Calculus and Analytical Geometry

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One Side Limit of a Function

25–32 Determine the infinite limit.

25. $\lim_{x \to -3^+} \dfrac{x+2}{x+3}$

26. $\lim_{x \to -3^-} \dfrac{x+2}{x+3}$

27. $\lim_{x \to 1} \dfrac{2-x}{(x-1)^2}$

28. $\lim_{x \to 5^-} \dfrac{e^x}{(x-5)^3}$

29. $\lim_{x \to 3^+} \ln(x^2-9)$

30. $\lim_{x \to \pi^-} \cot x$

31. $\lim_{x \to 2\pi^-} x \csc x$

32. $\lim_{x \to 2^-} \dfrac{x^2-2x}{x^2-4x+4}$

LIMIT LAWS Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \qquad \text{and} \qquad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$
4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

11. $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

12. $\lim_{x \to -4} \dfrac{x^2 + 5x + 4}{x^2 + 3x - 4}$

13. $\lim_{x \to 2} \dfrac{x^2 - x + 6}{x - 2}$

14. $\lim_{x \to 4} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$

15. $\lim_{t \to -3} \dfrac{t^2 - 9}{2t^2 + 7t + 3}$

16. $\lim_{x \to -1} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$

17. $\lim_{h \to 0} \dfrac{(4 + h)^2 - 16}{h}$

18. $\lim_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1}$

19. $\lim_{x \to -2} \dfrac{x + 2}{x^3 + 8}$

20. $\lim_{h \to 0} \dfrac{(2 + h)^3 - 8}{h}$

21. $\lim_{t \to 9} \dfrac{9 - t}{3 - \sqrt{t}}$

22. $\lim_{h \to 0} \dfrac{\sqrt{1 + h} - 1}{h}$

23. $\lim_{x \to 7} \dfrac{\sqrt{x + 2} - 3}{x - 7}$

24. $\lim_{x \to -1} \dfrac{x^2 + 2x + 1}{x^4 - 1}$

25. $\lim_{x \to -4} \dfrac{\dfrac{1}{4} + \dfrac{1}{x}}{4 + x}$

26. $\lim_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$

27. $\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

28. $\lim_{h \to 0} \dfrac{(3 + h)^{-1} - 3^{-1}}{h}$

29. $\lim_{t \to 0} \left(\dfrac{1}{t\sqrt{1 + t}} - \dfrac{1}{t} \right)$

30. $\lim_{x \to -4} \dfrac{\sqrt{x^2 + 9} - 5}{x + 4}$

39–44 Find the limit, if it exists. If the limit does not exist, explain why.

39. $\lim_{x \to 3} \left(2x + |x - 3|\right)$

40. $\lim_{x \to -6} \dfrac{2x + 12}{|x + 6|}$

41. $\lim_{x \to 0.5^-} \dfrac{2x - 1}{|2x^3 - x^2|}$

42. $\lim_{x \to -2} \dfrac{2 - |x|}{2 + x}$

43. $\lim_{x \to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{|x|} \right)$

44. $\lim_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{|x|} \right)$

48. Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leqslant 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

(a) Evaluate each of the following limits, if it exists.

(i) $\lim_{x \to 1^-} g(x)$ (ii) $\lim_{x \to 1} g(x)$ (iii) $g(1)$

(iv) $\lim_{x \to 2^-} g(x)$ (v) $\lim_{x \to 2^+} g(x)$ (vi) $\lim_{x \to 2} g(x)$

(b) Sketch the graph of g.