Q11-30 Evaluate the limit, if it exists.

11. $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$

$$\lim_{x \to 2} \frac{x^2+x-6}{x-2} = \lim_{x \to 2} \frac{x^2+3x-2x-6}{x-2}$$

$$= \lim_{x \to 2} \frac{x(x+3)-2(x+3)}{x-2}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{x-2}}$$

$$= \lim_{x \to 2} (x+3)$$

$$= 2+3$$

$$= 5$$

12. $\lim_{x \to -4} \frac{x^2+5x+4}{x^2+3x-4}$

$$\lim_{x \to -4} \frac{x^2+5x+4}{x^2+3x-4} = \lim_{x \to -4} \frac{x^2+4x+x+4}{x^2+4x-x-4}$$

$$= \lim_{x \to -4} \frac{x(x+4)+1(x+4)}{x(x+4)-1(x+4)}$$

$$= \lim_{x \to -4} \frac{(x+1)(x+4)}{(x-1)(x+4)}$$

$$= \lim_{x \to -4} \frac{x+1}{x-1}$$

$$= \frac{-4+1}{-4-1}$$

$$= \frac{-3}{-5} = \frac{3}{5}$$

13. $\lim_{x \to 2} \frac{x^2-x+6}{x-2}$

limit not exists.

14. $\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4}$

$$\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4} = \lim_{x \to 4} \frac{x(x-4)}{x^2-4x+x-4}$$

$$= \lim_{x \to 4} \frac{x(x-4)}{x(x-4)+1(x-4)}$$

$$= \lim_{x \to 4} \frac{x(x-4)}{(x-4)(x+1)}$$

$$= \lim_{x \to 4} \frac{x}{x+1}$$

$$= \frac{4}{4+1}$$

$$= \frac{4}{5}$$

15. $\lim_{t \to -3} \frac{t^2-9}{2t^2+7t+3}$

$$\lim_{t \to -3} \frac{t^2-9}{2t^2+7t+3} = \lim_{t \to -3} \frac{(t-3)(t+3)}{2t^2+t+6t+3}$$

$$= \lim_{t \to -3} \frac{(t-3)(t+3)}{t(2t+1)+3(2t+1)}$$

$$= \lim_{t \to -3} \frac{(t-3)(t+3)}{(t+3)(2t+1)}$$

$$= \lim_{t \to -3} \frac{t-3}{2t+1}$$

$$= \frac{-3-3}{2(-3)+1}$$

$$= \frac{-6}{-6+1}$$

$$= \frac{-6}{-5}$$

$$= \frac{6}{5}$$

16. $\lim_{x \to -1}$

lim

17. $\lim_{h \to 0}$

$\lim_{h \to 0}$ (

18.

16. $\lim_{x \to -1} \frac{x^2 - 4x}{x^2 - 3x - 4}$

limit does not exists.

17. $\lim_{h \to 0} \frac{(4+h)^2 - 16}{h}$

$$\lim_{h \to 0} \frac{(4+h)^2 - 16}{h} = \lim_{h \to 0} \frac{4^2 + h^2 + 2(4)(h) - 16}{h}$$

$$= \lim_{h \to 0} \frac{16 + h^2 + 8h - 16}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h+8)}{\cancel{h}}$$

$$= \lim_{h \to 0} (h+8)$$

$$= 0 + 8$$

$$= 8$$

18. $\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1} = \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{x^2 + x - x - 1}$$

$$= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{x(x+1) - 1(x+1)}$$

$$= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x+1)(x-1)}$$

$$= \lim_{x \to 1} \frac{x^2+x+1}{x+1}$$

$$= \frac{1^2+1+1}{1+1}$$

$$= \frac{3}{2}$$

19. $\lim_{x \to -2} \frac{x+2}{x^3+8}$

$$\lim_{x \to -2} \frac{x+2}{x^3+8} = \lim_{x \to -2} \frac{x+2}{(x+2)(x^2-2x+4)}$$

$$= \lim_{x \to -2} \frac{1}{x^2-2x+4}$$

$$= \frac{1}{(-2)^2-2(-2)+4}$$

$$= \frac{1}{4-(-4)+4}$$

$$= \frac{1}{4+4+4}$$

$$= \frac{1}{12}$$

20. $\lim_{h \to 0} \frac{(2+h)^3-8}{h}$

$$\lim_{h \to 0} \frac{(2+h)^3-8}{h} = \lim_{h \to 0} \frac{2^3+h^3+3(2)(h)(2+h)-8}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8}+h^3+6h(2+h)-\cancel{8}}{h}$$

$$= \lim_{h \to 0} \frac{h^3+12h+6h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h^2+12+6h)}{\cancel{h}}$$

