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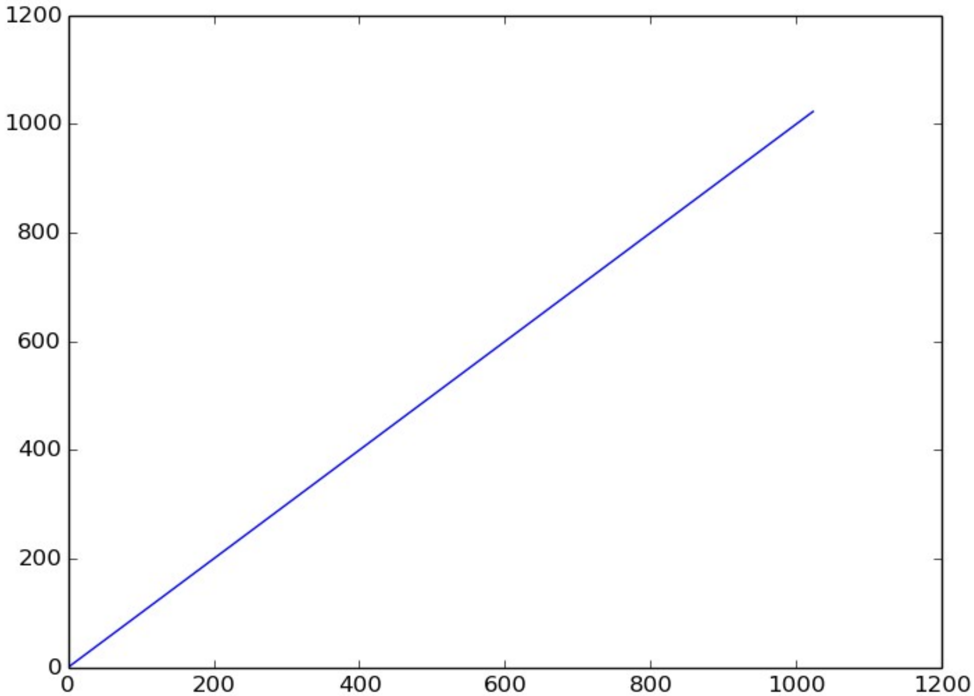
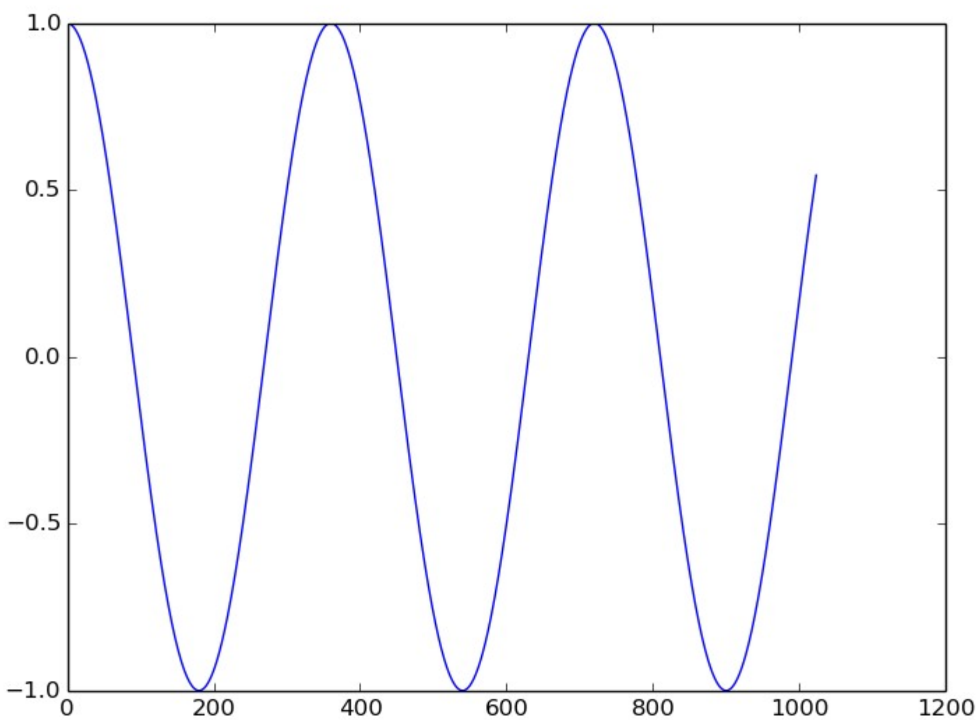
Midterm Signal Process
Butterworth Filter

