**Problem Set 3**

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**solutions**

**Problem 1**

Consider a government bond that promises to pay $100 in one year from now.

A) What is the interest rate on the bond if its price today is $75? $85? $95?

The interest rate is computed as

Hence the interest rate is:

* 33.3%, if the price today is $75
* 17.6% if the price today is $85
* 5.3% if the price today is $95

B) What is the relation between the price of the bond P and the interest rate i? Explain.

There is an inverse relation between the two: given that the amount payed by the bond at the end of the period is determined and fixed, the lower the price at which the bond is sold today, the higher is the financial return on that bond: hence, the higher the interest rate.

C) If the interest rate is 8%, what is the price of the bond today?

To answer the question, we rewrite the formula

as P = 100 / (1+i). With an interest rate of i = 8%, the price therefore is

**Problem 2**

This problem examines the implications of allowing investment to depend on output in the IS submodel. (Note that the effect of the interest rate on investment is not examined here). Suppose the IS submodel is characterized by the following equations:

* C = c0 + c1 × D
* D = Y - T
* I = b0 + b1 × Y
* G and T are constant.

The notation is the following: C is consumption, D is disposable income, I is investment, G is government spending, and T is taxes net of transfers. The parameters satisfy c0 > 0, b0 > 0, 1 > c1 > 0, and 1 > b1 > 0.

A) Compute the expenditure function Z(Y).

The expenditure function is given by

Which can be rewritten as

where the first term in brackets is the *autonomous expenditure* and the second is the *consumers’* *expenditure out of income*.

B) Solve for equilibrium output Y\*.

The equilibrium is obtained by substituting Z=Y into Z=Z(Y), as usual

C) What is the value of the multiplier? Is the multiplier smaller or larger than the typical multiplier? How does the relation between investment and output affect the value of the multiplier? Explain.

The multiplier is

Given that , then , which means that (as long as ) we have 1/. Thus, the multiplier here is larger than the typical multiplier in the IS submodel, .

The more investment responds to output (higher b1), the higher the multiplier. This is because a high response of investment amplifies the multiplier effect after an increase in autonomous expenditure: when income increases, firms reinject a high fraction of this additional income as investment expenditure, which leads to higher income in the economy, and so on.

D) For the multiplier to be positive, what condition must (c1 + b1) satisfy? Then, is the multiplier smaller or larger than 1?

The condition for the multiplier to be positive is , which yields

If the above holds, then the multiplier will be larger than one, as the denominator will be (strictly) between zero and one.

E) Suppose that the parameter b0, sometimes called business confidence, increases. How will equilibrium output be affected? Will investment change by more or less than the change in b0? Explain.

The parameter enters the expenditure function in the autonomous expenditure part

which, graphically, is the intercept of the expenditure curve. The result of an increase in business confidence is an upward shift in the expenditure curve, causing an increase in the equilibrium output.



Y\*\*

Investment will increase by more than the increase in :

* The increase in output from the old Y\* to Y\*\* is due to the initial increase in , which is an increase in expenditure (A–B) and thus in income (B–C)
* The increase in output from Y\*\* to the new Y\* is due to the multiplier effect: the initial increase in income (B–C) leads to a further increase in expenditure (C–D), and hence new income, that translate in higher expenditure, and so on until the new equilibrium is reached.

This result can also be seen algebraically from the formula for Y\* derived in B). The formula shows that Y\* = autonomous expenditure × spending multiplier, where the spending multiplier > 1. So when autonomous expenditure increases due to the increase in equilibrium income will increase by more because the spending multiplier is larger than 1.

F) After the increase in business confidence b0, what will happen to aggregate saving (the sum of private saving and public saving)? Explain.

Aggregate saving is going to increase following an increase in business confidence. As shown in class, starting from the income = expenditure equation,

Y = C + I + G

then we subtract T from both sides of the equation to get

Y – T – C = I + (G – T).  
Using the definition of private saving,

S = I + (G – T)  
I = S + (T – G)

which means that investment has to equal aggregate saving (private + public). As investment increases here, both because is higher and because income is higher, aggregate saving has to increase by the same amount.

