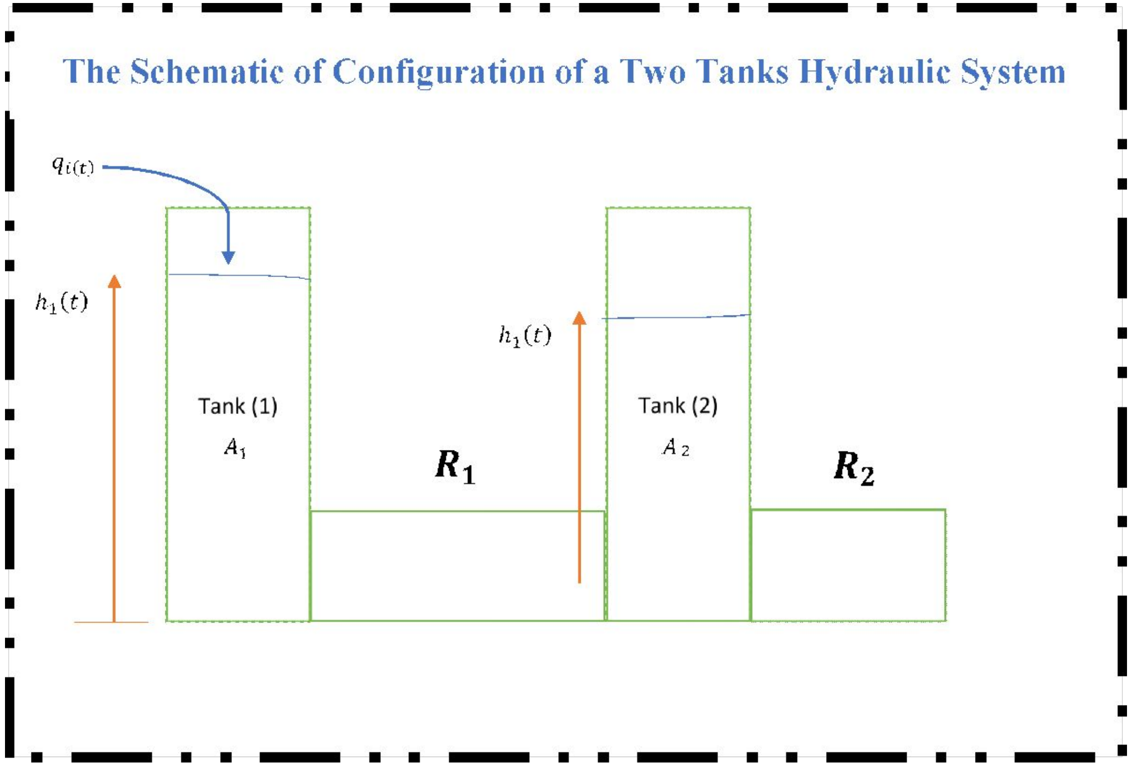
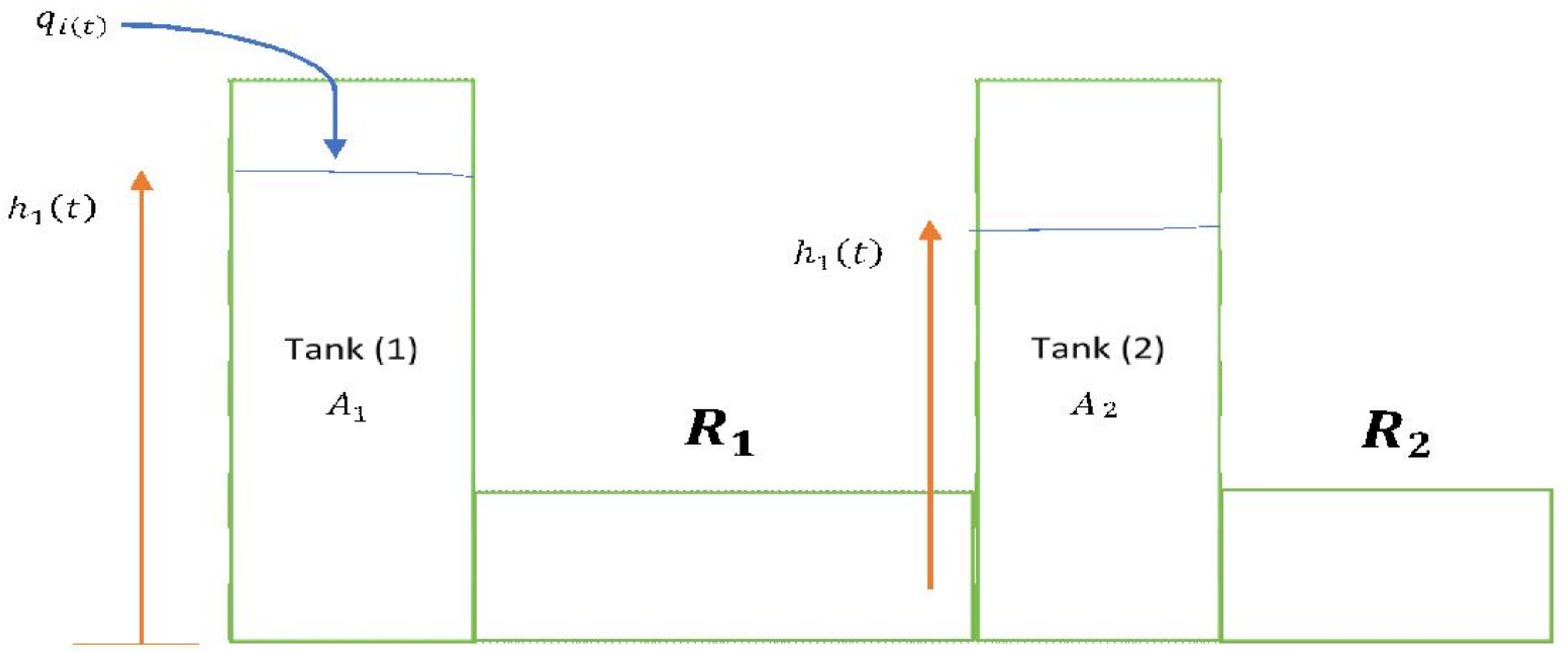
MODELING OF A TWO TANKS HYDRAULIC SYSTEM

