**Problem Set 5**

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**solutions**

**Problem 1**

Consider a matching model of unemployment with labor force of size H, a matching function b × U2/3 × V1/3 (where b > 0, U is number of unemployed workers, and V is number of vacancies posted by firms), a recruiting cost of r > 0 recruiters per vacancy, a job-separation rate s > 0, a fixed wage W > 0, and a production function a × Nα  (where a > 0, N is the number of producers in the firm, and 0 < α < 1). Assume that firms must pay a payroll tax T > 0. As a consequence, the after-tax wage paid by firms is (1+T) × W, and the labor cost incurred by the firm is (1+T) × W × L, where L is the number of workers in the firm.

A) Compute the job-finding rate f(θ). Is f(θ) increasing or decreasing in θ? What happens to f(θ) when θ = 0 and when θ = +∞? Interpret.

The job-finding rate is

Which is increasing in . When labor market tightness is lower, it takes longer to find a job because there are a lot of jobseekers relative to vacancies, and competition for jobs among workers is strong.

When , there are no vacancies or a very high number of unemployed workers, so the job-finding rate is zero. When , the job-finding rate tends to infinity, everyone will find a job almost immediately because there are infinitely many vacancies or almost no other jobseekers.

B) Compute the vacancy-filling rate q(θ). Is q(θ) increasing or decreasing in θ? What happens to q(θ) when θ = 0 and when θ = +∞? Interpret.

The vacancy-filling rate is

Which is decreasing in . When labor market tightness is higher, it takes longer to fill a vacancy. The reason is that there are a lot of vacancies posted relative to jobseekers, and competition for workers among firms is strong.

When , There are no vacancies or a very high number of unemployed workers, so the vacancy-filling rate tends to infinity: it’s easy for a firm to fill a vacancy as soon as there is one, given the high number of unemployed people. When , The vacancy-filling rate tends to zero: there are too many vacancies with respect to the number of unemployed workers, and the competition for firms to find a worker will be very high.

C) Using the assumption that labor market flows are balanced, compute the recruiter-producer ratio τ(θ). Is τ(θ) increasing or decreasing in θ? What happens to τ(θ) when θ = 0? Interpret.

s × L jobs are destroyed each month; assuming balanced flows, s × L jobs need to be created. We know that vacancies are filled with probability q(θ), so if V vacancies are posted, q(θ) × V jobs are created. To fill s × L jobs, it is therefore necessary to post a number V = L × s / q(θ) of vacancies. V vacancies require r × V recruiters, so the number of recruiters is R = r × s × L/ q(θ).

Starting from this last equality, we can derive:

The recruiter-producer ratio τ(θ) is increasing in θ. The reason is that when tightness is higher, it is more difficult to fill vacancies, so firms have to allocate more workers to recruiting. When , : vacancies are filled almost immediately so almost no recruiters are needed to fill the firm’s vacancies.

D) Let θm be the value of the labor market tightness such that q(θm) = s × r. Compute θm . What happens to τ(θ) when θ = θm? What happens in the labor market when θ = θm? Would policymakers want to stimulate the labor market so much that θ = θm?

This particular value of is the one that makes the fraction of vacancies filled in a month equal to the fraction of jobs lost times the number of recruiters needed to fill a vacancy (recruiting costs). We can obtain it as

At this level of tightness,

As tends to infinity, all the workers in firms are recruiters: there are no producers left in the labor market. At this tightness, it takes so long to fill vacancies that firms have to allocate all their workers to recruiting in order to maintain their size. Then, firms are composed of recruiters who recruit other recruiters.

Of course, policymakers would not want to increase tightness so much that the situation θ = θm is reached. When all workers are recruiters, there are no producers, so the economy is not producing anything, and people have nothing to consume (although many people are employed). This explains why policymakers do not want to push the unemployment rate all the way to 0%: they are happy with unemployment at 3% or 4% instead.

E) If b increases, what happens to f(θ)? If b increases, what happens to q(θ)? If b increases, what happens to τ(θ)? Interpret.

As b increases, both f(θ) and q(θ) will increase, while τ(θ) will decrease. The parameter b captures the efficacy of matching on the labor market. A market with better matching will have higher vacancy-filling rate and higher job-finding rates. Such a market will also have lower recruiter-producer ratio as firms will need fewer recruiters to fill the vacancies.

F) If s increases, what happens to τ(θ) and to θm? If r increases, what happens to τ(θ) and θm? Interpret.

When the job-separation rate s (fraction of employed workers who lose their jobs in a month) increases, τ(θ) will increase and will decrease. Indeed, when more workers leave the firm, more vacant jobs need to be replaced, which requires more recruiters.

