Name: ____________________

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ECON 1210 | Fall 2017

**FINAL EXAM**

**Duration: 3 hours**
**Total: 100 points**
**A simple calculator is allowed.**
**Phone, laptop, notes, and textbook are not allowed.**

**Multiple-choice questions: 1 point each | 10 points total**

Select **the one alternative** that best answers the question or completes the statement. Write your answers in the designated places. **Answers not in the designated places will not receive any credit.**

1) Consider the Solow model without technological progress or population growth, and assume that the saving rate decreases permanently. In the long run, which of the following variables is bound to be lower?
A) consumption per worker
B) capital per worker
C) the growth rate of capital per worker
D) the golden-rule level of capital per worker
E) none of the above

ANSWER:_______

2) Consider the Solow model without technological progress or population growth, and assume that the depreciation rate decreases permanently. In the long run, which of the following variables is bound to be lower?
A) investment per worker
B) capital per worker
C) the growth rate of capital per worker
D) the amount of capital that depreciates each period
E) none of the above

ANSWER:_______

3) In the Solow model without technological progress or population growth, starting from the steady state, a decrease in the saving rate will have which of the following effects?
A) make the growth of output per worker temporarily positive
B) increase the steady-state growth of output per worker
C) make the growth of output per worker temporarily negative
D) decrease the steady-state growth of output per worker
E) decrease temporarily the depreciation rate

ANSWER:_______

4) In the Malthusian model, an increase in how much a child eats will have which of the following effects?
A) an increase in population growth in the long run
B) an increase in population in the long run
C) an increase in output per capita in the long run
D) a decrease in output per capita in the long run
E) a decrease in fertility in the long run

ANSWER:_______

5) Consider the Solow model with technological progress and population growth. Suppose the saving rate is initially above the golden-rule saving rate. We know that in the long run, an increase in the saving rate will cause
A) a decrease in output per worker
B) a decrease in the consumption per worker
C) an increase in consumption per worker
D) an increase in technological progress
E) an increase in population growth

ANSWER:_______

6) In the matching model, a decrease in the job-separation rate leads to
A) an outward shift of the labor supply curve and an outward rotation of the labor demand curve
B) an inward shift of the labor supply curve and an outward rotation of the labor demand curve
C) an outward shift of the labor supply curve and an inward rotation of the labor demand curve
D) an inward shift of the labor supply curve and an inward rotation of the labor demand curve
E) an outward shift of the labor supply curve but no effect on the labor demand curve

ANSWER:_______

7) Since approximately 1970, the most stable Phillips-type relationship for the United States has involved which of the following variables?
A) the rate of inflation and the change in the unemployment rate
B) the unemployment rate and the change in the rate of inflation
C) the change in the unemployment rate and the change in the rate of inflation
D) the unemployment rate and the rate of inflation
E) the rate of inflation and the growth rate of GDP

ANSWER:_______

8) In the Malthusian model, which of the following will cause a decrease in output per worker in the short run?
A) an increase in the level of technology
B) the discovery of new arable land
C) a one-time wave of immigration
D) a one-time plague that kills 30% of the working population
E) none of the above

ANSWER:_______

9) In the IS module, an equal and simultaneous reduction in government spending G and tax revenue T will cause
A) an increase in consumption
B) no change in consumption
C) a reduction in consumption
D) an increase in government deficit
E) a decrease in government deficit

ANSWER:_______

10) In the LM module, which of the following will cause a reduction in the amount of money that one wishes to hold?
A) an increase in the interest rate
B) a reduction in the interest rate
C) an increase in government spending
D) an increase in income
E) a reduction in taxes

ANSWER:_______

Name: ______________________

**Problems: 30 points each | 90 points total**

For each question, provide all derivations in the space provided. **Answers that are correct but without derivation will only receive very partial credit.**

Problem 1: Minimum wage in the matching model

Consider the matching model of the labor market with a labor force of size H, a matching function $M = m \times U^{1/3} \times V^{2/3}$, a recruiting cost of r recruiters per vacancy, a job-separation rate s, and a revenue function for firms $Y = a \times N^{1/2}$. Assume that all workers are paid at a minimum wage W.

A) Compute the job-finding rate $f(\theta)$. How does $f(\theta)$ depend on $\theta$? What happens to $f(\theta)$ when $\theta=0$? Interpret.

