3.2

Bayes's Formula

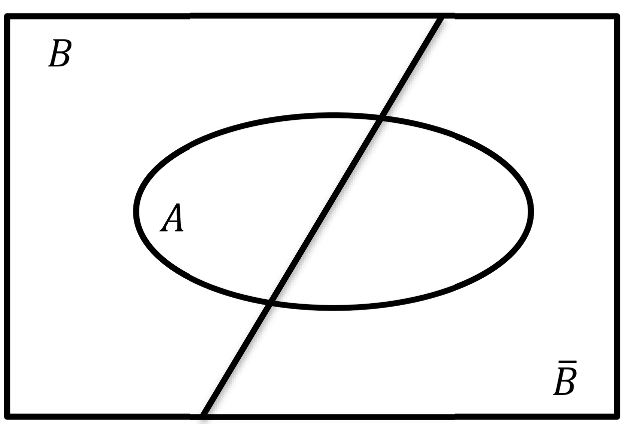
3.2. Bayes's Formula

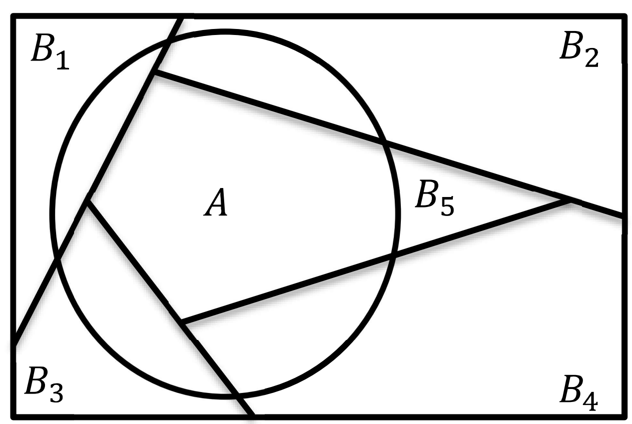
Theorem 1 (The total probability theorem)

Let $B_1, B_2, \dots, B_k$ be a partition of Ω, and A be an event, then:

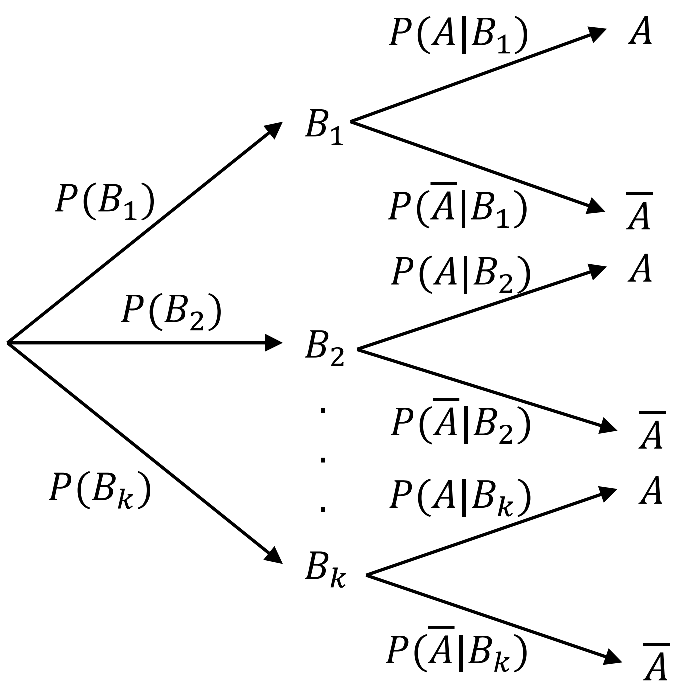
$$P(A) = \sum_{i=1}^{k} P(A \cap B_i)$$

$P(A) = P(A \cap B) + P(A \cap \bar{B})$





3.2. Bayes's Formula



3.2. Bayes's Formula

Let A and B be two integers, then:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Longrightarrow P(A \cap B) = P(A|B)P(B)$$

Bayes's Formula

Let $B_1, B_2, \dots, B_k$ be a partition of Ω, and A be an event, then:

$$P(A) = \sum_{i=1}^{k} P(A \cap B_i) \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

3.2. Bayes's Formula

Example 5

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability decreases to 0.2 for a person who is not accident prone. Assume that 30 percent of the population is accident prone

i. What is the probability that a new policyholder will have an accident within a year of purchasing a policy?
ii. Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

3.2. Bayes's Formula

Solution

i. $P(A) = P(A \cap P) + P(A \cap \overline{P}) = P(A|P)P(P) + P(A|\overline{P})P(\overline{P}) = 0.4 \times 0.3 + 0.2 \times 0.7 = 0.26$

ii. $P(P|A) = \frac{P(P \cap A)}{P(A)} = \frac{0.12}{0.26} = \frac{6}{13}$