MEE 4071-Thermal System Design

Heat Exchanger to Recover the Heat from Jacuzzi's Drained Water

Submitted to Dr. Darshan Pahinkar

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Introduction

Heat Exchangers play an important role in designing heat recovery thermal systems for the various applications in residential and industrial facilities. Heat loss by hot tubs is extremely significant, whether this loss is because of a poorly manufactured cover or through draining the wastewater that contains valuable thermal energy. Heating swimming pools requires enormous amounts of energy, especially the outdoor pools. Statistics highlight that swimming pools that are exposed to ambient air would lose 75 % or more of its energy. The latest shades light on the need to invest in new methods to minimize wasted energy in pools and hot tubs which would reflect in increasing the thermal efficiency and decreasing the carbon footprint. Having said that, this project team is tasked to design a heat exchanger to re-utilize heat waste of the water drained from 90 square feet indoor jacuzzi with a capacity of 600 gallons. This heat recovery process includes designing a pre-heater that will drive up the temperature value of the water entering the heater unit of the jacuzzi. Increasing the temperature entering the heater will achieve the principal goal of this project which is increasing the efficiency of the Jacuzzi's heater and reducing energy consumption.

The design of the heat exchanger will be approached using the LMTD method, and an Engineering Equation Solver code will be used for many analytical sets. The code will be established based on the equations and correlations imported from the thermal design system textbooks. The best type of heat exchanger will be used for this application is a cross flow tube-in-tube heat exchanger. The given dimensions, boundaries and assumptions are generated based on research conducted as well as self-stated assumptions and boundaries for the sake of analysis.

State of Art

Heating and cooling and hot water account for the largest share of residential energy (Torio and Schmidt 2010). Energy efficiency has become one of the indicators of economic development in recent years and rationalization of its use has been the subject of numerous scientific studies (Turner and Doty 2007, Tsioliaridou and Bakos 2006, Schaumann 2007). Globally, it is about minimizing the negative impact of energy on the environment. Evidence that much of the energy used to heat domestic water is wasted in drainage systems through applications such as showers, bathtubs and dishwashers. For example, a typical dishwasher heats water during the wash cycle, and then above 80°C the discharges this hot gray water at a slightly cooler temperature down the drain. A typical washing machine heats water to 60°C and discharges similarly hot gray water. In a typical shower, the water is heated to over 40 °C and hot gray water, around 30-38 °C depending on the ambient temperature, is discharged into the drainage system (Wong et al. 2010).

The issue of energy savings for heat recovery from wastewater use has been addressed in the UK (Wong et al. 2010), Ireland (Boait et al.2012), Spain (Hernandez and Kenny, 2012), Italy (Torras et al. 2016), Netherlands (Cipolla and Maglionico, 2014), Australia (Hobbi and Siddiqui 2009), and Brazil (Beal et al. 2012) analyzed the effectiveness of using different technologies to save energy, from residential buildings to municipal wastewater. A Swiss study (Schmid 2009) showed that 15% of the thermal energy supplied to buildings is lost through sewage systems. In well-insulated buildings with low consumption, this value increases by up to 30%. As a result, sewers are currently the largest source of heat loss in buildings (Schmid 2009). In Lithuania, even though cost of treating water to the right temperature is relatively high, the use of wastewater heat recovery in residential buildings is still controversial, and wastewater heat exchangers are Not widely available.

This issue may be due to a lack of data on the financial performance of the proposed investment. Information on the economics of heat recovery systems from sanitary wastewater is usually obtained from the manufacturer's literature. As a result, they are not very reliable for potential users. The purpose of this project was to review and analyze different types of wastewater heat exchangers in different building types, in terms of energy savings and temperature effects, and to show practical performance differences.

Section (1): Current System of the Jacuzzi

Figure (1) shows the detailed system of the indoor jacuzzi that this paper will be focusing on establishing a method to utilize the wasted heat from it. The water is supplied at 25 C and a mass flow rate of 0.25 m/s to a heater unit in which it works on heating the water to the maximum temperature set for the jacuzzi which is 55 C. The drained water is assumed to be recycled at 45 C with a mass flow rate of 0.5 m/s. Table (1) shows all the dimensions, boundaries and assumptions established for the jacuzzi system.

Table (1): Boundaries and Assumptions of the jacuzzi system

Boundaries and Dimensions	
Ambient temperature	25 C
Free Convective Heat transfer coefficient	25 W/m^2-K
Emissivity	0.95
Maximum Temperature of the Jacuzzi (T_set)	55 C
Temperature of the Drained water (T-old)	45 C
Temperature of supplied water	25 C
Mass flow rate of supplied water	0.25 kg/s
Mass flow rate of drained water	0.50 kg/s
Capacity Volume of the Jacuzzi	2.27 m^3 (600 Gallons)
Upper Surface Area of the Jacuzzi	8.36 m^2
Total surface area of the Jacuzzi's walls	14.5 m^2
Thermal Conductivity of the walls (Acrylic)	0.2 W/m-K
Jacuzzi's walls thickness	10 cm
Heat Capacity of the water in jacuzzi; obtained at T_mean =50 C	4.181 kJ/K-kg
Density of the water in the jacuzzi, Obtained at T_mean = 50 C	988.02 kg/m^3
Total water mass in the jacuzzi	2242.8 kg(rho*V)
Assumptions	
1-D heat transfer	
Constant material properties	
Evaporation losses are neglected	
Water is Still in the jacuzzi (not moving)	

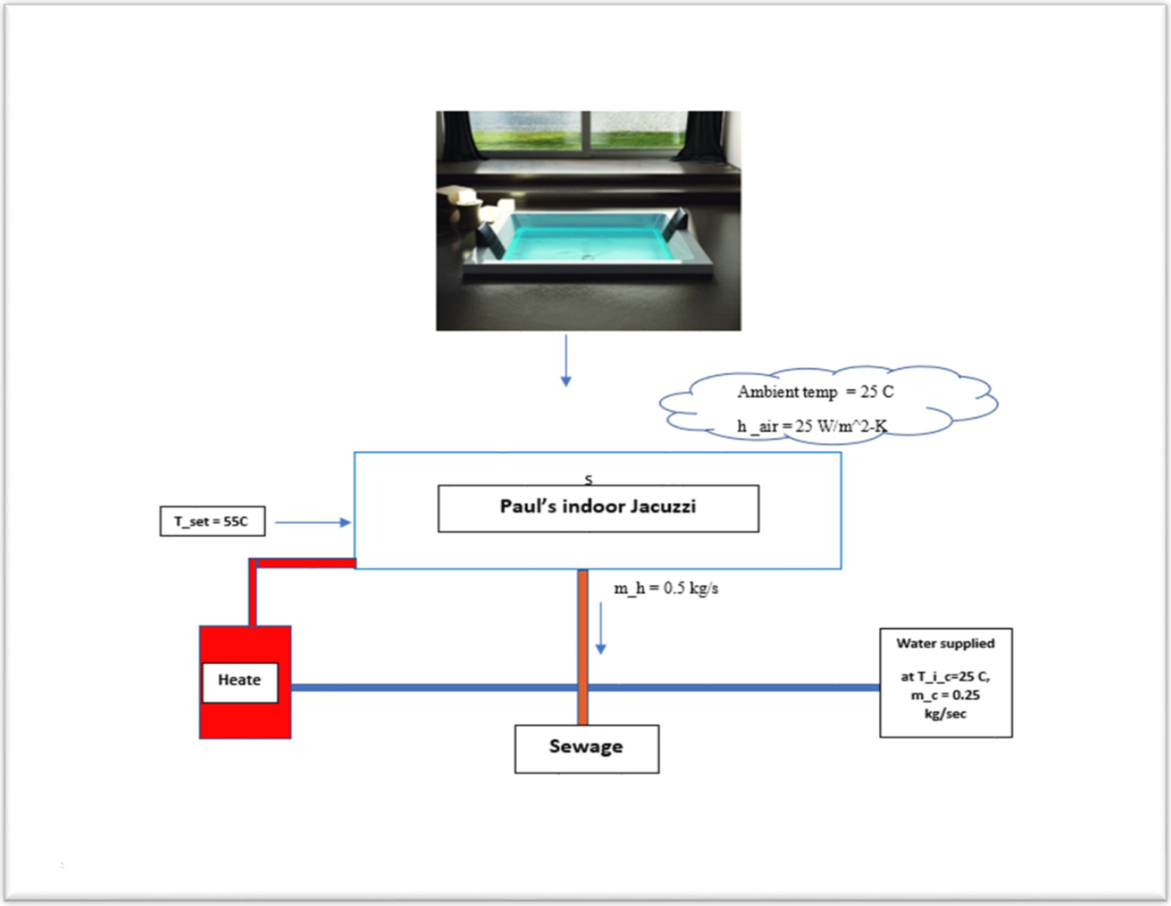

Figure (1): Illustration of the Jacuzzi system and its Boundaries

1) Cooling Time

For the sake of giving a complete idea of the jacuzzi system, it's important to evaluate the time it takes the jacuzzi to cool down from 55 C to 45 C (the undesirable Jacuzzi Temperature) in order to know when exactly the water should be charged and discharged as well as installing any new unit to the system accordingly, such as pumps, heater and ect. Therefore, an energy balance was established for the jacuzzi to evaluate the cooling time based on the three heat losses modes which are conduction, convection and radiation. The time is evaluated based on the assumptions and boundaries stated in table (1):

