2.2

Axioms of Probability

2.2. Axioms of probability

Definition 8

A probability measure is a function that assigns to each event in a random experiment a value between 0 and 1. This value describes the chance that this event will occur. The probability of a given event A is denoted by $P(A)$.

Axioms of probability

1. For any event A, we have $0 \leq P(A) \leq 1$.
2. The probability of the sample space Ω is $P(\Omega) = 1$.
3. For any two **disjoint** events A and B, we have $P(A \cup B) = P(A) + P(B)$.

2.2. Axioms of probability

Remark

If the sample space is discrete, then $P(A) = \frac{|A|}{|\Omega|}$

Example 5

We flip a coin. Calculate the probabilities $P(\{H\})$ and $P(\{T\})$

Example 6

If the coin were biased and we believed that a head were twice likely as likely to appear as a tail, calculate $P(\{H\})$ and $P(\{T\})$.

Example 7

If a fair die is rolled, find $P(\{1\})$, $P(\{2\})$, $P(\{3\})$, $P(\{4\})$, $P(\{5\})$, $P(\{6\})$ and $P(\{1,2,4\})$.

2.3

Simple propositions

2.3. Simple propositions

Rules

1. $P(A^c) = 1 - P(A)$, for any event A.
2. $P(\emptyset) = 0$.
3. If $A \subseteq B$, then $P(A) \leq P(B)$.
4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
5. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$

2.3. Simple propositions

Example 8

Sara is taking two books along on her holiday vacation. With probability 0.5, she will like the first book; with probability 0.4, she will like the second book; and with probability 0.3, she will like both books. What is the probability that she likes neither book?

2.4

Sample spaces having equally likely outcomes

2.4. Sample spaces having equally likely outcomes

Activity 1

We roll a die. What is the probability of each singleton event?

2.4. Sample spaces having equally likely outcomes

In many experiments, it is natural to assume that all outcomes in the sample space are equally likely to occur.

That is, consider a sample space consisting of n outcomes $\Omega = \{a_1, a_2, \ldots, a_n\}$, then it is often natural to assume that $P(\{a_1\}) = P(\{a_2\}) = \cdots = P(\{a_n\}) = \frac{1}{n}$.

So, for every event A, we have $P(A) = \frac{|A|}{|\Omega|}$.

2.4. Sample spaces having equally likely outcomes

Example 9

If two dice are rolled, what is the probability that the sum of the upturned faces will equal 7?

Example 10

If 3 balls are “randomly drawn” from a bowl containing 6 white and 5 black balls, what is the probability that exactly one of the balls is white?

2.4. Sample spaces having equally likely outcomes

Example 11

5 people are to be randomly selected from a group of 20 individuals consisting of 10 married couples. Determine $P(A)$, the probability that the 5 chosen are all unrelated. (That is, no two are married to each other.)

Example 12

A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?

2.4. Sample spaces having equally likely outcomes

Example 13

An urn contains n balls, one of which is special. If k of these balls are withdrawn one at a time, with each selection being equally likely to be any of the balls that remain at the time, what is the probability that the special ball is chosen?

Example 14

A 5-card poker hand is said to be a full house if it consists of 3 cards of the same denomination and 2 other cards of the same denomination (of course, different from the first denomination). Thus, one kind of full house is three of a kind plus a pair. What is the probability that one is dealt a full house?