**Problem Set 4**

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**solutions**

**Problem 1**

Consider the IS-LM model with the following consumption and investment functions:

C = c0 + c1 × (Y – T)

I = b0 + b1 × Y – b2 × i

Assume c1 + b1 < 1, c1 > 0, b1 > 0, and b2 > 0. Also assume that government spending G > 0 and taxes T > 0 are constant.

A) Compute the expenditure function Z(Y,i). What are the autonomous expenditure and spending multiplier?

The expenditure function is given by

Which can be rewritten as

where the first term in brackets is the *autonomous expenditure* and the second term is *expenditure out of income*.

B) Solve for the equilibrium levels of output, investment, and consumption when the interest rate is i = i\*.

The equilibrium is obtained by substituting Z = Y into Z = Z(Y), as usual

Equilibrium investment is

Which can be rewritten, substituting in the expression equilibrium output, as

Equilibrium consumption is

Which can be rewritten, substituting in the expression equilibrium output, as

C) Using an IS diagram and an IS-LM diagram, show the effect on output of a decrease in government spending. Explain how the diagrams are constructed.

We can first have a look at the IS submodel diagram. The decrease in government spending will affect the autonomous expenditure part of the expenditure function, which is the intercept in the graph. The effect is shown below



This is the diagram we used in the past problem sets, with expenditure and income on the two axis and the equilibrium condition Z=Y. In the IS-LM diagram, we replace the expenditure with the interest rate. The curve will be:

* downward sloping, because while higher income was leading to higher expenditure (in the IS submodel), higher interest rates lead to lower income
* this can be seen also by looking at the autonomous expenditure term of the expenditure function, featuring the interest rate with a negative sign: higher interest leads to lower expenditure at any level of income

The construction of curve is demonstrated below.



Looking now at the IS-LM model, we can look at how the decrease in government spending affects the IS curve. The IS curve will shift left, as output will decrease holding interest rate constant.



D) We now go behind the scene in the LM submodel. Assume that the money demand is Md(Y,i) = d1 × Y – d2 × i. Use the equilibrium condition in the LM submodel to determine the level of money supply such that i = i\*. How does the money supply change when government spending decreases?

Given the equilibrium income Y\* we found above, and given that we want i=i\*, the money supply should be such that

When government spending decreases, money supply will decrease as well.

**Problem 2**

Consider a numerical example of the IS-LM model. The consumption function is C = 200 + 0.25 × (Y–T), the investment function is I = 150 + 0.25 × Y – 1000 × i, government spending is G = 250, taxes net of transfers are T = 200, and the interest rate set by the central bank is i = 5%.

1. Derive the expression Y(i) for the IS curve. Interpret.

We are looking for an equation giving output as a function of the interest rate. This equation is obtained by finding equilibrium output in the IS submodel. As usual, equilibrium output satisfies:

As expected, equilibrium output in the IS submodel is a decreasing function of the interest rate – so the IS curve will be downward sloping in the typical IS-LM diagram.

B) What is the equilibrium level of output Y\*?

The equilibrium level of output can be obtained by substituting in the above expression the interest rate set by the central bank:

C) The central bank sets an interest rate of 5%. What is the level of money supply Ms(Y) required to reach that interest rate when output is Y? Use the following money demand: Md(Y,i) = 2 × Y – 8000 × i. What is the level of money supply M\* when the IS-LM model is in equilibrium?

The money supply can be found by using the fact that in equilibrium money supply equals money demand. With an interest rate of 5% and income Y, the money demand is

Hence, to achieve an interest rate of 5% for an income of Y, the money supply must be Ms (Y) = 2 × Y – 400.

When the IS-LM model is in equilibrium, output is Y\* = 1000, so the money supply must be M\* = Ms (Y\*) = 2 × Y\* – 400 = 2 × 1000 – 400 = 1600.

D) Solve for the equilibrium values of consumption C\* and investment I\*, and verify the value you obtained for Y by adding C, I, and G.

The equilibrium values of C and I are

And indeed .

E) Now suppose that the central bank cuts the interest rate to 3%. How does this change the LM curve? Illustrate on an IS-LM diagram.

Cutting the interest rate will shift the LM curve downwards:



F) Solve for the new equilibrium values of output, consumption, and investment. Describe in words the effects of such monetary policy. Is the monetary policy expansionary or contractionary?