$E_{st} = E_{in} - E_{out}$ = m *cp * dT/dt = - Q_radiation – Q_convection – Q_conduction

Q_ Radiation = $A\sigma\varepsilon$ ((T^4)_set - (T^4)_ambiant

Q_Convection= $hA(T_{set} - T\_ambiant)$

Q_conduction= $\frac{\Delta T}{R} = \frac{T_{set} - T_{ambiant}}{\frac{L_{wall}}{k * A\_wall}}$

Q_c = 13584 [W]
Q_{conv} = 6270 [W]
Q_{rad} = 1663 [W]

Figure (2) illustrates the results found for each heat loss mode as well as the cooling time from 55 C to 45 C (dT).

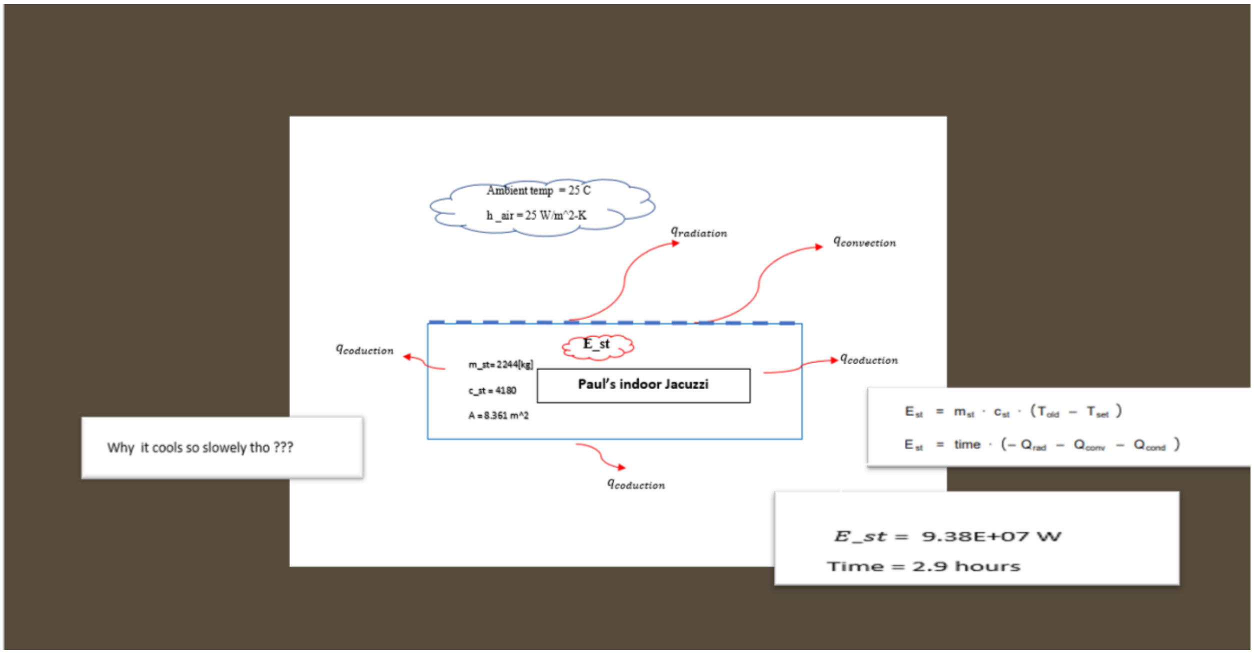

Figure (2): Energy balance for the Jacuzzi

The time required was found to be about 3 hours. This time seems to be overestimated. The justification is that we assumed the heat loss due to evaporation to be negligible which is a major loss contribution that would have altered the obtained time value. Also, the loss due to conductivity was found to be relatively minimal (6270 W) because the jacuzzi is said to be made from Acrylic which is a low conductive material (k =0.2 W/m^2-K). The later would increase the thermal resistances inside of Jacuzzi's wall and k; thus, increasing the efficacy of the jacuzzi to keep the water warm.

2) Power required by the Heater

Jacuzzi's heater is an important component in the overall system of the jacuzzi and the most major source for energy consumption in the system. The project's team is looking to base their ultimate objective on reducing the power required by the heater. Figure (3) summarizes the evaluation of the power required the heater to heat the supplied water from 25 C to 55 C at a flow rate of 0.25 m/s.

$$\dot{Q}_{heater} = \dot{m} \cdot c_{p_c} \cdot \left(T_{out_{set}} - T_{in_c}\right)$$

Where c_{p_c} is given at $T_{m\_heater} = \frac{T_{out_{set}} + T_{in_c}}{2} = \frac{55+25}{2} = 40\ C$

$$\dot{Q}_{heater} = 0.25 \cdot 4.179 \cdot (55 - 25) = 31.1\text{kW}$$

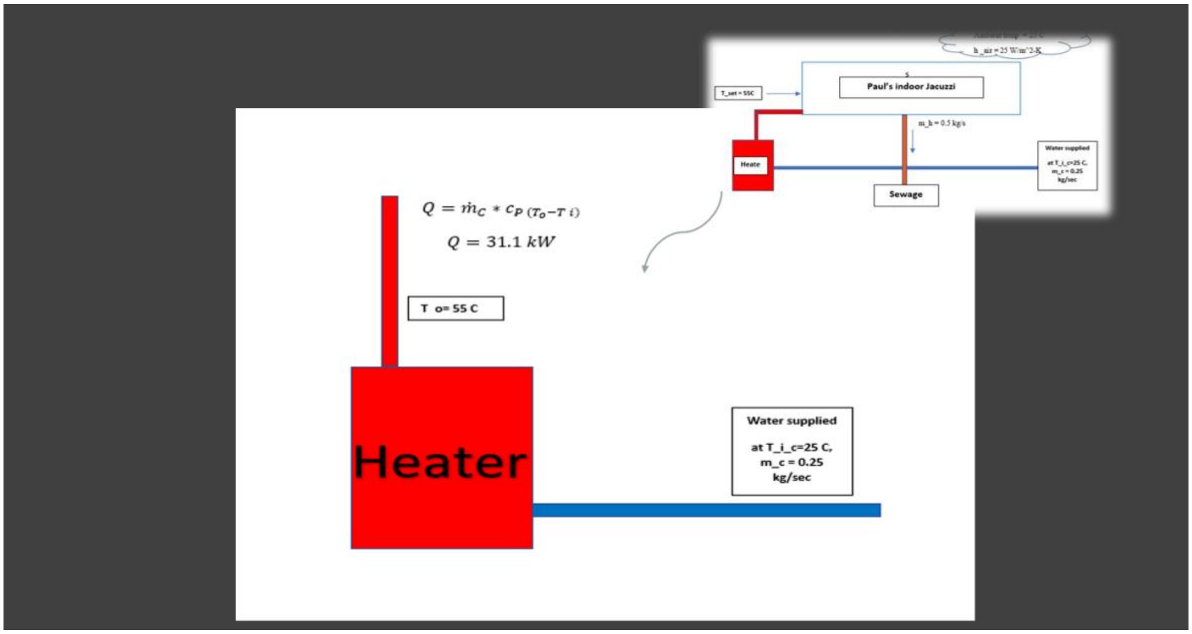

Figure (3): The Power Required by the Heater for Given set of Boundaries

Section (2): Problem Statement

The power required by the heater was found to be 31.1 kW. The goal of this project is to figure out a way to save up the power required by 40 percent, so the new power required by the heater that the project team is looking to get is:

$$\dot{Q}_{new} = \dot{Q}_{heater} * (1 - 0.4) = 18.8\ kW$$

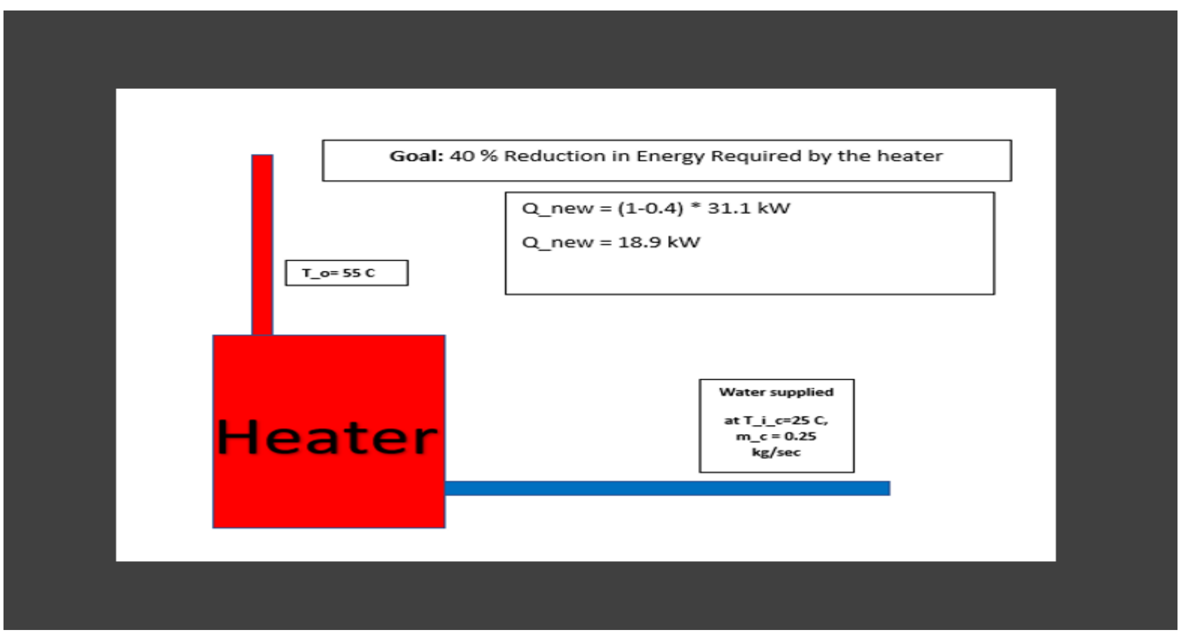

Figure (4): The new heater power goal

The 40 % saving goal is to be accomplished by designing a heat exchanger and adding it to the Jacuzzi's overall system to preheat the supplied water prior to entering the heater. The supplied water will be preheated by utilizing the heat of drained water discharged from the jacuzzi at 45 C. Figure (5) shows how the overall jacuzzi system looks like after adding the preheater unit to it. Moreover, the scope of designing this heat exchanger will be on finding its required length to satisfy the main saving goal as well as satisfying the geometry restriction and requirements. The go-to type of this exchanger is selected to be a cross flow tube-in-tube heat exchanger. Table (2) outlies the main design's goal and the heat exchanger design restrictions.

Table (2): Goals and Design Restrictions

Main goal: Save up 40 % of the power required by the heater such that the goal power required is $\dot{Q}_{new}$ = 17.8 kW.
Solution: Utilizing the heat of the drained water from the jacuzzi and recover it using a heat a tube-in-tube heat exchanger
Main Heat Exchanger Design Goal: Find tube length required by the heat exchanger so it achieves the goal stated.
Geometry Restrictions: The proposed design for the heat exchanger shall fit an installation compartment of 1 m^2 and height of 0.5 m meters (Volume of the compartment = 0.5 m^3).
Technical core requirement: The pressure drop inside the heat exchanger tube(s) shall not exceed AN Allowable pressure drop of 10 psi.

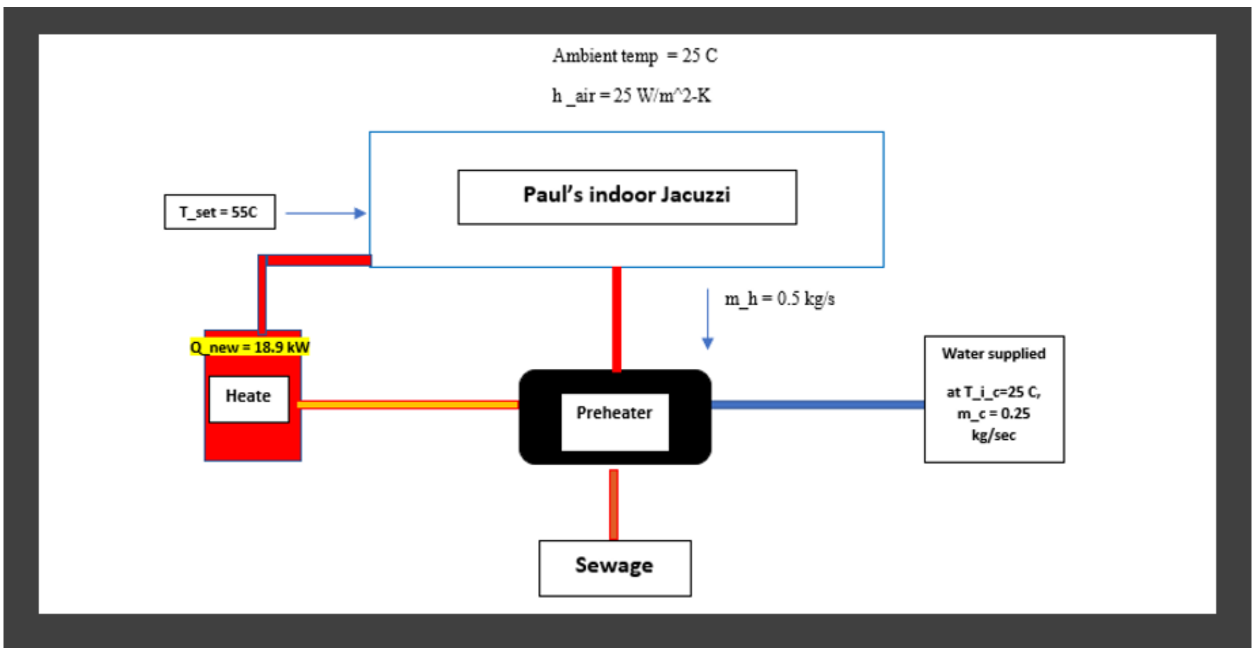

Figure (5): Overall Jacuzzi System after adding the preheater unit

Section (3): Design Methodology

1) Design Approach in General

Figure (6) shows a detailed scheme of the heat exchanger that the design process will be based on as well as the mass flow rates and inlet/outlet temperatures of the cold and hot water. The LMTD method will be used to design for the length required so it satisfies the boundary restrictions stated in the problem statement section. The method states that the heat transfer between the cold and hot flows is given as follows:

$$\dot{Q} = U \cdot A_{surface} \cdot LMTD$$

Where U is the overall heat transfer coefficient and is obtained after evaluating the total thermal resistance inside, in which the heat transfer occurs between the two fluids in the heat exchanger. $A_{surface}$ is the surface area of the inner tube which could be given based on the outer diameter of the inner tube or the its inner diameter. LMTD is the logarithmic mean temperature difference and is obtained as shown below:

$$LMTD = \frac{\left(T_{out_{hot}} - T_{out_{cold}}\right) - \left(T_{in_{hot}} - T_{in_{cold}}\right)}{ln ln \left(\frac{T_{out_{hot}} - T_{out_{cold}}}{T_{in_{hot}} - T_{in_{cold}}}\right)}$$

As seen in the above LMTD formula, all inlet and outlet temperatures of the two fluids shall be known to use the method. After obtained all unknowns, the heat exchanger length required can be derived from the following formula:

$$L_{required} = \frac{\dot{Q}}{U\_o \cdot \pi \cdot D_{io} \cdot LMTD \cdot n_{tubes}}$$

Using the above approach, number of calculations design sets will be implemented using EES until reaching the optimal design.

