Musical Analysis: Analyzing Models to Compare Features of Songs using Spotify's API

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I. ABSTRACT

Music is a field all three of us are very interested in. The main objective of our project is to find out how similar a given set of songs are. We plan to quantify a list of *key characteristics* of the songs, then compile a matrix of relationships between each song in the set based on each characteristic; these qualities namely, ***Key, Mode, Tempo, Energy, Instrumentalness and Speechiness.*** The result will be **six** 24x24 Adjacency Matrices comparing the ***ith*** to the ***jth*** song for each of the *key characteristics,* containing entries greater than 1. Out of six we will choose four of the song qualities as similarity metrics. By adding the four *characteristic (Adjacency) matrices*, we will arrive at an interpretable table of degrees of similarity between each song. There will be a total of fifteen different combinations of these characteristic matrices (*Out of six, choose four = 15*). Through analyzing the Dominance and Power Matrix of each combination, we create a ranking for each song, and compare it to the "genre" groupings of the selected Spotify songs.

II. INTRODUCTION

From the day of Pythagoras, as early as 500 BC (and likely even before), music has held a deeply mathematical structure. Today there are mathematical tools, programming and editing softwares, and even physically-manifested sound altering "pedals" (Peter Frampton's *Wah Wah pedal*!) that allow us to produce very structured and *appealing* songs. For a given set of twenty-four songs (four songs from six different "genres" listed on Spotify) we will extract the six *key* *characteristic* values of each song. If two songs are not similar, then we will analyze the matrices to determine how different those songs are. Our values will range from 0 to 1, with 0 being not at all similar and 1 being very similar, i.e. the value represents a sort of ratio between the values. When the ratios produced are either too small or too large, we will categorize the values by range within 0 to 1. That way, with each entry scaled down in the four characteristic matrices that make up the combination, the entries of the sum will range from 0 to 4. In this way, we can more easily compare the solution sets (the rankings) to the information we know about the songs (genre, characteristic values, music theory).

III. MODEL

We are modeling our data off of public API (developer programs) for individual songs. We will select and quantify six song features to characterize each song, listed below:

- **Key** (0=C, 1=C#, 2=D, 3=D#, 4=E, 5=E#, 6=F, 7=F#, 8=G, 9=G#, 10=A, 11=A#. 12=B)
- **Mode** (Major = 1 or minor = 0)
- **Tempo** (measure of beat; BPM)
- **Energy** (0.0-1.0, intensity)
- **Instrumentalness** (0.0-1.0) [(no vocals) - “ooh, ahh” - rap (strictly vocal)]
- **Speechiness** (0.0-1.0) (less speechy - straight podcast)

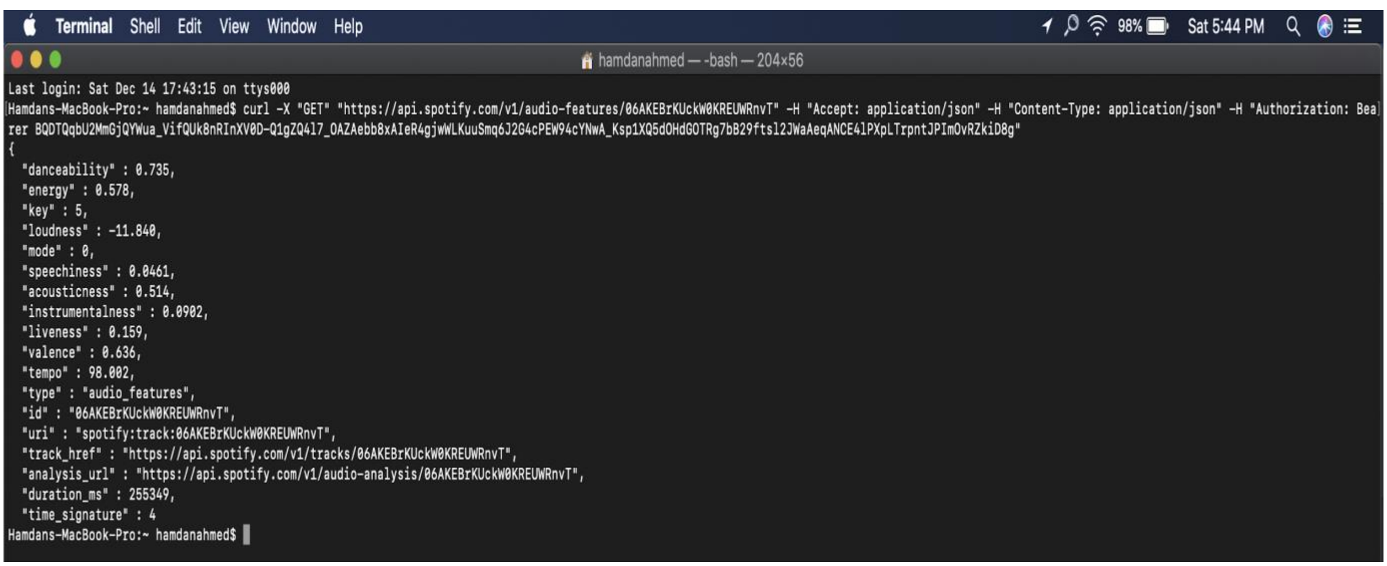
For our song inputs, we will choose four songs from six of Spotify’s predetermined *genres*, as follows:

	"Rock"		**"Classical"**
1	For What It's Worth, Buffalo Springfield	**13**	4 Seasons, Antonio Vivaldi
2	Purple Haze, Jimi Hendrix	**14**	Beethoven 7th Symphony
3	Pinball Wizard, The Who	**15**	Canon in D major

4	Stairway to Heaven, Led Zeppelin	**16**	Toccata and Fugue in D
	"EDM/House"		**"Jazz/Blues"**
5	Piece of my Heart	**17**	Alternate One, Clark Terry
6	In My Mind	**18**	Take Five, The Dave Brubeck Quartet
7	Lose Control (Meduza)	**19**	Sleeping on the Blacktop, Colter Wall
8	Summer Days	**20**	Mia and Sebastian's Theme, Kyle Landry
	"Hip-Hop/Rap"		**"Pop"**
9	One Dance, Drake	**21**	Closer, The Chainsmokers
10	Own It, Stormzy	**22**	Senorita, Shawn Mendes
11	HUMBLE., Kendrick Lamar	**23**	I Like It, Cardi B
12	Numb, Linkin Park	**24**	I Gotta Feeling, The Black Eyed Peas

By selecting from "distinguished" genres, we will be able to compare our results to *both* our opinions and Spotify's.

<u>EXTRACTION OF FEATURES OF EACH SONG</u>



We used Spotify's API and Spotify's Desktop Client to extract features of the songs and wrote RESTFUL bash commands to extract the data for each song

The aim of our calculations is to find relative similarities for each song to the remaining songs in the set. By choosing four characteristics of the six, we may find four corresponding 24 x 24 adjacency matrices for each of the fifteen combinations ("out of 6, choose 4" = 15). After summing the four character matrices of each combination we will arrive with a matrix of entries comparing the *ith* and *jth* songs by those four metrics of "communication". Using the idea of Dominance matrices, we will rank each song to the remaining 23 songs, and then compare the 15 rankings to determine the differences from our method to Spotify's "opinion" on any given songs being alike. The 15 combinations of characteristics are as follows:

Combinations				
I	**II**	**III**	**IV**	**V**
Key	Key	Key	Key	Key
Mode	Mode	Mode	Mode	Mode
Tempo	Tempo	Tempo	Energy	Energy
Energy	Instr.	Speech.	Instr.	Speech.
VI	**VII**	**VIII**	**IX**	**X**
Key	Key	Key	Key	Key
Mode	Tempo	Tempo	Tempo	Energy
Instr.	Energy	Energy	Instr.	Instr.
Speech	Instr.	Speech.	Speech.	Speech.
XI	**XII**	**XIII**	**XIV**	**XV**
Mode	Mode	Mode	Mode	Tempo
Tempo	Tempo	Tempo	Energy	Energy
Energy	Energy	Instr.	Instr.	Instr.
Instr.	Speech.	Speech.	Speech.	Speech.

Figure 2

From a selection of 24 songs, we can produce adjacency matrices for each of the six characteristic sets of relationships, where an *(i,j)the entry* closer to **"1"** represents a stronger similarity between the *ith* and *jth* song of a certain characteristic.

Adding these four "Characteristic" adjacency matrices will show which *(i,j)th* entry has the greatest number of relationships -- or in other words, which two songs share the most characteristics, and are thus more alike.

Let [**A]n** be the matrix that represents the sum of the four characteristic matrices for **n=15** combinations. Thereby, no entry in any matrix **[A]n** is greater than four, and there will be fifteen different "methods of ranking" for each of the 24 songs. To dispute any ties we may first examine the **A^2** matrix corresponding to the row of the song. This will show us the number of two-step similarities between the *ith* song and the *jth* song of each column entry. When there remains a tie, we may examine the same *ith* row of the "Power Matrix" **A + A^2.**

We can wonder: Are the characteristics we chose the strongest correlations of similarity between songs? Are there more significant or impactful *key characteristics* that describe a song.

IV. SOLUTION TO THE MODEL

Our solution set will consist of *fifteen* 24*24 adjacency matrices, wherein each entry should indicate the significance of similarities of the selected two songs. That is, the greater the entry, the more similar we *expect* the *ith* and *jth* song to be. The purpose of varying the combination of these six key features is to analyze which characteristics of music as a whole are more significant in comparing two pieces of work. The fifteen summed adjacency matrices, corresponding to the roman numerals **I-XV** of 'Figure 2', are as follows:

I.