If the recruiting cost r increases, τ(θ) will increase again, and will decrease. Indeed, when the recruiting cost goes up, firms need more recruiters per vacancy, so the ratio of recruiters relative to producers will necessarily increase.

G) Using the assumption that labor market flows are balanced, compute the labor supply Ls(θ). What happens to Ls(θ) when θ=0 and when θ=+∞?

Starting from our assumption that labor market flows are balanced, we can derive the labor supply as

When θ = 0, : the labor supply to zero. When θ = +∞ , f(θ) = H: the labor supply will tend to H, the whole labor force.

H) Take the derivative of Ls(θ) with respect to θ. Is Ls(θ) increasing or decreasing in θ? Interpret.

The derivative indicates the change in labor supply for a unit change in θ:

Which simplifies to

We know that the derivative is positive because H > 0, s > 0, f is increasing in θ so f’(θ) > 0, and the denominator is of course positive. We infer that Ls is increasing in θ. The reason is that when tightness is higher, unemployed workers find jobs more easily, which leads to more workers being employed.

Although it is not necessary, we can also compute the derivative of the labor supply in the specific case here. Given that and ,

This confirms again that labor supply is increasing in tightness.

I) If the payroll tax T increases, what happens to Ls(θ)? If s increases, what happens to Ls(θ)? If b increases, what happens to Ls(θ)? Interpret.

The labor supply does not depend on the payroll tax. If the job-separation rate s goes up, the labor supply decreases. Indeed, when workers lose their jobs at a higher rate, fewer people are being employed. If b increases, the labor supply increases. (You can see that by writing the labor supply as follows:

When b goes up, s/b in the denominator goes down, so Ls goes up.) The intuition is that when matching is more efficacious on the labor market, people find jobs at a faster rate, so more people are being employed.

J) Express the profits of a firm as a function of the number of producers N, the recruiter-producer ratio τ(θ), the wage W, and the payroll tax T. Then compute the optimal number of producers that a firm would hire to maximize profits.

The profits of a firm can be computed as the output minus the labor costs:

We maximize the profits by computing the first derivative of the profit function with respect to N, and setting it equal to zero:

K) Using your answer to J), compute the labor demand Ld(θ). Is Ld(θ) increasing or decreasing in θ? What happens to Ld(θ) when θ = 0 and when θ = θm? Interpret.

The labor demand is just

Using the results that we found in the previous parts of the problem, given that is increasing in , the labor demand will be decreasing in . When , will tend to zero as well and the labor demand will be

On the other hand, when θ = θm, will tend to infinity and the labor demand will tend to zero.

The labor demand is decreasing in tightness because when the tightness is higher, it is more difficult to fill vacancies, so firms need to employ more recruiters, which makes it less profitable to employ workers.

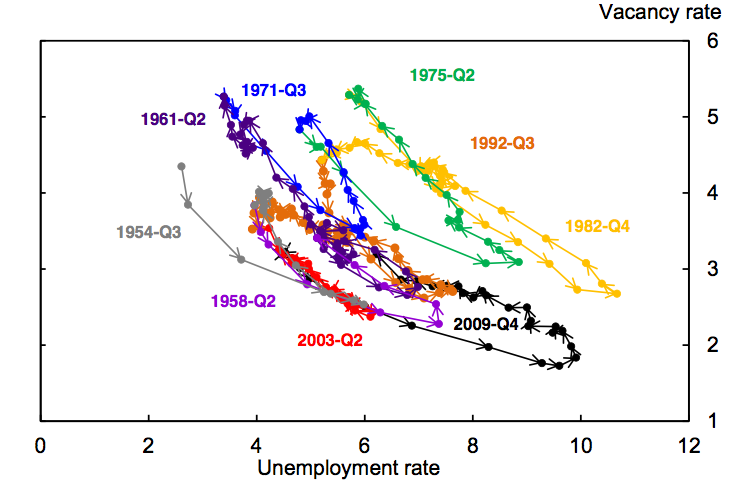
L) If the payroll tax T increases, what happens to Ld(θ)? If s increases, what happens to Ld(θ)? If r increases, what happens to Ld(θ)? If b increases, what happens to Ld(θ)? If W increases, what happens to Ld(θ)? Interpret.

An increase in T would lead to a decrease in labor demand, as firm would face higher labor costs with the higher taxes (the post-tax wage paid by the firm is (1+T)W). An increase in s leads to an increase in (see above) and hence to a decrease in the labor demand; the same happens with an increase in r; an increase in s or r forces firms to have a higher recruiter-producer ratio, which reduces the profitability of employing workers, and depresses labor demand. When matching is better and b increases, decreases and the labor demand increases; an increase in b allows firms to have a lower recruiter-producer ratio, which raises the profitability of employing workers and stimulates labor demand. Finally, an increase in wages leads to a decrease in labor demand. This is simply because a higher wage raises labor costs and makes it less profitable to employ workers.