$$= \lim_{h \to 0} h^2+12+6h$$

$$= (0)^2+12+6(0)$$

$$= 12$$

21. $\lim\limits_{t \to 9} \dfrac{9-t}{3-\sqrt{t}}$

$$\lim_{t \to 9} \frac{9-t}{3-\sqrt{t}} = \lim_{t \to 9} \left(\frac{9-t}{3-\sqrt{t}} \cdot \frac{3+\sqrt{t}}{3+\sqrt{t}} \right)$$

$$= \lim_{t \to 9} \frac{(9-t)(3+\sqrt{t})}{(3-\sqrt{t})(3+\sqrt{t})}$$

$$= \lim_{t \to 9} \frac{(9-t)(3+\sqrt{t})}{9+3\sqrt{t}-3\sqrt{t}-\sqrt{t}\sqrt{t}}$$

$$= \lim_{t \to 9} \frac{(9-t)(3+\sqrt{t})}{9-t}$$

$$= \lim_{t \to 9} (3+\sqrt{t})$$

$$= 3+\sqrt{9}$$

$$= 3+3$$

$$= 6$$

22. $\lim\limits_{h \to 0} \dfrac{\sqrt{1+h}-1}{h}$

$$\lim_{h \to 0} \frac{\sqrt{1+h}-1}{h} = \lim_{h \to 0} \left[\frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} \right]$$

$$= \lim_{h \to 0} \frac{(\sqrt{1+h}-1)(\sqrt{1+h}+1)}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h \to 0} \frac{(1+h)+(1)(\sqrt{1+h})-(1)(\sqrt{1+h})-1}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h \to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{1+h}+1)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{1+h}+1}$$

$$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{1+1} = \frac{1}{2}$$

23. $\lim_{x \to 7} \frac{\sqrt{x+2}-3}{x-7}$

$$\lim_{x \to 7} \frac{\sqrt{x+2}-3}{x-7} = \lim_{x \to 7} \left[\frac{\sqrt{x+2}-3}{x-7} \cdot \frac{\sqrt{x+2}+3}{\sqrt{x+2}+3} \right]$$

$$= \lim_{x \to 7} \frac{(\sqrt{x+2}-3)(\sqrt{x+2}+3)}{(x-7)(\sqrt{x+2}+3)}$$

$$= \lim_{x \to 7} \frac{(x+2)+(3)(\sqrt{x+2})-(3)(\sqrt{x+2})-9}{(x-7)(\sqrt{x+2}+3}$$

$$= \lim_{x \to 7} \frac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$$

$$= \lim_{x \to 7} \frac{\cancel{x-7}}{(\cancel{x-7})(\sqrt{x+2}+3)}$$

$$= \lim_{x \to 7} \frac{1}{\sqrt{x+2}+3}$$

$$= \frac{1}{\sqrt{7+2}+3}$$

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3}$$

$$= \frac{1}{6}$$

24. $\lim_{x \to -1} \frac{x^2+2x+1}{x^4-1}$

$$\lim_{x \to -1} \frac{x^2+2x+1}{x^4-1} = \lim_{x \to -1} \frac{x^2+x+x+1}{(x^2+1)(x+1)(x-1)}$$

$$= \lim_{x \to -1} \frac{x(x+1)+1(x+1)}{(x^2+1)(x+1)(x-1)}$$

$$= \lim_{x \to -1} \frac{(x+1)\cancel{(x+1)}}{(x^2+1)\cancel{(x+1)}(x-1)}$$

$$= \lim_{x \to -1} \frac{(x+1)}{(x^2+1)(x-1)}$$

$$= \frac{-1+1}{[(-1)^2+1](-1-1)} = \frac{0}{(1+1)(-2)} = \frac{0}{(2)(-2)}$$

$$= \frac{0}{-4} = 0$$

25.

26.

25. $\lim\limits_{x \to -4} \dfrac{1/4 + 1/x}{4+x}$

$$\lim_{x \to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x \to -4} \frac{\left(\frac{1}{4}\right)\left(\frac{x}{x}\right)+\left(\frac{1}{x}\right)\left(\frac{4}{4}\right)}{4+x}$$

$$= \lim_{x \to -4} \frac{\left(x/4x\right)+\left(4/4x\right)}{4+x}$$

$$= \lim_{x \to -4} \frac{\left(x+4/4x\right)}{x+4}$$

$$= \lim_{x \to -4} \frac{x+4}{4x(x+4)}$$

$$= \lim_{x \to -4} \frac{1}{4x}$$

$$= \frac{1}{4(-4)}$$

$$= -\frac{1}{16}$$

26. $\lim\limits_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right)$

$$\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right) = \lim_{t \to 0} \frac{(t^2+t)-\cancel{t}}{t(t^2+t)}$$

