USAL

جامعة العلوم والآداب اللبنانية

UNIVERSITY OF SCIENCES & ARTS IN LEBANON

Chapter 2:
Axioms of Probability

MATH204: Probability and Statistics I

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Axioms of Probability

Section 1

Sample space and events

Section 2

Axioms of probability

Section 3

Simple propositions

Section 4

Sample spaces having equally likely outcomes

2.1

Sample space and events

2.1. Sample space and events

Definition 1

A random experiment is a phenomenon whose outcome cannot be predicted with certainty

Example 1

Roll a die → 1 – 2 – 3 …

Roll a die three times → 1 2 3 – 2 2 2 – 2 1 3 …

Roll three dice once → {1,2,3} – {2,2,2} – {1,6,6} …

Flip a coin → T – F

Definition 2

An outcome is a result of a random experiment

2.1. Sample space and events

Definition 3

The sample space of a random experiment is the set of all its possible outcomes

Example 2

Roll a die

$$\boldsymbol{\Omega = \{1, 2, 3, 4, 5, 6\}}$$

Roll a die three times

$$\boldsymbol{\Omega = \{111, 112, 113, 114, 115, 116, 121, 122, \dots, 666\}}$$

Roll three dice once

$$\boldsymbol{\Omega = \{\{1, 1, 1\}, \{1, 1, 2\}, \{1, 1, 3\}, \dots, \{6, 6, 6\}\}}$$

Flip a coin

$$\boldsymbol{\Omega = \{T, F\}}$$

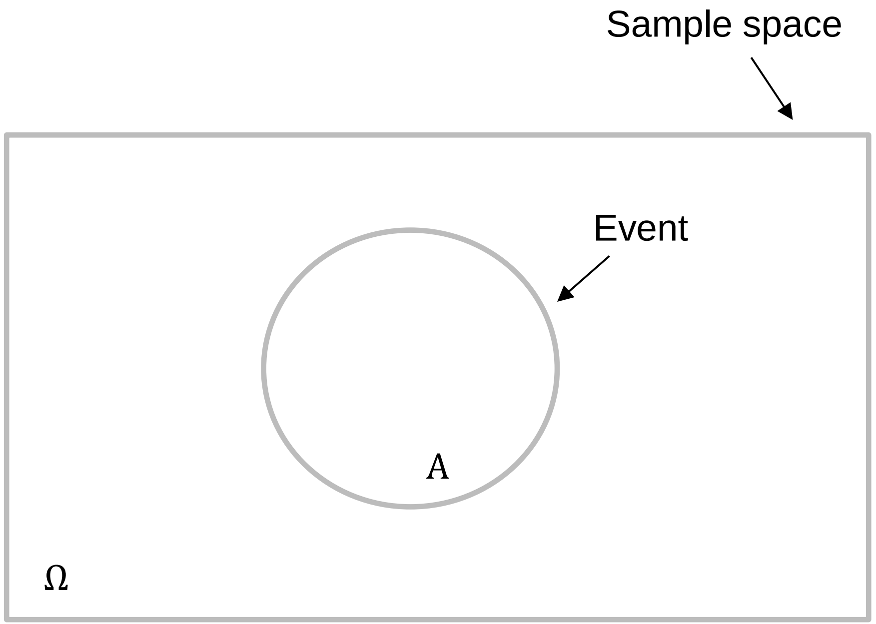
2.1. Sample space and events

Definition 4

An event is a subset of the sample space. We say that an event A occurs if the outcome of the experiment is an element of A.

Example 3

Experiment: Roll a die
Sample space: $\Omega = \{1,2,3,4,5,6\}$
Events: $A_1 = \{1,2,5\}$
$A_2 = \{2\}$
$A_3 = \{4,6\}$



2.1. Sample space and events

Definition 5

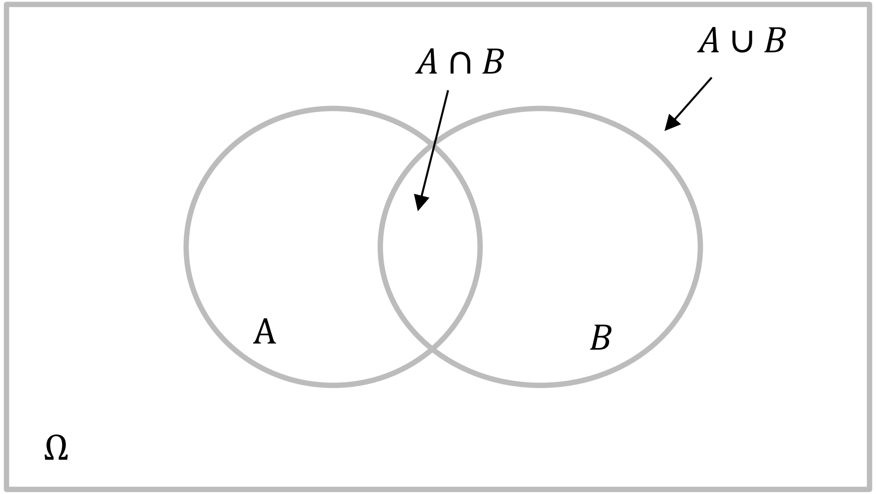
For any two events A and B, we define the events:

- A union B, denoted by $A \cup B$, that consists of all outcomes that are either in A or in B.
- The intersection of A and B, denoted by $A \cap B$, that consists of all outcomes that are in both A and B.

Example 4

We flip successively two coins. Consider the events:
A: “The first coin lands a head”
B: “The second coin land a head”

i. Write the sample space.
ii. Write all outcomes of A and B.
iii. Find $A \cup B$ and $A \cap B$.



2.1. Sample space and events

Laws

Commutative laws: $A \cup B = B \cup A$
$A \cap B = B \cap A$

Associative laws: $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

Distributive laws: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's laws: $(A \cup B)^c = A^c \cap B^c$
$(A \cap B)^c = A^c \cup B^c$

2.1. Sample space and events

Definition 7

- The events $A_1, A_2, \dots, A_n$ are said to be mutually exclusive (or disjoint) if $A_i \cap A_j = \emptyset$ for all $i \neq j$
- A partition of Ω is a collection of mutually exclusive events $A_1, A_2, \dots, A_n$ such that $\bigcup_{i=1}^{n} A_i = \Omega$