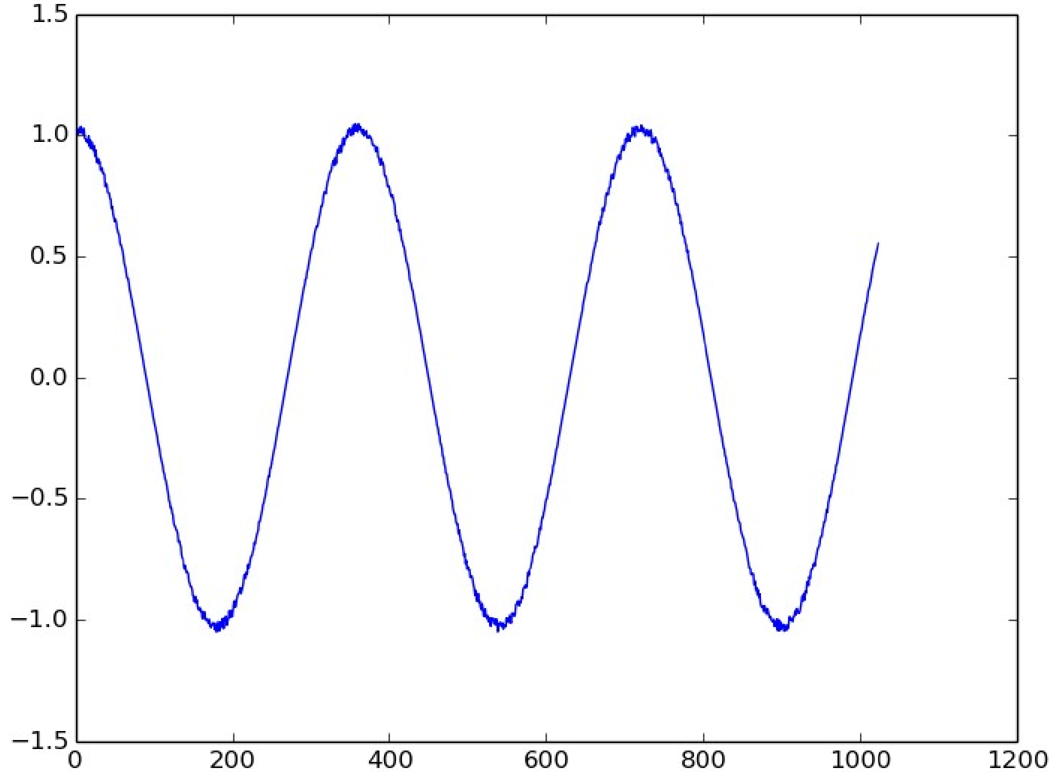
First the sample rate is stated as 1024(it needs to be even for the reasons of using discrete fourier transform.) Then time domain created:



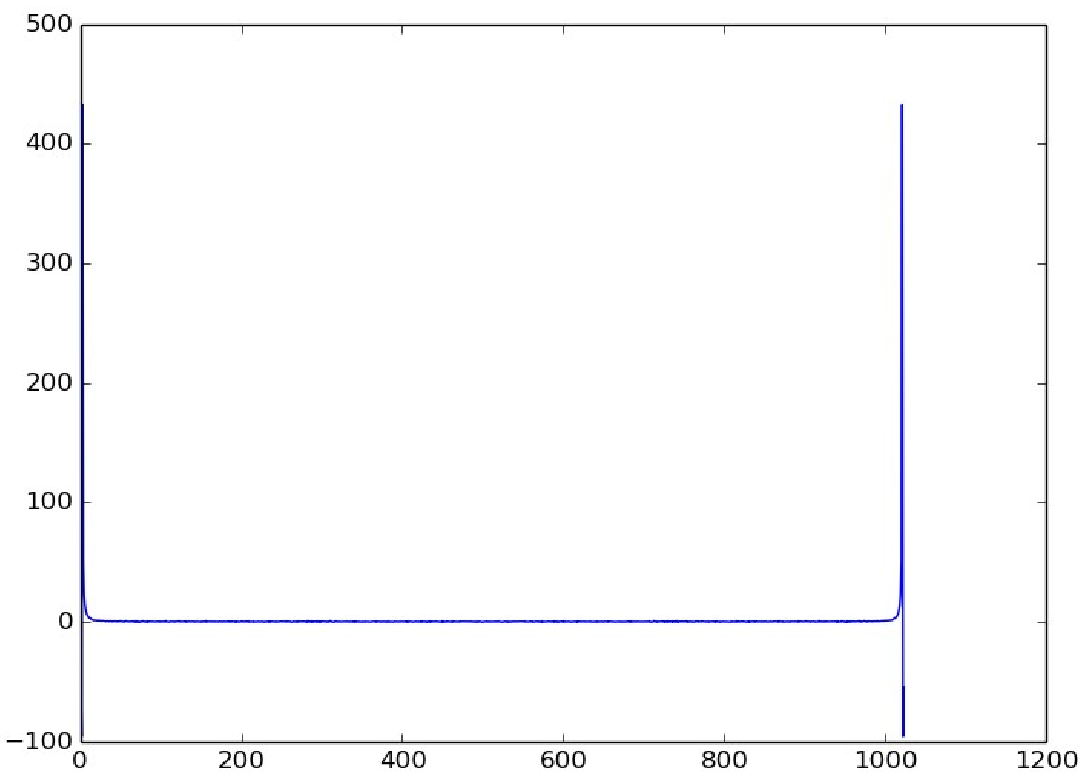
Then a cosine signal is created:



