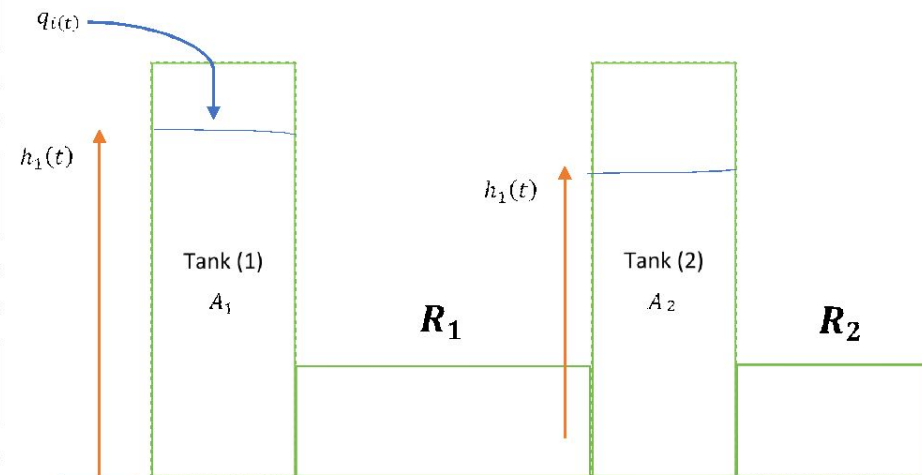


MODELING OF A TWO TANKS HYDRAULIC SYSTEM

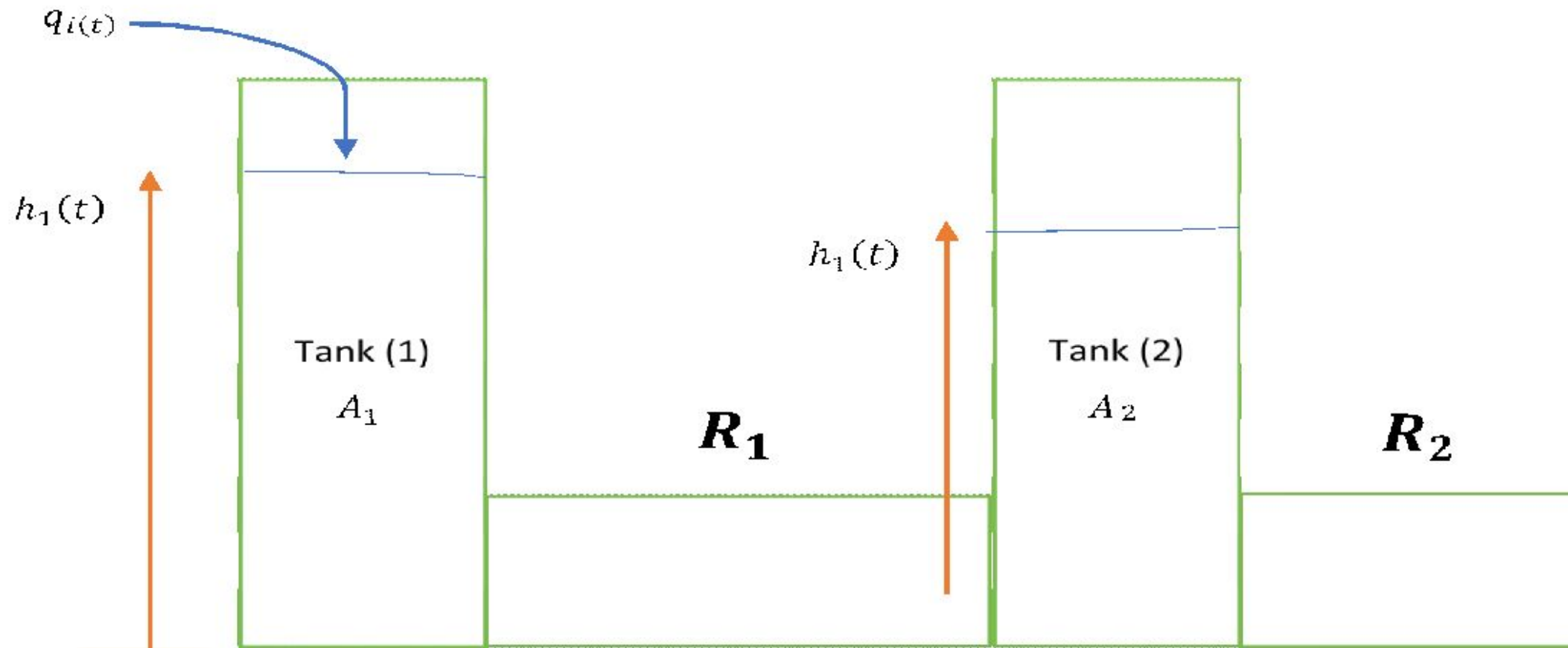
Presented By:

Hamed Alsubhi

The Schematic of Configuration of a Two Tanks Hydraulic System



The Schematic of Configuration of a Two Tanks Hydraulic System

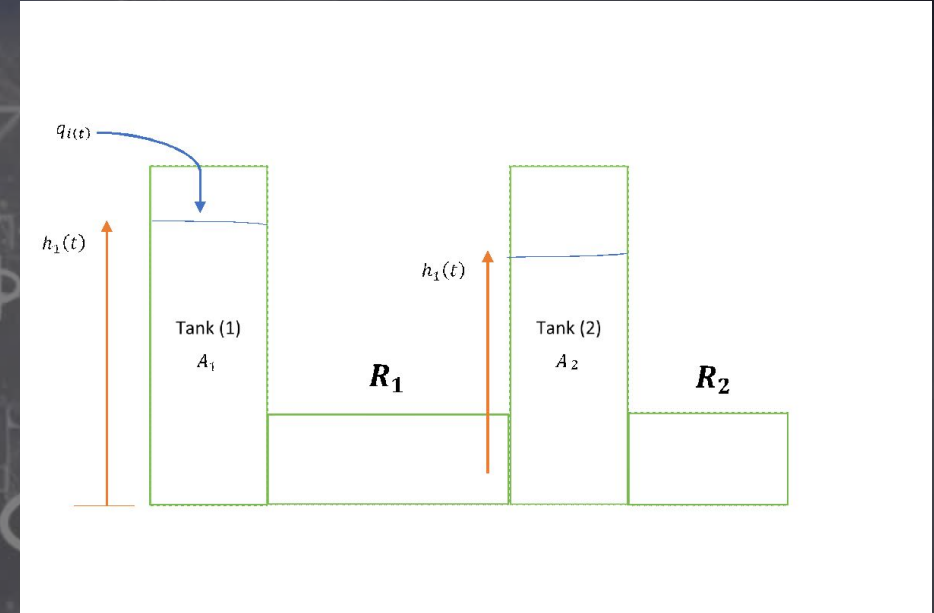


THE DIFFERENTIAL EQUATIONS' REPRESENTATION OF THE SYSTEM

$$h_1^*(t) = \frac{1}{A_1} q_i(t) - \frac{\rho g}{A_2} h_1(t) + \frac{\rho g}{A_1 R_1} h_2(t)$$

$$h_2^*(t) = \frac{\rho g}{A_2 R_1} h_1(t) - \frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) h_2(t)$$

- A_k = Cross Sectional Area of Tank k (m^2)
- R_k = Resistance of Pipe k ($\frac{Ns}{m^5}$)
- h_k = Water Height in Tank k (m)
- $q_i(t)$ = Volumetric Flow Rate ($\frac{m^3}{s}$)
- g = Gravity ($\frac{m}{s^2}$)
- ρ = Density ($\frac{Kg}{m^3}$)

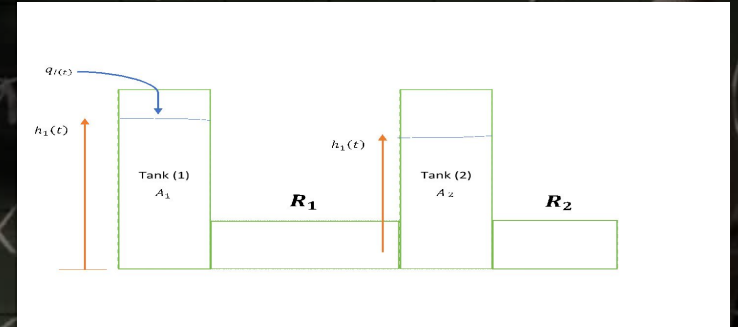


$$\frac{df}{dt} = \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h}$$

STATE-SPACE REPRESENTATION OF THE SYSTEM

$$\dot{h}_1(t) = \frac{1}{A_1} q_i(t) - \frac{\rho g}{A_1 R_1} h_1(t) + \frac{\rho g}{A_1 R_1} h_2(t)$$

$$\dot{h}_2(t) = \frac{\rho g}{A_2 R_1} h_1(t) - \frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) h_2(t)$$