4.0	3.5	3.5	2.7	1.5	2.5	1.8	1.6	2.8	1.6	1.5	3.6	5.3	69.4	7.5	5.3	5.4	2.6	3.9	5.3	3.0	1.8	1.3	2.5
4.8	4.0	4.1	4.0	2.1	3.0	2.6	2.2	3.5	2.2	2.2	4.2	7.9	120.5	11.0	7.9	8.0	4.1	5.2	7.5	3.9	2.6	1.8	3.0
4.6	3.9	4.0	3.6	2.0	2.9	2.4	2.1	3.3	2.1	2.0	4.1	6.8	95.5	9.5	6.8	7.0	3.5	4.8	6.7	3.7	2.4	1.7	2.9
1.5	1.1	1.1	4.0	2.1	1.1	2.3	2.2	1.3	2.2	2.1	1.2	3.0	46.6	3.8	4.0	3.1	2.8	3.1	4.1	1.5	3.3	2.0	1.1
2.7	2.0	2.0	4.7	4.0	2.0	4.4	3.1	2.4	3.1	3.0	2.1	6.0	100.4	7.7	7.0	6.1	4.6	4.9	5.9	2.7	3.4	3.8	1.9
3.8	3.0	3.1	3.8	2.0	4.0	2.5	2.2	3.4	2.2	2.1	3.1	7.2	102.8	9.0	6.2	7.3	3.7	4.0	7.0	3.8	2.5	1.8	3.0
2.3	1.7	1.7	4.1	3.7	1.7	4.0	2.8	2.0	2.9	2.7	1.8	4.6	71.5	5.8	5.6	4.8	3.7	4.2	4.7	2.3	3.0	3.5	1.7
2.5	1.8	1.9	4.5	2.9	1.8	3.3	4.0	2.3	3.0	2.9	2.0	5.8	97.5	7.5	6.8	6.9	4.4	4.7	5.6	2.6	3.3	2.7	1.8
3.3	2.6	2.7	3.1	1.7	2.6	2.0	1.8	4.0	1.8	2.7	2.8	6.0	83.5	7.5	5.1	6.2	3.0	3.3	5.9	3.3	2.0	1.5	2.6
2.6	1.8	1.9	4.6	2.9	1.8	3.3	3.0	2.2	4.0	3.0	2.0	6.1	105.1	7.9	7.0	6.1	4.6	4.8	5.8	2.6	3.3	2.6	1.8
2.7	2.1	2.1	4.6	3.0	2.0	3.4	3.2	3.4	3.2	4.0	2.2	5.4	84.3	6.9	6.5	5.7	4.2	4.8	5.6	2.8	3.4	2.8	2.0
4.6	3.9	4.0	3.6	1.9	2.9	2.4	2.0	3.3	2.0	2.0	4.0	7.2	105.7	10.0	7.2	7.2	3.7	4.8	6.9	3.6	2.4	1.7	2.9
2.5	2.2	2.1	1.8	1.1	2.1	1.2	1.2	2.3	1.3	1.0	2.3	4.0	21.8	3.4	2.1	3.3	1.2	1.9	3.6	2.5	1.3	1.1	2.1
1.3	1.2	1.1	2.6	2.1	1.1	2.1	2.2	1.3	2.3	1.9	1.2	2.2	4.0	1.3	2.4	1.6	1.8	2.6	2.1	1.4	2.1	2.0	1.0
3.3	3.1	3.0	1.6	1.0	2.0	1.1	1.1	2.2	1.2	0.9	3.1	2.7	15.8	4.0	2.8	3.0	1.0	2.6	3.2	2.3	1.1	0.9	2.0
2.3	2.1	2.0	2.6	2.0	1.0	2.1	2.1	1.2	2.1	1.9	2.1	1.9	22.5	3.3	4.0	2.1	2.2	3.7	2.3	1.3	2.1	1.9	1.0
2.1	1.9	1.9	1.5	0.9	1.9	1.0	2.0	2.1	1.0	0.8	2.0	2.8	21.8	3.2	1.9	4.0	1.1	1.5	3.1	2.2	1.0	0.8	1.9
2.3	1.9	1.8	3.9	2.8	1.7	2.9	2.9	2.1	3.0	2.6	2.0	3.2	36.7	3.9	4.5	3.7	4.0	3.9	4.1	3.3	3.0	2.6	1.7
2.4	2.1	2.1	2.9	2.1	1.1	2.3	2.2	1.3	2.2	2.1	2.2	2.8	42.8	4.5	4.8	2.9	2.7	4.0	2.9	1.5	2.3	2.0	1.1
2.0	1.8	1.8	2.3	0.8	1.8	0.9	0.8	1.9	0.9	0.7	1.8	2.8	25.2	3.3	1.9	3.0	1.1	1.4	4.0	2.0	1.9	0.7	1.8
3.0	2.5	2.5	2.7	1.5	2.4	1.8	1.6	2.8	1.6	1.5	2.5	5.3	70.0	6.5	4.3	5.4	3.6	2.8	5.3	4.0	1.8	1.3	2.4
2.2	1.7	1.7	5.0	2.7	1.6	3.0	2.8	2.0	2.8	2.6	1.8	4.6	73.3	5.9	5.6	4.8	3.7	4.1	5.7	2.3	4.0	2.5	1.6
3.1	2.2	2.4	5.3	4.3	2.2	4.8	3.4	2.7	3.4	3.4	2.4	7.3	126.3	9.5	8.3	7.4	5.4	5.6	7.0	3.2	3.8	4.0	2.2
3.8	3.0	3.1	3.8	2.1	3.0	2.5	2.2	3.5	2.2	2.1	3.2	7.2	102.3	9.0	6.2	7.3	3.7	4.0	7.1	3.8	2.5	1.8	4.0

II.

4.0	2.9		7.7	131.4	1901.8		1700.1	13.1	4012.5	388.4		1.9	1.1	2.9	2.0	2.2	27.4	2.3	2.5				
30.8	4.0		181.9	3613.4	52547.3		46992.8	309.5	110941.5	10724.3		2.9	9.9	3.6	2.8	3.8	741.7	4.8	3.3				
3.3	3.1		1.5	1.0	2.0		1.1	2.2	1.2	0.8		2.1	0.9	3.2	2.3	2.5	0.7	2.5	2.9				

1.0	0.8		4.0	21.7	291.6		261.9	2.5	616.0	60.9		0.7	1.7	0.8	1.9	1.0	5.6	2.0	2.3				
1.3	1.1		2.6	4.0	15.5		15.1	1.3	32.9	4.8		1.1	1.9	1.2	2.3	1.5	1.9	2.5	1.9				
2.3	2.2		1.5	1.1	4.0		2.0	2.2	3.3	1.0		2.1	1.0	2.2	1.3	2.5	0.7	1.5	2.9				
1.3	1.1		2.5	3.0	1.0		2.1	1.2	2.2	1.8		1.1	1.9	1.2	2.3	1.5	1.7	2.5	1.9				
1.2	1.0		2.4	2.0	2.0		4.0	1.1	4.5	2.0		1.0	1.9	1.1	2.2	2.4	1.7	2.4	1.7				
2.1	2.0		1.8	12.6	172.7		153.8	4.0	361.8	36.6		1.9	0.8	2.0	1.1	2.3	3.0	1.3	2.6				
1.1	1.0		2.3	1.9	1.3		2.3	1.0	4.0	1.8		0.9	1.8	1.0	2.1	1.3	1.6	2.3	1.6				
1.5	1.4		2.8	2.5	6.1		6.7	2.5	12.8	4.0		1.3	2.1	1.4	2.5	1.8	1.9	2.8	2.3				
3.1	3.0		1.3	0.9	1.9		0.9	2.0	1.0	0.7		1.9	0.8	3.0	2.1	2.3	0.6	2.3	2.6				
31.2	3.1		191.1	3794.7	55183.7		49350.6	325.0	116507.8	11262.4		4.0	11.4	2.7	1.9	3.9	778.9	4.0	3.4				
4.4	1.3		22.6	401.4	5810.1		5197.3	35.3	12267.1	1187.4		2.3	4.0	1.3	2.4	1.7	83.7	2.9	2.1				
48.1	4.6		295.4	5882.1	85547.3		76505.0	502.6	180615.2	17459.0		3.5	15.5	4.0	3.2	4.7	1207.0	6.4	3.7				
43.4	3.4		272.5	5408.0	78637.1		70327.1	461.0	166028.8	16049.9		2.3	15.3	2.8	4.0	3.4	1110.5	6.9	2.4				
20.7	2.4		124.4	2469.4	35910.8		32115.6	211.9	75816.5	7328.9		2.4	6.8	2.2	1.3	4.0	506.9	2.7	2.7				
1.8	1.6		3.4	7.3	72.3		65.9	2.1	152.4	16.6		1.5	2.3	1.6	2.8	2.1	4.0	3.1	2.7				
12.8	2.2		73.9	1439.2	20909.8		18700.9	123.1	44147.7	4268.7		1.1	5.2	2.0	3.1	1.6	296.3	4.0	1.5				
45.0	3.2		284.9	5663.0	82365.2		73659.1	483.5	173897.0	16809.3		3.1	14.7	2.6	1.7	4.1	1161.9	4.7	4.0				
2.0	1.9		1.2	0.8	1.8		0.8	1.9	0.9	0.6		1.8	0.7	1.9	1.0	2.1	1.5	1.1	2.5				
1.2	1.1		3.4	1.9	0.9		2.0	1.1	2.1	1.8		1.0	1.9	1.1	2.2	1.4	1.7	2.4	2.8				
1.3	1.2		2.6	3.0	1.0		2.1	1.2	2.2	1.9		1.1	2.0	1.2	2.3	1.6	1.7	2.6	2.0				
2.3	2.2		1.6	1.0	2.0		1.1	2.2	1.2	0.9		2.1	1.0	2.2	1.3	2.5	0.7	1.5	3.0				

III.