Presented By:

Hamed Alsubhi



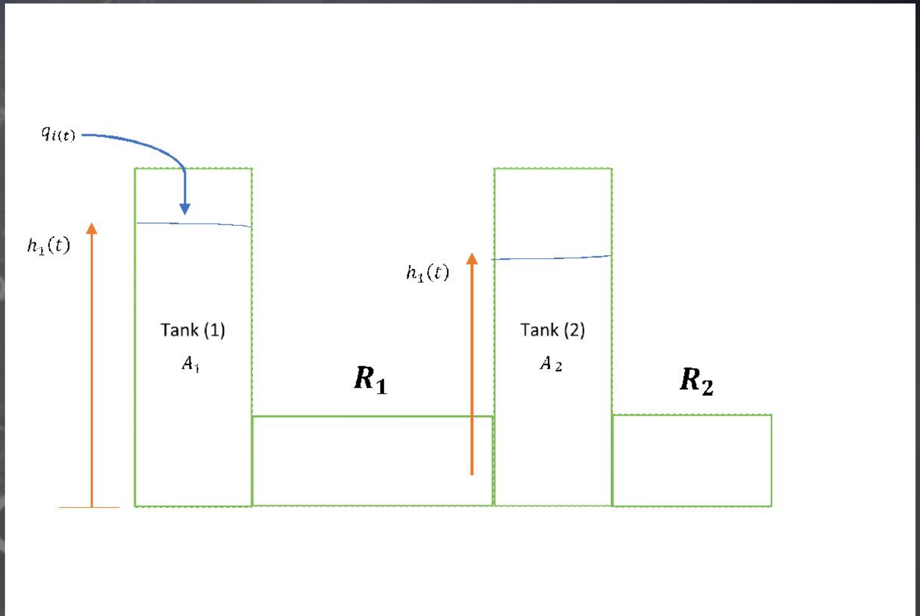
The Schematic of Configuration of a Two Tanks Hydraulic System



THE DIFFERENTIAL EQUATIONS' REPRESENTATION OF THE SYSTEM

$$h_1^*(t) = \frac{1}{A_1} q_i(t) - \frac{\rho g}{A_2} h_1(t) + \frac{\rho g}{A_1 R_1} h_2(t)$$

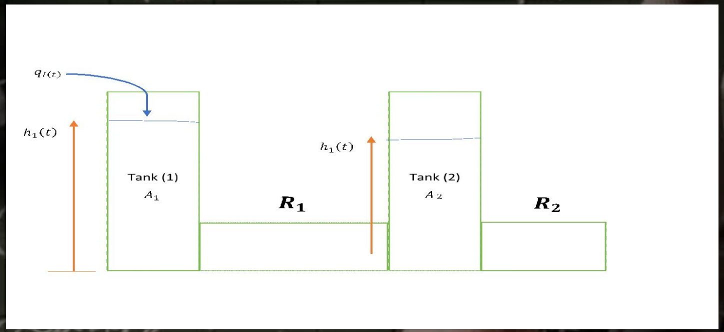
$$h_2^*(t) = \frac{\rho g}{A_2 R_1} h_1(t) - \frac{\rho g}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) h_2(t)$$