Then noise added to signal using random functions:



Then taking fast fourier transform of the signal. With the argument of 1024.



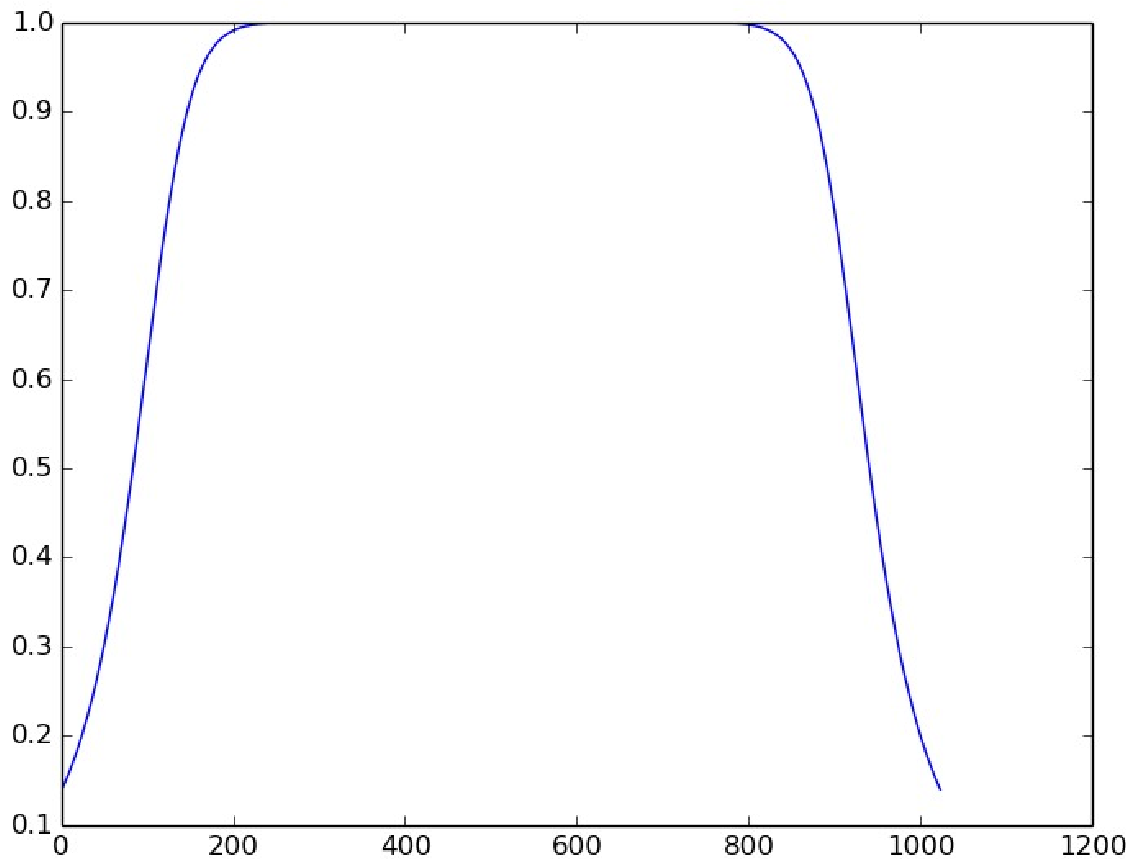
Then we will use the butterworth filter.

