Pascal Michaillat | Brown University ECON 1210 | Fall 2017

#### **MIDTERM EXAM 2**

**Duration: 50 minutes** 

Total: 100 points

A calculator is allowed. Phones, laptops, notes, and textbook are not allowed.

#### Multiple-choice questions [4 points each | 36 points total]

Select **the one alternative** that best answers the question or completes the statement. Write your answers in the designated places. **Answers not in the designated places** will not receive any credit.

- 1) During 2005 in the United States, consumer confidence increased significantly. Which of the following occurred as a result?
- A) The LM curve shifted up.
- B) The LM curve shifted down.
- C) The IS curve shifted rightward.
- D) The IS curve shifted leftward.
- E) Both IS and LM curve shifted up.

ANSWER:
---------

- 2) Suppose policymakers reduce taxes. Which of the following will occur?
- A) The LM curve shifts up and the economy moves along the IS curve.
- B) The IS curve shifts rightward and the economy moves along the LM curve.
- C) The IS curve shifts leftward and the economy moves along the LM curve.
- D) Both the IS and LM curves shift.
- E) Neither the IS nor the LM curve shifts.

ANSWER:	

- 3) Consider the IS-LM model in which the investment function is  $I = b_0 + b_1 \times Y b_2 \times I$ . Suppose there is a fiscal consolidation, so that T increases and G decreases. Which of the following is a complete list of the variables that must decrease?
- A) consumption
- B) consumption and investment
- C) consumption and output
- D) consumption, output, and the interest rate
- E) consumption, output, and investment

ANSWER:
---------

- 4) Efficiency wage theories suggest that
- A) workers are paid less than their reservation wage.
- B) productivity drops if the wage rate is too low.
- C) firms are resistant to increase wages as unemployment insurance goes up.
- D) unskilled workers have a lower turnover rate than skilled workers.
- E) firms are resistant to increase wages as the labor market tightens.

ANSWER:	

- 5) Imagine that the government implements training programs to increase the skills and productivity of workers. In the matching model, this policy would
- A) shift the labor demand curve outward.
- B) shift the labor demand curve inward.
- C) shift the labor supply curve outward.
- D) shift the labor supply curve inward.
- E) improve the matching function.

ANSWER:
---------

- 6) In the matching model the labor demand is decreasing in the labor market tightness because
- A) a lower tightness increases the job-separation rate.
- B) a lower tightness makes it more difficult to fill vacancies.
- C) a lower tightness makes it more difficult to find jobs.
- D) a lower tightness reduces the recruiter-producer ratio.
- E) none of the above.

ANSWER:	
---------	--

- 7) In the matching model the labor supply is increasing in the labor market tightness because
- A) a lower tightness makes it more expensive to hire producers.
- B) a lower tightness makes it easier to find jobs.
- C) a lower tightness makes it easier to fill vacancies.
- D) a lower tightness reduces the job-finding rate.
- E) a lower tightness reduces the job-separation rate.

- 8) Henry Ford's experiment with efficiency wages resulted in
- A) a dramatic drop in output per worker.
- B) a dramatic reduction in unionization.
- C) a dramatic increase in the layoff rate.
- D) a dramatic reduction in the turnover rate.
- E) no noticeable effects.

- 9) If the production function is  $Y = a \times N$  in the matching model, what is shape of the labor demand curve in the usual (employment, tightness) diagram?
- A) horizontal
- B) vertical
- C) downward sloping
- D) upward sloping
- E) none of the above

ANSWER:	
---------	--

# Problems [32 points each | 64 points total]

For each question, provide all the derivations in the space provided. Answers that are correct but without derivation will only receive very partial credit.

## **Problem 1: Rationing and Frictional Unemployment**

Consider the matching model with a labor force of size H = 1, a matching function  $m = b \times U^{1/2} \times V^{1/2}$ , a recruiting cost of r recruiters per vacancy, a jobseparation rate s, a fixed wage W, and production function a  $\times N^{\alpha}$ .

A) Compute the job-finding rate  $f(\theta)$  and the vacancy-filling rate  $q(\theta)$ . How do  $f(\theta)$  and  $q(\theta)$  depend on  $\theta$ ? What happens to  $f(\theta)$  and  $q(\theta)$  when  $\theta$ =0? Interpret the results.