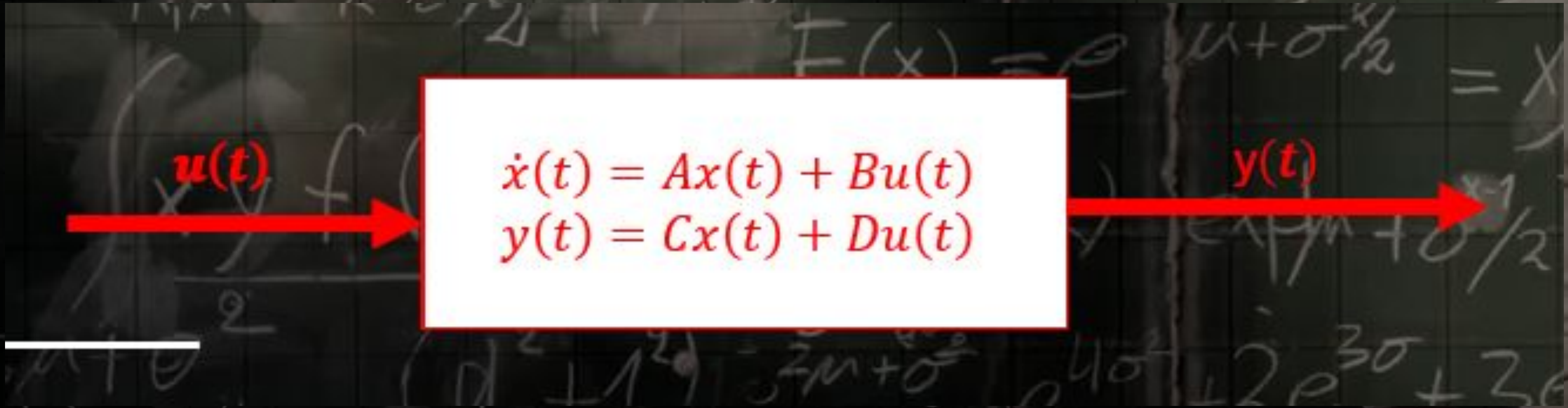


- $A_k = Cross\ Secional\ Arae\ of\ Tank\ k\ (m^2)$
- R_k = Resistance of Pipe k $\left(\frac{Ns}{m^5}\right)$
- h_k = Water Height in Tank k (m)
- $q_i(t)$ = Volumetric Flow Rate ($\frac{m^3}{s}$)
- g =Gravity ($\frac{m}{s^2}$)
- ρ =Density ($\frac{Kg}{m^3}$)

STATE-SPACE REPRESENTATION OF THE SYSTEM

- $h_1^*(t) = \frac{1}{A_1} q_i(t) - \frac{\rho g}{A_1 R_1} h_1(t) + \frac{\rho g}{A_1 R_1} h_2(t)$
- $h_2^*(t) = \frac{\rho g}{A_2 R_1} h_1(t) - \frac{\rho g}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) h_2(t)$





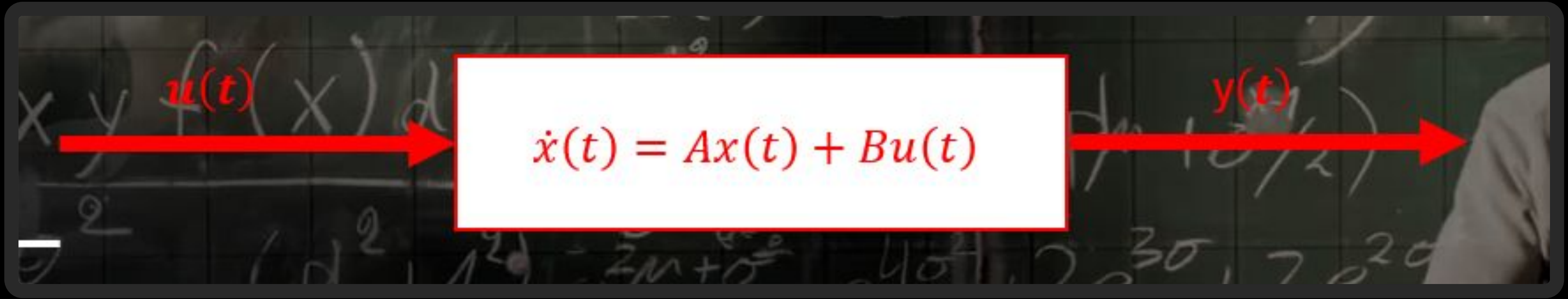
STATE-SPACE REPRESENTATION OF THE SYSTEM

$$h_1^*(t) = \frac{1}{A_1} q_i(t) - \frac{\rho g}{A_1 R_1} h_1(t) + \frac{\rho g}{A_1 R_1} h_2(t)$$

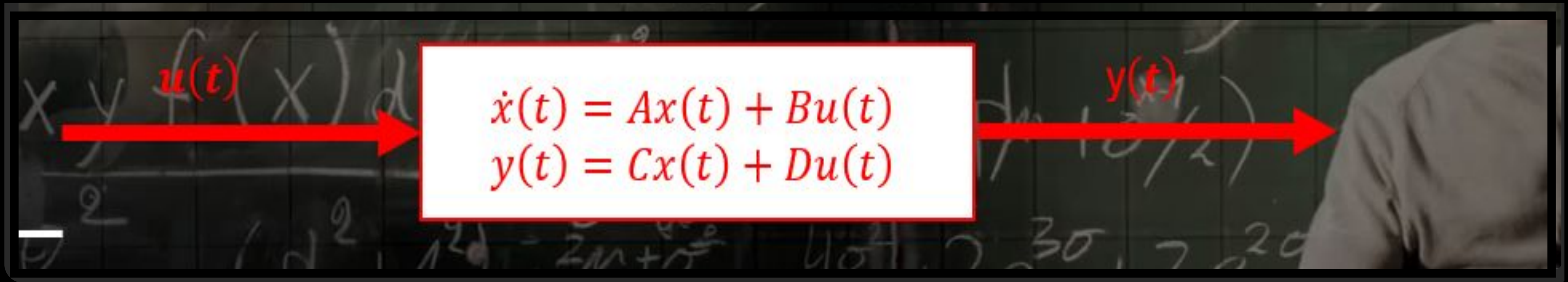
$$h_2^*(t) = \frac{\rho g}{A_2 R_1} h_1(t) - \frac{\rho g}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) h_2(t)$$