4.0	3.6	4.2	2.7	2.5	2.1	2.0	1.7	2.9	1.4	1.1	3.2	3.0	2.0	4.4	3.5	3.0	1.8	3.3	3.2	3.5	2.6	1.4	3.6
4.6	4.0	5.0	3.5	3.4	2.4	2.7	2.3	3.5	1.7	1.5	3.5	3.7	2.7	5.2	4.4	3.6	2.5	4.0	3.7	4.4	3.6	1.7	4.7
4.0	3.6	4.0	2.6	2.2	2.2	1.9	1.7	2.9	1.5	1.2	3.4	2.9	1.8	4.2	3.3	3.1	1.6	3.3	3.4	3.4	2.3	1.4	3.3
1.5	1.2	1.6	4.0	2.8	0.9	2.5	2.3	1.4	2.1	1.9	1.0	1.5	2.5	1.7	2.9	1.6	2.3	2.7	2.7	1.9	3.9	2.0	1.9
1.8	1.5	1.8	3.4	4.0	1.2	3.7	2.6	1.7	2.5	2.1	1.3	1.8	2.7	2.0	3.2	2.0	2.4	3.1	2.3	2.2	3.1	3.3	2.1
5.3	4.1	6.2	5.9	6.1	4.0	4.6	3.7	5.0	2.5	2.3	3.1	5.6	4.7	6.5	5.7	5.1	4.4	4.8	4.9	6.7	6.3	2.8	7.6
2.1	1.7	2.2	3.7	4.4	1.3	4.0	2.8	2.0	2.6	2.2	1.4	2.1	3.0	2.4	3.5	2.2	2.7	3.4	2.4	2.5	3.5	3.5	2.5
2.3	1.8	2.5	4.0	3.8	1.3	3.3	4.0	2.2	2.6	2.3	1.4	2.3	3.3	2.7	3.9	3.3	3.1	3.6	2.5	2.9	3.9	2.6	3.0
3.1	2.7	3.3	2.8	2.7	2.2	2.1	1.9	4.0	1.5	2.2	2.3	3.2	2.1	3.5	2.7	3.2	1.9	2.4	3.3	3.7	2.8	1.4	3.8
3.3	2.4	4.0	5.6	5.6	1.6	4.5	3.9	3.1	4.0	2.8	1.6	3.6	4.6	4.2	5.4	3.2	4.4	4.7	3.1	4.4	5.8	3.1	5.0
3.6	2.7	4.1	5.8	5.7	1.9	4.7	4.1	4.3	3.4	4.0	2.0	3.7	4.7	4.3	5.6	3.6	4.4	5.0	3.6	4.6	5.8	3.4	5.0
6.4	5.2	7.5	6.2	6.5	3.0	4.9	3.9	5.1	2.5	2.3	4.0	5.9	5.0	7.8	7.1	5.2	4.8	5.9	4.8	7.0	6.7	2.8	8.1
3.0	2.6	3.1	2.6	2.4	2.2	1.9	1.8	2.9	1.5	1.2	2.3	4.0	2.9	3.3	2.4	3.1	1.7	2.3	3.3	3.5	2.5	1.4	3.5
2.1	1.7	2.2	3.8	3.4	1.3	3.0	2.9	2.0	2.6	2.3	1.5	3.1	4.0	2.4	3.5	2.3	2.7	3.5	2.5	2.6	3.5	2.5	2.5
3.8	3.4	3.8	2.3	2.0	2.1	1.7	1.6	2.7	1.3	1.1	3.2	2.8	1.7	4.0	3.1	2.9	1.5	3.0	3.1	3.1	2.1	1.2	3.1
2.7	2.3	2.7	3.2	2.9	1.0	2.6	2.4	1.6	2.2	2.0	2.1	1.6	2.6	2.9	4.0	1.7	2.4	3.9	1.9	2.0	3.0	2.2	2.0
3.0	2.5	3.3	2.7	2.6	2.0	2.1	2.8	2.9	1.3	1.1	2.1	3.1	2.1	3.4	2.6	4.0	1.9	2.3	3.0	3.6	2.7	1.3	3.8
2.6	2.1	2.5	4.3	3.8	1.7	3.4	3.2	2.4	3.0	2.6	1.9	2.5	3.3	2.8	4.0	2.8	4.0	4.0	3.2	4.0	3.9	2.8	2.9
2.8	2.4	2.9	3.4	3.2	1.0	2.8	2.5	1.6	2.2	2.0	2.1	1.8	2.8	3.1	4.2	1.8	2.6	4.0	1.9	2.2	3.3	2.2	2.4
3.2	2.6	3.7	4.0	3.1	2.0	2.4	1.9	3.0	1.3	1.2	2.1	3.4	2.4	3.8	2.9	3.1	2.3	2.5	4.0	3.9	4.2	1.4	4.4
2.6	2.3	2.7	2.1	1.9	2.0	1.6	1.4	2.5	1.2	1.0	2.1	2.6	1.6	2.9	2.0	2.7	2.4	1.9	2.9	4.0	2.0	1.1	3.0
1.8	1.5	1.7	4.3	2.9	1.1	2.6	2.5	1.7	2.4	2.1	1.3	1.7	2.6	1.9	3.1	1.9	2.4	3.0	3.2	2.1	4.0	2.2	2.0
3.0	2.3	3.4	5.0	5.9	1.6	5.1	3.6	2.8	3.0	2.7	1.7	3.1	4.1	3.6	4.8	3.0	3.8	4.4	3.1	3.8	5.0	4.0	4.2
2.8	2.5	2.8	2.3	1.9	2.2	1.7	1.6	2.7	1.5	1.1	2.4	2.7	1.6	3.0	2.1	3.0	1.4	2.1	3.3	3.1	2.0	1.3	4.0

IV.

4.0	2.6		8.1	131.3	1901.7		1699.9	12.9	4012.2	388.6		4.5	69.0	6.6	4.3	4.3	28.8	2.8	3.8				

31.4	4.0		183.3	3613.7	52547.6		46993.1	309.9	110941.7	10725.0		7.9	128.8	10.6	7.4	8.1	744.5	6.4	6.5				
3.4	2.8		2.1	1.0	1.9		1.0	2.1	0.9	1.2		5.7	94.6	8.3	5.6	5.5	2.8	3.3	4.8				
0.8	0.4		4.0	21.5	291.4		261.6	2.2	615.6	60.9		2.3	46.0	3.0	3.2	2.1	6.4	2.1	2.8				
1.4	0.8		3.2	4.0	15.5		15.0	1.3	32.7	5.2		4.9	99.4	6.6	5.7	4.7	4.1	3.4	4.0				
2.5	1.9		2.3	1.1	4.0		2.0	2.2	3.1	1.4		6.1	101.9	7.8	4.9	5.8	3.0	2.5	5.1				
1.0	0.6		2.5	2.7	0.7		1.7	0.8	1.7	1.8		3.5	70.6	4.7	4.4	3.3	3.0	2.7	2.8				
1.4	0.8		3.1	2.0	2.1		4.0	1.2	4.3	2.4		4.8	96.6	6.4	5.6	5.5	3.8	3.3	3.9				
2.3	1.7		2.4	12.6	172.7		153.7	4.0	361.6	36.9		5.1	82.7	6.5	4.0	4.9	4.8	2.0	4.3				
1.5	0.9		3.3	2.1	1.5		2.5	1.3	4.0	2.4		5.2	104.3	6.9	6.0	4.9	4.0	3.5	4.2				
1.2	0.7		2.8	2.2	5.7		6.2	2.0	12.1	4.0		4.1	83.1	5.5	5.0	3.9	3.5	3.0	3.3				
3.5	2.9		2.3	1.1	2.0		1.1	2.3	1.0	1.3		6.3	104.9	9.0	6.1	6.0	3.1	3.5	5.2				
30.3	2.2		190.1	3794.0	55183.0		49349.8	324.1	116506.9	11261.8		4.0	30.5	3.0	1.7	3.5	778.8	3.1	2.5				
3.1	0.1		21.0	400.4	5809.1		5196.1	34.0	12265.9	1186.5		1.2	4.0	0.1	1.1	0.2	83.0	1.3	0.1				
47.2	3.8		294.4	5881.4	85546.6		76504.2	501.7	180614.3	17458.4		3.3	29.7	4.0	2.8	4.1	1206.8	5.5	2.6				
42.7	2.7		271.8	5407.5	78636.6		70326.4	460.4	166028.1	16049.5		2.5	35.3	3.3	4.0	3.2	1110.6	6.3	1.8				
20.2	1.9		123.9	2469.0	35910.3		32115.0	211.4	75815.9	7328.6		2.7	27.3	2.8	1.5	4.0	507.0	2.2	2.3				
0.5	0.3		2.0	6.2	71.2		64.8	0.8	151.0	15.9		1.7	35.4	2.3	2.7	1.6	4.0	1.8	1.4				
12.6	1.7		73.8	1438.9	20909.5		18700.6	122.8	44147.3	4268.7		2.4	45.7	4.0	4.2	2.5	297.1	4.0	1.9				
44.7	2.8		284.7	5662.8	82364.9		73658.8	483.2	173896.6	16809.2		3.7	38.9	3.6	2.2	4.5	1162.3	4.5	4.0				
2.0	1.6		1.5	0.7	1.7		0.7	1.8	0.7	0.8		4.5	69.3	5.6	3.3	4.3	3.0	1.7	3.8				
1.0	0.6		3.6	1.7	0.7		1.7	0.9	1.7	1.9		3.6	72.4	4.8	4.4	3.4	3.1	2.7	3.9				

1.8	1.0		3.8	3.3	1.2		2.3	1.5	2.2	2.5		6.2	125.3	8.3	7.0	5.9	4.6	4.0	5.0				
2.5	1.8		2.3	1.0	2.0		1.1	2.2	1.0	1.2		6.1	101.3	7.8	4.9	5.8	2.9	2.5	5.1				

V.