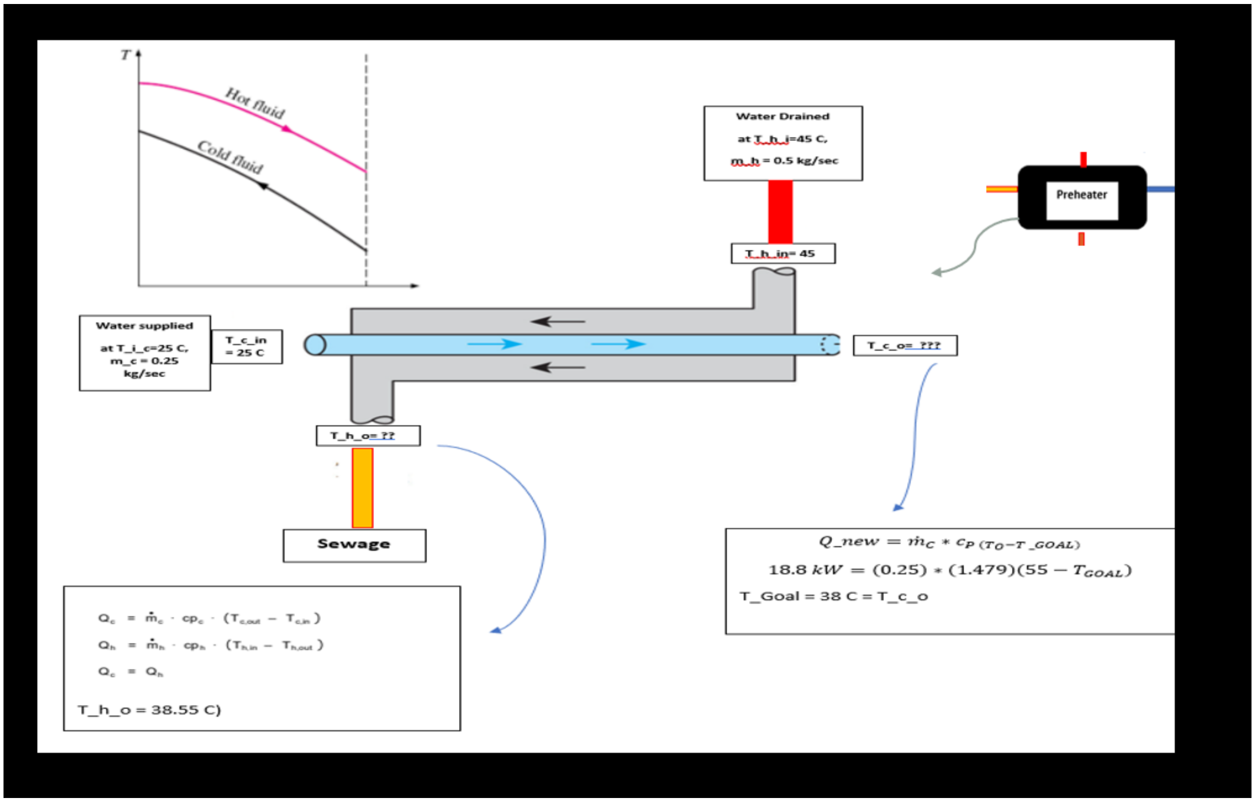

Figure (6): Illustrated scheme of the heat exchanger with set of Known boundaries

2) General Basic Boundaries

The inlet temperature of the cold fluid in the inner tube of the heat exchanger is the temperature of the supplied water which is given at 25 C, and the outlet temperature of the cold flow is corresponding to the flow temperature entering the heater that set as new power value of 18.8 kW. The outlet temperature of the cold flow is found as follows:

$$T_{in_{goal(to\ heater)}} = T_{o_{cold}} = T_{out_{heater}} - \frac{\dot{Q}_{new}}{\dot{m} \cdot c_{p_h}}$$

$$T_{o_{cold}} = 55 - \frac{18.8\ kW}{0.25 \cdot 4.179 \dfrac{kJ}{kg * K}} = 38\ °C$$

The above obtained value ($T_{o_{cold}} = 38\ C$) is supplied water temperature after heating using the heat exchanger, and the corresponding to the new input temperature of the heater. Moreover, the inlet temperature of hot flow is the temperature of the drained water which is set at 45 C, and the outlet temperature is obtained by equating the thermal energy held by the cold flow and hot flow inside the heat exchanger as shown down below:

$$\dot{Q}_c = \dot{Q}_h$$

$$\dot{m\_c} \cdot c_{p_c} \cdot \left(T_{out_c} - T_{in_c}\right) = \dot{m\_h} \cdot c_{p_h} \cdot \left(T_{out_h} - T_{in_h}\right)$$

$$0.25*995.2*(38\text{-}25) = 0.5*991.5*(T_{out_h}\text{-}45)$$

$$T_{out_h} = 38.5C$$

Therefore, the temperature of the drained will drops to 38.85 after reutilizing its heat content to preheat the supplied water from 25 C to 38 C. Table (3) shows the properties of cold and hot fluids inside the heat exchanger that will be carried out for all thermal analysis and design sets.

Table (3): Goals and Design Restrictions

Flow Property	**Cold flow (supplied water)**	**Hot Flow (Drained water)**
Mass Floe Rate	0.25 m/s	0.5 m/s
Inlet temperature	25 C	45 C
Outlet Temperature	38 C	38.5 C
Density, ρ	995.2kg/m^3	991.5kg/m^3
Viscosity, μ	Pa-s	Pa-s
Conductivity	0.6166W/m-K	0.6307W/m-K
Specific heat, c_p	4.181kJ-kg-K	4.180kJ/kg-K
Prandtl Number-Pr	5.236	4.188

3) Primary Design Set Along with Results and Explanation)

a) Tube size Selection:

Tube Size Selection for the Primary Set	
Inner Tube Diameter	16 mm
Outer Tube Diameter	19 mm
Pipe's Diameter	25 cm
Number of tubes	1

The size of the tube in which the cold water is flowing as well as the size of the pipe in which the annulus hot flow is going through is given in the above table. It's worth mentioning that tube sizes are imported from published standardized table for copper tube sizes.

b) Evaluate LMTD:

$$LMTD = \frac{\left(T_{out_{hot}} - T_{out_{cold}}\right) - \left(T_{in_{hot}} - T_{in_{cold}}\right)}{ln\left(\frac{T_{out_{hot}} - T_{out_{cold}}}{T_{in_{hot}} - T_{in_{cold}}}\right)} = LMTD = \frac{(45-38)-(38.5-25)}{lnln\left(\frac{45-38}{38.5-25}\right)} = 9.9 \text{ K}$$

c) Evaluate Thermal Resistances:

Ignoring the fouling resistance, the total thermal resistance is given as follows:

$$R_{total} = \frac{1}{h_c \cdot A_i} + \frac{\ln\left[\frac{D_o}{D_i}\right]}{2 \cdot 3.142 \cdot k_{wall}} + \frac{1}{h_h \cdot A_o}$$

Where the first term is the convective thermal resistance due to the cold flow, and the second term is the conductive thermal resistance due to the copper tube thickness, and the third term is the convective thermal resistance due to the annulus flow of the hot fluid.

c-1: The cold flow convective thermal resistance:

$$\frac{1}{h_c \cdot A_i}$$

- The surface Area of the inner tube is

$$A_i = 3.142 \cdot D_i$$ =0.0016*3.142=0.05027m

- The convictive heat transfer coefficient of the cold flow, h_c is given as follows:

$$Re_c = \rho_c \cdot v_c \cdot \frac{D_i}{visc_c}$$

$$\rho_c = \rho\left(water , T = T_{mean,c} , P = 101325 \; [Pa]\right)$$

$$visc_c = \mathbf{Visc}\left(water , T = T_{mean,c} , P = 101325 \; [Pa]\right)$$

$$Pr_c = \mathbf{Pr}\left(water , T = T_{mean,c} , P = 101325 \; [Pa]\right)$$

$$k_c = \mathbf{k}\left(water , T = T_{mean,c} , P = 101325 \; [Pa]\right)$$

$$A_{cross,i} = 0.25 \cdot 3.142 \cdot D_i^2$$

$$v_c = \frac{\dot{m}_c}{A_{cross,i} \cdot \rho_c}$$

$$f = 8 \cdot \left[\left[\frac{8}{Re_c}\right]^{12} + \left[\left[\frac{37530}{Re_c}\right]^{16} + \left[-2.467 \cdot \mathbf{ln}\left[\left[\frac{7}{Re_c}\right]^{0.9} + 0.27 \cdot \frac{eps}{D_i}\right]\right]^{16}\right]^{-1.5}\right]^{\left[\frac{1}{12}\right]}$$

$$Nusselt_c = \frac{\frac{f}{8} \cdot (Re_c - 1000) \cdot Pr_c}{1 + 12.7 \cdot \left[\frac{f}{8}\right]^{(1/2)} \cdot Pr_c^{(2/3)} - 1}$$

$$h_c = Nusselt_c \cdot \frac{k_c}{D_i}$$

The above equations were used to find the h_c based on the non-dimensional Nusselt number correlation. The latter is a function of the Prandtl number (given in table 3 at the mean temperature between the inlet and outlet temperature the cold flow, Pr_c = 5.236). The Reynold number, Re_c depends on viscosity Hydraulic Diameter, Density (which are given in table 3) and the cross-sectional velocity of cold flow inside the tube). The Reynold identify the flow regime type, and based on it, the appropriate Nusselt Number Correlation is selected. The Reynold number was found to be Re_c = 25752. The values of Re_c and Pr_c identify the flow regime inside the inner tube turbulent fully developed flow. Therefore, the Gnielinski Nusselt Number correlation is selected accordingly

$$Nusselt_c = \frac{\frac{f}{8} \cdot (Re_c - 1000) \cdot Pr_c}{1 + 12.7 \cdot \left[\frac{f}{8}\right]^{(1/2)} \cdot Pr_c^{(2/3)} - 1}$$