Name:	

B) Using the assumption that labor market flows are balanced, compute the recruiter-producer ratio  $\tau(\theta)$ . How does  $\tau(\theta)$  depend on  $\theta$ ? What happens to  $\tau(\theta)$  when  $\theta$ =0? Interpret the results.

C) Using the assumption that labor market flows are balanced, compute the labor supply  $L^s(\theta)$ . How does  $Ls(\theta)$  depend on  $\theta$ ? Interpret the results.

D) Express the profits of firms as a function of employment L, the recruiter-producer ratio  $\tau(\theta)$ , and the wage W. Compute the labor demand  $L^d(\theta)$ . How does  $L^d(\theta)$  depend on  $\theta$ ? What happens to  $L^d(\theta)$  when  $\theta$ =0? Interpret the results.

Name:		

E) Find a condition on the parameter a such that  $L^d(0) < 1$ . What happens to rationing unemployment when the condition is satisfied and when the condition is not satisfied?

F) Assume that  $L^d(0) < 1$ . Plot the labor demand and labor supply curve in the usual labor market diagram. Illustrate rationing and frictional unemployments.

G) Using the diagram, determine the effect of a decrease in the parameter a on rationing unemployment, frictional unemployment, total unemployment, and labor market tightness. Interpret the results.

H) Using the diagram, determine the effect of an increase in the parameter r on rationing unemployment, frictional unemployment, total unemployment, and labor market tightness. Interpret the results.

Name:	

- extra space -

## **Problem 2: Policy in IS-LM Model**

Consider the IS-LM model. The government spends G and collects a tax revenue T. Consumption is a linear function of disposable income:  $C(Y-T) = a + b \times (Y-T)$ , where a > 0 and 0 < b < 1. The parameter b is the marginal propensity to consume and the parameter a is autonomous consumption. Investment is a linear function of the interest rate:  $I(i) = c - d \times i$ , where c > 0 and d > 0. The parameter d is the sensitivity of investment to the interest rate, and the parameter c is autonomous investment.

A) Compute total expenditure, *Z*, as a function of Y, i, G, T, and the parameters of the model. What is autonomous spending?

B) Compute equilibrium output in the IS module,  $Y^{\rm IS}$ , as a function of i, G, T, and the parameters of the model. What is the multiplier?

C) How does the slope of the IS curve depend on the parameter d? Explain.

Next suppose that demand for money balances is a linear function of income and the interest rate:  $M^d(Y,i) = e \times Y - f \times i$ , where e > 0 and f > 0. The parameter e is the sensitivity of money demand to income, and the parameter f is the sensitivity of money demand to the interest rate.

D) Suppose that the money supply is fixed at M > 0. Compute equilibrium interest rate in the LM module as a function of Y, M, and the parameters of the model. Interpret the results.

Name:			

E) Now imagine that the central bank wants to keep the interest rate at  $i^{LM} > 0$  for any level of output. Compute the required money supply as a function of Y,  $i^{LM}$ , and the parameters of the model. Interpret the results.

Name:			

F) Imagine that the central bank wants to raise the interest rate  $i^{LM}$  by 1 percentage point. Compute the required change in money supply as a function of Y and the parameters of the model.

G) Plot the IS curve and LM curve in the usual IS-LM diagram. Compute the equilibrium level of output, investment, and consumption as a function of  $i^{LM}$ , G, T, and the parameters of the model.

H) Assume that G increases by \$10. How much should the central bank change  $i^{LM}$  to keep output constant? What happens to consumption and investment after these changes in G and  $i^{LM}$ ?

Name:			

- extra space -

Name:			

- extra space -

#### ECON 1210 MIDTERM EXAM 2 - SOLUTION

### 1. Multiple-Choice Questions

- 1. During 2005 in the United States, consumer confidence increased significantly. Which of the following occurred as a result?
  - C) The IS curve shifted rightward
- 2. Suppose policymakers reduce taxes. Which of the following will occur?
  - B) The IS curve shifts rightward and the economy moves along the LM curve
- 3. Consider the IS-LM model in which the investment function is  $I = b_0 + b_1 \times Y b_2 \times i$ . Suppose there is a fiscal consolidation, so that T increases and G decreases. Which of the following is a complete list of the variables that must decrease?
  - E) consumption, output, and investment
- 4. Efficiency wage theories suggest that
  - B) productivity drops if the wage rate is too low
- 5. Imagine that the government implements training programs to increase the skills and productivity of workers. In the matching model, this policy would
  - A) shift the labor demand curve outward
- 6. In the matching model, the labor demand is decreasing in the labor market tightness because **D**) a lower tightness reduces the recruiter-producer ratio
- 7. In the matching model, the labor supply is increasing in the labor market tightness because **D**) a lower tightness reduces the job-finding rate
- 8. Henry Ford's experiment with efficiency wages resulted in
  - D) a dramatic reduction in the turnover rate
- 9. If the production function is  $Y = a \times N$  in the matching model, what is the shape of the labor demand curve in the usual (employment, tightness) diagram?
  - A) horizontal