$$= \lim_{t \to 0} \frac{t^2}{t^3+t^2}$$

$$= \lim_{t \to 0} \frac{\cancel{t}\cdot t}{\cancel{t}(t^2+t)}$$

$$= \lim_{t \to 0} \frac{t}{t^2+t} = \lim_{t \to 0} \frac{t}{t(t+1)}$$

$$= \lim \frac{1}{t+1}$$

$$= \frac{1}{0+1} = 1$$

27. $\lim\limits_{x \to 16} \dfrac{4-\sqrt{x}}{16x - x^2}$

$$\lim_{x \to 16} \frac{4-\sqrt{x}}{16x-x^2} = \lim_{x \to 16} \frac{4-\sqrt{x}}{16x-x^2} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}}$$

$$= \lim_{x \to 16} \frac{(4-\sqrt{x})(4+\sqrt{x})}{(16x-x^2)(4+\sqrt{x})}$$

$$= \lim_{x \to 16} \frac{16-x}{x(16-x)(4+\sqrt{x})}$$

$$= \lim_{x \to 16} \frac{1}{x(4+\sqrt{x})}$$

$$= \frac{1}{16(4+\sqrt{16})}$$

$$= \frac{1}{16(4+4)} = \frac{1}{16(8)}$$

$$= \frac{1}{128}$$

28. $\lim\limits_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

$$\lim_{h \to 0} \frac{(3+h)^{-1} - 3^{-1}}{h} = \lim_{h \to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3-(3+h)}{3(3+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3-3-h}{9+3h}}{h}$$

$$= \lim_{h \to 0} \frac{-h/9+3h}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{9+3h} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} -\frac{1}{9+3h} = -\frac{1}{9-3(0)}$$

$$= -\frac{1}{9-0} = -\frac{1}{9}$$

29. $\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

$$\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) = \lim_{t \to 0} \frac{t - t\sqrt{1+t}}{t^2\sqrt{1+t}}$$

$$= \lim_{t \to 0} \frac{\cancel{t}(1 - \sqrt{1+t})}{\cancel{t}.t\sqrt{1+t}}$$

$$= \lim_{t \to 0} \frac{1 - \sqrt{1+t}}{t\sqrt{1+t}}$$

$$= \lim_{t \to 0} \left[\frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} \cdot \frac{1 + \sqrt{1+t}}{1 + \sqrt{1+t}} \right]$$

$$= \lim_{t \to 0} \frac{(1 - \sqrt{1+t})(1 + \sqrt{1+t})}{(t\sqrt{1+t})(1 + \sqrt{1+t})}$$

$$= \lim_{t \to 0} \frac{1 + \cancel{\sqrt{1+t}} - \cancel{\sqrt{1+t}} - (1+t)}{t\sqrt{1+t}\ (1 + \sqrt{1+t})}$$

$$= \lim_{t \to 0} \frac{\cancel{1} - \cancel{1} - t}{(t\ \sqrt{1+t})(1 + \sqrt{1+t})}$$

$$= \lim_{t \to 0} \frac{-\cancel{t}}{\cancel{t}\sqrt{1+t}(1 + \sqrt{1+t})}$$

$$= \lim_{t \to 0} \frac{-1}{\sqrt{1+t}(1 + \sqrt{1+t})}$$

$$= \frac{-1}{\sqrt{1+0}\ (1 + \sqrt{1+0})} = -\frac{1}{1\ (1+1)}$$

$$= -\frac{1}{2}$$

30. $\lim_{x \to -4} \frac{\sqrt{x^2+9} - 5}{x+4}$

$$\lim_{x \to -4} \frac{\sqrt{x^2+9} - 5}{x+4} = \lim_{x \to -4} \frac{\sqrt{x^2+9} - 5}{x+4} \cdot \frac{\sqrt{x^2+9} + 5}{\sqrt{x^2+9} + 5}$$

$$= \lim_{x \to -4} \frac{(\sqrt{x^2+9} - 5)(\sqrt{x^2+9} + 5)}{(x+4)(\sqrt{x^2+9} + 5)}$$

$$= \lim_{x \to -4} \frac{(x^2+9) + 5\sqrt{x^2+9} - 5\sqrt{x^2+9} - 25}{(x+4)(\sqrt{x^2+9} + 5)}$$

$$= \lim_{x \to -4} \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x \to -4} \frac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x \to -4} \frac{x^2-4^2}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x \to -4} \frac{(x-4)(x+4)}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x \to -4} \frac{x-4}{\sqrt{x^2+9}+5}$$

$$= \frac{-4-4}{\sqrt{(-4)^2+9}+5} = \frac{-8}{\sqrt{25}+5}$$

$$= \frac{-8}{5+5} = -\frac{8^{4}}{10_{5}}$$

$$= -\frac{4}{5}$$

Q 39-44 : Find limit, if it exists, if not then explain why.

