MultiVerse: Causal Reasoning Using Importance Sampling in Probabilistic Programming

Presented by:
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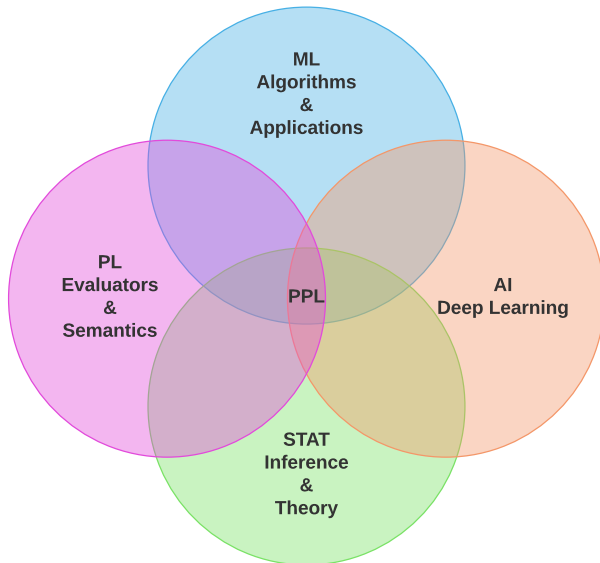https://haluk.github.io/
hdogan@vivaldi.net
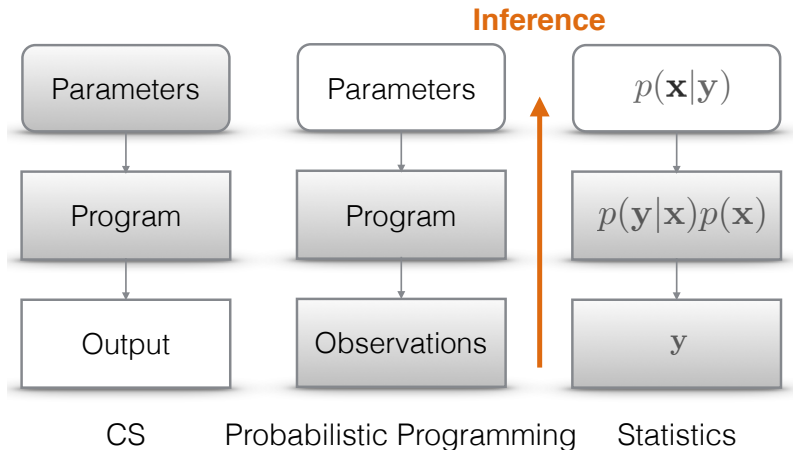Some slides copied from
Frank Wood

Department of Computer Science
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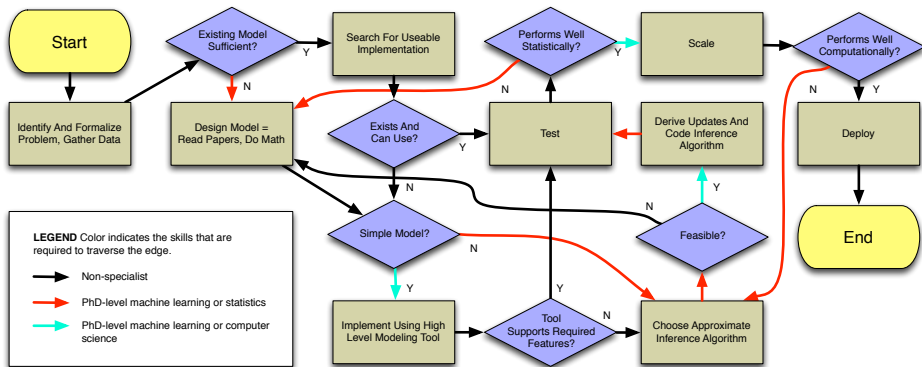
March 4, 2020

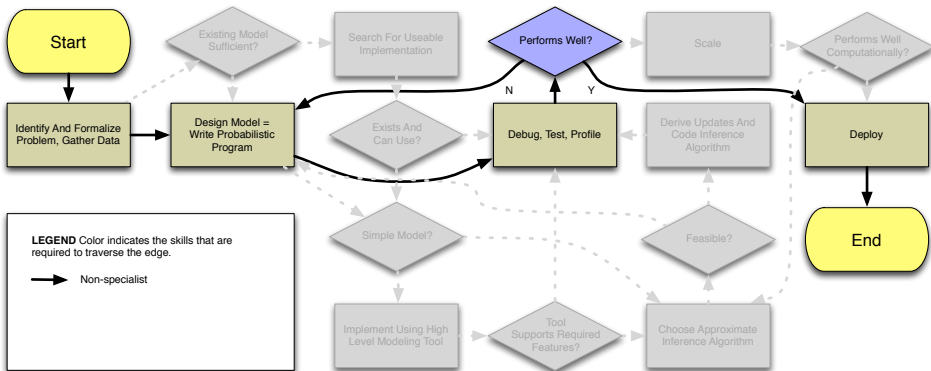# Probabilistic Programming Languages (PPL)