M) Plot labor demand and labor supply curves in an employment-tightness diagram. Indicate what happens at θ = 0 and θ = θm. Also display the size of the labor force, H, and unemployment.



**Problem 2**

This is the Beveridge curve for the US labor market:

1. Describe the relationship that you see between vacancy rate and unemployment rate? Is the Beveridge curve stable over time or is it shifting a lot?

The curve describes a negative relationship between unemployment rate and vacancy rate: when the unemployment rate is high, the vacancy rate is low.

The Beveridge curve is fairly stable, although it has shifted over time. Furthermore, during each business cycle (indicated by a specific color), it looks like the Beveridge curve follows a counter-clockwise loop, instead of being on a straight line.

B) Consider a matching model of unemployment with labor force H > 0 and a Cobb-Douglas matching function m(U,V) = a × U1/2 × V1/2 , where a > 0, U is number of unemployed workers, and V is number of vacancies posted by firms. Assuming that labor market flows a balanced, compute an equation that relates the unemployment rate u = U/H to the vacancy rate v = V/H.

The unemployment rate can be computed, starting from the balanced market flows assumption, as

And using the specific functional form given, we get so that

C) According to the equation that you derived in B), what happens to the vacancy rate when the unemployment rate goes up? Is that consistent with your description of the Beveridge curve in A)?

When the unemployment rate u goes up, the left-hand side of the equation goes down, so the vacancy rate v must go down. This is exactly what happens along the Beveridge curve: when the unemployment rate goes up, the vacancy rate goes down. In fact, the equation in B) is the equation for the Beveridge curve in our matching model of unemployment.

D) According to the equation that you derived in B), what type of shocks would lead to shifts of the Beveridge curve? Combining this result with your description of the Beveridge curve in A), what do you learn about the types of shocks that influence the US labor market?

The equation from B) implies that shocks to the job-separation rate s and matching efficacy a would lead to shifts of the Beveridge curve.

Give that the Beveridge curve is stable over a business cycle but moves slowly over longer periods, we infer that changes in the job-separation rate of matching efficacy do not occur during a business cycle but may occur over longer time periods (say 10 or 20 years).

**Problem 3** [each question: 5 points|total: 15 points]

Consider a matching model of unemployment with labor force of size H, a matching function b × U1/2 × V1/2 (where b > 0, U is number of unemployed workers, and V is number of vacancies posted by firms), a recruiting cost of r > 0 recruiters per vacancy, a job-separation rate s > 0, a fixed wage W > 0, and a linear production function a × N (where a > 0 is productivity and N is the number of producers in the firm).

A) Following the usual steps, compute the job-finding rate f(θ), vacancy-filling rate q(θ), and recruiter-producer ratio τ(θ).

Steps are shown in problem 1 above. Given the functional form provided,

B) Express the profits of a firm as a function of the number of producers N, the recruiter-producer ratio τ(θ), and the wage W. Take the derivative of the profits with respect to N and set the derivative to 0; compute the value of θ implied by this zero-derivative condition.

The profits of a firm can be computed as the output minus the labor costs:

We maximize the profits by computing the first derivative of the profit function with respect to N, and setting it equal to zero. We obtain

Which can be written as τ(θ)= W/a – 1. Using the expression for the recruiter-producer ratio, we obtain

Hence the tightness implied by the zero-derivative condition is θ = [b × (1-W/a)/(r × s )]2.

C) The value of θ computed in B) corresponds to the labor demand when the production function is linear. How would the labor-demand curve look like in the typical labor-market diagram with employment L on the x-axis and tightness θ on the y-axis (draw the diagram and explain)? How is this labor-demand curve different from the labor-demand curve that we saw in lecture (plot the typical labor-demand curve in the same diagram and explain)? How would the labor-demand curve shift if productivity a increased? How would the labor-demand curve shift if the wage W increased? Interpret these shifts.

Since the labor demand determines a fixed level of tightness for any employment level, the labor-demand curve would be horizontal. This is different from what we saw in lecture, where the labor-demand curve was downward sloping in the labor-market diagram with employment L on the x-axis and tightness θ on the y-axis. If productivity a increased, 1-W/a would increase so the labor-demand curve would shift up. In this case, it is more profitable to employ workers so firms tolerate a higher tightness. If the wage W increased, 1-W/a would decrease so the labor-demand curve would shift down. In this case, it is less profitable to employ workers so firms only tolerate a lower tightness.



