USAL
جامعة العلوم والآداب اللبنانية
UNIVERSITY OF SCIENCES & ARTS IN LEBANON

Chapter 1: Combinatorial Analysis

MATH204: Probability and Statistics I

Dr. Ayman El Zein

Combinatorial Analysis

Section 1

The basic principle of counting

Section 2

Permutations

Section 3

Combinations

Section 4

Multinomial coefficients

1.1

The basic principle of counting

1.1. The basic principle of counting

Activity 1

A small community of 10 women, each of whom has 3 children. If one woman and one of her children are to be chosen as mother and child of the year, how many different choices are possible?

1.1. The basic principle of counting

Activity 2

How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 places by numbers (digits)?

ABC 7901

FLL 0003

1.1. The basic principle of counting

Rule

If r experiments that are to be performed are such that:

- The 1^{st} one may result in any of n_1 possible outcomes
- The 2^{nd} one may result in any of n_2 possible outcomes

  .

  .

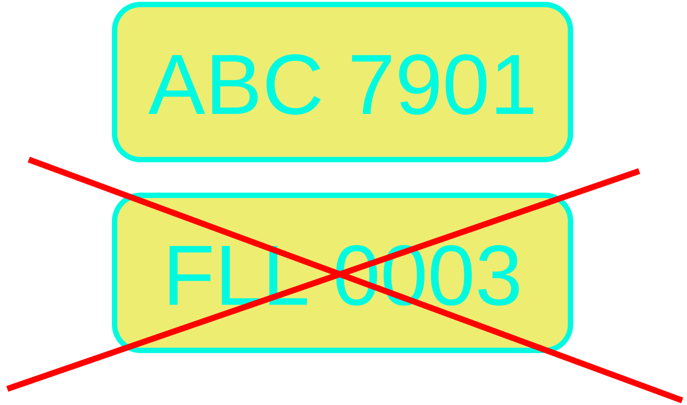
  .
- The r^{th} one may result in any of n_r possible outcomes

Then, there is a total of $n_1 n_2 \dots n_r$ possible outcomes of the r experiments

1.1. The basic principle of counting

Example 1

In the previous activity, how many license plates would be possible if the repetition among letters or numbers were prohibited?



1.1. The basic principle of counting

Example 2

How many bit strings of length 8 can we make?

0110 1100
1101 0000
1010 0101
1111 0000
1100 1111

1.2

Permutations

1.2. Permutations

Activity 3

How many ordered arrangements of the letters A, B and C are possible?

1.2. Permutations

Definition 1

A permutation of n objects is an ordered arrangement of them.
The number of permutations of n objects is $n!$

1.2. Permutations

Example 3

Sami has 10 books that he is going to put on his bookshelf. Of these,

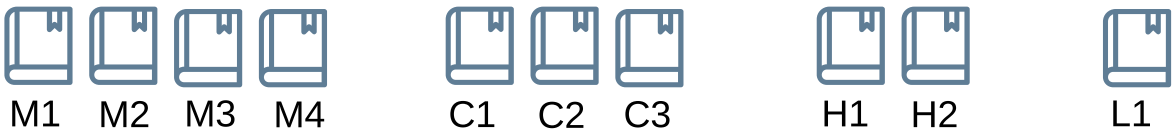
- 4 are mathematics books,
- 3 are chemistry books,
- 2 are history books,
- And, 1 is a language book.

How many different arrangements are possible if:

i. They are arranged randomly.

ii. All the books dealing with the same subject are together on the shelf such that mathematics books are at the beginning, then the chemistry books, then the history books, and finally the language book.

iii. All the books dealing with the same subject are together on the shelf.



1.2. Permutations

Activity 4

How many different letter arrangements can be formed from the letters $AABBB$?

1.2. Permutations

Rule 2

The number of the different permutations of n objects, of which n_1 are alike, n_2 are alike, …, n_r are alike, is:

$$\frac{n!}{n_1!\,n_2!\ldots n_r!}$$

1.2. Permutations

Example 4

A chess tournament has 10 competitors, of which 4 are Russian, 2 are American, 2 are British and 2 are Brazilian. If the tournament result lists just the nationalities of the players in the order which they placed, how many outcomes are possible?

- Russia
- Russia
- Brazil
- USA
- Russia
- Britain
- USA
- Brazil
- Russia
- Britain

1.2. Permutations

Activity 5

How many different words of 3 letters can be formed from the letters $ABCDEFG$?

1.2. Permutations

Rule 3

The number of the different arrangements of r objects chosen from n objects, is:

$$n(n-1)\ldots\big(n-(r-1)\big)=\frac{n!}{(n-r)!}=P_r^n$$

1.2. Permutations

Example 5

A president and a treasurer are to be chosen from a club of 50 students. How many different choices of officers are possible if:

i. There are no restrictions?

ii. A student X will serve only if he is president?

iii. Two students X and Y will serve together or not at all?

iv. Two students X and Y will not serve together?