Intuition

The Way Machine Learning Is

The Way Machine Learning Will Be

Causality
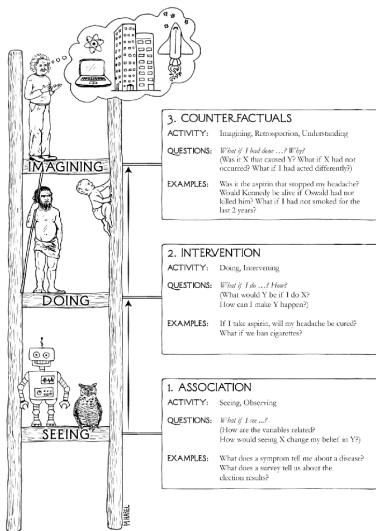
Ladder of Causation (Judea Pearl 2018)

Causal Model $M(X, Y, F)$:

- X: Exogenous variables (latent)
- Y: Endogenous variables (observed)
- F: Set of structural equations $\{F_K \mid K \in Y\}$

Counterfactual inference query:

$$P(Y' \mid Y = e; \mathsf{do}(D = d))$$

Evaluating this query in 3 steps:

- Abduction (observational inference)
- Intervention (action)
- Prediction

Abduction and Counterfactual Inference

Abduction

- The hardest part of the counterfactual procedure
  - Exact inference is possible but either difficult or intractable
- $M' \leftarrow P(X \mid Y = e)$
  - It has the same structure as model M
  - X is replaced by $X \mid Y = e$
  - Y is not observed anymore

Counterfactual inference

- Twin-net approach, i.e., loopy belief propagation (Balke and Pearl 1994)
- Single-world intervention graphs (Richardson and Robins 2013)
- Matching (Li 2012)

Importance Sampling

- Approximate inference technique

$$\mathbb{E}[f(x)] = \int f(x)p(x)dx \approx \frac{1}{n}\sum_i f(x_i)$$

- If sampling from $p(x)$ is very hard

$$\mathbb{E}[f(x)] = \int f(x)p(x)dx = \int f(x)\frac{p(x)}{q(x)}q(x)dx \approx \frac{1}{n}\sum_i f(x_i)\frac{p(x_i)}{q(x_i)}$$

- $\sigma^2(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

Continuous Variable Example in MultiVerse

```python
#!/usr/bin/env python3

import timeit

from multiverse import (DeltaERP, NormalERP, ObservableNormalERP, do, observe,
                        predict, run_inference)
from utils import calculate_expectation

NUM_SAMPLES = 1000


def base_program():
    X = NormalERP(0, 1)
    Z = NormalERP(0, 1)
    Y = ObservableNormalERP(X.value + Z.value, 2, depends_on=[X, Z],)

    return X, Z, Y


def program_with_data():
    X, Z, Y = base_program()
    observe(Y, 1.2342)
    do(Z, -2.5236)

    predict(Y.value, predict_counterfactual=True)


start = timeit.default_timer()
results = run_inference(program_with_data, NUM_SAMPLES)
stop = timeit.default_timer()

print("Time:", stop - start)
result = calculate_expectation(results)
print("Prediction:", result)
```

$X \sim \mathcal{N}(0,1) \quad Z \sim \mathcal{N}(0,1) \quad \epsilon \sim \mathcal{N}(0,2)$
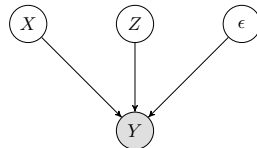
$$Y = X + Z + \epsilon$$

$$\mathbb{E}(Y' \mid Y = 1.2342, \operatorname{do}(Z = -2.5236))$$

Continuous Variable Example in Pyro

```python
#!/usr/bin/env python3

import timeit

import numpy
import pyro
import torch

NUM_SAMPLES = 1000   # for both abduction and intervention/prediction
ROUND_DIGIT_APPR = (
    1   # discretisation to avoid 'infinite rejection sampling' for continuous variables
)
GUIDE_TO_USE = None
latent_procedure_sites = ["X", "Z", "Y_epsilon"]


def rounder(val):
    if ROUND_DIGIT_APPR is None:
        # Don't round:
        return torch.tensor(float(val))
    else:
        return torch.tensor(round(float(val), ROUND_DIGIT_APPR))


def extract_obs_if_any(data, var_name):
    if data is not None and var_name in data:
        return data[var_name]
    else:
        None
```

