İSTANBUL TEKNIK ÜNIVERSITESI, MATEMATIK BÖLÜMÜ

KISMI TÜREVLI DIFERANSIYEL DENKLEMLER IÇIN SAYISAL ANALIZ I

Ödev III

Haydar Altuğ Yıldırım

509161108

October 18, 2016

1)Analitik çözümü bul ve çizdir

2)Euler şeması=20

$$u_t = u_x x \tag{1.1}$$

$$u(0, t) = u(1, t) = 0 \tag{1.2}$$

$$u^0(x) = 2x \, if \, 0 \leq x \leq 1/2 \tag{1.3}$$

$$u^0(x) = 2 - 2x \, if \, 1/2 \leq x \leq 1 \tag{1.4}$$

$$\tag{1.5}$$

Nümerik ve analitik çözüm için yazılmış kod;

```
clear all;
clc;
%numerical solution
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%defining the Dx
dx=1/20;
t=40;
%defining Dt
dt=0.0012;
%dt=0.0013;

%defining mu
mu=dt/(dx)^2;
```

```matlab
%creating the mesh grid for 1D and defining the boundary conditions for x
x_num=0:dx:1;

%creating array for
u= zeros(1,length(x_num));

%initial conditions for u(x)
for k=1:length(x_num)
  if (x_num(k)<0.5)
    u(k)=2*x_num(k);
  else (x_num(k)>=0.5);
    u(k)=2-2*x_num(k);
  end
end

%boundary conditions for u(x)
u(1)= 0;
u(length(x_num))=0;

%calculating Euler scheme
for n=1:t
  for j=2:(length(x_num)-1)
    u(j)=u(j)+mu*(u(j+1)-2*u(j)+u(j-1));
   end
  u(1)= 0;
  u(length(x_num))=0;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%analytical solution
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x_space=0:dx:1; %creating space for analytical solution
u_analytical=zeros(1,length(x_space)); %solution u(x,t)
g=0; %single point for u(x,t)
syms x;
for m=1:length(x_space) %calculating every single u(x,t) value for plotting
    for k=1:6 %this loop represents the sum in the analytical solution
        %fourier coefficient symbolic solution
        F=int(4*x*sin(pi*k*x),0,0.5)+int(2*(2-2*x)*sin(pi*k*x),0.5,1);
        %calculating u(x,t) for single point in mesh
        g=g+double(F*sin(k*pi*x_space(m))*exp(-((k*pi)^2)*t));
    end
    %creating the array for plotting
    u_analytical(m)=g;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%The analytical part simply do not work but I could not understand the
%reason thus there is only numerical part that is plotted.
plot (x_num,u);
```

Nümerik çözümün farklı Δt ve zaman aralıkları için grafikleri;

at t=1, Δt=0,0012

at t=15, Δt=0,0012

at t=40, Δt=0,0012

t=1, Δt=0,0013

t=15, Δt=0,0013

t=40, Δt=0,0013