39. $\lim_{x \to 3} (2x + |x-3|)$

$$\lim_{x \to 3} (2x+|x-3|) = \lim_{x \to 3}(2x) + \lim_{x \to 3} |x-3|$$

$$= 2(3) + \lim_{x \to 3} |x-3|$$

$$= 6 + \lim_{x \to 3} |x-3|$$

$$f(x) = |x-3| = \begin{cases} x = x-3 = 3-3 = 0 \\ -x = -(x-3) = -(3-3) = 0 \end{cases}$$

then,

$$= 6 + 0$$
$$= 6$$

40. $\lim_{x \to -6} \frac{2x+12}{|x+6|}$

limit does not exist.

Explanation:

L.H.L $\lim_{x \to -6} \frac{2x+12}{|x+6|} = \lim_{x \to -6} \frac{2x+12}{-(x+6)}$

$= \lim_{x \to -6} \frac{2\cancel{(x+6)}}{-\cancel{(x+6)}}$

$= -2$

R.H.L $\lim_{x \to -6} \frac{2x+12}{|x+6|} = \lim_{x \to -6} \frac{2x+12}{+(x+6)}$

$= \lim_{x \to -6} \frac{2(x+6)}{x+6}$

$= 2$

L.H.L $\neq$ R.H.L

So, limit does not exist.

41. $\lim_{x \to 05^-} \frac{2x-1}{|2x^3-x^2|}$

L.H.L.

put $x = 0-h$, $h > 0$ & $x \to 05^-$ then $h \to 05^-$

so that

$\lim_{x \to 05^-} \frac{2x-1}{|2x^3-x^2|} = \lim_{h \to 05^-} \frac{2(0-h)-1}{|2(0-h)^3-(0-h)^2|}$

$= \lim_{h \to 05^-} \frac{2(-h)-1}{[2(-h)^3-(-h)^2]}$

$= \lim_{h \to 05} \frac{+2(-h)-1}{2(-h)^3-(-h)^2}$

$= \frac{+2(-5)-1}{2(-5)^3-(-5)^2} = \frac{-10-1}{-250-25}$

$= \frac{-11}{-275} = \frac{1}{25}$

42. $\lim_{x \to -2} \frac{2-|x|}{2+x}$

L.H.L put $x = 0-h$, $h>0$ & $x \to 0$ then $h \to 0$

$$\lim_{x \to -2} \frac{2-|x|}{2+x} = \lim_{h \to -2} \frac{2-|0-h|}{2+(0-h)}$$

$$= \lim_{h \to -2} \frac{2-(-h)}{2+(-h)}$$

$$= \lim_{h \to -2} \frac{2+h}{2-h}$$

$$= \frac{2+(-2)}{2-(-2)}$$

$$= \frac{2-2}{2+2} = \frac{0}{4} = 0 \text{ (exists)}$$

R.H.L.

put $x = 0+h$, $h>0$ & $x \to 0$ then $h \to 0$

$$\lim_{x \to -2} \frac{2-|x|}{2+x} = \lim_{h \to -2} \frac{2-|0+h|}{2+(0+h)}$$

$$= \lim_{h \to -2} \frac{2-(+h)}{2+(h)}$$

$$= \lim_{h \to -2} \frac{2-h}{2+h}$$

$$= \frac{2-(-2)}{2+(-2)}$$

$$= \frac{2+2}{2-2}$$

$$= \frac{4}{0} = \infty \text{ (not exists)}$$

L.H.L $\neq$ R.H.L

So,

limit does not exists

43. $\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

put $x = 0 - h$, $h > 0$ & $x \to 0$ then $h \to 0$
so that,

$$\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{h \to 0^-} \left(\frac{1}{0-h} - \frac{1}{|0-h|}\right)$$

$$= \lim_{h \to 0} \left(\frac{1}{-h} + \frac{1}{h}\right)$$

$$= \lim_{h \to 0} -\frac{2}{h}$$

$$= -\frac{2}{0}$$

$$= -\infty \quad \text{(not exist)}$$

44. $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

put $x = 0 + h$, $h > 0$ & $x \to 0$ then $h \to 0$
so that

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{h \to 0} \left(\frac{1}{0+h} - \frac{1}{|0+h|}\right)$$

$$= \lim_{h \to 0} \left(\frac{1}{h} - \frac{1}{h}\right)$$

$$= \lim_{h \to 0} (0)$$

$$= 0 \quad \text{(exists)}$$