Continuous Variable Example in Pyro (cont'd)

```python
def model(data=None, posterior_distribution=None):
    if posterior_distribution is not None and "X" in posterior_distribution:
        # If we are re-using a sample from a posterior,
        # we just should use the value for this variable
        # directly:
        X = posterior_distribution["X"]
    else:
        X = pyro.sample(
            "X", pyro.distributions.Normal(0, 1), obs=extract_obs_if_any(data, "X")
        )
    if posterior_distribution is not None and "Z" in posterior_distribution:
        Z = posterior_distribution["Z"]
    else:
        Z = pyro.sample(
            "Z", pyro.distributions.Normal(0, 1), obs=extract_obs_if_any(data, "Z")
        )
    if posterior_distribution is not None and "Y_epsilon" in posterior_distribution:
        Y_epsilon = posterior_distribution["Y_epsilon"]
    else:
        Y_epsilon = pyro.sample("Y_epsilon", pyro.distributions.Normal(0, 2))
    discrete_Y = rounder(X + Z + Y_epsilon)
    # We must (re-)evaluate deterministic variables in any case:
    Y = pyro.sample(
        "Y",
        pyro.distributions.Delta(torch.tensor(discrete_Y)),
        obs=extract_obs_if_any(data, "Y"),
    )
    return X, Z, Y_epsilon, Y
```

Continuous Variable Example in Pyro (cont'd)

```
62    data = {"Y": rounder(1.2342), "X": None, "Z": None}
63    start = timeit.default_timer()
64
65    # 1. Abduction
66    posterior = pyro.infer.Importance(
67        model, guide=GUIDE_TO_USE, num_samples=NUM_SAMPLES
68    ).run(data=data)
69    print("AbductionESS:", posterior.get_ESS())
70    posterior = pyro.infer.EmpiricalMarginal(posterior, sites=latent_procedure_sites)
71
72    # 2. Intervention
73    intervention = {"Z": -2.5236}
74    intervened_posterior = pyro.do(model, intervention)
75
76    # 3. Prediction
77    predictions_Y = []
78    for sample_index in range(NUM_SAMPLES):
79        # We are drawing a sample from the posterior world:
80        posterior_sample_vector = posterior.sample()
81        # We drew that sample in a vector form;
82        # now we need to transform
83        # it to a dictionary of variables.
84        posterior_sample = {}
85        for index, var_name in enumerate(latent_procedure_sites):
86            if var_name in intervention:
87                # We must ensure that we don't
88                # use intervened variables
89                # from its posterior:
90                pass
91            else:
92                posterior_sample[var_name] = posterior_sample_vector[index]
93        X, Z, Y_epsilon, Y = intervened_posterior(posterior_distribution=posterior_sample)
94        predictions_Y.append(Y)
95
96    stop = timeit.default_timer()
97
98    print("Time:", stop - start)
99    expected_Y = numpy.mean(predictions_Y)
100   print("Prediction:", expected_Y)
```
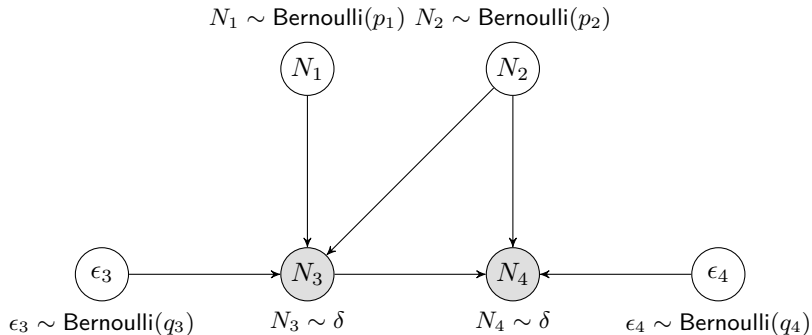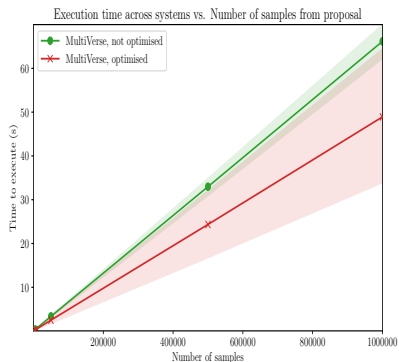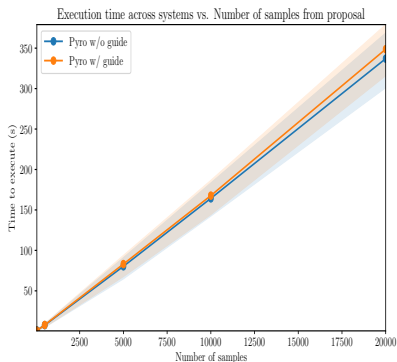
Experiments