4.0	3.2	4.1	3.0	2.4	2.0	2.2	1.6	2.8	1.1	1.3	3.0	5.6	69.9	8.0	5.8	5.1	3.2	3.8	4.4	3.5	2.7	1.1	3.6
5.3	4.0	5.4	4.9	3.8	2.7	3.5	2.6	3.9	1.8	2.2	3.6	8.8	121.6	12.2	9.0	8.0	5.4	5.6	6.8	5.0	4.3	1.9	5.0
4.1	3.3	4.0	3.2	2.2	2.2	2.2	1.6	2.8	1.2	1.5	3.1	6.6	95.5	9.4	6.6	6.1	3.6	4.1	5.3	3.4	2.6	1.2	3.3
1.3	0.8	1.4	4.0	2.6	0.7	2.5	2.1	1.2	1.7	1.9	0.6	3.1	46.8	4.0	4.2	2.7	3.2	2.8	3.3	1.7	3.8	1.8	1.7
2.0	1.2	1.9	4.1	4.0	1.2	4.1	2.6	1.7	2.2	2.5	1.1	5.6	100.2	7.4	6.6	5.2	4.6	4.0	4.4	2.3	3.4	3.1	2.1
5.5	3.8	6.3	6.7	6.1	4.0	5.1	3.7	5.0	2.3	2.7	2.9	9.6	105.6	12.1	9.4	8.4	6.7	5.8	7.1	6.9	6.6	2.6	7.6
1.8	1.1	1.9	3.8	4.1	1.0	4.0	2.5	1.6	2.0	2.3	0.9	4.5	71.6	5.9	5.6	4.0	4.1	3.6	3.4	2.2	3.4	3.1	2.3
2.5	1.5	2.6	4.8	3.9	1.3	3.7	4.0	2.2	2.4	2.7	1.3	6.1	98.1	8.0	7.3	6.5	5.2	4.6	4.6	3.1	4.3	2.4	3.1
3.3	2.4	3.4	3.4	2.7	2.2	2.5	1.8	4.0	1.3	2.5	2.1	6.4	84.0	8.1	5.6	5.8	3.7	3.2	5.0	3.8	3.0	1.3	3.8
3.8	2.3	4.2	6.6	5.8	1.8	5.2	4.1	3.3	4.0	3.4	1.7	7.8	107.1	10.1	9.3	6.8	6.8	6.0	5.7	4.8	6.3	3.2	5.2
3.2	2.0	3.7	5.8	5.3	1.5	4.6	3.7	3.9	2.7	4.0	1.4	6.5	85.7	8.4	8.0	5.6	5.9	5.2	4.7	4.2	5.7	2.9	4.7
6.9	5.1	7.8	7.2	6.7	3.1	5.5	4.0	5.4	2.5	2.9	4.0	10.2	109.1	13.8	11.0	8.8	7.2	7.2	7.4	7.4	7.3	2.8	8.3
2.1	1.7	2.4	1.7	1.6	1.5	1.3	1.0	2.0	0.6	0.7	1.4	4.0	22.0	3.5	2.2	2.7	1.6	1.4	2.4	2.5	1.7	0.7	2.8
0.8	0.5	1.1	2.2	2.4	0.3	2.0	1.7	0.8	1.4	1.4	0.2	2.0	4.0	1.2	2.2	0.7	2.0	1.9	0.6	1.2	2.4	1.5	1.5
2.9	2.6	3.1	1.4	1.3	1.4	1.1	0.8	1.8	0.5	0.5	2.4	2.6	15.8	4.0	2.8	2.3	1.3	2.1	2.1	2.2	1.4	0.5	2.5
2.0	1.6	2.2	2.4	2.3	0.4	2.1	1.8	0.9	1.5	1.6	1.4	1.8	22.6	3.4	4.0	1.6	2.4	3.2	1.3	1.3	2.5	1.6	1.5
2.5	1.9	2.8	2.2	2.2	1.6	1.7	2.2	2.3	0.7	0.8	1.5	3.4	22.6	4.1	2.7	4.0	2.1	1.8	2.6	3.0	2.3	0.9	3.4
1.3	0.8	1.5	2.9	2.7	0.6	2.5	2.1	1.2	1.7	1.8	0.6	2.7	36.4	3.5	3.9	2.3	4.0	2.7	1.9	2.7	2.9	1.8	1.9
2.5	1.9	2.7	3.3	3.0	0.7	2.7	2.2	1.3	1.8	1.9	1.7	3.1	43.3	5.1	5.3	2.7	3.3	4.0	2.3	1.9	3.2	1.9	2.1
2.9	2.2	3.4	3.8	2.8	1.8	2.2	1.6	2.7	0.9	1.0	1.7	4.0	26.6	4.8	3.5	3.5	2.6	2.3	4.0	3.6	4.0	1.1	4.1
2.7	2.0	2.7	2.5	1.9	1.9	1.8	1.3	2.5	1.0	1.2	1.9	5.3	70.2	6.6	4.3	4.9	3.9	2.4	4.2	4.0	2.1	1.0	3.0
1.6	1.0	1.6	4.4	2.7	0.9	2.7	2.3	1.4	1.9	2.2	0.9	4.3	73.1	5.6	5.3	3.9	3.8	3.4	4.3	1.9	4.0	1.9	1.8
3.5	2.2	3.7	6.2	6.1	1.8	5.8	3.8	3.1	3.0	3.3	1.7	8.2	127.4	10.7	9.5	7.3	6.7	5.9	6.1	4.3	5.7	4.0	4.4
3.0	2.2	2.8	3.0	1.9	2.2	2.1	1.5	2.7	1.2	1.5	2.1	6.7	102.0	8.5	5.7	6.2	3.6	3.0	5.4	3.2	2.3	1.1	4.0

VI.

4.0	2.7		8.0	132.3	1901.3		1700.1	13.0	4012.0	388.2		2.2	1.6	3.5	2.5	1.9	28.0	2.2	1.7				
31.2	4.0		182.8	3615.1	52547.0		46993.2	309.9	110941.2	10724.3		3.7	10.9	4.8	3.9	3.8	742.9	5.2	2.6				
2.7	2.5		1.1	1.2	1.2		0.6	1.7	0.3	0.3		1.8	0.9	3.0	2.1	1.6	0.9	1.8	1.5				
0.8	0.5		4.0	22.1	291.1		261.8	2.3	615.5	60.7		0.8	1.9	1.0	2.0	0.6	5.9	1.8	1.5				
0.6	0.4		1.9	4.0	14.7		14.5	0.6	32.0	4.3		0.7	1.7	0.9	1.9	0.5	1.9	1.6	0.4				
4.0	3.0		4.4	5.1	4.0		3.5	3.8	3.4	1.7		4.5	3.7	5.3	4.4	3.6	3.7	3.3	3.0				
0.8	0.6		2.2	3.4	0.3		1.7	0.8	1.4	1.4		1.0	2.0	1.2	2.2	0.7	2.0	1.9	0.5				
1.1	0.8		2.7	3.0	1.5		4.0	1.1	3.9	1.8		1.3	2.4	1.6	2.7	2.0	2.4	2.2	0.7				
2.2	1.7		2.2	13.6	172.3		153.8	4.0	361.3	36.4		2.3	1.4	2.5	1.6	1.9	3.7	1.2	1.7				
2.3	1.5		4.3	4.8	1.2		3.4	2.1	4.0	2.2		2.7	3.8	3.2	4.3	1.9	3.8	3.5	1.5				
2.1	1.4		4.0	4.8	5.6		7.2	2.9	12.3	4.0		2.4	3.6	2.9	4.0	1.8	3.6	3.2	1.3				
5.3	4.2		4.9	5.6	2.1		2.9	4.1	1.5	1.6		4.9	4.2	6.8	6.0	3.9	4.1	4.6	3.2				
30.9	2.6		190.9	3795.2	55183.1		49350.3	324.7	116507.1	11262.0		4.0	11.6	2.9	2.0	3.3	779.3	3.6	2.2				
3.9	0.6		22.1	401.7	5809.4		5196.8	34.7	12266.2	1186.9		2.1	4.0	1.2	2.3	0.9	83.9	2.2	0.6				
47.7	4.1		295.1	5882.4	85546.7		76504.7	502.2	180614.5	17458.6		3.4	15.6	4.0	3.1	4.0	1207.3	5.9	2.5				
43.1	2.9		272.3	5408.4	78636.6		70326.8	460.7	166028.2	16049.6		2.2	15.4	2.9	4.0	2.8	1110.8	6.5	1.4				
21.1	2.4		125.1	2470.7	35910.5		32115.8	212.2	75816.3	7329.0		3.0	7.6	3.1	2.2	4.0	507.9	3.0	2.2				
0.8	0.5		2.4	7.2	71.2		65.1	1.2	151.1	15.9		1.0	2.0	1.2	2.2	0.7	4.0	1.9	0.5				
12.9	2.0		74.2	1440.0	20909.4		18701.0	123.2	44147.3	4268.6		1.5	5.7	2.6	3.6	1.4	297.0	4.0	0.9				
45.9	3.6		286.4	5665.1	82365.1		73659.9	484.3	173897.0	16809.6		4.3	16.1	4.2	3.3	4.6	1163.4	5.6	4.0				
1.7	1.4		1.0	1.1	1.2		0.6	1.6	0.3	0.3		1.8	0.8	2.0	1.0	1.6	1.8	0.7	1.4				

0.6	0.4		2.8	2.0	0.2		1.5	0.5	1.3	1.3		0.7	1.7	0.8	1.9	0.5	1.7	1.6	1.4				
1.7	1.1		3.5	4.8	0.6		2.5	1.6	1.7	1.8		2.0	3.1	2.4	3.5	1.4	3.1	2.8	1.1				
1.5	1.4		0.8	0.9	1.2		0.5	1.5	0.2	0.3		1.6	0.7	1.8	0.8	1.5	0.7	0.6	1.3				

VII.

4.0	2.5		9.3	132.1	1901.5		1700.8	12.9	4013.1	389.3		4.3	69.7	6.5	5.3	4.5	29.4	3.9	4.3				
31.5	4.0		184.6	3614.6	52547.5		46994.1	309.9	110942.7	10725.7		7.9	129.6	10.6	8.6	8.4	745.1	7.7	7.1				
3.6	2.9		3.6	2.0	1.9		2.1	2.3	2.1	2.0		5.8	95.5	8.5	6.8	6.0	3.5	4.8	5.7				
1.6	1.1		4.0	21.1	292.0		261.4	3.0	615.4	60.5		3.0	45.6	3.8	3.0	3.1	5.9	2.1	4.1				
2.7	2.0		3.7	4.0	16.5		15.1	2.5	32.9	5.0		6.0	99.4	7.7	6.0	6.1	3.8	3.9	5.9				
2.8	2.0		3.8	2.1	4.0		3.1	2.4	4.3	2.3		6.2	102.8	8.0	6.2	6.3	3.7	4.0	6.0				
2.3	1.7		3.1	2.7	1.7		1.8	2.0	1.9	1.7		4.6	70.5	5.8	4.6	4.8	2.7	3.2	4.7				
2.5	1.8		3.5	2.0	3.0		4.0	2.3	4.4	2.2		5.8	96.5	7.5	5.8	6.9	3.4	3.7	5.6				
2.3	1.6		3.7	13.4	172.5		154.6	4.0	362.6	37.6		5.0	83.5	6.5	5.1	5.2	5.4	3.3	4.9				
2.6	1.8		3.6	1.9	2.3		2.4	2.3	4.0	2.0		6.1	104.1	7.9	6.0	6.1	3.6	3.8	5.8				
2.7	2.1		3.7	2.4	6.9		6.6	3.5	12.6	4.0		5.4	83.3	6.9	5.5	5.7	3.3	3.8	5.6				
3.6	2.9		3.6	1.9	1.9		2.0	2.3	2.0	2.0		6.2	105.7	9.0	7.2	6.2	3.7	4.8	5.9				
30.5	2.3		191.5	3794.9	55182.9		49350.8	324.2	116508.0	11262.6		4.0	31.3	3.1	2.9	3.9	779.4	4.5	3.2				
4.4	1.3		21.6	400.4	5810.1		5196.3	35.3	12266.2	1186.4		2.3	4.0	1.4	1.5	1.8	82.7	1.9	2.1				
47.3	3.7		295.7	5882.3	85546.4		76505.2	501.7	180615.4	17459.1		3.2	30.5	4.0	3.9	4.4	1207.5	6.7	3.3				
43.7	3.6		272.0	5407.2	78637.3		70326.3	461.3	166028.0	16049.1		3.3	35.0	4.2	4.0	4.3	1110.1	6.4	3.3				
20.0	1.6		124.9	2469.6	35910.0		32115.8	211.2	75816.7	7329.2		2.4	28.0	2.6	2.3	4.0	507.5	3.2	2.6				

2.3	1.9		3.1	6.6	72.6		65.3	2.5	151.7	16.1		3.2	35.7	3.9	3.5	3.7	4.0	2.9	4.1				
13.4	2.5		73.8	1438.6	20910.2		18700.3	123.6	44147.1	4268.2		3.2	45.4	4.8	4.1	3.5	296.5	4.0	3.2				
44.4	2.4		285.5	5663.3	82364.4		73659.4	482.8	173897.2	16809.6		3.3	39.4	3.2	2.9	4.2	1162.6	5.3	4.0				
2.0	1.5		2.7	1.5	1.4		1.6	1.8	1.6	1.5		4.3	70.0	5.5	4.3	4.4	3.6	2.8	4.3				
2.2	1.7		4.0	1.7	1.6		1.8	2.0	1.8	1.6		4.6	72.3	5.9	4.6	4.8	2.7	3.1	5.7				
3.1	2.2		4.3	3.3	2.2		2.4	2.7	2.4	2.4		7.3	125.3	9.5	7.3	7.4	4.4	4.6	7.0				
2.8	2.0		3.8	2.1	2.0		2.2	2.5	2.2	2.1		6.2	102.3	8.0	6.2	6.3	3.7	4.0	6.1				

VIII.

