} = 1/2$ of getting either heads or tails in each trial. You toss the coin for the total of N-times and you want to determine the probability of getting heads exactly (0, 1, 2, ..., N) times out of those N trials. In order to obtain an estimate of such probability, you have to toss the coin N times, count the number of trials $N_{\text{heads,1}}$ in which you got heads in this particular series of N tosses, and then repeat the whole procedure M times, recording the counts of heads ($N_{\text{heads,2}}$, ..., $N_{\text{heads,M}}$) in these M repeated series of N tosses. Final estimate of the probability distribution can be then found from the histogram of all recorded $N_{\text{heads,i}}$ where $i = (1, \ldots, M)$. In this scheme, an individual series of N coin tosses corresponds to a single experimental data point and $M \gg N$ to obtain probability estimates with low noise. Instead of performing a direct experiment which can be quite time consuming for large N and M, you can find the answer using a computer simulation of the problem.

(a) Using Matlab or another computer programming language of your choice, write a program which simulates the above described repeated coin-tossing experiment. To simulate coin tosses, use a suitable random number generator providing two possible outcomes with identical probability. Input parameters of your program will be the number N of coin tosses in a single coin-tossing series and the number M of repetitions of the coin-tossing series. As the output, the program will generate a histogram describing the distribution of counts of heads N_{heads} for a given M, N. Normalize the histogram so that the sum of values of all its bins is equal to 1. How does the probability distribution estimate change for increasing N?

(b) Compare the simulated estimate of coin-tossing probability distribution obtained in part (a) to the expected values given by the binomial distribution which predicts the probability $P(N; N_{\text{heads}})$ of getting heads N_{heads}-times in N trials as:

$$P(N; N_{\text{heads}}) = \frac{1}{2^N} \frac{N!}{N_{\text{heads}}!\,(N - N_{\text{heads}})!}$$

How does this comparison between the simulations and theory change for increasing M?

The homework will be handed in by actively presenting the program to the instructor and explaining the obtained results. In case of any questions related to the assignment, please, contact the instructor.