**Problem 3**

Suppose that a person’s wealth is W, which she allocates between money and bonds. Suppose that her yearly income is Y. Also suppose that her money demand function is given by Md(Y,i)= Y × (0.35 – i), where i is the interest rate.

A) Derive the person's demand for bonds Bd as a function of her wealth W, income Y, and the interest rate i. Interpret.

The demand for bonds would be

We interpret the above equation as:

* Wealth is allocated between money and bonds, such that (like a budget constraint). Hence the demand for bonds is Bd = W – Md. When the money demand increases, the bond demand will decrease, given fixed .
* The amount of money demanded is increasing in income Y, because income determines the amount of transactions conducted by the person and money is used to conduct transactions. This then explains why the bond demand is decreasing in income.
* The higher the interest rate on bonds, the more appealing it is to hold bonds, which explains why the bond demand is increasing in the interest rate.

B) What are the effects of an increase in wealth on her demand for money and her demand for bonds? Explain.

The increase in wealth is not affecting the demand for money (which only depends on income and interest rate), but it is affecting the demand for bonds, which will be higher.

C) What are the effects of an increase in income on her demand for money and her demand for bonds? Explain.

The increase in income will increase the demand for money. Since wealth is fixed, the increase in income will thus decrease the demand for bonds.

D) Consider the statement “When people earn more money, they obviously will hold more bonds.” Is this statement correct or incorrect? Explain.

The statement refers to earning more income; looking at our demand function, having higher income results in a higher demand for money and a lower demand for bonds, as wealth doesn’t change with income. Hence the statement is incorrect.

The idea behind our model of money demand is that more income implies more transactions and thus a higher need for money. Since wealth is kept fixed here, the higher need for money implies a lower demand for bonds.

E) Suppose that the person’s wealth is W = $50,000 and her yearly income is Y = $60,000. Suppose the interest rate increases by 10 percentage points. What is the effect on her demand for money? What is the effect on her demand for bonds?

When the interest rate increases by 10 percentage points, the demand for money will decrease by , while the demand for bonds will increase by the same amount.

**Problem 4**

Consider a LM submodel with financial intermediaries (banks). Assume the following:

* The public holds no currency.
* The ratio of reserves to checkable deposits is 0.1.
* The demand for money is given by Md(Y,i)= Y × (0.8 – 4 × i)

Initially, the supply of central-bank money is Hs = $100 billion and income is Y = $5 trillion.

A) What is the demand for central-bank money, Hd(Y,i)?

The demand for central bank money depends on the reserve-deposit ratio (), and on the demand for money:

B) Find the equilibrium interest rate by setting the demand for central-bank money equal to the supply of central-bank money.

C) Illustrate the equilibrium using a typical LM equilibrium diagram (with central-bank money H on the x-axis and interest rate i on the y-axis).



Demand of central bank money

0.2

= 0.15

= $100 billion

The supply of money is fixed at $100 billion, while the demand is

Note that using we get i=15%.

D) What is the overall supply of money? Is it equal to the overall demand for money at the interest rate you found in part B)?

We assumed that the public holds no currency (C = 0); the money multiplier can be computed as

The overall supply of money is equal to , which is $1 trillion. Since supply and demand of central-bank money are equal, and since demand and supply of money equal the money multiplier times central-bank money, then supply and demand of money are also equal.

E) How would the interest rate change if central-bank money increased to Hs = $300 billion?

The new equilibrium interest rate can be obtained as above

F) If the overall money supply increased to Ms = $4 trillion, what would be the effect on the interest rate i? What would happen to the interest rate if the overall money supply increased even further? Explain.

We saw above that the money multiplier is 10, so the supply of central-bank money is $400 billion when the overall money supply is $4 trillion.

We can now replicate the above computations, obtaining

A further increase in the money supply does not change the interest rate as the zero lower bound (ZLB) has been reached; the equilibrium interest rate couldn’t be negative, or there won’t be any reason to hold bonds. The situation is depicted in the graph below, where the point B corresponds exactly to .