4.0	3.1	3.9	4.2	3.2	1.8	3.0	2.5	2.7	2.1	2.0	2.9	5.5	70.6	8.0	6.8	5.3	3.8	5.0	4.9	3.5	3.5	1.9	3.3
5.4	4.0	5.3	6.2	4.6	2.5	4.4	3.5	3.9	2.9	2.9	3.6	8.7	122.4	12.2	10.1	8.3	6.0	6.9	7.5	5.1	5.2	2.7	4.9
4.3	3.4	4.0	4.7	3.2	2.2	3.2	2.7	3.0	2.4	2.3	3.3	6.7	96.4	9.5	7.9	6.6	4.3	5.6	6.2	3.7	3.6	2.1	3.2
2.2	1.6	2.1	4.0	2.3	1.3	2.1	1.8	2.0	1.5	1.4	1.4	3.8	46.5	4.8	4.0	3.7	2.6	2.8	4.5	2.5	3.5	1.4	2.4
3.3	2.4	2.9	4.6	4.0	2.1	4.1	2.6	2.9	2.4	2.3	2.3	6.7	100.1	8.6	6.9	6.7	4.3	4.5	6.3	3.6	3.5	3.1	3.1
5.8	4.0	6.3	8.2	7.1	4.0	6.1	4.8	5.2	3.5	3.5	3.0	9.7	106.5	12.3	10.7	8.9	7.4	7.3	8.0	7.2	7.7	3.6	7.6
3.1	2.3	2.9	4.3	4.1	1.9	4.0	2.6	2.8	2.2	2.1	2.1	5.5	71.6	7.0	5.9	5.5	3.8	4.1	5.3	3.5	3.5	3.0	3.2
3.7	2.6	3.5	5.2	3.8	2.2	3.7	4.0	3.3	2.5	2.5	2.3	7.1	97.9	9.1	7.5	7.9	4.8	4.9	6.4	4.3	4.3	2.3	4.0
3.3	2.4	3.2	4.7	3.5	2.0	3.3	2.7	4.0	2.3	3.2	2.1	6.3	84.8	8.1	6.7	6.1	4.3	4.4	5.6	3.9	3.9	2.1	3.7
4.8	3.3	5.1	6.9	5.7	2.6	5.0	4.0	4.3	4.0	3.0	2.6	8.7	106.9	11.1	9.4	8.1	6.4	6.2	7.2	5.9	6.2	3.0	6.1
4.8	3.4	4.9	6.7	5.5	2.7	4.8	4.0	5.3	3.1	4.0	2.8	7.8	85.8	9.8	8.5	7.4	5.8	6.0	7.0	5.8	6.0	3.0	5.8
7.0	5.1	7.6	8.5	7.6	3.0	6.4	5.0	5.4	3.5	3.6	4.0	10.1	109.9	13.8	12.1	9.1	7.8	8.5	8.1	7.6	8.2	3.7	8.1
2.3	1.8	2.3	3.1	2.6	1.4	2.2	2.0	2.1	1.7	1.4	1.5	4.0	22.9	3.6	3.4	3.1	2.3	2.8	3.1	2.7	2.7	1.6	2.7
2.1	1.7	2.2	2.8	2.4	1.3	2.0	1.9	2.0	1.6	1.3	1.5	3.1	4.0	2.5	2.6	2.3	1.8	2.5	2.6	2.6	2.5	1.5	2.5
3.0	2.6	3.0	2.7	2.2	1.2	1.9	1.7	1.9	1.5	1.2	2.3	2.5	16.6	4.0	3.9	2.6	1.9	3.4	2.7	2.4	2.3	1.4	2.3
3.0	2.5	2.9	2.6	2.1	1.2	1.9	1.7	1.8	1.4	1.2	2.3	2.7	22.3	4.3	4.0	2.7	2.0	3.4	2.8	2.3	2.3	1.3	2.2
2.3	1.7	2.5	3.2	2.8	1.3	2.4	3.0	2.1	1.5	1.4	1.3	3.2	23.2	3.9	3.6	4.0	2.5	2.8	2.9	2.9	3.0	1.5	3.0
3.1	2.4	2.9	4.1	3.1	2.0	2.9	2.6	2.8	2.4	2.0	2.2	4.2	36.7	5.1	4.7	4.4	4.0	3.8	4.6	4.5	3.4	2.1	3.2
3.4	2.7	3.4	3.3	2.6	1.4	2.4	2.0	2.1	1.6	1.5	2.4	3.9	42.9	5.8	5.2	3.7	2.8	4.0	3.5	2.8	2.9	1.5	2.8

2.5	1.8	2.9	4.6	3.4	1.3	2.7	2.2	2.3	1.5	1.5	1.3	3.6	27.1	4.5	4.1	3.3	3.0	3.1	4.0	3.3	4.5	1.6	3.6
2.7	1.9	2.4	3.7	2.6	1.7	2.6	2.2	2.4	1.9	1.8	1.8	5.1	70.9	6.5	5.3	5.0	4.4	3.6	4.7	4.0	3.0	1.7	2.7
2.8	2.1	2.5	4.9	2.6	1.8	2.7	2.3	2.5	2.1	1.9	1.9	5.3	73.0	6.7	5.5	5.3	3.5	3.8	6.1	3.1	4.0	1.8	2.7
4.8	3.3	4.7	6.8	6.1	2.8	5.8	3.9	4.3	3.2	3.2	2.9	9.3	127.4	11.9	9.8	8.9	6.4	6.4	8.1	5.6	5.8	4.0	5.4
3.3	2.4	2.8	4.6	3.0	2.2	3.1	2.7	3.0	2.4	2.3	2.3	6.8	103.0	8.7	7.0	6.8	4.3	4.6	6.4	3.6	3.4	2.1	4.0

IX.

4.0	2.6		9.2	133.1	1901.1		1700.9	13.0	4012.9	388.9		2.1	2.3	3.4	3.5	2.1	28.6	3.4	2.2				
31.3	4.0		184.2	3615.9	52546.8		46994.2	309.9	110942.2	10725.0		3.7	11.8	4.8	5.0	4.1	743.5	6.5	3.3				
3.0	2.6		2.6	2.2	1.2		1.7	1.9	1.5	1.2		1.9	1.8	3.2	3.3	2.1	1.6	3.3	2.4				
1.7	1.2		4.0	21.8	291.8		261.5	3.1	615.3	60.3		1.5	1.5	1.8	1.9	1.6	5.4	1.8	2.7				
1.9	1.5		2.4	4.0	15.7		14.6	1.8	32.2	4.1		1.8	1.7	2.0	2.2	2.0	1.7	2.1	2.3				
4.3	3.1		5.9	6.1	4.0		4.6	4.0	4.7	2.5		4.6	4.7	5.5	5.7	4.1	4.5	4.8	3.9				
2.1	1.7		2.7	3.4	1.3		1.8	2.0	1.6	1.2		2.1	2.0	2.4	2.5	2.2	1.7	2.4	2.4				
2.3	1.8		3.1	2.9	2.4		4.0	2.2	4.0	1.5		2.3	2.3	2.7	2.9	3.3	2.1	2.6	2.5				
2.2	1.7		3.4	14.4	172.1		154.7	4.0	362.3	37.1		2.2	2.2	2.5	2.7	2.2	4.3	2.4	2.3				
3.3	2.4		4.6	4.7	2.1		3.3	3.1	4.0	1.9		3.6	3.6	4.2	4.4	3.2	3.4	3.7	3.1				
3.6	2.7		4.8	5.0	6.8		7.5	4.4	12.7	4.0		3.7	3.7	4.3	4.6	3.6	3.5	4.0	3.6				
5.4	4.2		6.2	6.5	2.0		3.9	4.1	2.5	2.3		4.9	5.0	6.8	7.1	4.2	4.8	5.9	3.8				
31.1	2.7		192.3	3796.1	55183.0		49351.3	324.8	116508.2	11262.8		4.0	12.4	2.9	3.1	3.6	779.9	4.9	3.0				
5.2	1.8		22.7	401.8	5810.4		5197.0	36.0	12266.5	1186.8		3.2	4.0	2.5	2.6	2.4	83.7	2.7	2.6				
47.8	4.1		296.4	5883.3	85546.5		76505.6	502.2	180615.5	17459.3		3.3	16.4	4.0	4.2	4.3	1207.9	7.1	3.1				
44.0	3.8		272.5	5408.2	78637.4		70326.6	461.7	166028.1	16049.2		3.1	15.1	3.8	4.0	3.9	1110.4	6.7	2.9				

20.9	2.2		126.2	2471.4	35910.1		32116.6	212.0	75817.1	7329.5		2.7	8.3	2.9	3.0	4.0	508.3	4.0	2.5				
2.6	2.1		3.5	7.6	72.6		65.7	2.8	151.7	16.0		2.5	2.3	2.8	3.0	2.8	4.0	3.0	3.2				
13.8	2.8		74.2	1439.7	20910.1		18700.7	124.0	44147.1	4268.2		2.2	5.4	3.3	3.5	2.4	296.5	4.0	2.1				
45.5	3.2		287.2	5665.6	82364.7		73660.5	484.0	173897.7	16810.1		3.9	16.6	3.8	4.0	4.4	1163.8	6.4	4.0				
1.6	1.3		2.1	1.9	1.0		1.4	1.5	1.2	1.0		1.6	1.6	1.9	2.0	1.7	2.4	1.9	1.9				
1.8	1.5		3.3	1.9	1.1		1.5	1.7	1.4	1.1		1.7	1.6	1.9	2.1	1.9	1.4	2.0	3.2				
3.0	2.3		4.0	4.9	1.6		2.6	2.8	2.0	1.7		3.1	3.1	3.6	3.8	3.0	2.8	3.4	3.1				
1.8	1.5		2.3	1.9	1.2		1.6	1.7	1.5	1.1		1.7	1.6	2.0	2.1	2.0	1.4	2.1	2.3				

X.