The associated Darcy friction factor due to the cold flow inside the inner tube, f, is given as follows:

$$f = 8 \cdot \left[\left[\frac{8}{Re_c} \right]^{12} + \left[\left[\frac{37530}{Re_c} \right]^{16} + \left[-2.467 \cdot \ln \left[\left[\frac{7}{Re_c} \right]^{0.9} + 0.27 \cdot \frac{eps}{D_i} \right] \right]^{16} \right]^{-1.5} \right]^{\left[\frac{1}{12} \right]}$$

and found to be equal, f = 0.02442, where Nusselt_c= 187 , then substituting everything to

$$h_c = Nusselt_c \cdot \frac{k_c}{D_i}$$

The convective heat transfer coefficent was obtained to be 7207 [W/m^2-K]

<u>c-2: Conductive thermal resistance:</u>

$$\frac{\ln \left[\frac{D_o}{D_i} \right]}{2 \cdot 3.142 \cdot k_{wall}}$$

Where D_o is the outer diameter of the iiner tube and D_i is the iiner diamter of the iner tuber, K_wall is the thermal conductivity of copper which is 398 [W/m-K]. For our case, we want always to monimize the conductive thermal resisatnc by choosing choosing a relatively good conductive material for the tube(Copper in our case), so you can enhance the heat tranfer betwee the working fluids.

<u>c-3: Convictive thermal resistance due to the annulus flow of the hot fluid in the pipe:</u>

$$\frac{1}{h_h \cdot A_o}$$

- A_o = 3.42 * D_o = 3.42 * 0.019 = 0.05969 m^2
- Convective heat transfer coefficient due to the annulus flow, h_h:

The Reynold number is found be:

$$Re_h = \rho_h \cdot v_c \cdot \frac{D_h}{visc_h}$$

$$\rho_h = \rho\left(water, T = T_{mean,h}, P = 101325 \text{ [Pa]}\right)$$

$$visc_h = \mathbf{Visc}\left(water, T = T_{mean,h}, P = 101325 \text{ [Pa]}\right)$$

$$Pr_h = \mathbf{Pr}\left(water, T = T_{mean,h}, P = 101325 \text{ [Pa]}\right)$$

$$k_h = \mathbf{k}\left(water, T = T_{mean,h}, P = 101325 \text{ [Pa]}\right)$$

$$A_{cross,h} = 0.25 \cdot 3.142 \cdot \left(D_{o,o}^2 - D_o^2\right)$$

$$v_h = \frac{\dot{m}_h}{A_{cross,h} \cdot \rho_h}$$

$$D_h = D_{o,o} - D_o$$

Re_h = 452746 . now, let'schecjk the flow regim by comparing Re_h with the critical Reynold numbe values foe annulus flow:

$$Re_{CL} = 2089.26 + 686.15 \cdot \frac{D_o}{D_{o,o}} \quad \boxed{= 2141}$$

$$Re_{CU} = 2963.02 + 334.16 \cdot \frac{D_o}{D_{o,o}} \quad \boxed{= 2988}$$

Since Re_h is way higher than Re_CU, the flow is turbulent, and the following Nusselt number correlation is to be used:

$$Nusselt_h = 0.025 \cdot Re_h^{0.78} \cdot Pr_h^{0.48} \cdot \left[\frac{D_o}{D_{o,o}}\right]^{-0.14} \quad \boxed{= 1840}$$

where Pr_h = 4.188

$$h_h = Nusselt_h \cdot \frac{k_h}{D_h} \quad \boxed{= 5024}$$

Where k_h = 0.6307 [W/m^2-K]

For later pressure drop analysis, the associated Darcy friction factor due to annulus flow of the hot fluid is:

$$f_h = 4 \cdot \left[1.7372 \cdot \ln\left[\frac{Re_h}{1.964 \cdot (\ln(Re_h) - 3.8215)}\right]\right]^{-2} \cdot \left[1 + 0.0925 \cdot \frac{D_o}{D_{o,o}}\right] \quad \boxed{= 0.01301}$$

c_4: Total Thermal Resistance:

By substituting the values found in (3-1to3-3) into

$$R_{total} = \frac{1}{h_c \cdot A_i} + \frac{\ln\left[\frac{D_o}{D_i}\right]}{2 \cdot 3.142 \cdot k_{wall}} + \frac{1}{h_h \cdot A_o}$$

The total thermal resistance for the primary set is found to be 0.006164 [K/W]

d) Calculating overall heat Transfer coefficient based on the outer diameter of the inner tube:

$$U_o \cdot A_o = \frac{1}{R_{total}}$$

= 2718 [W/m^2-K]

e) Calculate the length of heat Exchanger required for the primary Set:

$$Q_c = U_o \cdot A_o \cdot L_{req} \cdot LMTD$$

Where $\dot{Q}_c = \dot{Q}_h = \dot{m}\_c \cdot c_{p_c} \cdot (T_{out_c} - T_{in_c}) = \dot{m}_h \cdot c_{p_h} \cdot (T_{out_h} - T_{in_h}) = 13584\ W$

L_required = 8.46 meters

f) Pressure drops in the tube and annulus:

$$\delta_{P,c} = 0.5 \cdot f \cdot \rho_c \cdot v_c^2 \cdot \frac{L_{req}}{D_i} \quad \boxed{= 10.03\ \text{kPa}}$$

$$\delta_{P,h} = 0.5 \cdot f_h \cdot \rho_h \cdot v_h^2 \cdot \frac{L_{req}}{D_h} \quad \boxed{= 0.02991\ \text{Pa}}$$

Since pressure drop inside the inner tube was found to be 10.03 kPa which is way less than the allowable pressure drop value which is 10 psi, Thus it satisfies the requirement stated in table (2)

g) Conclusion of the Primary Set:

The length required was found to be 8.48 meters which violates the geometry limit requirement stated in table (2). A second set is to be done to find the length required if 20 tubes with smaller sizes are to be used.

4) Second Design Set Along with Results and Explanation

a) Tube size Selection:

Tube Size Selection for the Primary Set	
Inner Tube Diameter	5 mm
Outer Tube Diameter	8 mm
Pipe's Diameter	25 cm
Number of tubes	20

The same design process of the primary design set is followed for the second one (Check out Appendix II to see code and formulas used to obtain the results for this set). Table (4) summaries the main Results of the this Set:

Table (4): Main Results of Second Design Set

Main Results of Second St	
h_h_new	172.7 W/m^2-K
h_c_new	20972 W/m^2-K
R_total_new	0.0005572 K/W
U_o_new	3570 W/m^2-K
L_required	**0.7647 meters**
v_c	**12.79 m/s**
ΔP_c_new	238 kPa
ΔP_h_new	0.869

b) Conclusion of the Second Set:

Using 20 tubes with smaller diameter size results in enhancing the overall heat transfer, U_o as the total thermal resistance got lowered. The new required length was found to be 0.7647 meters which is satisfactory based on the geometry restrictions. However, as a result of using smaller tube sizes and 20 of them, the velocity inside the inner tubes increased from 1.249 m/s (in the primary set) to 12.79 m/s. The latter caused an increase in pressure drop to 238 kPa which extremely beyond the allowable value (10 psi).

$$\delta_{P,c} = 0.5 \cdot f \cdot \rho_c \cdot v_c^{\ 2} \cdot \frac{L_{req}}{D_i}$$

$$\delta_{P,h} = 0.5 \cdot f_h \cdot \rho_h \cdot v_h^{\ 2} \cdot \frac{L_{req}}{D_h}$$

As you can see from the above pressure drop formula, when velocity decreases, the pressure drop inside the tube decrease too as there is an inverse relation between the two parameters. To find the appropriate velocity inside the inner tubes so that we have an acceptable pressure drop for the same number of tubes, we can adjust play with tube size by running a parametric study and then select the appropriate size that satisfies the geometry and pressure drop requirements. Figure (7) shows the parametric study done in this regard:

1..30	$D_{i,new}$ [m]	$D_{o,new}$ [m]	$L_{req,new}$ [m]	$\delta P_{c,new}$ [Pa]	$\delta P_{h,new}$	$h_{h,new}$ [W/m2-K]
Run 1	0.001	0.003	1.713	1.304E+09	0.015	101.1
Run 2	0.0015	0.0035	1.493	1.575E+08	0.01344	107.3
Run 3	0.002	0.004	1.329	3.465E+07	0.01228	113.6
Run 4	0.0025	0.0045	1.202	1.063E+07	0.01141	120.2
Run 5	0.003	0.005	1.101	4.026E+06	0.01072	127
Run 6	0.0035	0.0055	1.017	1.768E+06	0.01017	134
Run 7	0.004	0.006	0.9479	864881	0.009724	141.2
Run 8	0.0045	0.0065	0.8891	459897	0.00936	148.7
Run 9	0.005	0.007	0.8387	261224	0.00906	156.4
Run 10	0.0055	0.0075	0.795	156544	0.008812	164.4
Run 11	0.006	0.008	0.7566	98071	0.008606	172.7
Run 12	0.0065	0.0085	0.7228	63780	0.008436	181.2
Run 13	0.007	0.009	0.6926	42824	0.008296	190.1
Run 14	0.0075	0.0095	0.6656	29558	0.008182	199.2
Run 15	0.008	0.01	0.6413	20898	0.008092	208.6
Run 16	0.0085	0.0105	0.6193	15092	0.008021	218.3
Run 17	0.009	0.011	0.5992	11105	0.007968	228.4
Run 18	0.0095	0.0115	0.5809	8310	0.007932	238.9
Run 19	0.01	0.012	0.564	6313	0.00791	249.7
Run 20	0.0105	0.0125	0.5485	4861	0.007902	260.8
Run 21	0.011	0.013	0.5342	3790	0.007907	272.4
Run 22	0.0115	0.0135	0.5209	2988	0.007924	284.4

Figure (7): Parametric study to select the appropriate tube size

According to the above parametric study the pressure drop gets corrected starting from run 12 with tube size of (D_i0.0065 and D_o 0.0085). From the published standard copper tube size, a new tube size was selected, and a third design set done to check the validity of the selection.

5) Third Design Set Along with Results and Explanation

a) Tube size Selection:

Tube Size Selection for the Primary Set	
Inner Tube Diameter	8 mm
Outer Tube Diameter	9.5 mm
Pipe's Diameter	25 cm
Number of tubes	20

The same design process of the primary and secondary sets is followed for the third one (Check out Appendix III to see code and formulas used to obtain the results for this set). Table (5) summaries the main Results of the this Set:

Table (5): Main Results of Third Design Set

Main Results of Second St	
h_h_new	199.5 W/m^2-K
h_c_new	13628 W/m^2-K
R_total_new	0.0004822 K/W
U_o_new	3467W/m^2-K
L_required	**0.6619 meters**
v_c	**4.998 m/s**
ΔP_c_new	21.6 kPa
ΔP_h_new	0.008145Pa

b) Conclusion of the Second Set:

The minor change in tube size seems to have a major impact in the pressure drop such that it decreased to 21.6 kPa which is less than the allowable pressure drops. Also, it satisfies the geometry limit as the obtained required length was found to be 0.6619 meters. All in all, the third set is the optimal design selection.

6) Summary of the Optimal Design

The Optimal Selection for Tube and Pipe Sizes Along with Major Results	
Inner Tube Diameter	8 mm
Outer Tube Diameter	9.5 mm
Pipe's Diameter	25 cm
Number of tubes	20
Length of the heat exchanger	**0.6619 meters**
Height of the heat exchanger	25 cm
Volume of the Heat exchanger	0.00325 m^3(Hence, it can be fitted in the 0.5 m^3 small compartment)

Economic Impact

To understand how to reduce operating costs, start by learning how Jacuzzis and hot barrels use energy. They generally use electric energy to heat and continuously circulate the water. Barrels are covered and unused further than 95 percent of the time, yet this is when they use 75 percent of their energy. Therefore, energy conservation starts at this “steady state” mode and at reducing heat losses from the cover and walls. While the utmost of the energy used in Jacuzzi goes into the heater, the energy for pumping is also significant.

According to a spa study by the Davis Energy Group, if you own a spa it is the biggest electrical consumer in the house. For many jacuzzi owners, heating their jacuzzi takes too long and is expensive. So, adding a heat exchanger that saves 40% of the power, and in less time that's a big deal. So, after calculations to heat up 600 gallons of water without the heat exchanger will cost almost 7.5$ every hour but after adding the heat exchanger the cost will reduce almost the half and that’s on the short term, on the other hand, on the long term, if we use the jacuzzi every day with the heat exchanger for one month that would save us 90$ which is 1080$ yearly.

Also, there are some factors to save energy and money. First covers; The sequestration value of the cover and the littleness of its seal to the jacuzzi are one of the most important construction details in terms of overall energy use. Designing a cover that's well insulated, provides a good air ocean, and is light enough for a single person to handle is a real challenge. The warm, sticky air trapped between the cover and the water face is rich in energy and a small air leak in the seal increases evaporation from the water face, bypassing the cover’s insulation and adding heat loss from the jacuzzi. The cover insulation should be good- quality, closed cell froth that won't absorb water. It should be supported adequately so it doesn't sag in the middle, and one person should be suitable to remove it alone. Secondly Pumps: The circular pump moves water through a sludge and heater continuously during steady- state operation. Some barrels have a two-speed motor, using the same pump for low speed rotation in steady state mode and for high speed operation when the spurts are on. These pumps aren't generally veritably effective in any mode, but particularly by steady state because the motor is veritably smoothly loaded and running at low effectiveness the utmost of the time. Since these are air cooled motors, getting rid of the waste heat from the motor in the summer is a problem too. Some manufacturers use separate pumps for rotation and spurts. While original costs are slightly advanced, this helps optimize the rotation pump and can yield good savings during steady state operation.

Project Conclusion

The main goal of this project is reducing the power required by an indoor 600-gallon Jacuzzi's heater from 31.1 kW to 18.8 kW. The goal is achieved by designing an appropriate cross flow heat tube-in-tube exchanger using the LMTD method. The optimal design shall satisfy two major restrictions, including fitting a compartment of 0.5 m^3 and keeping the pressure drop inside the heat exchanger tube lower than 10 psi. A total of three design sets done to reach the optimal design. In each of which, the required length of heat exchanger the pressure drop was obtained and compared with acceptable values. All in all, the optimal selection is to design a cross flow heat exchanger that consists of 1 bigger pipe of 25 cm diameter (the annulus hot flow), 20 tube of inner and outer diameter 8 and 9.5 mm respectively and a length of 0.6619 meters. Appendices. V and IV state the results of the three design sets.