$u(t)$

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t)\end{aligned}$$

$y(t)$

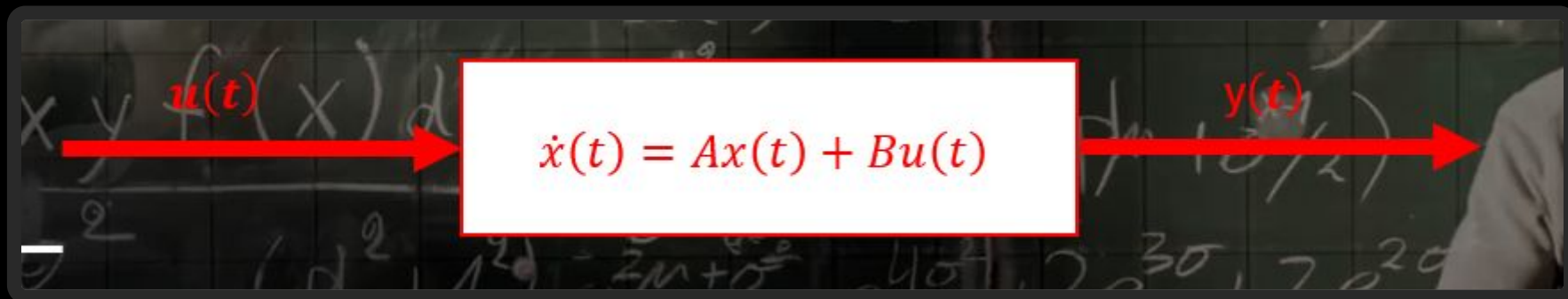
STATE-SPACE REPRESENTATION OF THE SYSTEM

$$h_1^*(t) = \frac{1}{A_1} q_i(t) - \frac{\rho g}{A_1 R_1} h_1(t) + \frac{\rho g}{A_1 R_1} h_2(t)$$

$$h_2^*(t) = \frac{\rho g}{A_2 R_1} h_1(t) - \frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) h_2(t)$$

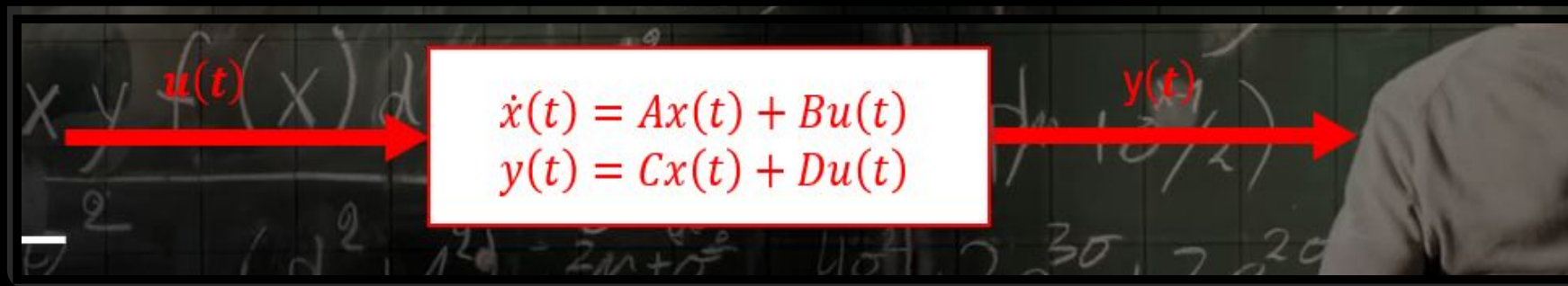
$$\begin{bmatrix} h_1^*(t) \\ h_2^*(t) \end{bmatrix} = \begin{bmatrix} -\frac{\rho g}{A_1 R_1} & \frac{\rho g}{A_1 R_1} \\ \frac{\rho g}{A_2 R_1} & -\frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ \mathbf{0} \end{bmatrix} [q_i(t)]$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [q_i(t)]$$



$$\begin{bmatrix} h_1^*(t) \\ h_2^*(t) \end{bmatrix} = \begin{bmatrix} -\frac{\rho g}{A_1 R_1} & \frac{\rho g}{A_1 R_1} \\ \frac{\rho g}{A_2 R_1} & -\frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} [q_i(t)]$$

Diagram illustrating the state-space representation of the hydraulic system. The state vector $\dot{x}(t)$ is defined by the derivative of the state variables $x(t)$. The system matrix A is the coefficient matrix in the state equation. The input matrix B is the coefficient matrix for the input $u(t)$. The output $y(t)$ is the measured variable, which is the first component of the state vector $x(t)$.