4.0	2.3		9.5	133.0	1901.0		1700.8	12.9	4012.6	389.1		4.7	70.2	7.0	5.8	4.2	30.0	3.9	3.5				
31.9	4.0		185.5	3616.3	52547.1		46994.5	310.3	110942.3	10725.8		8.7	130.6	11.8	9.7	8.4	746.4	8.1	6.5				
3.1	2.3		3.2	2.2	1.2		1.6	1.8	1.2	1.5		5.6	95.5	8.4	6.6	5.1	3.6	4.1	4.3				
1.5	0.8		4.0	21.6	291.6		261.2	2.9	614.9	60.2		3.1	45.9	4.0	3.2	2.7	6.3	1.9	3.3				
2.0	1.2		3.1	4.0	15.7		14.6	1.8	31.9	4.5		5.6	99.2	7.4	5.6	5.2	3.8	3.0	4.4				
4.5	2.8		6.7	6.2	4.0		4.6	4.0	4.4	2.9		8.6	105.6	11.1	9.4	7.4	6.7	5.8	6.1				
1.8	1.1		2.8	3.1	1.0		1.5	1.6	1.0	1.3		4.5	70.6	5.9	4.6	4.0	3.1	2.6	3.4				
2.5	1.5		3.8	3.0	2.4		4.0	2.2	3.8	1.9		6.1	97.1	8.0	6.3	6.5	4.2	3.6	4.6				
2.4	1.4		4.0	14.4	172.1		154.7	4.0	362.1	37.4		5.4	84.0	7.1	5.6	4.8	6.2	3.2	4.1				
3.8	2.3		5.6	4.9	2.2		3.5	3.3	4.0	2.5		7.8	106.1	10.1	8.3	6.8	5.8	5.0	5.7				
3.3	2.0		4.9	4.6	6.4		7.0	3.9	12.1	4.0		6.5	84.7	8.4	7.0	5.6	5.0	4.2	4.7				
5.9	4.1		7.2	6.7	2.1		4.0	4.4	2.5	2.9		9.2	109.1	12.8	11.0	7.8	7.2	7.2	6.4				

30.2	1.8		191.4	3795.4	55182.3		49350.5	323.9	116507.3	11262.3		4.0	31.5	3.2	2.9	3.2	779.8	4.0	2.0				
3.9	0.6		21.2	400.7	5809.4		5195.8	34.7	12265.2	1185.9		2.1	4.0	1.3	1.3	0.9	82.9	1.2	0.6				
46.9	3.2		295.4	5882.6	85545.8		76504.8	501.4	180614.7	17458.8		3.1	30.5	4.0	3.8	3.7	1207.7	6.2	2.1				
43.4	3.1		271.8	5407.6	78636.8		70326.0	461.0	166027.4	16048.8		3.3	35.1	4.3	4.0	3.8	1110.4	6.0	2.2				
20.4	1.6		125.6	2470.9	35909.7		32116.1	211.4	75816.5	7329.2		3.1	28.8	3.5	3.2	4.0	508.5	3.5	2.1				
1.3	0.8		2.2	6.6	71.5		64.5	1.6	150.4	15.3		2.7	35.4	3.5	2.9	2.3	4.0	1.7	1.9				
13.5	2.3		74.1	1439.5	20909.8		18700.4	123.7	44146.7	4268.1		3.5	45.9	5.3	4.6	3.3	297.2	4.0	2.5				
45.2	2.8		286.9	5665.3	82364.4		73660.1	483.6	173897.3	16810.0		4.5	40.8	4.8	4.5	4.8	1164.2	6.2	4.0				
1.7	1.0		2.5	1.9	0.9		1.3	1.5	1.0	1.2		4.3	70.2	5.6	4.3	3.9	3.9	2.4	3.2				
1.6	1.0		3.4	1.7	0.9		1.3	1.4	0.9	1.2		4.3	72.1	5.6	4.3	3.9	2.8	2.4	4.3				
3.5	2.2		5.2	5.1	1.8		2.8	3.1	2.0	2.3		8.2	126.4	10.7	8.5	7.3	5.7	4.9	6.1				
2.0	1.2		3.0	1.9	1.2		1.5	1.7	1.2	1.5		5.7	102.0	7.5	5.7	5.2	3.6	3.0	4.4				

XI.

4.00	2.52		9.26	132.12	1902.46		1700.77	13.90	4013.13	389.25		5.33	69.73	6.54	4.35	5.49	29.36	2.95	5.31				
31.50	4.00		184.61	3614.59	52548.49		46994.08	310.94	110942.70	10725.75		8.89	129.58	10.64	7.56	9.43	745.13	6.73	8.15				
3.63	2.93		3.61	1.96	2.91		2.08	3.34	2.11	1.98		6.81	95.52	8.49	5.83	6.96	3.46	3.79	6.72				
1.64	1.14		4.00	22.12	292.01		262.36	3.04	616.43	61.47		2.97	46.65	3.78	4.02	3.12	6.88	3.10	3.08				
2.70	1.96		4.74	4.00	16.50		16.13	2.47	33.86	5.99		6.01	100.36	7.75	7.02	6.14	4.78	4.89	5.88				
3.76	3.01		3.80	2.12	4.00		3.07	3.45	4.31	2.28		7.19	102.81	8.99	6.20	7.33	3.70	3.99	7.05				
2.27	1.72		4.05	2.71	1.67		2.82	2.03	2.87	2.67		4.56	71.52	5.82	5.63	4.78	3.73	4.18	4.71				

2.54	1.84		4.51	2.97	2.96		4.00	2.26	5.38	3.15		5.78	97.50	7.46	6.78	5.89	4.45	4.69	5.61				
3.35	2.65		3.69	13.43	173.55		154.62	4.00	362.64	36.58		6.05	83.49	7.51	5.06	6.16	5.41	3.27	5.94				
2.56	1.82		4.56	2.92	2.31		3.42	2.25	4.00	3.05		6.08	105.10	7.89	7.05	6.13	4.61	4.76	5.77				
2.72	2.06		4.66	3.38	6.90		7.56	2.47	13.58	4.00		5.42	84.28	6.90	6.50	5.69	4.32	4.80	5.62				
3.61	2.86		3.63	1.93	2.88		2.04	3.30	2.04	1.99		7.18	105.71	9.02	6.16	7.24	3.66	3.84	6.88				
31.50	3.27		191.53	3794.88	55183.93		49350.81	325.22	116508.02	11262.61		4.00	30.34	4.06	2.85	4.86	779.45	4.51	4.24				
4.41	1.33		22.59	401.45	5810.15		5197.29	35.27	12267.15	1187.42		1.30	4.00	1.37	2.48	1.80	83.71	2.89	2.13				
47.32	3.73		295.69	5882.26	85547.45		76505.16	502.74	180615.37	17459.15		4.23	30.48	4.00	2.91	5.37	1207.46	5.74	4.28				
42.67	2.56		272.95	5408.24	78637.34		70327.27	461.29	166029.00	16050.14		3.31	36.04	3.22	4.00	4.34	1111.13	6.43	3.28				
21.05	2.62		124.92	2469.64	35910.96		32114.77	212.16	75816.74	7329.20		3.43	27.98	3.62	2.33	4.00	507.50	3.23	3.57				
2.30	1.89		4.12	7.63	72.63		66.31	2.51	152.72	17.05		3.24	36.73	3.94	4.46	3.72	4.00	3.94	4.06				
12.44	1.50		74.80	1439.59	20910.15		18701.35	123.64	44148.07	4269.22		3.16	46.37	3.78	4.10	3.53	297.54	4.00	3.19				
45.37	3.37		284.47	5663.28	82365.40		73659.37	483.84	173897.22	16809.64		4.30	39.41	4.23	2.91	5.25	1162.63	5.33	4.00				
2.97	2.45		2.69	1.47	2.44		1.56	2.75	1.58	1.48		5.30	70.03	6.53	4.32	5.42	2.56	2.83	5.26				
2.22	1.67		4.01	2.67	1.63		2.77	1.99	2.82	2.65		4.59	73.31	5.87	5.65	4.78	3.75	4.14	4.68				
3.12	2.22		5.33	3.30	2.25		3.43	2.74	3.44	3.37		7.35	126.32	9.53	8.32	7.43	5.36	5.57	7.00				
3.77	3.02		3.81	2.06	3.01		2.18	3.46	2.21	2.09		7.19	102.29	8.98	6.20	7.33	3.68	4.00	7.06				

XII.

4.00	3.14	3.92	4.19	3.18	2.79	2.98	2.47	3.71	2.06	1.98	2.88	6.48	70.65	7.95	5.81	6.29	3.80	3.95	5.94	4.50	3.54	1.91	4.33
5.36	4.00	5.26	6.21	4.65	3.55	4.42	3.54	4.90	2.88	2.92	3.61	9.73	122.43	12.20	9.14	9.27	5.99	5.88	8.50	6.10	5.23	2.71	5.88

