PHSX815_Project4: Fermion Spin Measurements

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1 Introduction

Throughout the semester, I have simulated dice rolling (i.e. sampling from a categorical distribution) for various tasks/experiments, but the simulations were largely non-physics based. For this project, I will use dice rolling to simulate spin measurements of fermions, specifically of leptons. For leptons (spin 1/2 particles), it is expected for there to be an equal chance of being in an up or down state. However, it has been demonstrated that some particles, like neutrinos, have a favored spin state, and do not have an equal chance of being in either state. Here, we develop a hypothesis test to test whether particles are unfavored in their spin state.

2 Overview of Code and Experimental Procedure

For this experiment, we must simulate spin measurements of a fermion (lepton). To do so, we will roll a 2-sided die (same as flipping a coin, just makes more sense to re-use previously written dice code).The details of this dice rolling are below, and the subsequent spin measurement are below:

2.1 Dice Rolling

To simulate dice rolling, we add a method to the Random class, `Random.roll_die(Nsides, weights)`, that returns an integer from 1 to Nsides. We include the ability to roll a biased die with the inclusion of "weights," which allow the user to specify the exact probability of each side of the die being rolled. The algorithm divides the space from 0 to 1 into slices corresponding to each side of the die (either by splitting 0 to 1 evenly, or using the weights as the boundaries for the 'slices'), and then generates a random float from 0 to 1, and returns the side that this random float corresponds to.

This method can be used for any N-sided die, and does not make use of any `Numpy.random` methods.

2.2 Spin Measurement

To simulate a spin measurement, we make use of the `Random.roll_die(Nsides, weights)` method above. If the rolled value is a 1, we return 0 for a down spin-state, and if the rolled value is a 2, we return 1 for an up spin-state. Additionally, we allow the fermion spin state bias to be specified, such that the bias refers to the probability of a down spin-state (0.5 by default).

2.3 Uncertainty

In an experimental setup like the Stern-Gerlach experiment (to measure particle spin), there are a few possible sources of uncertainty in the spin measurements. Primarily, exposure to external electric or magnetic fields can cause one spin state to be favored. As long as the experiment is carried out in a location away from sources of potentially strong electric/magnetic fields, the effects of this should be small and unnecessary to model [1].

2.4 Experimental Design

For this project, we choose to simulate 10^4 experiments for both true fermions and one-sided fermions, but we will vary the the number of measurements per experiment, simulating for 50 measurements and 500 measurements per experiment. We aim to determine the power of a log-likelihood test using each of these sample sizes, where we test the following hypotheses:

Hypothesis $\mathbf{H_0}$: The particles are fermions without any handedness or favored spin state

Hypothesis $\mathbf{H_1}$: The particles are fermions with handedness and a favored spin state

2.5 log-likelihood ratios

For the analysis in this project, we choose to make use of log-likelihood ratios (LLRs) to quantitatively compare the hypotheses. In our analysis, we calculate the LLRs using the following for the Null hypothesis (fair 2-sided die):

$$P(x|H_0) = 1/2 \tag{1}$$

For the probability of data X, given $H1$, we iterate through the simulated data and total the frequency counts of each spin state and weight each total by the total number of measurements:

$$P(x|H_1) = \frac{\text{Num of x in test data}}{\text{total number of measurements}} \tag{2}$$

Then, for any given experiment with N measurements, the LLR becomes:

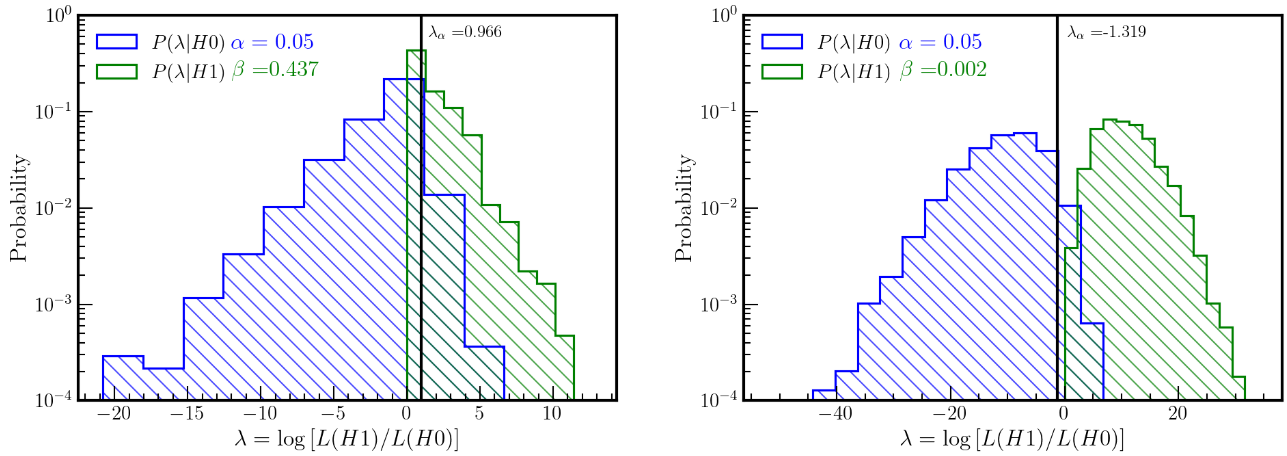
$$LLR = \sum_{i=0}^{N} \ln\left(\frac{P(x_i|H_1)}{P(x_i|H_0)}\right) = \sum_{i=0}^{N} \ln\left(2P(x_i|H_1)\right) \tag{3}$$

Where $P(x_i|H_1)$ is found from Equation (2).

3 Results and Conclusions

We adopt a significance level of $\alpha = 0.05$, and will find the corresponding β value for each set of experiments. We present the results in Figure (1). For both experiment sets, we find the critical likelihood λ_α, which allows the results of future experiments to be assessed. If the log-likelihood ratio of a future experiment is greater than λ_α, we can reject H0 in favor of H1 with 95% confidence. The value of λ_α does not change much with increasing number of measurements.

We note that for large numbers of measurements per experiment,β approaches 0, and thus the test becomes increasingly more powerful.This result makes sense, since for many measurements, the distribution of spin measurements will either look much like the distribution for a unbiased fermion or much different. However, for the experiment set with 50 measurements per experiment, we find $\beta = 0.437$, so the test power is now only 56.3%. With this test then, we would fail to correctly accept that the particles are fermions without handedness 43.7% of the time.



(a) 50 Rolls per Experiment

(b) 500 Rolls per Experiment

Figure 1: Log-Likelihood ratio plots for sets of 10^4 spin experiments with 50 measurements per experiment (left) and 500 measurements per experiment (right). It is evident that, for large numbers of measurements per experiment, β quickly approaches 0, and the LLR test becomes 100% powerful.

References

[1] W. E. Baylis, R. Cabrera, and D. Keselica. "Quantum/Classical Interface: Fermion Spin". In: *arXiv:0710.3144 [quant-ph]* (Oct. 19, 2007). arXiv: `0710.3144`.