B) Compute the vacancy-filling rate $q(\theta)$. How does $q(\theta)$ depend on $\theta$? What happens to $q(\theta)$ when $\theta=0$? Interpret.

C) Using the assumption that labor market flows are balanced, compute the recruiter-producer ratio $\tau(\theta)$. How does $\tau(\theta)$ depend on $\theta$? What happens to $\tau(\theta)$ when $\theta=0$? Interpret.

D) Using the assumption that labor market flows are balanced, compute the labor supply $L^s(\theta)$. How does $L^s(\theta)$ depend on $\theta$? Interpret.

E) Express the profits of firms as a function of employment L, the recruiter-producer ratio $\tau(\theta)$, and the minimum wage W. Compute the labor demand $L^d(\theta,W)$. How does $L^d(\theta,W)$ depend on $\theta$? How does $L^d(\theta,W)$ depend on W? What happens to $L^d(\theta,W)$ when $\theta=0$? Interpret.

F) Find a condition on the minimum wage such that $L^d(\theta=0,W) < H$. What happens to rationing unemployment when the condition is satisfied?

G) Assume that $L^d(\theta=0,W) < H$. Plot the labor demand and labor supply curves in the usual labor market diagram. Illustrate rationing and frictional unemployments.

H) Using the diagram, determine the effect of an increase in the minimum wage W on labor market tightness, total unemployment, rationing unemployment, and frictional unemployment.

I) From a social perspective, what is the benefit from lower unemployment? What is the cost from lower unemployment? Given cost and benefit, would it be efficient for the government to bring unemployment all the way to 0%?

J) Imagine that the goal of the government is to maintain unemployment at its efficient level. Under which circumstances should the government raise the minimum wage? Under which circumstances should it lower the minimum wage?

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Name: ____________________

Problem 2: Solow model with technological progress and population growth

Consider a Solow model with production function $Y = K^{1/2} (AN)^{1/2}$. The saving rate is 10%, the depreciation rate is 6%, the rate of population growth is 2%, and the rate of technological progress is 2%. The economy is in steady state.

A) At what rates do output, output per worker, and output per effective worker grow?

B) At what rates do consumption, consumption per worker, and consumption per effective worker grow?

C) Draw the equilibrium diagram of this Solow model. Show where the steady state is.

D) Derive the equations for all the curves that you plotted on the diagram.

E) Solve for capital per effective worker and output per effective worker in steady state.

F) Solve for consumption per effective worker and investment per effective worker in steady state.

G) Solve for the marginal product of capital, MPK, in steady state.

H) Does the economy have more or less capital than at the golden-rule steady state? To achieve the golden-rule steady state, does the saving rate need to increase or decrease?

I) Suppose the change in saving rate described in question H) occurs. During the transition to the golden-rule steady state, will the growth rate of output be higher or lower than the rate derived in question A)?

J) Draw the evolution of log consumption over time: in the old steady state, after the change in saving rate, and in the golden-rule steady state. Explain.

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Name: ____________________

Problem 3 : Malthusian model

Consider a Malthusian model with production function $Y = (AX)^{1/2} L^{1/2}$. The parameter A is technology and the parameter X is land. The variable Y is output and the variable L is working population. People live two periods. Each family has one parent and N children. Each child consumes 1 unit of food. Each parent maximizes his or her utility $U = N^{1/2} C^{1/2}$ , where C is the consumption of the parent.

A) Compute output per worker y. How does y depend on technology, land, and the size of the working population?

B) Give the budget constraint of a family with N children.

C) Compute the optimal consumption of a parent -- that is, compute the consumption maximizing utility U subject to the budget constraint derived in B).

D) Compute the optimal number of children for a parent.

E) Combining your answers to questions A) and D), express the working population at time t+1, L(t+1), as a function of the working population at time t, L(t), and parameters of the model.

F) What is the steady-state level of the working population, L*? How does L* depend on technology and land? Imagine that a new irrigation technique is discovered and A doubles. Does L* increase or decrease? In which proportion does L* change?

G) Draw the usual population diagram. Describe each curve and show the steady state. Use the diagram to show the effect of the new irrigation technique. Explain what happens to population once the new irrigation technique is discovered.