$$\begin{bmatrix} h_1^*(t) \\ h_2^*(t) \end{bmatrix} = \begin{bmatrix} -\frac{\rho g}{A_1 R_1} & \frac{\rho g}{A_1 R_1} \\ \frac{\rho g}{A_2 R_1} & -\frac{\rho g}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} [q_i(t)]$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_1(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [q_i(t)]$$



$$\underbrace{\begin{bmatrix} h_1^*(t)\\ h_2^*(t)\end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} -\dfrac{\rho g}{A_1R_1} & \dfrac{\rho g}{A_1R_1}\\ \dfrac{\rho g}{A_2R_1} & -\dfrac{\rho g}{A_2}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)\end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_1(t)\\ h_2(t)\end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix}\dfrac{1}{A_1}\\ 0\end{bmatrix}}_{B}\underbrace{[q_i(t)]}_{u(t)}$$



$$\underbrace{\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}}_{y(t)} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \boldsymbol{h_1(t)} \\ \boldsymbol{h_1(t)} \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} \underbrace{[\boldsymbol{q_i(t)}]}_{u(t)}$$

$u(t)$

$$\begin{bmatrix} h_1^*(t) \\ h_2^*(t) \end{bmatrix} = \begin{bmatrix} -\frac{\rho g}{A_1 R_1} & \frac{\rho g}{A_1 R_1} \\ \frac{\rho g}{A_2 R_1} & -\frac{\rho g}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} [q_i(t)]$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_1(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [q_i(t)]$$

$y(t)$

- $A_1 = 0.785$
- $A_2 = 0.635$
- $R_1 = 100$
- $R_2 = 50$
- $g = 9.81$
- $\rho = 988$

$$A = \begin{bmatrix} -\dfrac{\rho g}{A_1 R_1} & \dfrac{\rho g}{A_1 R_1} \\ \dfrac{\rho g}{A_2 R_1} & -\dfrac{\rho g}{A_2}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{1}{A_1} \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix} \qquad B = \begin{bmatrix} 1.27 \\ 0 \end{bmatrix}$$

TIME-DOMAIN SOLUTIONS

$$x(t) = e^{At}x(\mathbf{0}) + \int_0^t e^{A(t-\tau)}\,\beta u(\tau)d\tau$$

Zero-Input Solution

Zero-State Solution

Zero-Input Solution

$$\mathbf{x(t) = e^{At}x(0)}$$

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix} \qquad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- Let's find the Eigenvalues of A

$\det(\lambda I - A) = 0$ λ_1 = -73.5 λ_2 =-509.2

- Let's find the Eigenvectors of A

$(\lambda_1 I - A)v_1 = 0$ $v_1 = \begin{bmatrix} 0.9281 \\ 0.3722 \end{bmatrix}$

$(\lambda_2 I - A)v_2 = 0$ $v_2 = \begin{bmatrix} -0.3152 \\ 0.9490 \end{bmatrix}$

- Check the Validity of the Eigenvectors:

$\lambda_1 v_1 = Av_1 = \begin{bmatrix} -69.3207 \\ -27.3546 \end{bmatrix}$

$\lambda_2 v_1 = Av_2 = \begin{bmatrix} 157.6457 \\ -483.2458 \end{bmatrix}$

- P and P^{-1}

$$P = \begin{bmatrix} 0.9281 & -0.3152 \\ 0.3722 & 0.9490 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0.9508 & 0.3158 \\ -0.3729 & 0.9299 \end{bmatrix}$$

- D matrix:

$$D = P^{-1}A \;\; P = \begin{bmatrix} 0.9508 & 0.3158 \\ -0.3729 & 0.9299 \end{bmatrix} \begin{bmatrix} \mathbf{-124.7} & 124.\mathbf{7} \\ \mathbf{154.2} & \mathbf{-458} \end{bmatrix} \begin{bmatrix} 0.9281 & -0.3152 \\ 0.3722 & 0.9490 \end{bmatrix} = \begin{bmatrix} -73.4887 & 0 \\ 0 & -509.2113 \end{bmatrix}$$

- $e^{\mathrm{D}t} = \begin{bmatrix} \mathrm{e}^{(-73.4887)t} & 0 \\ 0 & \mathrm{e}^{(-509.2113)t} \end{bmatrix}$

- $e^{\mathrm{At}} = \mathrm{P}\, e^{\mathrm{D}t}\, P^{-1}$

$$e^{\mathrm{At}} = \begin{bmatrix} 0.9281 & -0.3152 \\ 0.3722 & 0.9490 \end{bmatrix} \begin{bmatrix} \mathrm{e}^{(-73.4887)\mathrm{t}} & 0 \\ 0 & \mathrm{e}^{(-509.2113)t} \end{bmatrix} \begin{bmatrix} 0.9508 & 0.3158 \\ -0.3729 & 0.9299 \end{bmatrix} =$$

$$\begin{bmatrix} [0.8824e^{-(73.4)t} + 0.1175e^{(-509.2)t}] & [0.29309e^{-(73.4)t} - 0.2931e^{(-509.2)t}] \\ [0.3539e^{(-73)t} - 0.3539e^{(-509.2)t}] & [0.1176e^{-(73.4)t} + 0.88247e^{(-509.2)t}] \end{bmatrix}$$