$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_1(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(t)$$

The equation is annotated with red curly braces and boxes to identify its components:

- A red curly brace under $\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$ is connected by a red line to a box labeled $y(t)$.
- A red curly brace under $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is connected by a red line to a box labeled C .
- A red curly brace under $\begin{bmatrix} h_1(t) \\ h_1(t) \end{bmatrix}$ is connected by a red line to a box labeled $x(t)$.
- A red curly brace under $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is connected by a red line to a box labeled D .
- A red curly brace under $u(t)$ is connected by a red line to a box labeled $u(t)$.

$u(t)$

$$\begin{bmatrix} h_1^*(t) \\ h_2^*(t) \end{bmatrix} = \begin{bmatrix} -\frac{\rho g}{A_1 R_1} & \frac{\rho g}{A_1 R_1} \\ \frac{\rho g}{A_2 R_1} & -\frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} [q_i(t)]$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [q_i(t)]$$

$y(t)$

➤ $A_1 = 0.785$

➤ $A_2 = 0.635$

➤ $R_1 = 100$

➤ $R_2 = 50$

➤ $g = 9.81$

➤ $\rho = 988$

$$A = \begin{bmatrix} -\frac{\rho g}{A_1 R_1} & \frac{\rho g}{A_1 R_1} \\ \frac{\rho g}{A_2 R_1} & -\frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix}$$

$$B = \begin{bmatrix} 1.27 \\ 0 \end{bmatrix}$$

TIME-DOMAIN SOLUTIONS

$$x(t) = e^{At}x(\mathbf{0}) + \int_0^t e^{A(t-\tau)}\beta u(\tau)d\tau$$

Zero-Input
Solution

Zero-State
Solution

Zero-Input Solution

$$x(t) = e^{At}x(0)$$

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix} \quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

□ Let's find the Eigenvalues of A

$$\det(\lambda I - A) = 0 \quad \lambda_1 = -73.5 \quad \lambda_2 = -509.2$$

□ Check the Validity of the Eigenvectors:

➤ Let's find the Eigenvectors of A

$$(\lambda_1 I - A)v_1 = 0$$

$$v_1 = \begin{bmatrix} 0.9281 \\ 0.3722 \end{bmatrix}$$

$$\lambda_1 v_1 = A v_1 = \begin{bmatrix} -69.3207 \\ -27.3546 \end{bmatrix}$$

$$(\lambda_2 I - A)v_2 = 0$$

$$v_2 = \begin{bmatrix} -0.3152 \\ 0.9490 \end{bmatrix}$$

$$\lambda_2 v_2 = A v_2 = \begin{bmatrix} 157.6457 \\ -483.2458 \end{bmatrix}$$

➤ P and P^{-1}

$$P = \begin{bmatrix} 0.9281 & -0.3152 \\ 0.3722 & 0.9490 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0.9508 & 0.3158 \\ -0.3729 & 0.9299 \end{bmatrix}$$

➤ D matrix:

$$D = P^{-1}A \quad P = \begin{bmatrix} 0.9508 & 0.3158 \\ -0.3729 & 0.9299 \end{bmatrix} \begin{bmatrix} -\mathbf{124.7} & 124.7 \\ \mathbf{154.2} & -\mathbf{458} \end{bmatrix} \begin{bmatrix} 0.9281 & -0.3152 \\ 0.3722 & 0.9490 \end{bmatrix} = \begin{bmatrix} -73.4887 & 0 \\ 0 & -509.2113 \end{bmatrix}$$

➤
$$e^{Dt} = \begin{bmatrix} e^{(-73.4887)t} & 0 \\ 0 & e^{(-509.2113)t} \end{bmatrix}$$

$$\triangleright e^{At} = P e^{Dt} P^{-1}$$

$$e^{At} = \begin{bmatrix} 0.9281 & -0.3152 \\ 0.3722 & 0.9490 \end{bmatrix} \begin{bmatrix} e^{(-73.4887)t} & 0 \\ 0 & e^{(-509.2113)t} \end{bmatrix} \begin{bmatrix} 0.9508 & 0.3158 \\ -0.3729 & 0.9299 \end{bmatrix} =$$

$$\begin{bmatrix} [0.8824e^{-(73.4)t} + 0.1175e^{(-509.2)t}] & [0.29309e^{-(73.4)t} - 0.2931e^{(-509.2)t}] \\ [0.3539e^{(-73)t} - 0.3539e^{(-509.2)t}] & [0.1176e^{-(73.4)t} + 0.88247e^{(-509.2)t}] \end{bmatrix}$$

