

**Problem Set 2**

**Issued:** Feb. 15, 2005.

**Due:** Feb. 25, 2005.

**Background for Problem Set 2:** Laser crystals and optical components made from glass, for example BK7 or Quartz (fused silica), exhibit positive second-order dispersion. For dispersion compensation in and outside laser resonators, one often uses a sequence of prisms or gratings, which deliver the required negative second-order dispersion through geometric effects (differences in transit time). Although the derivation process might involve quite lengthy calculations, it will be helpful to analyze them yourself for future design of laser resonators or external compression experiments.

The following references will be helpful:

E. B. Treacy, "Optical pulse compression with diffraction gratings", IEEE J. Quant. Electron. **5**, 454 (1969).

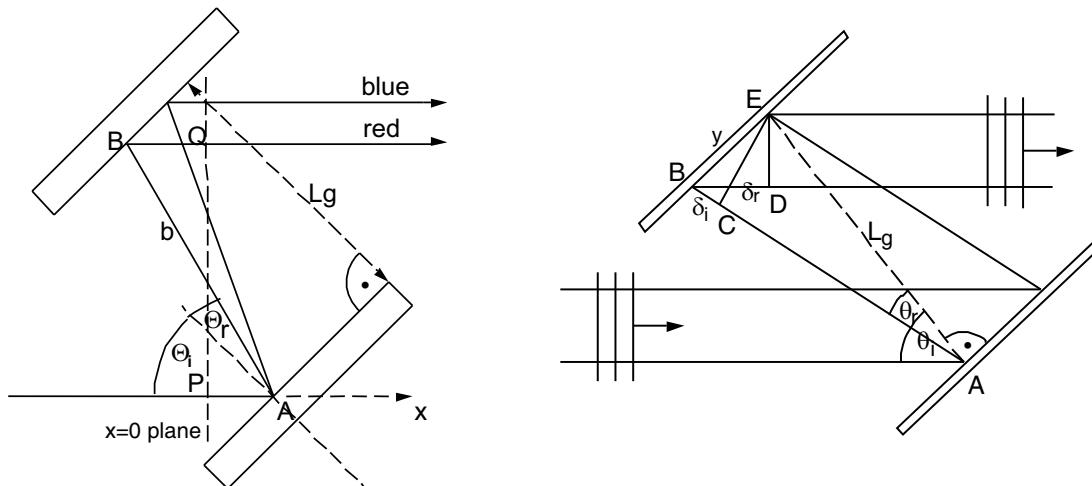
Stuart D. Brorson and Hermann A. Haus, "Geometrical limitations in grating pair pulse compression", Appl. Opt. **27**, 23 (1988).

H. A. Haus, *Waves and Fields in Optoelectronics*, Chapter 6.6.

R. L. Fork, O. E. Martinez, and J. P. Gordon, "Negative dispersion using pairs of prisms", Opt. Lett. **9**, 150 (1984).

**Problem 2.1: Diffraction Grating Pair**

- (a) Derive from simple geometric principles a condition for the angle  $\theta_r$  of maximum diffraction, as a function of the wavelength  $\lambda$ , the incident angle  $\theta_i$ , the grating period  $\Lambda$ , and the diffraction order  $m$ . Based on your answer, for which diffraction orders can the grating pair be used for generation of negative dispersion?

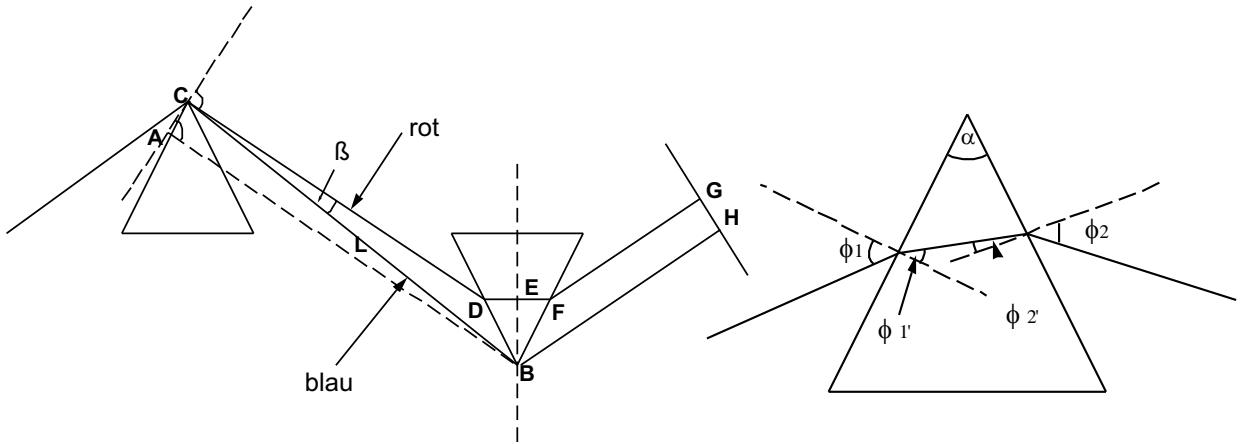


- (b) In schematic 1 the x-plane is defined by beginning and end of the optical path  $p_{opt} = \overline{PABQ}$  of an infinitely thin laser beam that is normal to the plane. Calculate this optical path as a function of  $\lambda$ ,  $\theta_r$ ,  $\theta_i$ , the diffraction order  $m$ , and the