The new equilibrium values can be obtained as we did above, using the new interest rate. They are: Y\* = 1040, C\* = 410, and I\* = 380.

Cutting the interest rate is an expansionary monetary policy: borrowing becomes cheaper, promoting investments and hence increasing output.

G) What is the new equilibrium value of the money supply?

The new money supply increases, and it can be obtained as above:

H) Return to the initial situation in which the interest rate set by the central bank is 5%. Now suppose that government spending increases to G = 400. Calculate the effects of such expansionary fiscal policy on output, consumption, investment, and money supply.

The new equilibrium values can be obtained as we did above, using the initial interest rate and the higher government spending. They are: Y\* = 1300, C\* = 475, and I\* = 425.

The new money supply is

**Problem 3**

Consider the IS-LM model. The government spends G and collects a tax revenue T. Consumption is a linear function of disposable income: C(Y–T) = a + b × (Y – T), where a > 0 and 0 < b < 1. The parameter b is the marginal propensity to consume and the parameter a is autonomous consumption. Investment is a linear function of the interest rate and income: I(Y,i) = c – d × i + e × Y, where c > 0, d > 0, and 0 < e < 1 – b. The parameter d is the sensitivity of investment to the interest rate, the parameter c is autonomous investment, and the parameter e is the marginal propensity to invest.

A) Compute total expenditure, Z, as a function of Y, i, G, T, and the parameters of the model. What is autonomous spending?

The expenditure function is given by

Which can be rewritten as

where the first term in brackets is the *autonomous expenditure* and the second term is *expenditure out of income*.

B) Compute equilibrium output in the IS module, YIS, as a function of i, G, T, and the parameters of the model. What is the spending multiplier?

The equilibrium is obtained by substituting Z=Y into Z=Z(Y), as usual

The spending multiplier is .

C) How does the slope of the IS curve depend on the parameter d? Interpret.

The slope is of the IS curve on the (Y,i) plane is . Note that to obtain the slope we can rewrite the equation as i(Y) and isolate the coefficient of Y. So when d increases the IS curve becomes flatter.

The parameter d indicates how investment reacts to a change in interest rate. When the parameter d is higher, changes in i will affect investment more, and hence they will affect equilibrium output in the IS submodel more. This will be reflected by a flatter IS curve.

Next suppose that demand for money balances is a linear function of income and the interest rate: Md(Y,i) = e × Y – f × i, where e > 0 and f > 0. The parameter e is the sensitivity of money demand to income, and the parameter f is the sensitivity of money demand to the interest rate.

D) Suppose that the money supply is fixed at M > 0. Compute equilibrium interest rate in the LM module as a function of Y, M, and the parameters of the model. Interpret the results.

The equilibrium interest rate can be obtained by solving money supply = money demand:

The equilibrium interest rate depends on the money supply and output in the usual way. Higher money supply leads to a lower interest rate, but higher income leads to a higher interest rate.

E) Now imagine that the central bank wants to keep the interest rate at iLM > 0 for any level of output. Compute the required money supply as a function of Y, iLM, and the parameters of the model. Interpret the results.

The required money supply is

Which is:

* Higher if income increases
* Lower if the interest rate increases.

F) Imagine that the central bank wants to raise the interest rate iLM by 1 percentage point. Compute the required change in money supply as a function of Y and the parameters of the model.

The increase can be expressed as

So, the new money supply will be lower by 0.01×f.

G) Plot the IS curve and LM curve in the usual IS-LM diagram. Compute the equilibrium level of output, investment, and consumption as a function of iLM, G, T, and the parameters of the model.

The equilibrium level of output when the central bank fixed the interest rate at is

And

H) Assume that T increases by $10. How much should the central bank change iLM to keep output constant? What happens to consumption and investment after these changes in T and iLM?

In order to keep output constant, we need a change in the interest rate that compensate the negative effect of the tax increase in the autonomous expenditure. Namely:

(To verify this, if we plug in this delta in the equilibrium output expression, we get

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which is the equilibrium output before the tax increase.)

As to consumption and investment, we need to recompute them using the new taxes and new interest rate (output does not change, however):

We have a decrease in consumption, caused by higher taxes and thus lower disposable income. And we have an increase in investment, caused by the lower interest rate. In the end, the increase in investment compensates the decrease in consumption so that output stays constant.