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Appendix I: Energy balance of jacuzzi & Primary Design Set EES Code

```
//Calculation of the time it takes the the teh jaccuzi to cool down from 55 C to 45 C
E_st = m_st *c_st*(T_old- T_set)
E_st= time* (-Q_rad- Q_conv-Q_cond)
m_st = 2243.8 [kg]
c_st = 4181 [J/ K-kg]
T_old = 318.15 [K]
T_set = 328.15 [K]
Q_rad = ( 5.67*10^(-8) ) *0.95 * 8.36 * ( T_set^(4)- T_c_in^(4) ) //(emissivity = 0.95)
Q_conv = 25*8.36 *(T_set-T_c_in) //( free conective heat transefer coefficintt of air h = 25 W/m^-K )
Q_cond = (T_set-T_c_in)/ ( 0.10 / (14.5*0.2)) // teh conductive area is the toltal surface area of the Jaccuzi's walls, K= 0.2 (
for Acrylic)
Time_h = time/3600
```

```
File:C:\Users\Dell\Desktop\Primary_design_HX_in_progress.EES                    12/7/2022 8:55:22 PM  Page 1
        EES Ver. 11.064: #6048: For use only by Adv. Manufacturing & Innovative Design, Florida Institute of Technolgy

// Select the length of the tube for a double pipe heat exchangers that has the following knowns:

//Cold water:
T_c_in = 298.15 [K]
T_c_out= 311.15 [K]
m_dot_c= 0.25 [kg/s]
T_mean_c =  (T_c_in +T_c_out)/2
cp_c =cp(Water,T=T_mean_c,P=Po#)
//Diameters of the tubes
D_i = 0.016 [m]
D_o= 0.019 [m]
D_o_o = 0.25 [m]
eps = 0.002/1000
// Hot water:
T_h_in= 318.15 [K]
m_dot_h = 0.5 [kg/s]
T_mean_h = (T_h_in +T_h_out)/2
cp_h = cp(Water,T= T_h_in,P=Po#)
//Find T_h_out:
Q_c = m_dot_c*cp_c*(T_c_out-T_c_in)
Q_h= m_dot_h*cp_h*(T_h_in-T_h_out)
Q_c=Q_h
//T_h_out =311.7 [K]
// h_c(heat tarnsfer coefficint inside the tube (D_i = 0.016m)
Re_c=  rho_c*v_c*D_i/visc_c
rho_c = density(Water,T=T_mean_c,P=Po#)
visc_c = viscosity(Water,T=T_mean_c,P=Po#)
Pr_c =prandtl(Water,T=T_mean_c,P=Po#)
k_c =conductivity(Water,T=T_mean_c,P=Po#)
A_cross_i= 0.25*pi#*(D_i^2)
v_c= m_dot_c/A_cross_i/rho_c
//Re_c = 26562 , Pr_c = 5.058   : Fully devoloped Turblent flow,  using Gnieliski eq with corretion factor f:

 f = 8* (  (8/Re_c)^12    +   (     (37530/Re_c)^16 + (-2.467  * ln(    (7/Re_c)^0.9 +(0.27*eps/D_i)  )      )^16
)^(-1.5)            )^(1/12)

Nusselt_c = (   (  (f/8)*(Re_c-1000) *Pr_c )   /(   1+12.7*(f/8)^(1/2) * (Pr_c^(2/3)) -1  )    )

h_c = Nusselt_c*k_c/D_i
// now let's find h_o
Re_h=  rho_h*v_c*D_h/visc_h
rho_h = density(Water,T=T_mean_h,P=Po#)
visc_h = viscosity(Water,T=T_mean_h,P=Po#)
Pr_h =prandtl(Water,T=T_mean_h,P=Po#)
k_h =conductivity(Water,T=T_mean_h,P=Po#)
A_cross_h= 0.25*pi#*(D_o_o^2 - D_o^2)
v_h= m_dot_h/A_cross_h/rho_h
D_h = D_o_o- D_o
//Check Flow Regim:
Re_CL = 2089.26 +686.15*(D_o/D_o_o)
Re_CU= 2963.02 +334.16 *(D_o/D_o_o)

//Since Re_h > Re_CU, the flow is turbluent , and we use the following correlation to find the Nusselt Number
Nusselt_h = 0.025 *Re_h^(0.78) *Pr_h^(0.48)*(  D_o/ D_o_o)^ (-0.14)

h_h = Nusselt_h*k_h/D_h
f_h = 4*  (1.7372*  ln(Re_h/  (1.964* (  ln(Re_h)-3.8215)     ) )      )^(-2) *(1+  (0.0925*(D_o/D_o_o) )    )
//Let's find R_total:
k_wall =398 [W/m-K]
A_i = pi#*D_i
A_o = pi#*D_o
R_total = 1/(h_c*A_i)+ ( ln(D_o/D_i)/(2*pi#*k_wall)  )+ 1/(h_h*A_o)
U_o*A_o=1/R_total
del_T_1= (T_h_in-T_c_out)
del_T_2 = (T_h_out-T_c_in)
LMTD =(del_T_1-del_T_2)/ln(  del_T_1  /       del_T_2 )
Q_c =  U_o*A_o *  L_req *LMTD
```

—

```
//Calculation of Pressure Drop for Case 1 (n_tubes = 1):
//1 )Pressure drop in the tube:
Delta_P_c= 0.5 * f * (rho_c) *v_c^(2)*(L_req/D_i)
 //2) pressure Drop in the annulus:
Delta_P_h= 0.5 * f_h * (rho_h) *v_h^(2)*(L_req/D_h)
```

Appendix II: Second and third Design Set EES Code **(They basically have the same code but with different D_i_new and D_o_new)**

```
same heat exchanger fuctinality
n_tubes = 20
D_i_new = 0.008[m]
D_o_new = 0.00952[m]
A_i_new= D_i_new*pi#
A_o_new = D_o_new *pi#
Re_c_new= rho_c*v_c_new*D_i_new/visc_c
A_cross_i_new= 0.25*pi#*(D_i_new^2)*n_tubes
A_cross_i_new_1 =0.25*pi#*(D_i_new^2)
v_c_new= m_dot_c/A_cross_i_new_1/rho_c
 f_new = 8* (  (8/Re_c_new)^12    +  (     (37530/Re_c_new)^16 + (-2.467  * ln(    (7/Re_c_new)^0.9 +(0.27*eps/D_i)  )
)^16                      )^(-1.5)           )^(1/12)
Nusselt_c_new = (   (  (f_new/8)*(Re_c_new-1000) *Pr_c )   /(   1+12.7*(f_new/8)^(1/2) * (Pr_c^(2/3)) -1  )    )
h_c_new = Nusselt_c_new*k_c/D_i
// now let's find h_o_new
Re_h_new=  4*m_dot_h/visc_h/pi#/D_h_new
v_h_new = m_dot_h/A_flow/rho_h
D_h_new = (A_flow / P_wetted)
A_flow = 0.25 *pi# * (D_o_o^(2) - (n_tubes* D_o_new^(2))  )
P_wetted = 0.25*pi# * (D_o_o + (n_tubes* D_o_new))
Nusselt_h_new = 0.023 *Re_h_new^(0.8)*Pr_h^(0.3)
h_h_new = Nusselt_h_new *k_h/D_h_new
f_h_new = 4*  (1.7372*  ln(Re_h_new/  (1.964* (  ln(Re_h_new)-3.8215)     ) )     )^(-2) *(1+  (0.0925*(D_o_new/D_o_o) )
 )


R_total_new = 1/(h_c_new*A_i_new*n_tubes)+ ( ln(D_o_new/D_i_new)/(2*pi#*k_wall*n_tubes)  )+ 1/(h_h*A_o_new*
n_tubes)
U_o_new*A_o_new*n_tubes=1/R_total_new

Q_c =  U_o_new*A_o_new *n_tubes*  L_req_new *LMTD


 //1)Pressure drop in the tube:
Delta_P_c_new= 0.5 * f_new* (rho_c) *v_c_new^(2)*(L_req_new/D_i_new)
 //2) pressure Drop in the annulus:
Delta_P_h_new= 0.5 * f_h_new * (rho_h) *v_h_new^(2)*(L_req_new/D_h_new)
```

Appendix III: Illustration of EES formulas used for all Design Sets

$$T_{mean,c} = \frac{T_{c,in} + T_{c,out}}{2}$$

$$cp_c = \mathbf{Cp}\left(water\ ,\ T = T_{mean,c}\ ,\ P = 101325\ [Pa]\right)$$

$$D_i = 0.016\ [m]$$

$$D_o = 0.019\ [m]$$

$$D_{o,o} = 0.25\ [m]$$

$$eps = \frac{0.002}{1000}$$

$$T_{h,in} = 318.15\ [K]$$

$$\dot{m}_h = 0.5\ [kg/s]$$

$$T_{mean,h} = \frac{T_{h,in} + T_{h,out}}{2}$$

$$cp_h = \mathbf{Cp}\left(water\ ,\ T = T_{h,in},\ P = 101325\ [Pa]\right)$$

$$Q_c = \dot{m}_c \cdot cp_c \cdot \left(T_{c,out} - T_{c,in}\right)$$

$$Q_h = \dot{m}_h \cdot cp_h \cdot \left(T_{h,in} - T_{h,out}\right)$$

$$Q_c = Q_h$$

$$Re_c = \rho_c \cdot v_c \cdot \frac{D_i}{visc_c}$$

$$\rho_c = \rho\left(water\ ,\ T = T_{mean,c}\ ,\ P = 101325\ [Pa]\right)$$

$$visc_c = \mathbf{Visc}\left(water\ ,\ T = T_{mean,c}\ ,\ P = 101325\ [Pa]\right)$$

$$Pr_c = \mathbf{Pr}\left(water\ ,\ T = T_{mean,c}\ ,\ P = 101325\ [Pa]\right)$$

$$k_c = \mathbf{k}\left(water\ ,\ T = T_{mean,c}\ ,\ P = 101325\ [Pa]\right)$$

$$A_{cross,i} = 0.25 \cdot 3.142 \cdot D_i^{\ 2}$$

$$v_c = \frac{\dot{m}_c}{A_{cross,i} \cdot \rho_c}$$

$$f = 8 \cdot \left[\left[\left[\frac{8}{Re_c}\right]\right]^{12} + \left[\left[\left[\frac{37530}{Re_c}\right]\right]^{16} + \left[-2.467 \cdot \mathbf{ln}\left[\left[\left[\frac{7}{Re_c}\right]\right]^{0.9} + 0.27 \cdot \frac{eps}{D_i}\right]\right]^{16}\right]^{-1.5}\right]^{\left[\frac{1}{12}\right]}$$

$$Nusselt_c = \frac{\frac{f}{8} \cdot \left(Re_c - 1000\right) \cdot Pr_c}{1 + 12.7 \cdot \left[\frac{f}{8}\right]^{(1/2)} \cdot Pr_c^{\ (2/3)} - 1}$$

$$h_c = Nusselt_c \cdot \frac{k_c}{D_i}$$

$$Re_h = \rho_h \cdot v_c \cdot \frac{D_h}{visc_h}$$

$\rho_h = \rho(\text{water}, T = T_{mean,h}, P = 101325 \text{ [Pa]})$

$visc_h = \textbf{Visc}(\text{water}, T = T_{mean,h}, P = 101325 \text{ [Pa]})$

$Pr_h = \textbf{Pr}(\text{water}, T = T_{mean,h}, P = 101325 \text{ [Pa]})$

$k_h = \textbf{k}(\text{water}, T = T_{mean,h}, P = 101325 \text{ [Pa]})$

$A_{cross,h} = 0.25 \cdot 3.142 \cdot (D_{o,o}^2 - D_o^2)$

$v_h = \dfrac{\dot{m}_h}{A_{cross,h} \cdot \rho_h}$

$D_h = D_{o,o} - D_o$

$Re_{CL} = 2089.26 + 686.15 \cdot \dfrac{D_o}{D_{o,o}}$

$Re_{CU} = 2963.02 + 334.16 \cdot \dfrac{D_o}{D_{o,o}}$

$Nusselt_h = 0.025 \cdot Re_h^{0.78} \cdot Pr_h^{0.48} \cdot \left[\dfrac{D_o}{D_{o,o}}\right]^{-0.14}$