H) Using your answers to questions A) and D), express the output per worker at time t+1, y(t+1), as a function of the output per worker at time t, y(t), and parameters of the model.

I) What is the steady-state level of output per worker, $y^*$? How does $y^*$ depend on technology and land? What is the implication of this result? What happens to $y^*$ when the new irrigation technique is discovered.

J) Draw the usual output-per-worker diagram. Describe each curve and show the steady state. Use the diagram to show the effect of the new irrigation technique. Explain what happens to output per worker once the new irrigation technique is discovered.

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**ECON 1210**
**FINAL EXAM - SOLUTION**

# 1. MCQ

1. B. capital per worker
2. E. none of the above
3. C. make the growth rate of output per worker temporarily negative
4. C. an increase in output per capita in the long run
5. B. a decrease in the consumption per worker
6. A. an outward shift of the labor supply curve and an outward rotation of the labor demand curve
7. B. the unemployment rate and the change in the rate of inflation
8. C. a one-time wave of immigration
9. B. no change in consumption
10. A. an increase in the interest rate

# 2. Problem 1

A. $f(\theta) = \frac{m(U,V)}{U} = m\theta^{2/3}$
$f(\theta)$ is increasing $\theta$
When $\theta = 0$, $f(\theta) = 0$

B. $q(\theta) = \frac{m(U,V)}{V} = m\theta^{-1/3}$
$q(\theta)$ is decreasing $\theta$
When $\theta = 0$, $q(\theta) = \infty$

C. With balanced flows, $s \times L = q(\theta)V$. From the lectures, $\tau(\theta) = \frac{rs}{m\theta^{-1/3} - rs}$
As $\theta$ increases, the denominator decreases and thus $\tau(\theta)$ increases
When $\theta = 0$, $\tau(\theta) = 0$

D. Similarly, with balanced flows, we have $s \times L = f(\theta) \times U$ and thus: $L^s(\theta) = \frac{f(\theta)H}{s+f(\theta)} = \frac{m\theta^{2/3}H}{s+m\theta^{2/3}}$
As $\theta$ increases, $f(\theta)$ increases. The increase in the numerator is greater than the increase in the denominator, so overall $L^s(\theta)$ increases.
When $\theta = 0$, $f(\theta) = 0 \Rightarrow L^s(\theta) = 0$.

E. The firm's profit is $aN^{1/2} - W \times L$
Using $L = (1 + \tau(\theta)) \times N$, we get:
$Profits = a\left(\frac{L}{1+\tau(\theta)}\right)^{1/2} - W \times L$
Labor demand comes from the first order condition of profit maximization:
$a\frac{1}{2}L^{-1/2}\left(\frac{1}{1+\tau(\theta)}\right)^{1/2} - W = 0$
$\implies L^d(\theta, W) = \frac{a^2}{4W^2(1+\tau(\theta))}$
$L^d(\theta, W)$ is decreasing in W and $\theta$. As $\theta$ increases, $\tau(\theta)$ increases and labor demand falls.
When $\theta = 0$, $L^d(\theta = 0, W) = (a/2W)^2$.

F. $L^d(\theta = 0, W) < H \Rightarrow \left(\frac{a}{2W}\right)^2 < H \Rightarrow W > \frac{a}{2\sqrt{H}}$.
When this condition is satisfied, rationing unemployment is positive.

G. See lecture notes.

H. An increase in minimum wage W reduces labor demand and has no impact on labor supply, so the labor demand curve shifts left and labor supply curve remains unchanged. In equilibrium, this lowers the equilibrium labor market tightness, $\theta$, increases total unemployment, increases rationing unemployment and increases frictional unemployment.

I. Benefit from lower unemployment: fewer workers remain idle.
Cost from lower unemployment: too many workers are allocated to recruiting.
Due to the decentralized nature of the labor market, there is always some inefficiency present in the matching process which causes frictional unemployment. In fact, some frictional unemployment is positive, since it is ultimately due to the cost of allocating resources (labor) towards their best match (a vacancy or job-opening). Hence, it is not necessarily a good thing to target zero unemployment.

J. The government should raise the minimum wage when unemployment is above the efficient level, and should lower minimum wage when unemployment is below its efficient level.