- Thee Zero Input Solution

$$\mathbf{e}^{\boldsymbol{At}}\boldsymbol{x}(\mathbf{0}) = \begin{bmatrix} [0.8824e^{-(73.4)t} + 0.1175e^{(-509.2)t}] & [0.29309e^{-(73.4)t} - 0.2931e^{(-509.2)t}] \\ [0.3539e^{(-73)t} - 0.3539e^{(-509.2)t}] & [0.1176e^{-(73.4)t} + 0.88247e^{(-509.2)t}] \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix} =$$

$$\begin{bmatrix} [0.8824e^{-(73.4)t} + 0.1175e^{(-509.2)t}] \\ [0.3539e^{(-73)t} - 0.3539e^{(-509.2)t}] \end{bmatrix}$$

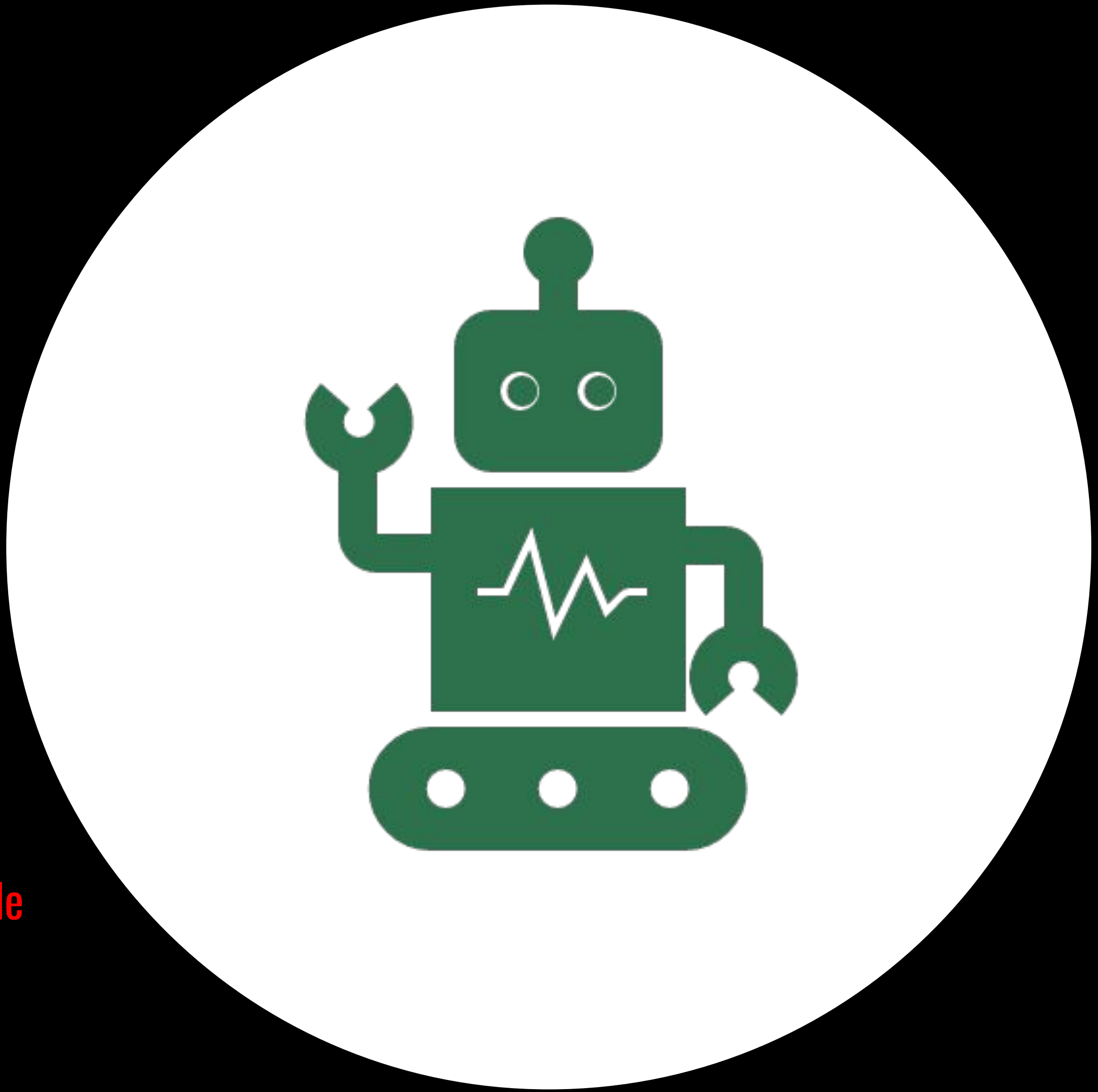
CHECK CONTROLLABILITY

Givens :

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix} \qquad B = \begin{bmatrix} 1.27 \\ 0 \end{bmatrix}$$

$$\text{C-Matric} = [\,B \;\; AB\,] = \begin{bmatrix} 1.27 & -158.4 \\ 0 & 195.8 \end{bmatrix}$$