\triangleright Thee Zero Input Solution

$$e^{At} \mathbf{x}(0) = \begin{bmatrix} [0.8824e^{-(73.4)t} + 0.1175e^{(-509.2)t}] & [0.29309e^{-(73.4)t} - 0.2931e^{(-509.2)t}] \\ [0.3539e^{(-73)t} - 0.3539e^{(-509.2)t}] & [0.1176e^{-(73.4)t} + 0.88247e^{(-509.2)t}] \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} [0.8824e^{-(73.4)t} + 0.1175e^{(-509.2)t}] \\ [0.3539e^{(-73)t} - 0.3539e^{(-509.2)t}] \end{bmatrix}$$

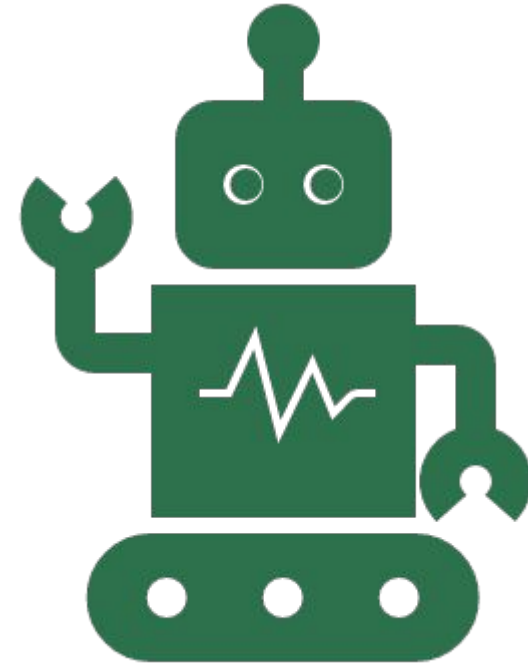
CHECK CONTROLLABILITY

Givens :

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix} \quad B = \begin{bmatrix} 1.27 \\ 0 \end{bmatrix}$$

$$\text{C-Matrix} = [B \ AB] = \begin{bmatrix} 1.27 & -158.4 \\ 0 & 195.8 \end{bmatrix}$$

$\text{Det}(C) = 248.7 \neq 0 \rightarrow$ **Full Rank** \rightarrow **Controllable**



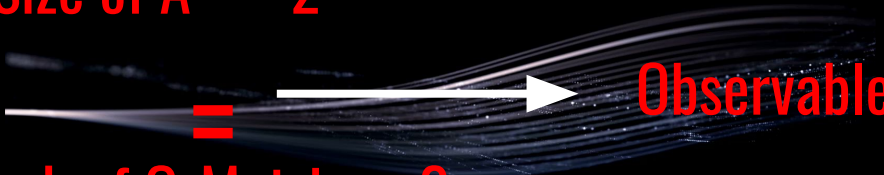
CHECK THE OBSERVABILITY

Given :

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{\text{O-Matrix}} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix}$$