### 2. Problem 1

(A) Job finding rate:

$$f(\theta) = \frac{m(U, V)}{U} = b\theta^{1/2}$$

Vacancy-filling rate (also possible to find using  $f(\theta) = \theta q(\theta)$ ):

$$q(\theta) = \frac{m(U, V)}{V} = b\theta^{-1/2}$$

f is increasing in  $\theta$  and q is decreasing in  $\theta$ . Moreover, f(0) = 0 and, as  $\theta \to 0$ ,  $q(\theta) \to \infty$ .

(B) With balanced flows,  $sL=q(\theta)V$ . Moreover, there are r recruiters per vacancy: R=rV. Use L=R+N to deduce

$$s(R+N) = q(\theta)R/r \iff sr(1+N/R) = q(\theta)$$

Using  $\tau(\theta) = R/N$ , we get:

$$q(\theta) - sr = sr/\tau(\theta) \iff \tau(\theta) = \frac{sr}{q(\theta) - sr}$$

Thus

$$\tau(\theta) = \frac{sr}{\theta^{-1/2} - sr}$$

As  $\theta$  increases, the denominator decreases and thus  $\tau(\theta)$  increases. Moreover,

$$\lim_{\theta \to 0} \tau(\theta) = 0$$

(C) With balanced flows (using  $f(\theta)U = q(\theta)V$ ), we have  $sL = f(\theta)U = f(\theta)(1-L)$  and thus:

$$L^{s}(\theta) = \frac{f(\theta)}{s + f(\theta)} = \frac{b\theta^{1/2}}{s + b\theta^{1/2}}$$

As  $\theta$  increases,  $L^s(\theta)$  increases:

$$\frac{\partial L^s(\theta)}{\partial \theta} = \frac{(1/2)\theta^{-1/2}sH}{(s+b\theta^{1/2})^2} > 0$$

(D) Firm's profit writes (use  $L = (1 + \tau(\theta))N$ )

$$a\left(\frac{L}{1+\tau(\theta)}\right)^{\alpha}-WL$$

Profit maximization yields

$$a\alpha \frac{L^{\alpha-1}}{[1+\tau(\theta)]^{\alpha}} = W$$

Thus

$$L^d(\theta) = \left(\frac{a\alpha}{W[1+\tau(\theta)]^{\alpha}}\right)^{1/(1-\alpha)}$$

Since  $\tau(\theta)$  is an increasing function of  $\theta$ ,  $L^d(\theta,T)$  is a decreasing function of  $\theta$ . Moreover, since  $\lim_{\theta\to 0} \tau(\theta) = 0$ 

$$\lim_{\theta \to 0} L^d(\theta) = \left(\frac{a\alpha}{W}\right)^{1/(1-\alpha)}$$

(E)

$$\left(\frac{a\alpha}{W}\right)^{1/(1-\alpha)} < 1 \iff a < \frac{W}{\alpha}$$

Recall the definition of rationing unemployment:

$$U^R = 1 - L^d(0)$$

So if the above condition is satisfied (that is if technology is not important enough in the production process) there is rationing unemployment.

(F) Lecture notes. Also, note that

$$1 + \tau(\theta) = \frac{\theta^{-1/2}}{\theta^{-1/2} - sr} = \frac{1}{1 - sr\theta^{1/2}} = (1 - sr\theta^{1/2})^{-1}$$

Hence we can rewrite labor demand as,

$$L^d(\theta) = \left(\frac{a\alpha[1+\tau(\theta)]^{-\alpha}}{W}\right)^{1/(1-\alpha)} = \left(\frac{a\alpha}{W}(1-sr\theta^{1/2})^{\alpha}\right)^{1/(1-\alpha)}$$