grating distance  $L_g$ . What is the phase delay  $\Phi_{opt}$  corresponding to this optical path?

- (c) Considering the finite beam cross-section, as indicated in schematic 2, an additional phase shift  $\Phi_c$  must be added to account for the phase-matching provided by the grating. Schematic 2 shows that this adjustment is necessary, because the planes (CE) and (DE) represent planes with constant phase, and the grating must compensate the extra paths  $\delta_i = \overline{CB}$  and  $\delta_r = \overline{BD}$  travelled by the lower part of the beam with an appropriate phase shift. Deduce, again from geometric considerations, similar to part (a), this phase shift  $\Phi_c$  as a result of phase matching! What is the total phase shift  $\Phi = \Phi_{opt} + \Phi_c$ ? That is, calculate the phase shift from the total path difference. (It is essential here to observe the correct choice of signs!). (For more details consult the book by H. A. Haus: Waves and Fields in Optoelectronics, Chapter 2.6).
- (d) Calculate the group delay  $T_g = \frac{\partial\Phi}{\partial\omega}$  and the second-order dispersion  $D_2 = \frac{\partial T_g}{\partial\omega}$ . What is the dispersion relation for a four-grating compressor? For what reason is a second grating pair used?
- (e) Example: Laser pulses with 200 fs duration at a center wavelength of 800 nm shall be delivered to a medical imaging system through a 1 m long quartz fiber. To avoid broadening of the pulse the second order dispersion of the fiber link shall be compensated with grating pairs. Two grating pairs with 600 grooves per mm are given and shall be used at an angle of incidence of 50 degrees. What grating distance is necessary to compensate the second-order dispersion of the fiber when the grating is optimized for the diffraction order  $m = -1$  (Blazed Grating)?

### Problem 2.2: Prism Pair



- (a) For calculation of the optical path difference in prism pairs, the path  $\overline{CB} = L$  is the reference path. Argue that the optical path  $\overline{CDE} = \overline{AB}$  and  $\overline{EFG} = \overline{BH}$ . The difference in the distance travelled by different frequency components arises therefore only from the length  $\overline{CDE}$ . Give an expression for the optical path length  $P = \overline{CDE}$  as a function of  $L$  and  $\beta$ .

- (b) Rewrite the second derivative of  $P$  with respect to  $\lambda$  in known quantities, by considering  $P$  as a function of  $\beta(n(\lambda))$  (Reprinted application of the chain rule!) Take the dispersion relation  $n(\lambda)$  as given.
- (c) Determine from the geometry of a prism (Prism schematic 2) the dependence of the angle of reflection  $\Phi_2$  of the transmitted beam on the index of refraction  $n$  and calculate the first and second derivatives. (Hint: Take the angle of incidence  $\Phi_1$  as fixed and calculate the angle  $\Phi_2$  as a function of the inner diffraction angles  $\Phi'_1$  and  $\Phi'_2$  as shown in Prism diagram 2.)
- (d) What is the first and second derivative of  $\beta$ , considering the symmetric beam path and operation at Brewster's angle (Hint: There is a simple relationship between the the first derivative of  $\beta$  and  $\Phi_2$ ) ?  
 Combine the results of parts (a) through (d) to find the dispersion formula  $\partial^2\Phi(\omega)/\partial\omega^2$  for a four prism compressor. What is it?
- (e) To a first approximation,  $l \sin \beta \approx 2\text{mm}$  (somewhat larger than the spot size of a typical beam), and  $\cos \beta \approx 1$ . What prism distance (from apex-to-apex) is necessary to compensate the positive dispersion from a Ti:Sapphire laser crystal of about  $300 \text{ fs}^2$ ?  
 (Quartz prism:  $n(0.8\mu\text{m}) = 1.4535$ ,  $\frac{\partial n}{\partial \lambda}(0.8\mu\text{m}) = -0.017\frac{1}{\mu\text{m}}$ ,  $\frac{\partial^2 n}{\partial \lambda^2}(0.8\mu\text{m}) = 0.04\frac{1}{\mu\text{m}^2}$ )