$\text{Det}(C) = 248.7 \neq 0 \rightarrow \boxed{\text{Full Rank}} \rightarrow$ Controllable



CHECK THE OBSERVABILITY

Givens :

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{\text{O-Matrix}} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix}$$

Size of A = 2

=

Rank of O-Matrix = 2

$\longrightarrow$ Observable

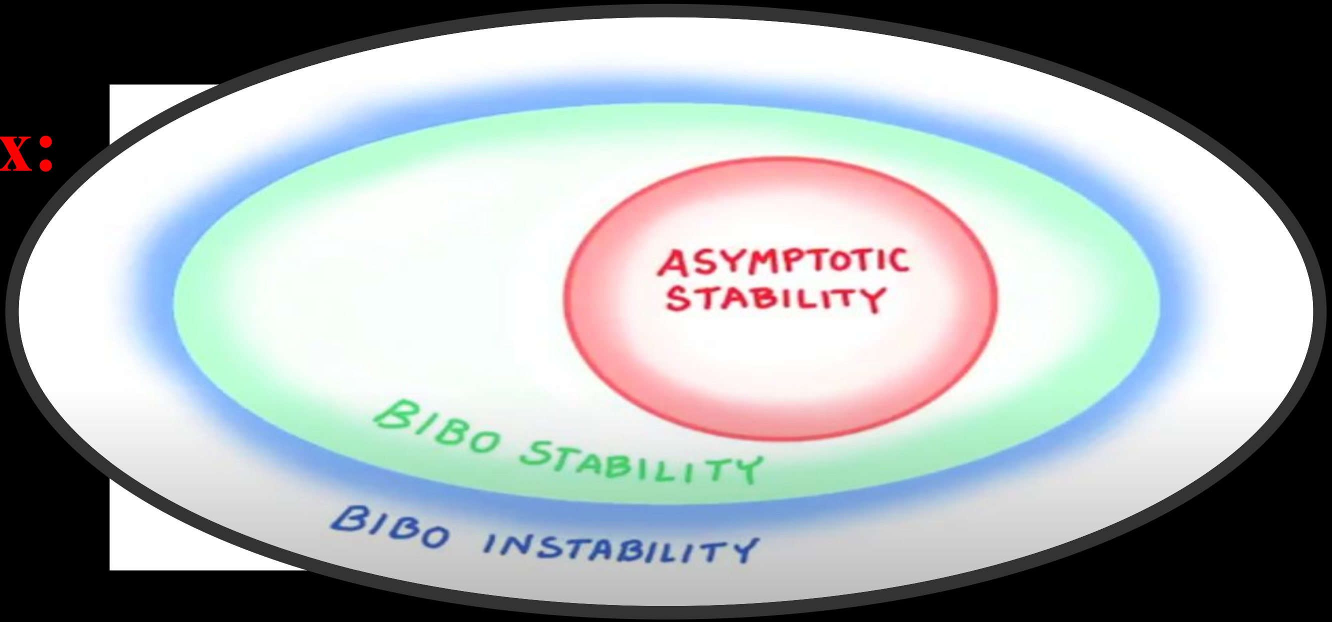
System Stability

$$A = \begin{bmatrix} -124.7 & 124.7 \\ 154.2 & -458 \end{bmatrix}$$

The eigenvalues of A-Matrix:

$$\lambda_1 = -73.5$$

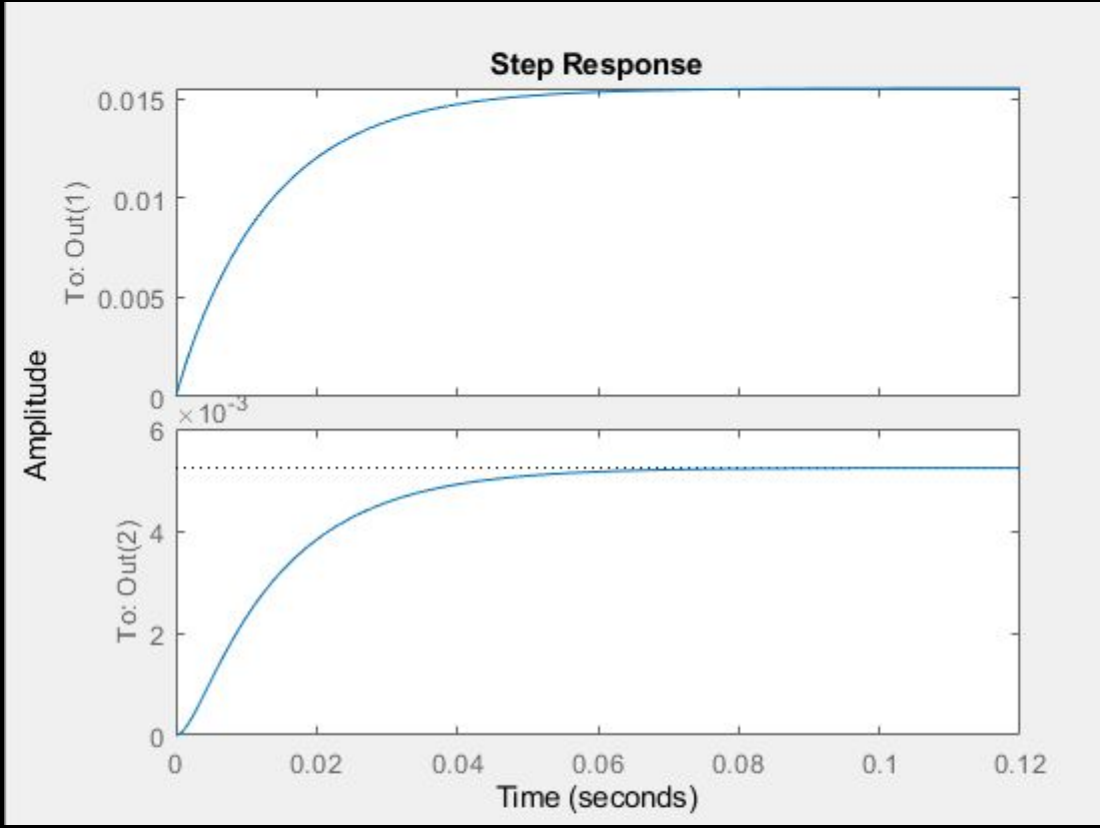
$$\lambda_2 = -509.2$$

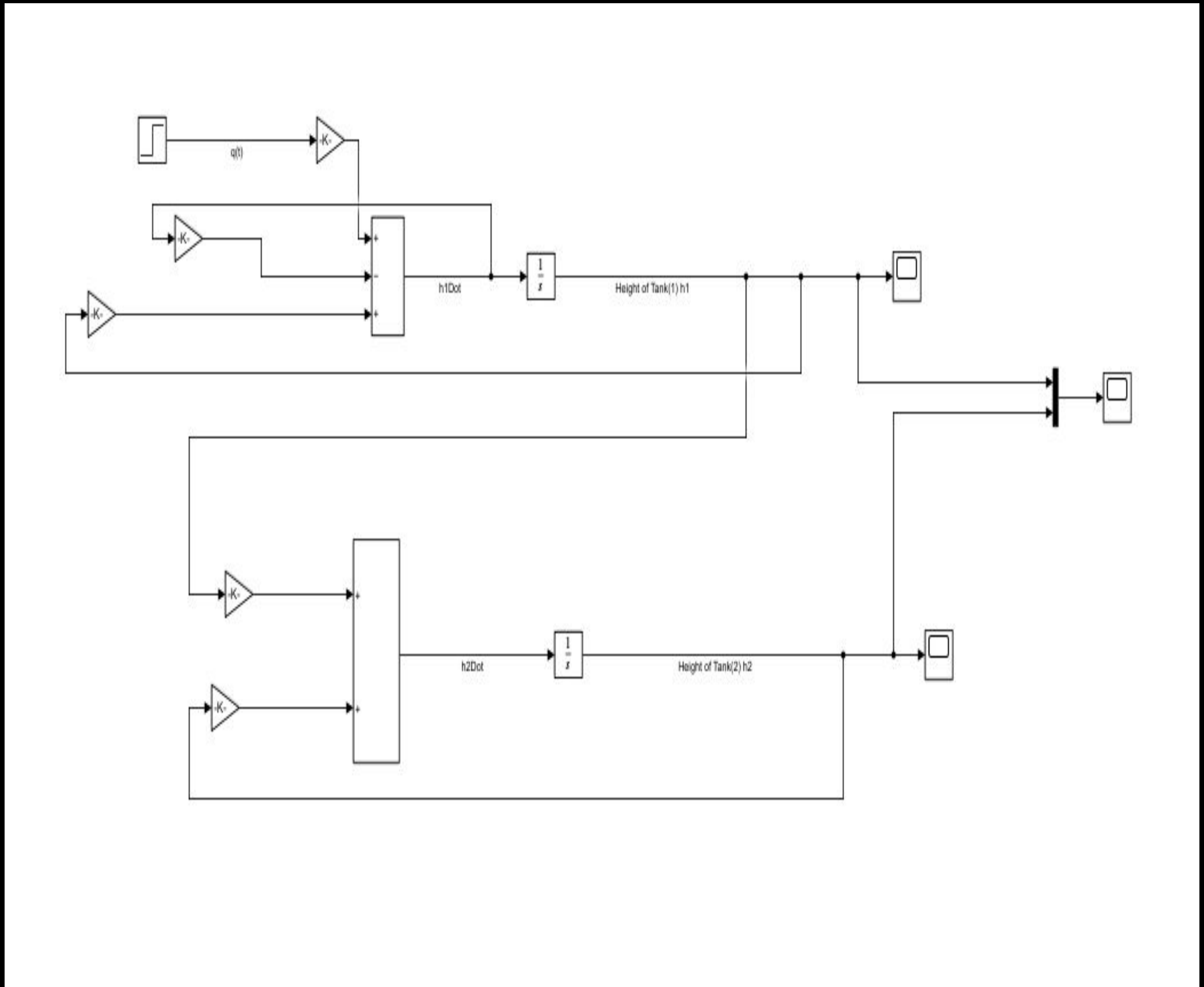


Asymptotic Stable $\xrightarrow{\text{System Solution} \rightarrow \text{Zero}, \quad t \rightarrow \text{Infinity}}$ BIBO Stable