## 3. Problem 2

(a) Let $y \equiv \frac{Y}{N}$, $\hat{y} \equiv \frac{Y}{AN}$ and $g_X \equiv \frac{X_{t+1}-X_t}{X_t}$

- In the steady state of a Solow model with technological progress we have

$$g_{\hat{y}} \equiv \frac{\hat{y}_{t+1} - \hat{y}_t}{\hat{y}_t} = 0$$

- Output per worker growth. Notice

$$y = \hat{y}A$$

Then

$$g_y \approx g_{\hat{y}} + g_A$$

$$g_y \approx 0 + 2\% = 2\%$$

- Output growth. Notice

$$Y = yN$$

Then

$$g_Y \approx g_y + g_N$$

$$g_Y \approx 2\% + 2\% = 4\%$$

(b) Let $C = (1-s)Y$, $c \equiv \frac{C}{N}$, $\hat{c} \equiv \frac{C}{AN}$.

- Consumption per effective worker growth. Since $c_t = (1-s)y_t$ and $g_{\hat{y}} = 0$

$$g_c = g_y = 0$$

- Consumption per worker growth. Notice

$$\hat{c} = cA$$

Then

$$g_{\hat{c}} \approx g_c + g_A$$

$$g_{\hat{c}} \approx 0 + 2\% = 2\%$$

- Consumption growth. Notice

$$C = cN$$

Then

$$g_C \approx g_c + g_N$$

$$g_C \approx 2\% + 2\% = 4\%$$

(c) See lecture notes/book

(d) First, derive the capital per effective worker law of motion

$$K_{t+1} - K_t = I_t - \delta K_t$$

Dividing by $K_t$

$$g_{K\,t} = \frac{K_{t+1} - K_t}{K_t} = \frac{I_t}{K_t} - \delta$$

In the other hand, using definition of $y$ and useful approximation for growth rates

$$g_{k\,t} = g_{K\,t} - g_A - g_N$$

Putting the last two equations together

$$g_k + g_A + g_N = \frac{I_t}{K_t} - \delta$$

$$K_t(g_k + g_A + g_N) = I_t - \delta K_t$$

$$K_t(g_k + g_A + g_N) = sY_t - \delta K_t$$

$$K_t(g_k + g_A + g_N) = s\sqrt{K_t}\sqrt{A_t N_t} - \delta K_t$$

Dividing by $A_t N_t$

$$\frac{K_t}{A_t N_t}(g_k + g_A + g_N) = s\sqrt{\frac{K_t}{A_t N_t}} - \delta \frac{K_t}{A_t N_t}$$

$$k_t(g_k + g_A + g_N) = s\sqrt{k_t} - \delta k_t$$

In the steady state $k_t = k_{t+1} \implies g_k = 0$. Then,

$$\bar{k}(g_A + g_N) = s\sqrt{\bar{k}} - \delta \bar{k}$$

$$\bar{k}(g_A + g_N + \delta) = s\sqrt{\bar{k}}$$

The previous equation is sufficient to characterize the steady-state level of capital per effective worker. Therefore, the latter is described graphically by the intersection of the following functions

$$h(k) = k(g_A + g_N + \delta)$$

$$l(K) = s\sqrt{k}$$

Additionally, to go from the capital per effective worker to out per effective worker we use the following function

$$y = f(k) = \frac{F(K, AN)}{AN} = \frac{\sqrt{K}\sqrt{AN}}{AN} = \sqrt{\frac{K}{AN}} = \sqrt{k}$$

(e) From previous question

$$\bar{k}(g_A + g_N + \delta) = s\sqrt{\bar{k}}$$

$$\bar{k}(g_A + g_N + \delta) = s\sqrt{\bar{k}}$$

$$\sqrt{\bar{k}} = \frac{s}{(g_A + g_N + \delta)}$$

$$\bar{k} = (\frac{s}{g_A + g_N + \delta})^2$$

Then,

$$\bar{y} = \sqrt{\bar{k}}$$

$$\bar{y} = \frac{s}{g_A + g_N + \delta}$$

(f) Consumption per effective worker in steady-state

$$\bar{c} = (1-s)\bar{y}$$

$$\bar{c} = \frac{(1-s)s}{g_A + g_N + \delta}$$

Investment per effective worker in steady-state

$$\bar{i} = s\bar{y}$$

$$\bar{i} = \frac{s^2}{g_A + g_N + \delta}$$

(g) Notice

$$MPK(k) = \frac{\partial F(K,N)}{\partial K} = \frac{1}{2}\sqrt{\frac{AN}{K}} = \frac{1}{2}k^{-1/2}$$