4.35	3.40	4.00	4.66	3.17	3.15	3.22	2.71	4.00	2.43	2.33	3.27	7.66	96.41	9.52	6.90	7.58	4.35	4.57	7.19	4.72	3.62	2.15	4.25
2.17	1.58	2.09	4.00	3.27	1.32	3.13	2.79	1.97	2.53	2.43	1.40	3.77	47.45	4.76	5.03	3.70	3.63	3.83	3.53	2.52	3.52	2.41	2.37
3.28	2.35	2.87	5.56	4.00	2.15	4.13	3.64	2.93	3.41	3.31	2.28	6.71	101.09	8.60	7.90	6.65	5.31	5.53	6.27	3.60	4.47	3.11	3.05
6.75	4.98	7.28	8.19	7.10	4.00	6.07	4.80	6.22	3.53	3.54	4.04	10.73	106.55	13.29	10.65	9.90	7.40	7.27	9.01	8.20	7.69	3.57	8.61
3.10	2.27	2.90	5.27	4.11	1.95	4.00	3.55	2.81	3.24	3.08	2.07	5.55	72.56	7.01	6.87	5.50	4.77	5.09	5.26	3.54	4.48	3.02	3.22
3.68	2.59	3.52	6.18	4.81	2.22	4.66	4.00	3.31	3.53	3.48	2.32	7.12	98.92	9.10	8.47	6.87	5.84	5.94	6.35	4.25	5.28	3.33	3.97
4.33	3.36	4.22	4.68	3.49	3.00	3.32	2.73	4.00	2.28	2.22	3.08	7.32	84.81	9.05	6.65	7.08	4.34	4.44	6.64	4.87	3.91	2.11	4.65
4.81	3.30	5.08	7.86	6.68	2.59	6.02	4.97	4.34	4.00	4.05	2.63	8.74	107.91	11.11	10.39	8.07	7.39	7.22	7.25	5.90	7.24	3.97	6.06
4.77	3.42	4.94	7.66	6.50	2.68	5.85	4.98	4.34	4.15	4.00	2.80	7.84	86.84	9.84	9.55	7.45	6.79	7.04	6.96	5.78	6.99	4.04	5.83
6.95	5.06	7.64	8.53	7.56	3.99	6.37	4.97	6.40	3.53	3.62	4.00	11.13	109.88	13.80	11.11	10.12	7.80	7.49	9.07	8.55	8.17	3.65	9.14
3.30	2.78	3.32	3.08	2.56	2.39	2.23	1.96	3.13	1.68	1.42	2.52	4.00	21.90	4.63	3.41	4.06	2.29	2.80	4.12	3.74	2.73	1.55	3.68
2.15	1.75	2.19	3.80	3.42	1.32	3.04	2.87	2.02	2.63	2.28	1.48	2.14	4.00	2.45	3.60	2.32	2.78	3.49	2.59	2.58	3.53	2.51	2.54
2.99	2.56	2.99	2.65	2.19	2.23	1.91	1.71	2.85	1.48	1.23	2.35	3.50	16.63	4.00	2.86	3.59	1.91	2.41	3.70	3.37	2.33	1.36	3.29
1.96	1.51	1.94	3.63	3.13	1.20	2.89	2.66	1.81	2.43	2.23	1.31	2.68	23.34	3.26	4.00	2.73	3.00	3.41	2.77	2.31	3.29	2.32	2.23
3.31	2.71	3.52	3.18	2.84	2.26	2.37	1.97	3.13	1.52	1.38	2.33	4.16	23.25	4.86	3.60	4.00	2.53	2.79	3.89	3.89	3.02	1.50	4.04
3.07	2.42	2.90	5.06	4.11	1.99	3.87	3.60	2.84	3.37	2.97	2.20	4.19	37.72	5.09	5.65	4.42	4.00	4.81	4.59	3.52	4.37	3.11	3.21
2.36	1.71	2.38	4.27	3.63	1.37	3.36	2.96	2.15	2.60	2.50	1.44	3.86	43.91	4.84	5.20	3.73	3.81	4.00	3.54	2.81	3.87	2.52	2.77
3.55	2.81	3.92	3.58	3.35	2.27	2.71	2.17	3.34	1.54	1.48	2.30	4.61	27.14	5.45	4.13	4.27	2.99	3.05	4.00	4.29	3.55	1.61	4.62
3.65	2.90	3.45	3.69	2.62	2.66	2.58	2.16	3.38	1.89	1.81	2.75	6.10	70.88	7.50	5.33	6.00	3.40	3.57	5.70	4.00	2.96	1.70	3.70
2.80	2.05	2.50	4.85	3.64	1.82	3.66	3.28	2.52	3.07	2.93	1.95	5.27	74.04	6.70	6.50	5.28	4.46	4.77	5.06	3.11	4.00	2.82	2.70
4.80	3.33	4.71	7.80	6.14	2.81	5.85	4.91	4.31	4.19	4.20	2.90	9.34	128.42	11.94	10.82	8.88	7.45	7.42	8.11	5.63	6.76	4.00	5.40
4.30	3.37	3.85	4.59	2.96	3.19	3.13	2.65	3.95	2.45	2.35	3.32	7.81	102.96	9.74	6.99	7.78	4.34	4.59	7.41	4.59	3.44	2.12	4.00

XIII.

4.00	2.60		9.20	133.11	1902.12		1700.93	14.00	4012.91	388.90		3.07	2.33	3.38	2.53	3.10	28.60	2.37	3.19				
31.27	4.00		184.17	3615.93	52547.83		46994.16	310.90	110942.22	10725.03		4.69	11.77	4.81	4.04	5.08	743.53	5.48	4.30				

2.97	2.61		2.55	2.21	2.22		1.72	2.86	1.51	1.18		2.92	1.83	3.19	2.34	3.11	1.60	2.27	3.37				
1.67	1.21		4.00	22.81	291.79		262.49	3.13	616.30	61.25		1.53	2.53	1.75	2.87	1.59	6.42	2.75	1.71				
1.86	1.53		3.43	4.00	15.73		15.62	1.83	33.17	5.08		1.78	2.68	2.01	3.16	2.00	2.65	3.14	2.29				
5.27	4.13		5.93	6.13	4.00		4.63	5.00	4.65	2.50		5.63	4.70	6.47	5.75	5.09	4.45	4.79	4.89				
2.09	1.69		3.73	3.41	1.26		2.82	1.97	2.56	2.23		2.06	2.98	2.36	3.52	2.21	2.75	3.40	2.45				
2.29	1.80		4.05	3.91	2.40		4.00	2.16	4.96	2.54		2.33	3.28	2.70	3.86	2.35	3.08	3.61	2.49				
3.22	2.67		3.43	14.40	173.09		154.71	4.00	362.32	36.10		3.18	2.16	3.52	2.67	3.18	4.34	2.44	3.30				
3.31	2.44		5.57	5.67	2.05		4.32	3.09	4.00	2.89		3.56	4.60	4.20	5.42	3.19	4.40	4.71	3.07				
3.57	2.73		5.85	6.00	6.78		8.50	3.37	13.70	4.00		3.73	4.70	4.35	5.59	3.57	4.47	5.05	3.64				
5.42	4.18		6.20	6.49	2.96		3.87	5.13	2.51	2.34		5.87	4.98	6.79	6.06	5.16	4.75	4.94	4.83				
32.05	3.66		192.32	3796.10	55184.01		49351.35	325.76	116508.20	11262.78		4.00	11.43	3.94	3.15	4.65	779.91	4.95	3.99				
5.19	1.85		23.74	402.79	5810.41		5197.98	36.00	12267.50	1187.80		2.20	4.00	2.45	3.62	2.44	84.67	3.74	2.62				
47.80	4.07		296.38	5883.29	85547.53		76505.62	503.21	180615.54	17459.31		4.30	16.41	4.00	3.22	5.27	1207.89	6.14	4.13				
43.04	2.83		273.48	5409.17	78637.36		70327.65	461.66	166029.10	16050.21		3.06	16.11	2.79	4.00	3.94	1111.37	6.66	2.88				
21.90	3.21		126.15	2471.38	35911.14		32115.57	212.98	75817.05	7329.51		3.75	8.26	3.86	3.04	4.00	508.33	3.99	3.47				
2.61	2.13		4.54	8.64	72.56		66.66	2.84	152.75	17.03		2.47	3.34	2.79	4.00	2.79	4.00	3.98	3.20				
12.76	1.76		75.23	1440.71	20910.06		18701.72	123.99	44148.08	4269.16		2.18	6.37	2.33	3.48	2.37	297.48	4.00	2.13				
46.54	4.17		286.16	5665.60	82365.66		73660.45	484.96	173897.66	16810.08		4.86	16.58	4.76	3.99	5.39	1163.81	6.39	4.00				
2.64	2.32		2.15	1.91	1.98		1.43	2.54	1.22	0.96		2.63	1.57	2.86	1.99	2.73	1.39	1.89	2.90				
1.76	1.45		3.27	2.92	1.12		2.53	1.66	2.38	2.06		1.70	2.61	1.92	3.06	1.90	2.39	3.04	2.17				
2.99	2.30		5.04	4.88	1.59		3.61	2.80	2.99	2.68		3.11	4.08	3.62	4.83	3.00	3.83	4.39	3.08				

2.83	2.53		2.33	1.93	2.19		1.59	2.73	1.47	1.11		2.74	1.64	2.96	2.11	3.00	1.39	2.12	3.31				

XIV.

4.00	2.27		9.52	133.01	1902.01		1700.78	13.87	4012.62	389.08		5.65	70.23	7.05	4.81	5.16	30.03	2.85	4.45				
31.92	4.00		185.51	3616.27	52548.14		46994.45	311.30	110942.33	10725.76		9.74	130.65	11.80	8.68	9.42	746.39	7.08	7.47				
3.09	2.26		3.15	2.17	2.17		1.62	2.81	1.23	1.50		6.58	95.47	8.35	5.62	6.08	3.64	3.08	5.29				
1.49	0.83		4.00	22.61	291.58		262.23	2.88	615.94	61.25		3.06	46.88	3.99	4.18	2.72	7.26	2.85	2.27				
2.03	1.21		4.11	4.00	15.71		15.56	1.83	32.93	5.46		5.63	100.16	7.43	6.62	5.16	4.80	4.04	4.37				
5.48	3.83		6.66	6.15	4.00		4.59	5.02	4.43	2.90		9.64	105.60	12.11	9.35	8.39	6.69	5.75	7.08				
1.85	1.13		3.77	3.11	0.96		2.46	1.62	2.04	2.25		4.47	71.62	5.85	5.59	4.01	4.06	3.60	3.36				
2.53	1.55		4.80	3.97	2.44		4.00	2.22	4.79	2.95		6.13	98.06	8.03	7.29	5.50	5.21	4.57	4.61				
3.37	2.40		4.01	14.41	173.08		154.66	4.00	362.12	36.41		6.42	84.05	8.08	5.58	5.84	6.15	3.20	5.05				
3.76	2.35		6.60	5.88	2.24		4.48	3.34	4.00	3.45		7.83	107.12	10.14	9.32	6.82	6.80	5.97	5.65				
3.25	2.04		5.85	5.63	6.39		8.04	2.93	13.05	4.00		6.54	85.71	8.44	8.00	5.65	6.00	5.24	4.66				
5.87	4.08		7.23	6.69	3.14		4.03	5.37	2.50	2.90		10.19	109.07	12.80	10.01	8.83	7.19	6.20	7.43				
31.18	2.78		191.37	3795.38	55183.30		49350.55	324.90	116507.29	11262.25		4.00	30.53	4.19	2.92	4.21	779.84	4.05	3.03				
3.87	0.65		22.17	401.73	5809.37		5196.84	34.74	12266.24	1186.93		1.10	4.00	1.28	2.31	0.90	83.94	2.18	0.64				
46.94	3.21		295.42	5882.58	85546.83		76504.83	502.36	180614.66	17458.78		4.12	30.55	4.00	2.84	4.69	1207.71	5.22	3.10				
42.36	2.11		272.76	5408.60	78636.79		70327.01	460.98	166028.37	16049.82		3.26	36.15	3.27	4.00	3.75	1111.41	6.00	2.24				
21.37	2.63		125.61	2470.92	35910.69		32115.06	212.44	75816.46	7329.21		4.08	28.80	4.50	3.20	4.00	508.47	3.50	3.05				

1.35	0.82		3.19	7.58	71.52		65.48	1.58	151.40	16.28		2.68	36.41	3.46	3.85	2.32	4.00	2.72	1.92				
12.52	1.35		75.13	1440.46	20909.80		18701.42	123.69	44147.68	4269.11		3.52	46.88	4.31	4.60	3.31	298.20	4.00	2.52				
46.24	3.78		285.91	5665.32	82365.39		73660.14	484.63	173897.27	16809.95		5.53	40.82	5.80	4.50	5.78	1164.15	6.21	4.00				
2.69	2.03		2.54	1.85	1.91		1.32	2.47	0.97	1.18		5.27	70.16	6.61	4.35	4.86	2.86	2.43	4.25				
1.62	0.98		3.43	2.70	0.89		2.25	1.40	1.95	2.15		4.26	73.15	5.61	5.30	3.87	3.79	3.37	3.27				
3.50	2.15		6.24	5.10	1.78		3.78	3.07	2.95	3.34		8.22	127.44	10.73	9.49	7.32	6.71	5.86	6.13				
3.01	2.20		3.03	1.93	2.17		1.53	2.72	1.22	1.49		6.70	101.99	8.54	5.67	6.25	3.61	3.05	5.45				

XV.

