# Calculus and Analytical Geometry

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# Continuity of a Function

**10–12** Use the definition of continuity and the properties of limits to show that the function is continuous at the given number $a$.

**10.** $f(x) = x^2 + \sqrt{7 - x}, \quad a = 4$

**11.** $f(x) = (x + 2x^3)^4, \quad a = -1$

**12.** $h(t) = \dfrac{2t - 3t^2}{1 + t^3}, \quad a = 1$

**13–14** Use the definition of continuity and the properties of limits to show that the function is continuous on the given interval.

**13.** $f(x) = \dfrac{2x + 3}{x - 2}, \quad (2, \infty)$

**14.** $g(x) = 2\sqrt{3 - x}, \quad (-\infty, 3]$

**15–20** Explain why the function is discontinuous at the given number $a$. Sketch the graph of the function.

**15.** $f(x) = \ln|x - 2|$ $\qquad a = 2$

**16.** $f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ $\qquad a = 1$

**17.** $f(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geqslant 0 \end{cases}$ $\qquad a = 0$

**18.** $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$ $\qquad a = 1$

**19.** $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$ $\qquad a = 0$

**20.** $f(x) = \begin{cases} \dfrac{2x^2 - 5x - 3}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$ $\qquad a = 3$

**29–30** Locate the discontinuities of the function and illustrate by graphing.

**29.** $y = \dfrac{1}{1 + e^{1/x}}$

**30.** $y = \ln(\tan^2 x)$

**42.** Find the values of $a$ and $b$ that make $f$ continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 < x < 3 \\ 2x - a + b & \text{if } x \geqslant 3 \end{cases}$$

**37–39** Find the numbers at which $f$ is discontinuous. At which of these numbers is $f$ continuous from the right, from the left, or neither? Sketch the graph of $f$.

**37.** $f(x) = \begin{cases} 1 + x^2 & \text{if } x \leqslant 0 \\ 2 - x & \text{if } 0 < x \leqslant 2 \\ (x - 2)^2 & \text{if } x > 2 \end{cases}$

**38.** $f(x) = \begin{cases} x + 1 & \text{if } x \leqslant 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x - 3} & \text{if } x \geqslant 3 \end{cases}$

**39.** $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^x & \text{if } 0 \leqslant x \leqslant 1 \\ 2 - x & \text{if } x > 1 \end{cases}$