**Problem Set 2**

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**solutions**

**Problem 1**

Suppose that the economy is modeled with the following behavioral equations:

* C = 160 + 0.6 × D
* I = 150
* G = 150
* T = 100
* NX = 0

This is a typical IS submodel. The notation is the following: C is consumption, D is disposable income, I is investment, G is government spending, T is taxes net of transfers, and NX are net exports.

A) Interpret the consumption function C(D).

The consumption function is a linear relation between disposable income and consumption. The propensity to consume is 0.6, so people would consume 60 cents more for each additional dollar of income. If the disposable income D is zero, people would consume $160: with or without income, people would still need to eat.

B) Compute the expenditure function Z(Y).

The expenditure function is given by

400 is autonomous expenditure, and 0.6 × Y is expenditure out of income.

C) The equilibrium of this IS submodel is described by two conditions: Z=Z(Y) and Z=Y. Using these two equations, solve for equilibrium income Y\*.

The equilibrium is obtained by substituting Z=Y into Z=Z(Y):

D) Solve for equilibrium disposable income D\* and equilibrium consumption C\*.

Now that we know the equilibrium level of Y, we can compute the equilibrium level for the disposable income as

For consumption, using the consumption function, we obtain

E) Plot the equilibrium diagram for this IS submodel (with Y on the x-axis and Z on the y-axis). Indicate in particular Y\* and Z(Y). Explain the construction of the diagram.

The diagram includes the two equilibrium conditions: Z=Y, in green, which is just the 45 degree line; and Z=Z(Y), which is the expenditure function, in red. The equilibrium is at the intersection of the two equilibrium conditions.



**Problem 2**

Recall that we define T as taxes net of transfers: T = taxes - transfer payments. Suppose that the government increases transfer payments to private households, but these transfer payments are not financed by tax increases, and they are not financed by reducing government spending. Instead, the government borrows to pay for the transfer payments. We denote by T' the new taxes net of transfer payments.

A) Is T' bigger or smaller than T?

T' will be smaller because transfer payments are higher.

B) In the IS submodel studied in lecture, what is the effect of the increase in transfer payments on the expenditure function Z(Y)? Explain.

The increase in transfer payments will increase autonomous expenditure (by increasing the disposable income), thus generating a shift upward of the expenditure function (see graph for the question D).

C) In the same IS submodel, what is the effect of the increase in transfer payments on equilibrium income Y\*? Explain.

An upward shift in the expenditure function increases equilibrium income Y\*, as the intersection of the two equilibrium conditions happen at a higher level of income (see graph for the question D).

D) Illustrate the effect of the increase in transfer payments in the usual IS equilibrium diagram. Explain in particular how and why the policy affects equilibrium income.



The policy affects equilibrium income because of the increase in disposable income, and hence the increase in autonomous expenditure. Furthermore, through the multiplier effect, the final increase in income is larger than the initial increase in autonomous expenditure:

* the initial increase corresponds to A–B in the diagram. Then, an increase in expenditure increases incomes by the same amount: this is B–C.
* thanks to the multiplier effect, the initial increase in income translates into higher consumption expenditure through the marginal propensity to consume: this is C–D in the diagram. This increase in expenditure again increases income by the same amount: this is D–E. And so on until the new Y\* is reached.

E) Suppose now that the government pays for the increase in transfer payments with a commensurate increase in taxes. How does the increase in transfer payments affect equilibrium income in this case?

In this case, the policy has no effect on the equilibrium: taxes net of transfer payments does not change, because the increase in transfer payments is perfectly compensated by the increase in taxes. Hence disposable income and autonomous expenditure do not change.

F) Now suppose that the population includes two kinds of people: one-half with high propensity to consume and one-half with low propensity to consume. Both groups receive the same income. Compute the aggregate expenditure function in that case. (The aggregate expenditure function is the sum of the expenditure functions for each group in the population.)

Consider first the part of the population with low propensity to consume. The consumption function for them will be

where is the level of consumption when the disposable income is zero, and the disposable income is . As stated in the problem, , i.e. half of the population has a lower propensity to consume.

For the other half of the population, the consumption function will be

where is the level of consumption when the disposable income is zero, and the disposable income is . As stated in the problem, , i.e. half of the population has a higher propensity to consume.

