3.3

Independent Events

3.3. Independent Events

Activity 3

Consider an experiment in which 2 cards are drawn in succession from an ordinary deck, with replacement. The events are defined as
A: the first card is an ace.
B: the second card is a spade.
Find $P(B|A)$ and $P(B)$. Conclude.

Solution

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{4}{52} \times \frac{13}{52}}{\frac{4}{52}} = \frac{1}{4}$$

$$P(B) = \frac{13}{52} = \frac{1}{4}$$

We conclude that $P(B|A) = P(B)$, we say that A and B are independent.

3.3. Independent Events

Definition 2

We say that two events A and B are independent if $P(A|B) = P(A)$ or $P(B|A) = P(B)$

Properties

1. Two events A and B are independent if and only if $P(A \cap B) = P(A) \times P(B)$
2. If A and B are independent, then A and $\overline{B}$ are independent

3.3. Independent Events

Example 10

A sequence of n independent trials is to be performed. Each trial results in a success with probability p, and a failure with probability $1 - p$. What is the probability that:

i. At least one trial results with success?
ii. Exactly k trials result with successes?
iii. All trials result with successes?

Solution

Consider the event

E_i: "The i^{th} trial results with success"

i. $P(\text{at least one success}) = 1 - P(\text{no success}) = 1 - P\left(\bigcap_{i=1}^{n} \overline{E}_i\right) = 1 - P\left(\overline{E}_i\right)P\left(\overline{E}_2\right) \dots P\left(\overline{E}_n\right) = 1 - (1-p)^n$

ii. $P(\text{exactly } k \text{ successes}) = \binom{n}{k} p^k (1-p)^{n-k}$

iii. $P\left(\bigcap_{i=1}^{n} E_i\right) = p^n$

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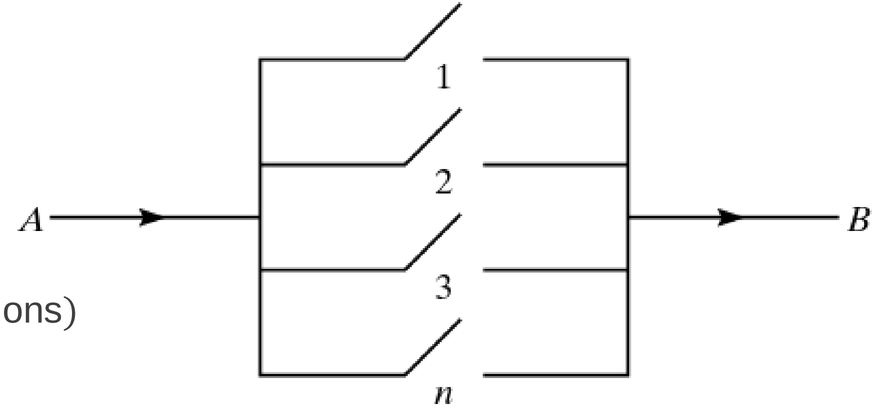
Example 11

A system composed of n separate components is said to be a parallel system if it functions when at least one of the components functions (See figure). For such a system, if component i, which is independent of the other components, functions with probability p_i, $i = 1, \dots, n$, what is the probability that the system functions?



Solution

Consider the event

E_i: "The i^{th} component functions"

$$\begin{aligned} P(\text{the system functions}) &= P(\text{at least one component functions}) \\ &= 1 - P(\text{no one functions}) \\ &= 1 - P\left(\bigcap_{i=1}^{n} \overline{E}_i\right) \\ &= 1 - (1-p_1)(1-p_2)\dots(1-p_n) \end{aligned}$$