$$G=\frac{1}{\sqrt{(1+(f/f_c)^{2n})}}$$

But in order to not discriminate against the negative frequencies we have to shift this from 0,1024 to -512,512. To do this we subtract the nyquist frequency, which we find with sample rate divide by 2. So our signal happens to be:

$$G=\frac{1}{\sqrt{(1+((f-nyquist)/f_c)^{2n})}}$$

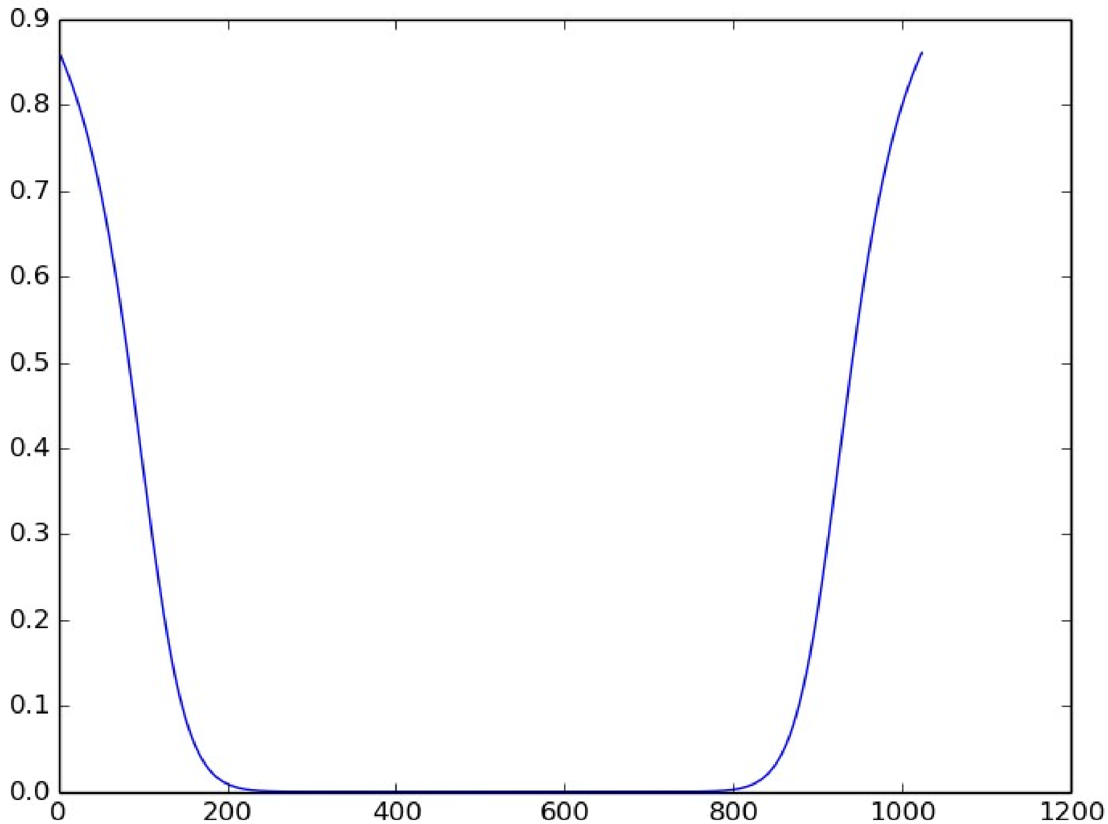
When we plot this:



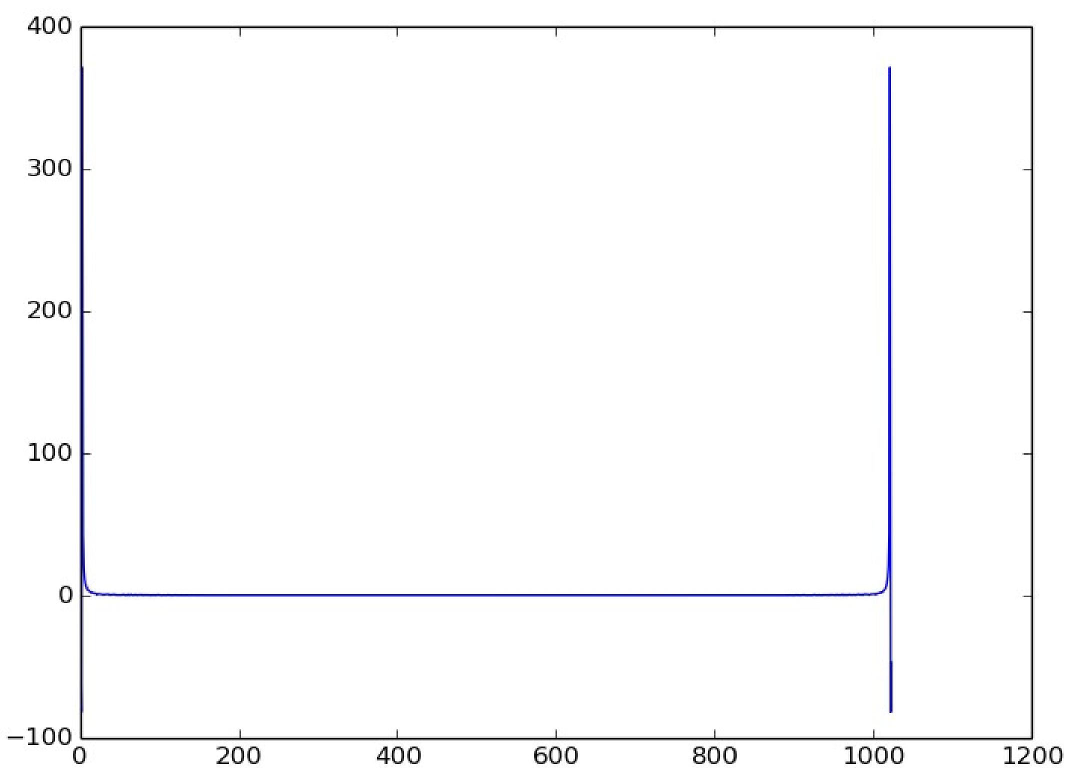
But this filter is not true for our signal because the needed frequencies for our signal is around the 0 frequency. To achieve this we have to invert this symmetrical to x-axis:

$$G=1-\frac{1}{\sqrt{(1+((f-nyquist)/f_c)^{2n})}}$$

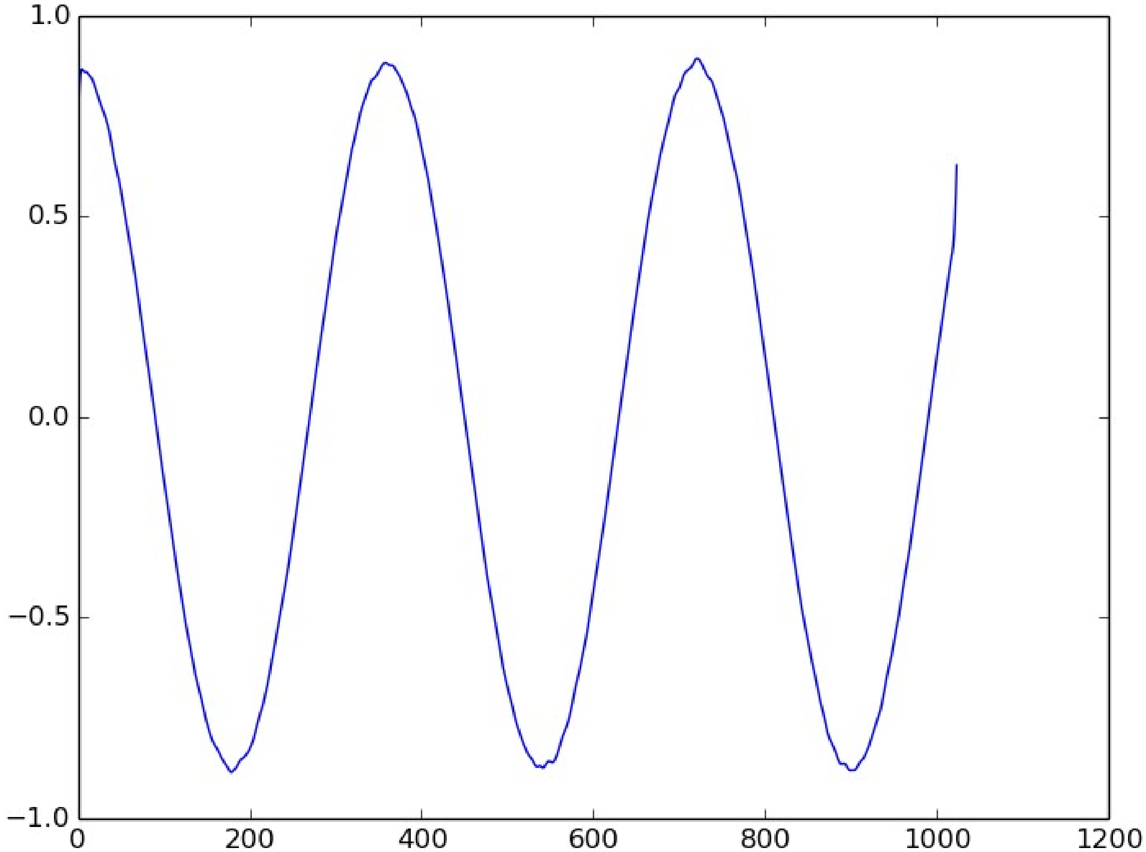
The order in use in here is 8 and the cut-off frequency is 400. Plotting this:



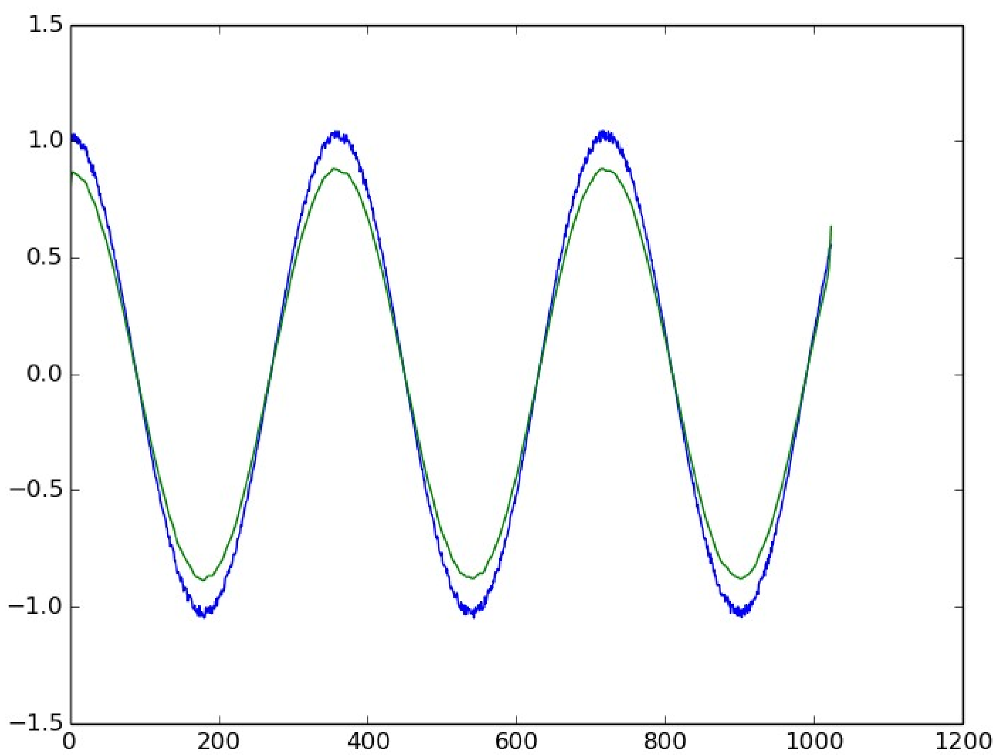
We apply this to our signal in frequency domain:



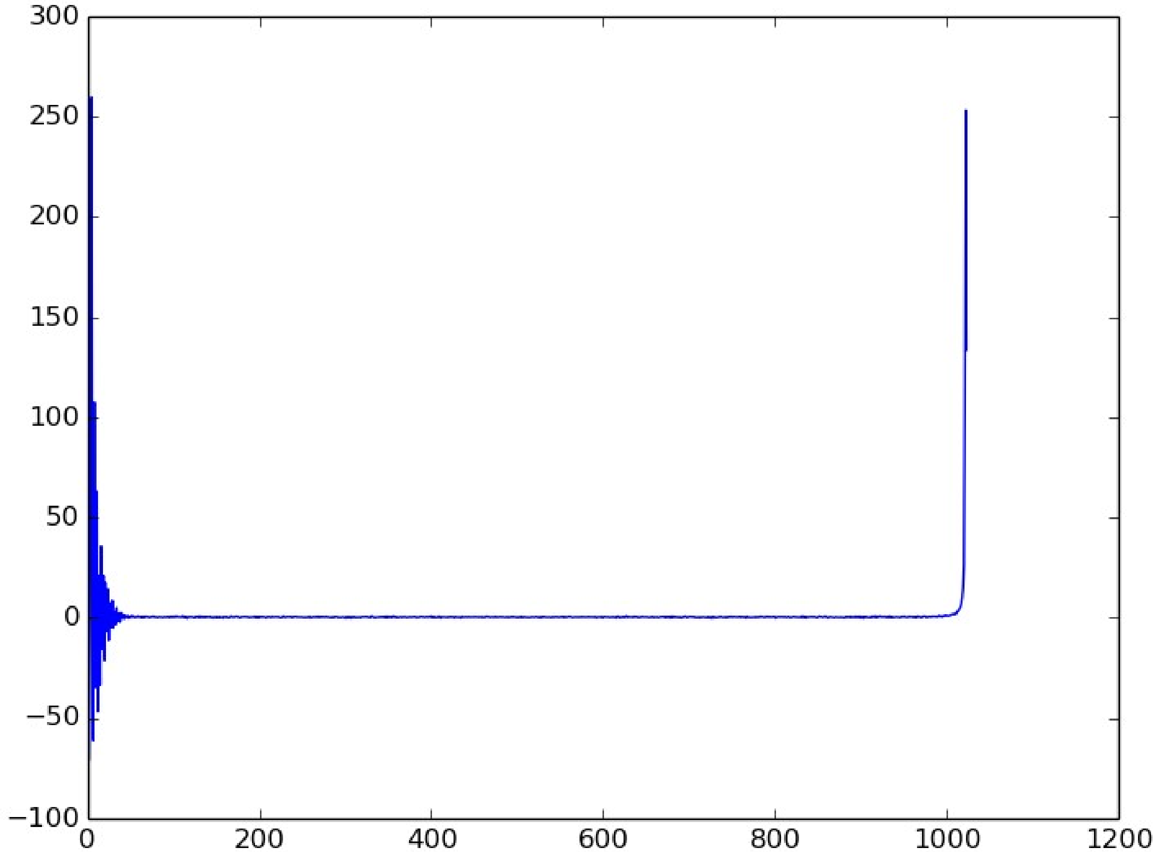
In graph it can be seen that noisy parts are gone with the filter. So taking the inverse fast fourier transform of the frequency domain signal we achieve:



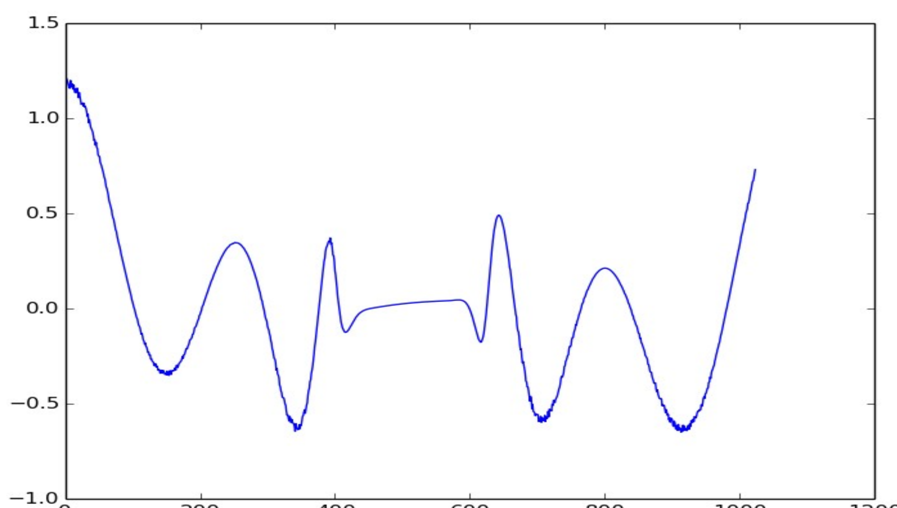
To compare the input signal and filtered signal:



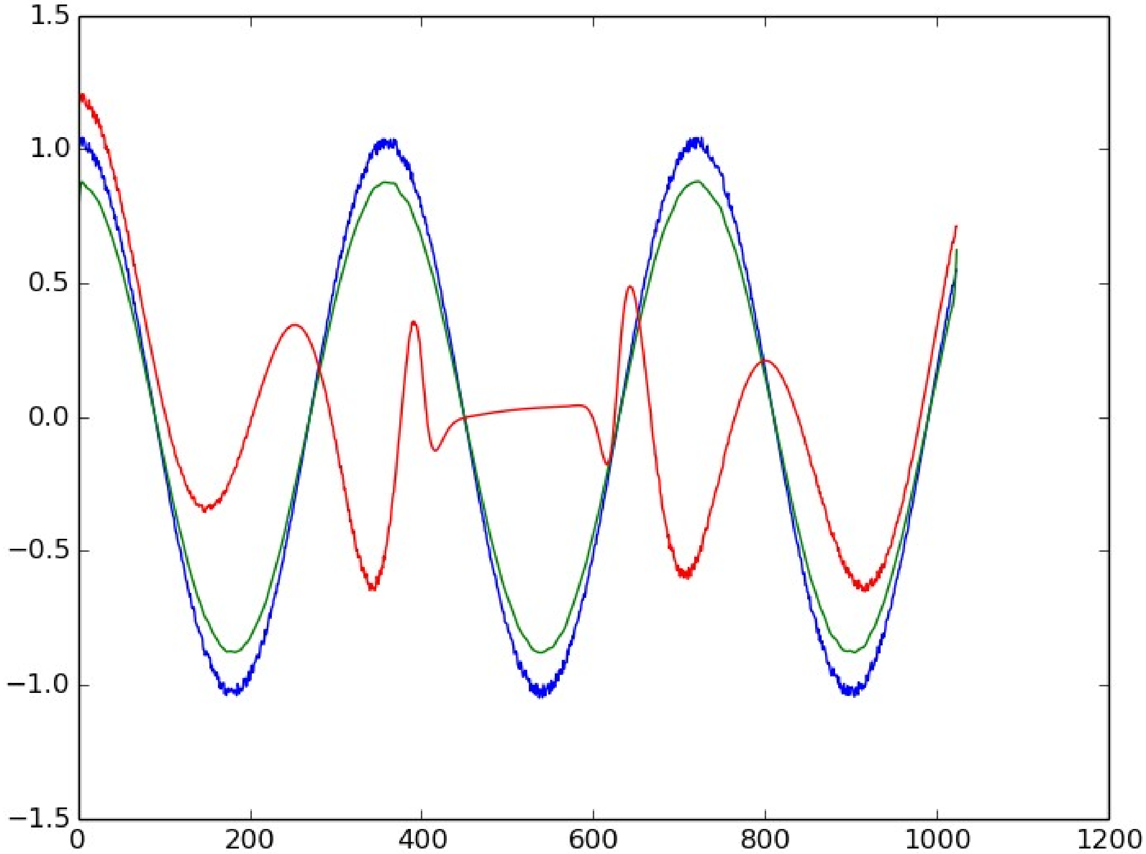
Additional Source: Also in order to have a reference I try to compare my results with the ready-to-use function of the SciPy library of the python but the results seems to be not very accurate. The frequency domain when we take use the function:



When we inverse fourier transform this:



Which seems very inaccurate. Comparing with the input signal, filtered signals with written filter and ready-to-use filter:



Our Code:

```
#libraries that needed for butterworth filter
from math import *
import numpy as np
from pylab import *
import random as ran
from scipy.signal import butter, lfilter

#Low-Pass Frequency Response Function
def lowpass_signal_func(t,frequency_domain_signal,f_c, nyq, n):
    h_lowpass=np.zeros(len(t))
    lowpass_signal=[]
    for i in range(len(frequency_domain_signal)):
        h_lowpass[i]=1-sqrt(1./(1+((i-nyq)/(f_c))**(2*n)))
        lowpass_signal.append(h_lowpass[i]*frequency_domain_signal[i])
    return lowpass_signal, h_lowpass

def scipy_lowpass_filter(frequency_domain_signal,f_c,nyq,n):
    cut=f_c/nyq
    b,a=butter(n,cut,btype='low')
    y=lfilter(b,a,frequency_domain_signal)
    return y


#Sampling Rate
f_s=1024.
#Nyquist Frequency
nyq=0.5 * f_s
#Generating Time Domain
t=linspace(0, f_s,f_s,endpoint=False)

#generating a clean signal
clean_signal = cos(t*pi/180)

#adding noise to clean signal
noise_signal=clean_signal
for i in range(len(clean_signal)):
    noise_signal[i]=noise_signal[i]*ran.uniform(1,1.05)

#Taking the Fast Fourier Transform of the signal
frequency_domain_signal=np.fft.fft(noise_signal,1024)

#Butterworth Filter
#Cutoff Frequency
f_c=400
#Order
n=8
frequency_domain_signal_filtered,G=lowpass_signal_func(t,frequency_domain_signal,f_c,nyq,n)
frequency_domain_signal_filtered_ready=scipy_lowpass_filter(frequency_domain_signal,f_c,nyq,n)
```

```
#Inverse fast fourier transform for filtered signal
filtered_signal=np.fft.ifft(frequency_domain_signal_filtered, 1024)
filtered_signal_ready=np.fft.ifft(frequency_domain_signal_filtered_ready, 1024)

#plot(t)
plot(noise_signal)
#plot(frequency_domain_signal)
#plot(G)
#plot(frequency_domain_signal_filtered)
#plot(frequency_domain_signal_filtered_ready)
plot(filtered_signal)
plot(filtered_signal_ready)

show()
```