In the steady state

$$MPK(\bar{k}) = \frac{1}{2}\bar{k}^{-1/2}$$

Using previous result

$$MPK(\bar{k}) = \frac{1}{2}((\frac{s}{g_A + g_N + \delta})^2)^{-1/2}$$

$$MPK(\bar{k}) = \frac{1}{2}\frac{g_A + g_N + \delta}{s}$$

(h) ALTERNATIVE 1: From previous question

$$MPK(\bar{k}) = \frac{1}{2}\frac{g_A + g_N + \delta}{s}$$

Using values given in instructions

$$MPK(\bar{k}) = \frac{1}{2}\frac{0.02 + 0.02 + 0.06}{0.1} = 0.5$$

Then,

$$MPK(\bar{k}) - \delta = 0.5 - 0.06 = 0.44$$

In the other hand,

$$g_A + g_N = 0.02 + 0.02 = 0.04$$

Then, $MPK(\bar{k}) - \delta > g_A + g_N$. To achieve the golden rule steady state, savings rate must increase (see Lecture 30). In turn, this implies that the new steady state level of capital per effective worker is higher.

ALTERNATIVE 2: directly computing golden rule
Golden rule ($s_{GR}$)

$$s_{GR} \equiv \underset{s}{\text{argmax}}\ \bar{c}(s)$$

$$s_{GR} = \underset{s}{\text{argmax}}\ \frac{(1-s)s}{g_k + g_A + \delta}$$

$$s_{GR} = \underset{s}{\text{argmax}}\ (1-s)s$$

F.O.C.

$$1 - 2s_{GR} = 0$$

Then,

$$s_{GR} = 1/2 = 50\%$$

Therefore, the saving rate must increase. Since a higher saving rate implies higher capital per effective worker, capital per effective worker must be higher in the steady state.

(i) It must be higher. Since the saving rate increased, the new steady state for output per effective worker is higher. Then, during the transition we must have a positive growth rate of output per effective worker. Let $\tilde{g_X}$ denote a growth rate during the transition. Then

$$\tilde{g_y} > 0$$

$$\tilde{g_y} > g_y$$

Since $g_y \approx g_{\hat{y}} + g_A$

$$\tilde{g_{\hat{y}}} + \tilde{g_A} > g_{\hat{y}} + g_A$$

Since technological progress is the same in both situations, $\tilde{g_A} = g_A$, then

$$\tilde{g_{\hat{y}}} > g_{\hat{y}}$$

(j) Notice,

$$c = \frac{C}{AN}$$

In any steady state, $g_c = 0$. Then, in any steady state $g_C = g_A + g_N$. In the other hand, $g_c = g_y$. Therefore, based on part i), we know that during the transition $g_c > 0$, which implies $g_C > g_A + g_N$. Additionally, we know that the level of consumption per effective worker is higher in the second steady state that in the first one ($\bar{c_0} < \bar{c_1}$).
In summary,

| Phase | $g_C$ | $c$ |
|---|---|---|
| Old steady state | $g_N + g_A$ | $\bar{c_0}$ |
| Transition | $> g_N + g_A$ | Drops below $\bar{c_0}$. Then grows at diminishing rates |
| New steady state | $g_N + g_A$ | $\bar{c_1}$ |

The full table allows to depict the evolution of log-consumption (See Figure 1).

(Just for completeness, this is not required) The third column of the table describes the dynamics of $c$ (see Figure 2).