4.00	2.18		10.72	133.80	1901.79		1701.65	13.83	4013.58	389.74		5.51	70.98	6.97	5.83	5.35	30.60	4.04	4.97				
32.02	4.00		186.83	3617.15	52548.00		46995.41	311.35	110943.38	10726.48		9.68	131.47	11.82	9.81	9.73	747.01	8.39	8.14				
3.35	2.40		4.66	3.17	2.15		2.71	3.00	2.43	2.33		6.66	96.41	8.52	6.90	6.58	4.35	4.57	6.19				
2.32	1.59		4.00	22.27	292.23		261.96	3.67	615.73	60.80		3.78	46.50	4.76	4.03	3.71	6.73	2.84	3.53				
3.29	2.35		4.61	4.00	16.70		15.65	3.02	33.12	5.28		6.71	100.10	8.60	6.90	6.65	4.51	4.53	6.27				
5.76	3.98		8.19	7.17	4.00		5.70	5.23	5.64	3.74		9.73	106.55	12.29	10.65	8.90	7.41	7.27	8.01				
3.10	2.27		4.27	3.11	1.95		2.55	2.81	2.24	2.08		5.55	71.56	7.01	5.87	5.50	3.77	4.09	5.26				
3.68	2.59		5.18	3.88	3.34		4.00	3.31	4.89	2.71		7.12	97.92	9.10	7.47	6.87	4.86	4.94	6.35				
3.42	2.36		5.27	15.24	172.91		155.57	4.00	363.12	37.10		6.32	84.84	8.05	6.66	6.09	6.75	4.45	5.64				
4.81	3.30		6.86	5.72	3.07		4.40	4.34	4.00	3.15		8.74	106.91	11.11	9.39	8.07	6.40	6.22	7.25				
4.77	3.42		6.67	5.84	7.58		8.36	4.37	13.49	4.00		7.84	85.84	9.84	8.55	7.45	5.86	6.04	6.96				
5.95	4.06		8.53	7.56	2.99		4.97	5.40	3.53	3.62		10.13	109.88	12.80	11.11	9.12	7.80	7.49	8.07				

31.34	2.83		192.77	3796.31	55183.21		49351.56	325.01	116508.39	11263.02		4.00	31.40	4.27	4.11	4.59	780.50	5.43	3.79				
5.21	1.86		22.77	401.80	5810.42		5197.00	36.01	12266.51	1186.81		2.25	4.00	2.52	2.67	2.48	83.70	2.76	2.66				
47.02	3.19		296.71	5883.44	85546.68		76505.77	502.39	180615.68	17459.49		4.05	31.35	4.00	3.94	4.97	1208.32	6.50	3.73				
43.34	3.00		272.94	5408.38	78637.56		70326.86	461.91	166028.30	16049.46		4.10	35.88	4.18	4.00	4.92	1110.97	6.17	3.72				
21.21	2.39		126.62	2471.59	35910.35		32115.79	212.24	75817.26	7329.76		3.81	29.43	4.28	4.06	4.00	508.94	4.50	3.33				
3.11	2.42		4.31	7.99	72.90		66.01	3.26	152.08	16.45		4.19	36.73	5.09	4.65	4.42	4.00	3.82	4.59				
13.36	2.11		75.14	1440.13	20910.46		18701.15	124.49	44147.48	4268.66		4.24	46.51	5.09	4.46	4.31	297.68	4.00	3.79				
45.90	3.38		286.71	5665.85	82364.91		73660.71	484.26	173897.90	16810.39		5.10	41.32	5.42	5.18	5.56	1164.52	6.99	4.00				
2.65	1.90		3.69	2.62	1.66		2.16	2.38	1.89	1.81		5.10	70.88	6.50	5.33	5.00	3.40	3.57	4.70				
2.80	2.05		3.85	2.64	1.82		2.28	2.52	2.07	1.93		5.27	73.04	6.70	5.50	5.28	3.46	3.77	5.06				
4.80	3.33		6.80	5.14	2.81		3.91	4.31	3.19	3.20		9.34	127.42	11.94	9.82	8.88	6.45	6.42	8.11				
3.30	2.37		4.59	2.96	2.19		2.65	2.95	2.45	2.35		6.81	102.96	8.74	6.99	6.78	4.34	4.59	6.41				

V. PREDICTIONS AND VARIABLE ANALYSIS

By examining the final fifteen matrices, it is easy to immediately exclude any matrix with entries ranging from very large (over about 10). This is a direct result of the rating system Spotify uses. For characteristics like Energy, Instrumentalness and Speechiness, the values commonly proceeded to the thousandth and ten-thousandth place - upon a ratio test, the quotients and their inverses ranged from inordinately large to “undefined”. The former explains for most entries greater than 10, while the latter explains the empty columns where a “#DIV/0!” error was displayed. Since no value is truly divided by 0, but rather just a very small number, the then extremely large ratio values can be further compared through logarithmic scaling. For sake of simplifying our method, we chose to exclude combination matrices with those two data anomalies. Thus we are left with matrices **I, III, V, VIII** and **XII** as our strongest and most interpretable ranking matrices.

We can predict that this method will, at least, provide a way to determine alikeness of the songs for each characteristic, but does that make the songs sound alike?

Since our experiment is testing variable combinations, and not variable songs or song features, it is the *rankings* of each matrix that we must compare. Hence, eliminating matrices based on any invalid entries is the most effective deductive process.

To maintain the interpretability of our solutions, we must remember that our question is a very open ended one. Musical preference, and the listeners affinity to categorize the song, is quite variable and highly interpretable itself. Thus, the real "test of success" will be whether us -- the listeners -- think the songs are alike, and affirm our opinions by Spotify's classifications.

VI. FURTHER STUDY & NEXT STEPS

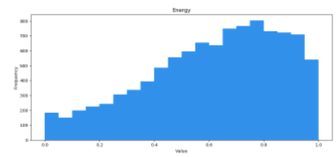
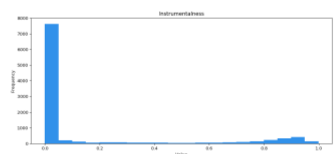
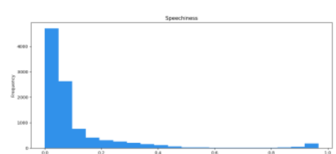
We can now rank which songs are similar in each chosen combination of features according to our model, and then hear those songs to determine the accuracy of each combination. Once we have the most accurate combination of features out of the chosen five, we can go back to Spotify API and compare our list of similar songs with the genres they belong to. If two songs are similar, then they would theoretically belong to the same genre and if they are different than they might belong to different genres. As suggested by Prof. Hao, we can also make our other non-chosen matrices of features useful by dividing each entry by log(10) which would create consistency in the values in the matrix. In addition to that, we used Spotify's API thus limiting our model to Spotify's interpretations and quantification of features of songs. It would be interesting to use Pandora's API and see if we find any similarities in the results from Spotify's API and Pandora's API.

VII. APPLICATIONS AND CONCLUSION

We started this project because of our common interests in the music field and our desire to do something with mathematics and music together. Our project was focused on creating a mathematical model and an algorithm to find a quantitative value of how similar and different a song ***i*** is to song ***j***, and thus has many applications in the music industry as well as the tech industry (Spotify, Pandora, Soundcloud). The model can be used to determine which features to use to find similarities between songs. In addition to that, we can also use this model in different DJ software to determine what features are similar in which songs which would be a game changer since mixing music is all about changing from one song to another smoothly and that can only be achieved when a feature of one song matches the feature of the other song. For example, from our model we can determine which songs have similar tempo or keys. To mix two songs, it is highly recommended that those songs have the similar tempo or BPM (beats per minute).

VOCABULARY

Key		Value Description
Key		The estimated overall key of the track. Integers map to pitches using standard . The estimated overall key of the track. Integers map to pitches using standard. Pitch Class notation . E.g. 0 = C, 1 = C♯/D♭, 2 = D, and so on. If no key was detected, the value is -1. E.g. 0 = C, 1 = C♯/D♭, 2 = D, and so on. If no key was detected, the value is -1.

Mode		Mode indicates the modality (major or minor) of a track, the type of scale from which its melodic content is derived. Major is represented by 1 and minor is 0.
Energy		Energy is a measure from 0.0 to 1.0 and represents a perceptual measure of intensity and activity. Typically, energetic tracks feel fast, loud, and noisy. For example, death metal has high energy, while a Bach prelude scores low on the scale. Perceptual features contributing to this attribute include dynamic range, perceived loudness, timbre, onset rate, and general entropy. The distribution of values for this feature look like this: 
Instrumentallness		Predicts whether a track contains no vocals. “Ooh” and “aah” sounds are treated as instrumental in this context. Rap or spoken word tracks are clearly “vocal”. The closer the instrumentalness value is to 1.0, the greater likelihood the track contains no vocal content. Values above 0.5 are intended to represent instrumental tracks, but confidence is higher as the value approaches 1.0. The distribution of values for this feature look like this: 
Speechiness		Speechiness detects the presence of spoken words in a track. The more exclusively speech-like the recording (e.g. talk show, audio book, poetry), the closer to 1.0 the attribute value. Values above 0.66 describe tracks that are probably made entirely of spoken words. Values between 0.33 and 0.66 describe tracks that may contain both music and speech, either in sections or layered, including such cases as rap music. Values below 0.33 most likely represent music and other non-speech-like tracks. The distribution of values for this feature look like this: 
Tempo		The overall estimated tempo of a track in beats per minute (BPM). In musical terminology, tempo is the speed or pace of a given piece and derives directly from the

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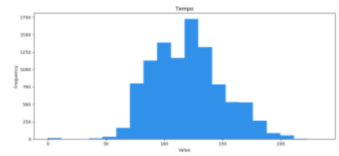
<tr><td></td><td></td><td>average beat duration. The distribution of values for this feature look like this:


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