- 16-core EC2 instance m4.4xlarge
- 1000 SCM with 15 probabilistic procedures
    - BNs in the form of probabilistic programs
- MultiVerse experiments
    - produce the same number of samples in less time than Pyro
    - have better inference convergence

Test Models

$N_1 \sim \mathsf{Bernoulli}(p_1)$ $N_2 \sim \mathsf{Bernoulli}(p_2)$

N_1

N_2

ϵ_3

N_3

N_4

ϵ_4

$\epsilon_3 \sim \mathsf{Bernoulli}(q_3)$ $\quad N_3 \sim \delta \quad\quad\quad N_4 \sim \delta \quad\quad \epsilon_4 \sim \mathsf{Bernoulli}(q_4)$

Time Performance Comparison

Convergence Test Performance

Absolute error across systems vs. Number of samples from proposal
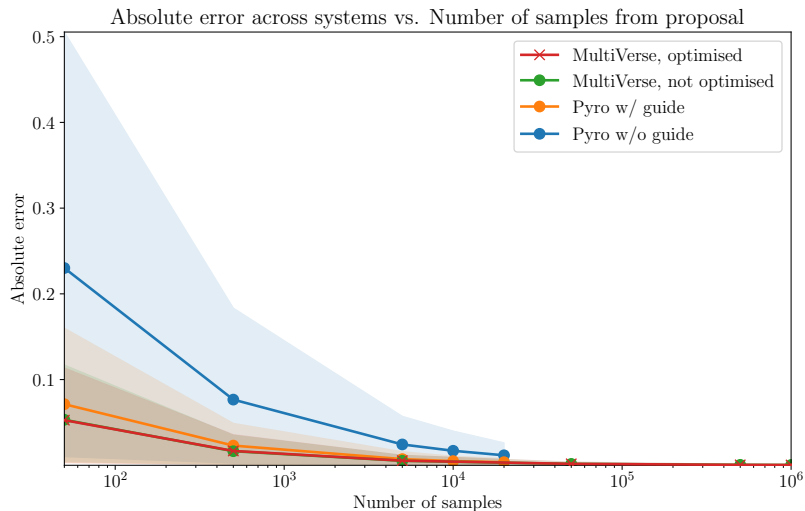
PPL Should Be Pure Functional

Anglican

```
[assume (a (normal 5 10))]
[assume (b (normal a 2))]
[assume (a (normal b 7))]
=> Error
```

Probabilistic-C

```
int a = normal(5, 10);
int b = normal(a, 2);
int a = normal(b, 7);
```

Birthday Paradox

Approximately, what's the probability that in a room filled with 23 people at least one pair of people have the same birthday?

```
[assume birthday (mem (lambda (i) (uniform-discrete 1 366)))]

[assume N 23]

[assume pair-equal
  (lambda (i j)
    (if (> i N)
      false
      (if (> j N)
        (pair-equal (+ i 1) (+ i 2))
        (if (= (birthday i) (birthday j))
          true
          (pair-equal i (+ j 1))))))]

[predict (pair-equal 1 2)]
```

Questions

Questions?

Judea Pearl, winner of Turing Award (2011)

References I

Balke, A. and J. Pearl (1994). "Probabilistic Evaluation of Counterfactual Queries". In: *Proceedings of the 12th National Conference on Artificial Intelligence, Seattle, WA, USA, July 31 - August 4, 1994, Volume 1*. Ed. by B. Hayes-Roth and R. E. Korf. AAAI Press / The MIT Press, pp. 230–237. URL: http://www.aaai.org/Library/AAAI/1994/aaai94-035.php (cit. on p. 7).

Judea Pearl, D. M. (May 15, 2018). *The Book of Why*. Hachette Book Group USA. 432 pp. ISBN: 046509760X. URL: https://www.ebook.de/de/product/30501615/judea_pearl_dana_mackenzie_the_book_of_why.html (cit. on p. 6).

Li, M. (June 2012). "Using the Propensity Score Method to Estimate Causal Effects". In: *Organizational Research Methods* 16.2, pp. 188–226. DOI: 10.1177/1094428112447816 (cit. on p. 7).

References II

Richardson, T. S. and J. M. Robins (2013). "Single world intervention graphs (SWIGs): A unification of the counterfactual and graphical approaches to causality". In: *Center for the Statistics and the Social Sciences, University of Washington Series. Working Paper* 128.30, p. 2013 (cit. on p. 7).