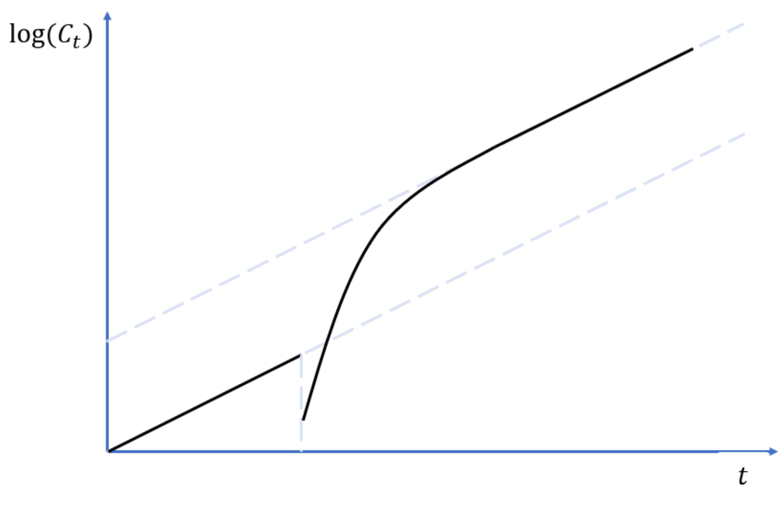


Figure 1: Log-consumption dynamics

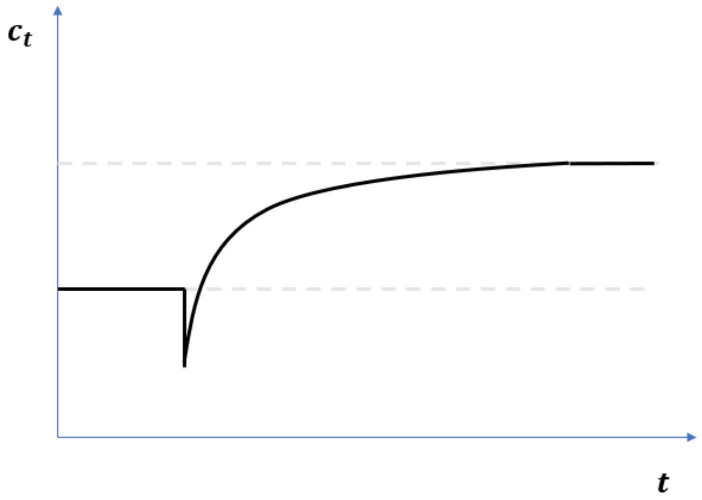


Figure 2: Consumption per effective worker dynamics

## 4. Problem 3

A. Output per worker in period t is

$$y(t) = \frac{Y(t)}{L(t)} = \sqrt{\frac{AX}{L(t)}}$$

B. the budget constraint writes

$$N(t) + C(t) = y(t)$$

C. Utility is Cobb-Douglas so optimal consumption is a fraction one half of income:

$$C(t) = \frac{1}{2}y(t) = \frac{1}{2}\sqrt{\frac{AX}{L(t)}}$$

D. Similarly,

$$N(t) = \frac{1}{2}y(t) = \frac{1}{2}\sqrt{\frac{AX}{L(t)}}$$

E. We know that

$$L(t+1) = N(t)L(t)$$

Thus

$$L(t+1) = \frac{1}{2}\sqrt{AXL(t)}$$

F. In the steady state, $L^* = L(t+1) = L(t)$, thus

$$L^* = \frac{AX}{4}$$

Technology and land have a positive effect on the steady state level of population. After a new technology is introduced, $L^*$ increases. Indeed, suppose $A' = 2A$. Then

$$(L^*)' = \frac{A'X}{4} = 2L^*$$

G. See lecture notes.

H.

$$y(t+1) = \sqrt{\frac{AX}{L(t+1)}} = \sqrt{\frac{AX}{N(t)L(t)}} = \frac{1}{\sqrt{N(t)}}y(t)$$

Thus,

$$y(t+1) = \sqrt{2y(t)}$$

I. The steady state level of output per worker is such that $y^* = y(t) = y(t+1)$, thus

$$y^* = 2$$

Technology and land have no impact on output per worker in the steady state: malthusian trap.

J. At the time of the shock and after, $A' = 2A$. If we were in steady state in $t$, income per worker was $y^* = 2$. Hence if the shock arises in $t+1$,

$$y'(t+1) = \sqrt{\frac{A(t+1)X}{L(t+1)}} = \sqrt{\frac{2AX}{N(t)L(t)}} = \sqrt{\frac{2}{N(t)}}y(t) = \sqrt{\frac{2}{N^*}}y^* = \sqrt{2} * 2$$

In the long run, income per worker goes back to the same steady state level since population increases:

$$N'(t+1) = \frac{1}{2}y(t+1) = \sqrt{2} > 1 = N^*$$

Hence

$$(y^*)' = 2 = y^*$$