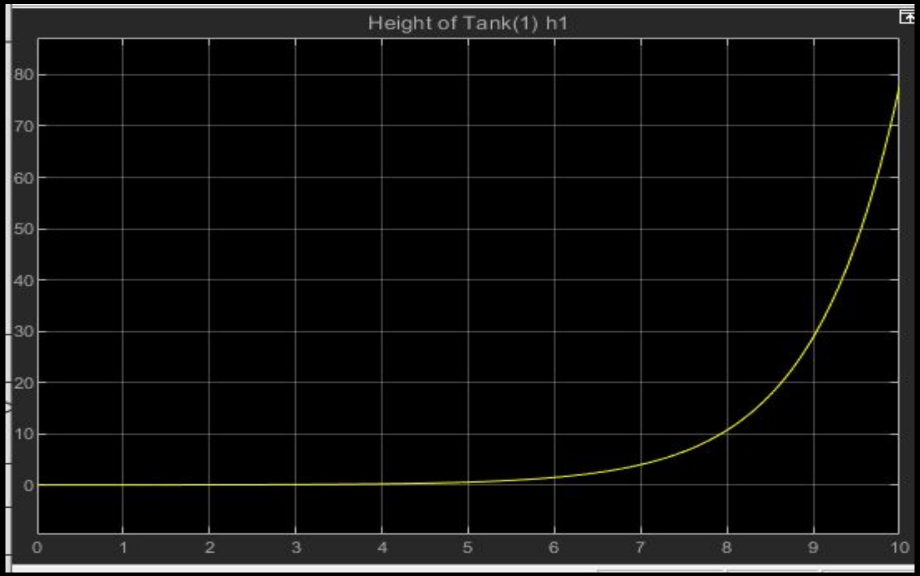
Step Response

```
p = 988;
A1= 0.785;
A2=0.635;
R1= 100;
R2 = 50;
g= 9.81;
A =[-124.7, 127.7; 154.2, -458];
B = [ 1.27;0];
C= [1,0; 0,1];
D = [0;0];
system = ss(A,B,C,D);
step(system)
```

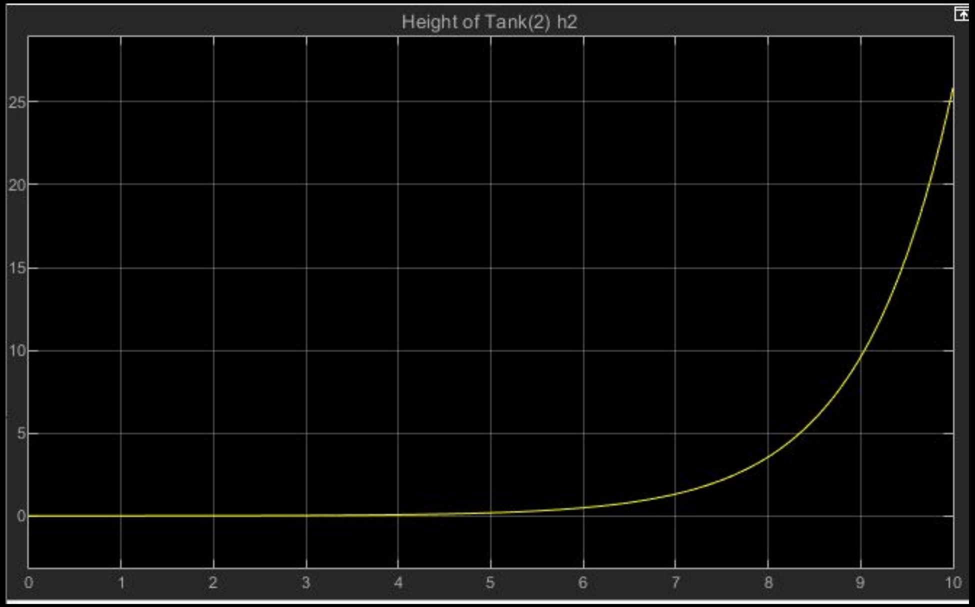




q(t)
-K-
h1Dot
Height of Tank(1) h1
h2Dot
Height of Tank(2) h2



Height of Tank(1) h1



Height of Tank(2) h2

CONCLUSION

CONCLUSION

- LTIC

➢ $h_1^*(t) = \frac{1}{A_1} q_i(t) - \frac{\rho g}{A_1 R_1} h_1(t) + \frac{\rho g}{A_1 R_1} h_2(t)$

$h_2^*(t) = \frac{\rho g}{A_2 R_1} h_1(t) - \frac{\rho g}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) h_2(t)$

➢ $\begin{bmatrix} h_1^*(t) \\ h_2^*(t) \end{bmatrix} = \begin{bmatrix} -\frac{\rho g}{A_1 R_1} & \frac{\rho g}{A_1 R_1} \\ \frac{\rho g}{A_2 R_1} & -\frac{\rho g}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} [q_i(t)]$

$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_1(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [q_i(t)]$

➢ $\begin{bmatrix} [0.8824e^{-(73.4)t} + 0.1175e^{(-509.2)t}] \\ [0.3539e^{(-73)t} - 0.3539e^{(-509.2)t}] \end{bmatrix}$

Controllability → Controllable

Observability → Observable

Stability → Asymptotic Stable → BIBO Stable