Integral of the week
- Week 2 -

Halvard Sutterud

March 14, 2017

$$I = \int_0^1 \left[\ln \frac{1}{x}\right]^n dx : n \in \natural$$

$$\begin{aligned} u &= \ln \frac{1}{x} \\ x &= e^{-u} \\ dx &= -e^{-u} du \\ u(0) &= \infty \\ u(1) &= 0 \end{aligned}$$

Using recursive partial integration:

$$\begin{aligned} I = \int_0^\infty u^n e^{-u} du = \Big[-u^n e^{-u} \Big]_0^\infty - \int_0^\infty -n u^{n-1} e^{-u} du \\ &= \int_0^\infty n u^{n-1} e^{-u} du \\ &= \int_0^\infty n(n-1) u^{n-2} e^{-u} du \\ &= \quad \dots \\ &= \int_0^\infty n! e^{-u} du \\ &= n! \Big[-e^{-u} \Big]_0^\infty \\ &= \underline{\underline{n!}} \end{aligned}$$