Size of A = 2


Rank of O-Matrix = 2

Observable

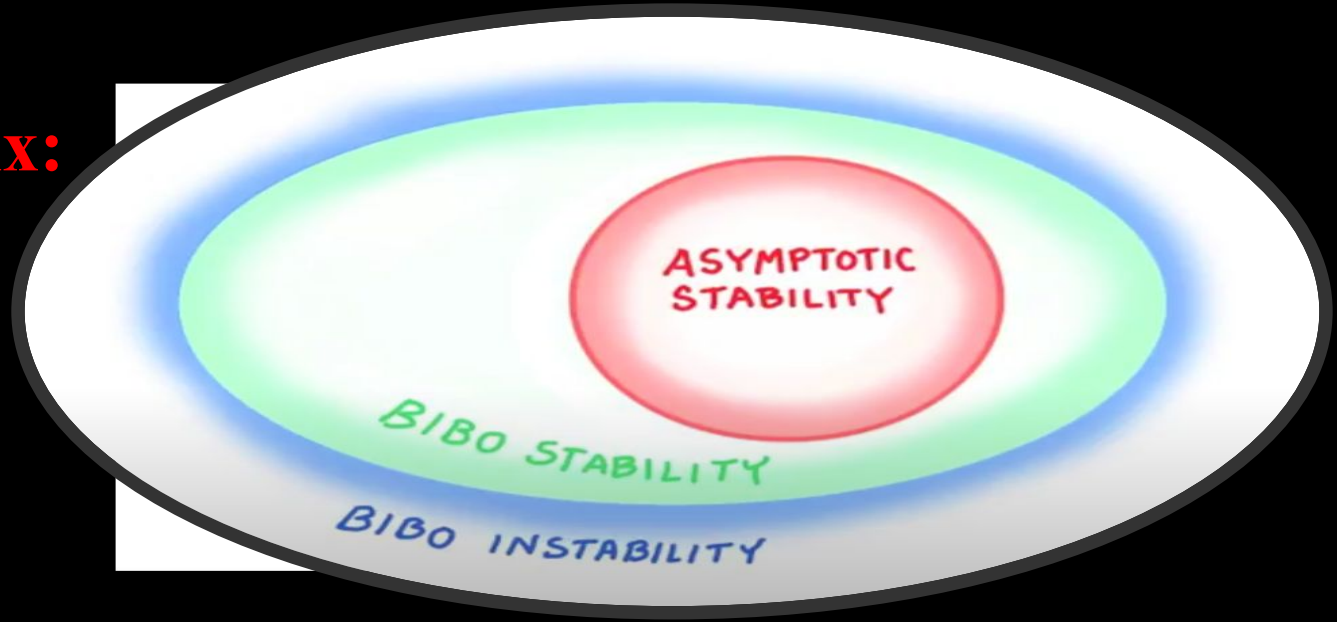
System Stability

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix}$$

The eigenvalues of A-Matrix:

$$\lambda_1 = -73.5$$

$$\lambda_2 = -509.2$$



Asymptotic Stable

System Solution

→ Zero

+

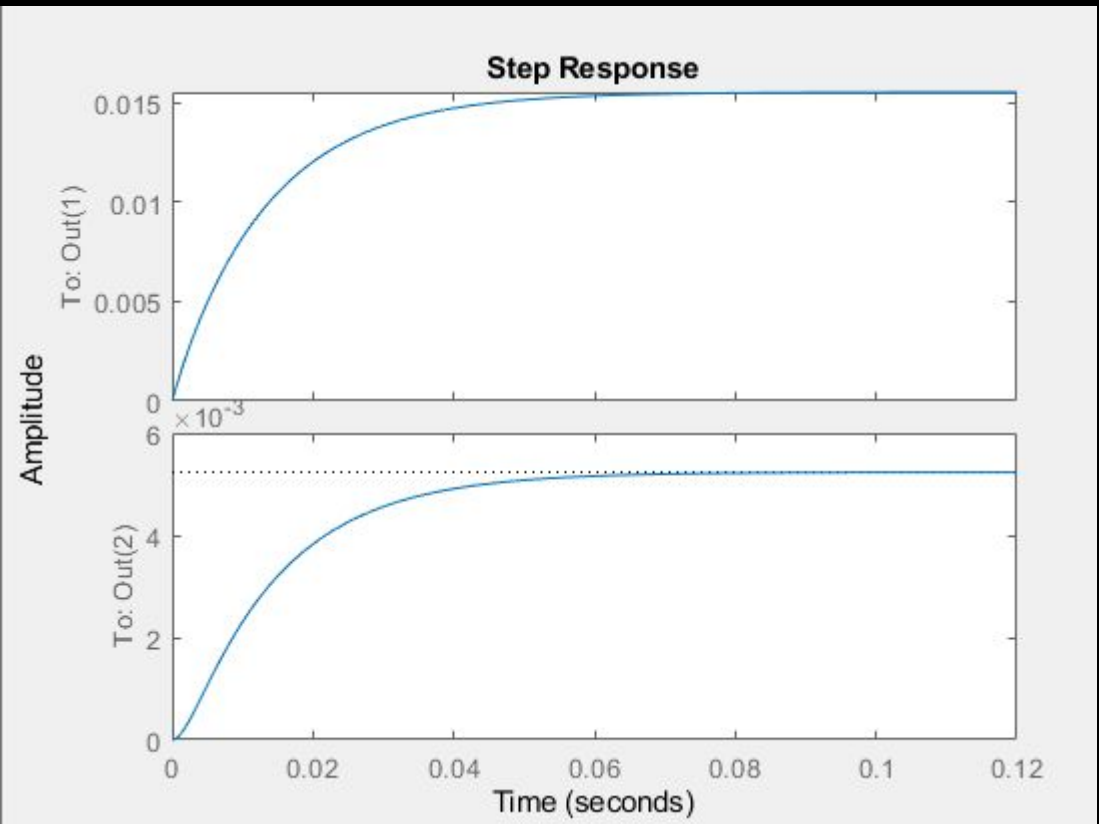
Infinity

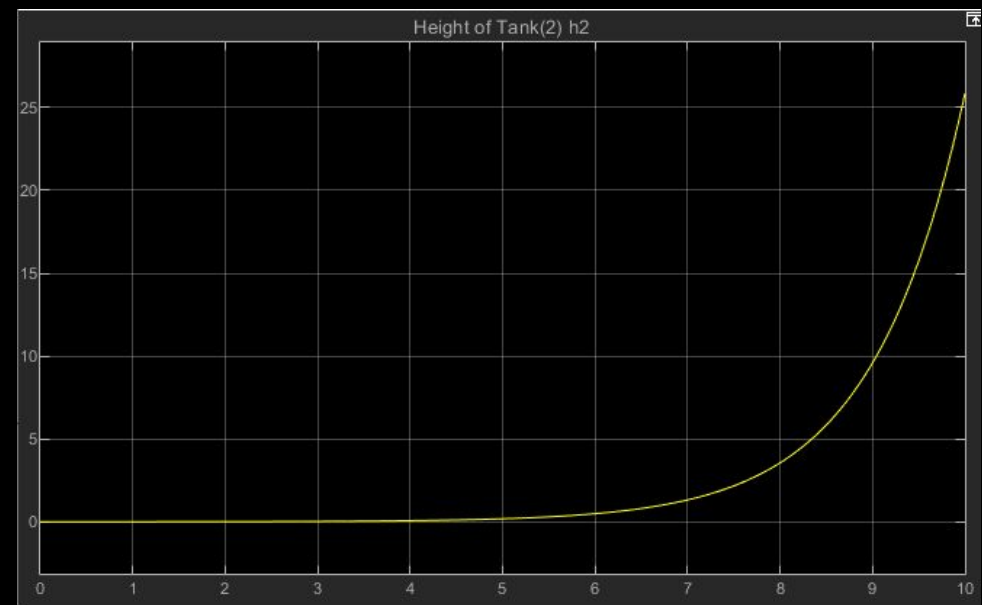
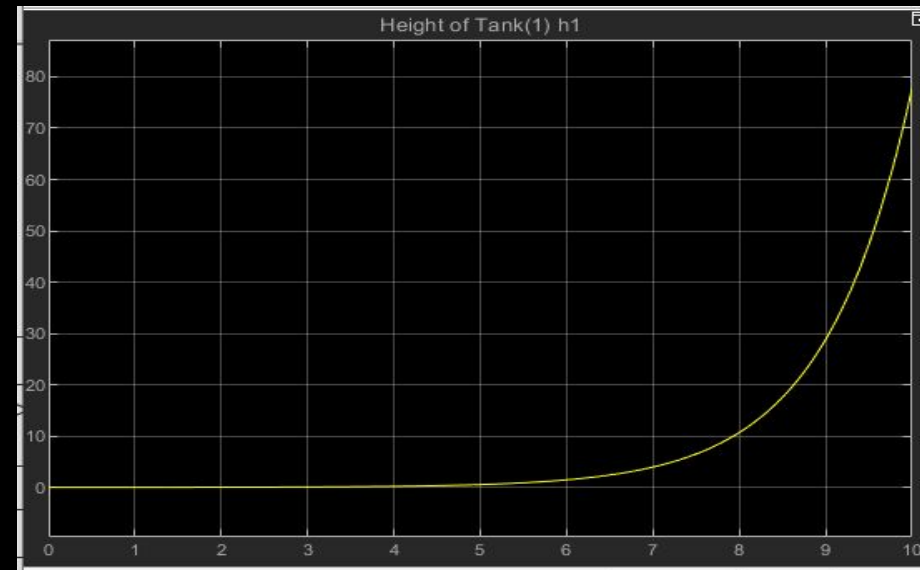
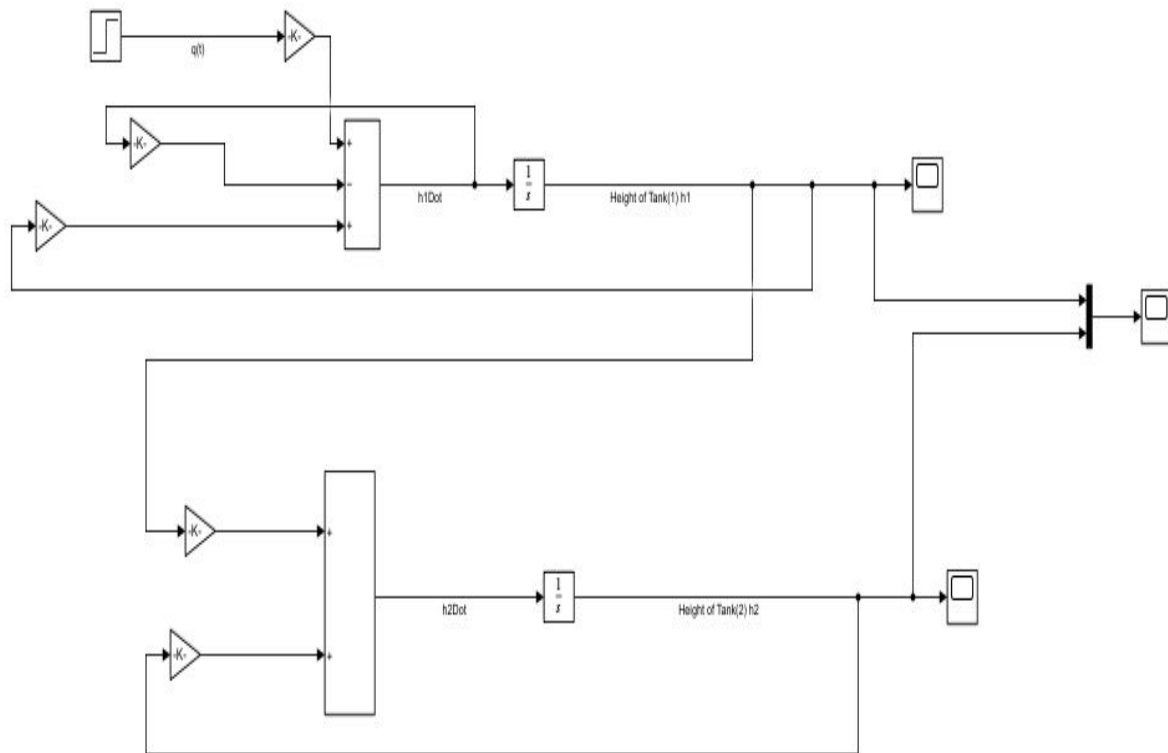
BIBO Stable