We deduce that

• The labor demand curve intersect the  $\theta$ -axis when  $L^d(\theta) = 0$ , that is when  $1 - sr\theta^{1/2} = 0$ :

$$\theta = \left(\frac{1}{sr}\right)^2$$

• The labor demand curve intersect the L-axis when  $\theta = 0$ , that is when (question D)

$$L = L^d(0) = \left(\frac{a\alpha}{W}\right)^{1/(1-\alpha)}$$

(G) Labor supply is not impacted. As of labor demand, only the position on the L-axis is impacted (see F), negatively. The labor demand rotates left but keeps the same position on the  $\theta$ -axis. Consequently, rationing unemployment increases. Frictional unemployment decreases. And total unemployment increases while tightness goes in the opposite direction. See Figure 1:

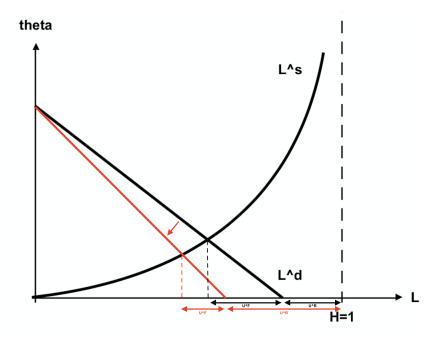


Figure 1: Decrease in a

(H) Labor supply is not impacted. As of labor demand, only the position on the  $\theta$ -axis is impacted (see F), negatively. The labor demand rotates left but keeps the same position on the L-axis. Consequently, rationing unemployment remains unchanged. Frictional unemployment increases. And total unemployment increases while tightness goes in the opposite direction. See Figure 2:

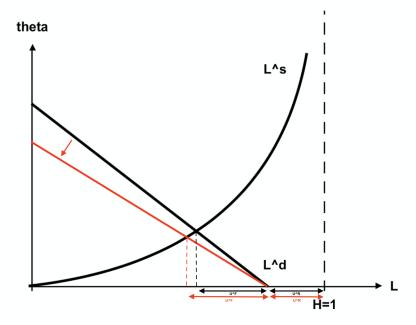


Figure 2: Increase in r

#### 3. Problem 2

(A) Z = C(Y-T) + G + I(i) Z = a + b(Y-T) + G + c - di) Z = a + c + G - bT - di + bY

Where (a + c + G - bT - di) corresponds to autonomous spending. It groups the elements of expenditure which doesn't depend on income.

(B) 
$$Y^{IS} = a + c + G - bT + bY^{IS} - di$$
 
$$Y^{IS}(1-b) = a + c + G - bT - di$$
 
$$Y^{IS} = \frac{a + c + G - bT - di}{1 - b}$$

Then,

$$\frac{\partial Y^{IS}}{\partial (a+c+G-bT)} = \frac{1}{1-b}$$

In words, the autonomous expenditure multiplier is  $\frac{1}{1-b}$ 

(C)  $Y^{IS} = \frac{a+c+G-bT-di}{1-b}$   $(1-b)Y^{IS} = a+c+G-bT-di$   $di = a+c+G-bT-(1-b)Y^{IS}$   $IS: i = \frac{a+c+G-bT-(1-b)Y^{IS}}{d}$ 

In words, the IS curve is a line in the plane (Y, i) with intercept  $\frac{(a+c+G-bT)}{d}$  and slope  $m \equiv \frac{-(1-b)}{d}$  Notice,

$$\frac{\partial m}{\partial d} = \frac{(1-b)}{d^2} > 0$$

Therefore, the IS line becomes flatter when the sensitivity of investment to the interest rate d increases. The slope m captures how much outcome must adjust when the interest rate changes such that the market of goods remains in equilibrium. Hence, our results shows that the higher the sensitivity of investment of the interest rate, the higher this adjustment must be for a given change in the interest rate.

(D)

$$M = M^{d}(Y, i^{*})$$

$$M = eY - fi^{*}$$

$$i^{*} = \frac{eY - M}{f}$$

Then,

$$\frac{\partial i^*}{\partial Y} = \frac{e}{f} > 0$$
$$\frac{\partial i^*}{\partial M} = -\frac{1}{f} < 0$$

Explanation: For a fixed money supply, an increase in income shifts up money demand, which rises the equilibrium interest rate  $i^*$ . In the other hand, an increase in money supply decreases the equilibrium interest rate (graphically, this movement occurs along the demand curve).