The aggregate expenditure can be computed as

where the first term in brackets is the *autonomous expenditure* and the second is the *consumers’* *expenditure out of income*.

G) Suppose the policy increases taxes on those with low propensity to consume by ∆ in order to increase transfers to people with high propensity to consume by ∆. How would this policy affect the aggregate expenditure function? Explain.

People with low propensity to consume will have lower disposable income, while people with higher propensity to consume will have more income to spend. As a result, the income transfer increases the propensity to consume for the economy as a whole.

To see this in the model, look at the consumption functions first, where the taxes net of transfer increase from to for low MPC people and decrease from to for high MPC people:

then sum them to obtain:

and it’s easy to see that

And given that and , then and .

H) How would the policy in G) affect equilibrium output Y\* in the IS submodel?

The equilibrium income will be higher. In the economy autonomous expenditure will be higher, causing a shift upward in the expenditure function.

On the contrary, if we would have taxed more people with high MPC, and taxed less people with lower MPC, we would have caused a downward shift of the expenditure function.

I) Represent the policy in G) and H) on the standard IS equilibrium diagram. Describe all the elements in the diagram.



The diagram features the two, standard equilibrium conditions (the 45 degree line Z=Y, in green, and the expenditure function Z=Z(Y), in red.

Note that the expenditure function is shifting entirely and not changing slope, as the policy is increasing autonomous expenditure, but it is not affecting the coefficient of Y in the expenditure function. This coefficient is (see above for the derivation). The transition dynamics are analogous to those described in section D above.

J) How do you think the propensity to consume might vary across individuals according to income? In other words, how do you think the propensity to consume compares for people with high income and people with low income?

We expect someone with above average income to consume less, as they would turn the extra income into savings. Someone with very low income would instead spend every extra dollar of disposable income. As shown in lecture 8 slides, people belonging to the lowest quintile of the income distribution have a median saving rate of 8.6%, while people belonging to the highest quintile of the income distribution (i.e. the wealthiest individuals) have a much higher saving rate (the median is 23%). For this reason, government transfers are often targeting low-income taxpayers, so to create a larger multiplier effect.

H) Given your answer, do you think tax cuts will be more effective at stimulating output when they are directed toward high-income or toward low-income taxpayers?

Tax cut would be more effective when directed toward low-income taxpayers, as they will spend more of the extra disposable income than high-income taxpayers.

**Problem 3**

In the typical IS submodel we assume that the fiscal-policy variables G and T are independent of income. In the real world, however, this is not the case. Taxes typically depend on income and so tend to be higher when income is higher. In this problem, we modify the IS submodel to examine how this automatic response of taxes can help reduce the impact of changes in autonomous spending on output.

Assume that the economy is modeled with the following equations:

* C = c0 + c1 × D
* T = t0 + t1 × Y, where t1 is between 0 and 1
* D = Y – T
* G and I are both constant.

The notation is the following: C is consumption, D is disposable income, I is investment, G is government spending, and T is taxes net of transfers.

A) Follow the steps presented in lecture to compute the expenditure function Z(Y) in this IS submodel.

The expenditure function is given by

which can be rewritten as

where the first term in brackets is the *autonomous expenditure* and the second is the *consumers’* *expenditure out of income*.

B) Solve for equilibrium income Y\* in this IS submodel.

The equilibrium is obtained by substituting Z=Y into Z=Z(Y), as usual

where the first term is the multiplier, and the rest of the expression is the autonomous expenditure.

C) What is the multiplier here? Is it bigger or smaller than the usual multiplier? Explain your result.

The multiplier is

Given that , then , , and finally

The economy responds less to changes in autonomous spending when is positive. After an increase in autonomous spending, the increase in total taxes (because of the increase in income) tends to reduce disposable income and lessen the increase in output.

D) Why is fiscal policy in this case called an automatic stabilizer?

Because of the automatic effect of taxes on the economy, the economy responds less to changes in autonomous spending than in the case where taxes are independent of income. Since output tends to fluctuate less (hence it is more stable), fiscal policy is called an automatic stabilizer.

Note also how that the stabilization happens through the multiplier: following, for example, a negative shock to autonomous expenditure (say because ), income would be affected in the same proportion . The decrease in income would be reflected in a decrease in expenditure (via a decrease in disposable income and consumption), but to a lesser extent than without income taxes, precisely because of the smaller multiplier.