Step Response

MATLAB
SIMULINK®

```
p = 988;  
A1= 0.785;  
A2=0.635;  
R1= 100;  
R2 = 50;  
g= 9.81;  
A =[-124.7, 127.7; 154.2, -458];  
B = [ 1.27;0];  
C= [1,0; 0,1];  
D = [0;0];  
system = ss(A,B,C,D);  
step(system)
```





CONCLUSION



CONCLUSION

□ LTIC

$$\begin{aligned} \Rightarrow h_1^*(t) &= \frac{1}{A_1} q_i(t) - \frac{\rho g}{A_1 R_1} h_1(t) + \frac{\rho g}{A_1 R_1} h_2(t) \\ h_2^*(t) &= \frac{\rho g}{A_2 R_1} h_1(t) - \frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) h_2(t) \end{aligned}$$

$$\begin{aligned} \Rightarrow \begin{bmatrix} h_1^*(t) \\ h_2^*(t) \end{bmatrix} &= \begin{bmatrix} -\frac{\rho g}{A_1 R_1} & \frac{\rho g}{A_1 R_1} \\ \frac{\rho g}{A_2 R_1} & -\frac{\rho g}{A_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} [q_i(t)] \\ \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [q_i(t)] \end{aligned}$$

$$\Rightarrow \begin{bmatrix} [0.8824e^{-(73.4)t} + 0.1175e^{(-509.2)t}] \\ [0.3539e^{(-73)t} - 0.3539e^{(-509.2)t}] \end{bmatrix}$$

Controllability



Controllable

Observability



Observable

Stability



Asymptotic Stable



BIBO Stable