(E) From question D we have that

$$i^* = \frac{eY - M}{f}$$

Setting  $i^* > 0$ 

$$\frac{eY-M}{f}>0$$

Then,

Explanation: If the central bank doesn't want to hit the zero lower bound, the money supply must be bounded above by eY. This upper bound for the money supply is increasing in Y. Higher income shifts the money demand up, leaving more space for the money supply to increase without hitting the zero lower bound.

(F) In equilibrium,

$$M^* = eY - fi^{LM}$$

$$\Delta M^* = e\Delta Y - f\Delta i^{LM}$$

$$\Delta M^* = -f\Delta i^{LM}$$

$$\Delta M^* = -0.01f$$

For  $\triangle i^{LM} = 0.01$ ,

Setting  $\triangle Y = 0$ ,

$$IS: i = \frac{a+c+G-bT-(1-b)Y^{IS}}{d}$$

Additionally, the LM curve is described by

$$LM: i = i^{LM}$$

Graph in the (Y,i) plane: IS curve is a line with intercept  $\frac{(a+c+G-bT)}{d}$  and slope  $m \equiv \frac{-(1-b)}{d}$ . LM curve is an horizontal line at  $i=i^{LM}$ .

To compute the equilibrium output  $Y^*$  we use IS and LM curves together.

$$i^{LM} = \frac{a+c+G-bT-(1-b)Y^*}{d}$$
 
$$di^{LM} = a+c+G-bT-(1-b)Y^*$$

$$Y^{*}(1-b) = a + c + G - bT - di^{LM}$$
$$Y^{*} = \frac{a + c + G - bT - di^{LM}}{1-b}$$

The consumption level in equilibrium is described by the following equation

$$C^* = C(Y^* - T)$$

$$C^* = a + b(Y^* - T)$$

$$C^* = a + b(\frac{a + c + G - bT - di^{LM}}{1 - b} - T)$$

$$C^* = a + b(\frac{a + c + G - bT - di^{LM} - T(1 - b)}{1 - b})$$

$$C^* = a + b(\frac{a + c + G - T - di^{LM}}{1 - b})$$

(H) From previous question we have

$$Y^* = \frac{a+c+G-bT-di^{LM}}{1-b}$$

Taking differences in endogenous variables  $(Y^*)$  and policy variables  $(G, T, i^{LM})$ ,

$$\Delta Y^* = \frac{\Delta G - b\Delta T - d\Delta i^{LM}}{1 - b}$$

The level of taxes doesn't change and the question requires no change in output. Then,  $\Delta T = 0$  and  $\Delta Y^*$ . Therefore,

 $0 = \frac{\Delta G - d\Delta i^{LM}}{1 - b}$ 

Then,

 $\triangle G = d \triangle i^{LM}$ 

Or

 $\Delta i^{LM} = \frac{\Delta G}{d}$ 

Since  $\triangle G = 10$ 

$$\Delta i^{LM} = \frac{10}{d}$$

To analyze the impact of this policy on consumption, it is sufficient to note that C = a + b(Y - T). Since Y and T don't change, consumption remains the same. We can also use the previous question result

$$C^* = a + b(\frac{a + c + G - T - di^{LM}}{1 - b})$$

Taking differences in endogenous variables  $(Y^*)$  and policy variables  $(G, T, i^{LM})$ ,

$$\Delta C^* = b(\frac{\Delta G - \Delta T - d\Delta i^{LM}}{1 - b})$$

Again,  $\triangle T = 0$ .

$$\Delta C^* = b(\frac{\Delta G - d\Delta i^{LM}}{1 - b})$$

Since  $\triangle G = d \triangle i^{LM}$ ,

$$\Delta C^* = b(\frac{d\Delta i^{LM} - d\Delta i^{LM}}{1 - b})$$
$$\Delta C^* = 0$$

To analyze change in investment, we start from investment function and take differences

$$I(i) = c - di^{LM}$$

Since 
$$\triangle i^{LM}=\frac{\triangle G}{d}$$
 
$$\triangle I(i)=-d\triangle i^{LM}$$
 
$$\triangle I(i)=-\triangle G$$
 Thus, 
$$\triangle I(i)=-10$$

Summary: to maintain income unchanged, the central bank must increase the interest rate by  $\frac{10}{d}$ . This produces a decrease in investment which completely offset the increase in government expenditure ( $\Delta I(i) = -\Delta G$ ). Consumption is not affected.