$h_h = Nusselt_h \cdot \dfrac{k_h}{D_h}$

$f_h = 4 \cdot \left[1.7372 \cdot \textbf{ln}\left[\dfrac{Re_h}{1.964 \cdot (\textbf{ln}(Re_h) - 3.8215)}\right]\right]^{-2} \cdot \left[1 + 0.0925 \cdot \dfrac{D_o}{D_{o,o}}\right]$

$k_{wall} = 398 \text{ [W/m-K]}$

$A_i = 3.142 \cdot D_i$

$A_o = 3.142 \cdot D_o$

$R_{total} = \dfrac{1}{h_c \cdot A_i} + \dfrac{\textbf{ln}\left[\dfrac{D_o}{D_i}\right]}{2 \cdot 3.142 \cdot k_{wall}} + \dfrac{1}{h_h \cdot A_o}$

$U_o \cdot A_o = \dfrac{1}{R_{total}}$

$del_{T,1} = T_{h,in} - T_{c,out}$

$del_{T,2} = T_{h,out} - T_{c,in}$

$LMTD = \dfrac{del_{T,1} - del_{T,2}}{\textbf{ln}\left[\dfrac{del_{T,1}}{del_{T,2}}\right]}$

$Q_c = U_o \cdot A_o \cdot L_{req} \cdot LMTD$

$n_{tubes} = 20$

$D_{i,new} = 0.008 \text{ [m]}$

$D_{o,new} = 0.00952 \text{ [m]}$

$$A_{i,new} = D_{i,new} \cdot 3.142$$

$$A_{o,new} = D_{o,new} \cdot 3.142$$

$$Re_{c,new} = \rho_c \cdot v_{c,new} \cdot \frac{D_{i,new}}{visc_c}$$

$$A_{cross,i,new} = 0.25 \cdot 3.142 \cdot D_{i,new}^2 \cdot n_{tubes}$$

$$A_{cross,i,new,1} = 0.25 \cdot 3.142 \cdot D_{i,new}^2$$

$$v_{c,new} = \frac{\dot{m}_c}{A_{cross,i,new,1} \cdot \rho_c}$$

$$f_{new} = 8 \cdot \left[\left[\frac{8}{Re_{c,new}}\right]^{12} + \left[\left[\frac{37530}{Re_{c,new}}\right]^{16} + \left[-2.467 \cdot \ln\left[\left[\frac{7}{Re_{c,new}}\right]^{0.9} + 0.27 \cdot \frac{eps}{D_i}\right]\right]^{16}\right]^{-1.5}\right]^{\left[\frac{1}{12}\right]}$$

$$Nusselt_{c,new} = \frac{\frac{f_{new}}{8} \cdot (Re_{c,new} - 1000) \cdot Pr_c}{1 + 12.7 \cdot \left[\frac{f_{new}}{8}\right]^{(1/2)} \cdot Pr_c^{(2/3)} - 1}$$

$$h_{c,new} = Nusselt_{c,new} \cdot \frac{k_c}{D_i}$$

$$Re_{h,new} = 4 \cdot \frac{\frac{\dot{m}_h}{visc_h}}{3.142 \cdot D_{h,new}}$$

$$v_{h,new} = \frac{\dot{m}_h}{A_{flow} \cdot \rho_h}$$

$$D_{h,new} = \frac{A_{flow}}{P_{wetted}}$$

$$A_{flow} = 0.25 \cdot 3.142 \cdot (D_{o,o}^2 - n_{tubes} \cdot D_{o,new}^2)$$

$$P_{wetted} = 0.25 \cdot 3.142 \cdot (D_{o,o} + n_{tubes} \cdot D_{o,new})$$

$$Nusselt_{h,new} = 0.023 \cdot Re_{h,new}^{0.8} \cdot Pr_h^{0.3}$$

$$h_{h,new} = Nusselt_{h,new} \cdot \frac{k_h}{D_{h,new}}$$

$$f_{h,new} = 4 \cdot \left[1.7372 \cdot \ln\left[\frac{Re_{h,new}}{1.964 \cdot (\ln(Re_{h,new}) - 3.8215)}\right]\right]^{-2} \cdot \left[1 + 0.0925 \cdot \frac{D_{o,new}}{D_{o,o}}\right]$$

$$R_{total,new} = \frac{1}{h_{c,new} \cdot A_{i,new} \cdot n_{tubes}} + \frac{\ln\left[\frac{D_{o,new}}{D_{i,new}}\right]}{2 \cdot 3.142 \cdot k_{wall} \cdot n_{tubes}} + \frac{1}{h_h \cdot A_{o,new} \cdot n_{tubes}}$$

$$U_{o,new} \cdot A_{o,new} \cdot n_{tubes} = \frac{1}{R_{total,new}}$$

$$Q_c = U_{o,new} \cdot A_{o,new} \cdot n_{tubes} \cdot L_{req,new} \cdot LMTD$$

$$E_{st} = m_{st} \cdot c_{st} \cdot (T_{old} - T_{set})$$

$$E_{st} = time \cdot (-Q_{rad} - Q_{conv} - Q_{cond})$$

$$m_{st} = 2243.8 \quad [kg]$$

$$c_{st} = 4181 \quad [J/\cdot K\text{-}kg]$$

$$T_{old} = 318.15 \quad [K]$$

$$T_{set} = 328.15 \quad [K]$$

$$Q_{rad} = 5.67 \cdot 10^{-8} \cdot 0.95 \cdot 8.36 \cdot (T_{set}^{\ 4} - T_{c,in}^{\ 4})$$

$$Q_{conv} = 25 \cdot 8.36 \cdot (T_{set} - T_{c,in})$$

$$Q_{cond} = \frac{\dfrac{T_{set} - T_{c,in}}{0.1}}{14.5 \cdot 0.2}$$

$$Time_h = \frac{time}{3600}$$

$$\delta_{P,c} = 0.5 \cdot f \cdot \rho_c \cdot v_c^{\ 2} \cdot \frac{L_{req}}{D_i}$$

$$\delta_{P,h} = 0.5 \cdot f_h \cdot \rho_h \cdot v_h^{\ 2} \cdot \frac{L_{req}}{D_h}$$

$$\delta_{P,c,new} = 0.5 \cdot f_{new} \cdot \rho_c \cdot v_{c,new}^{\ 2} \cdot \frac{L_{req,new}}{D_{i,new}}$$

$$\delta_{P,h,new} = 0.5 \cdot f_{h,new} \cdot \rho_h \cdot v_{h,new}^{\ 2} \cdot \frac{L_{req,new}}{D_{h,new}}$$

Appendix IV: Results of Primary and Second Design Sets (note: any parameter with subscript '_new' refers to the second design set)

SOLUTION

Unit Settings: SI K Pa J mass rad

$A_{cross,h}$ = 0.0488 [m^2]

$A_{cross,i,new}$ = 0.0003927 [m^2]

A_{flow} = 0.04808 [m^2]

$A_{i,new}$ = 0.01571 [m]

$A_{o,new}$ = 0.02513 [m]

cp_h = 4180 [J/kg-K]

$\delta_{P,c}$ = 10031 [Pa]

$\delta_{P,h}$ = 0.02522 [Pa]

$del_{T,1}$ = 7 [K]

D_h = 0.231 [m]

D_i = 0.016 [m]

D_o = 0.019 [m]

$D_{o,o}$ = 0.25 [m]

E_{st} = -9.381E+07 [W]

f_h = 0.01301

f_{new} = 0.01912

$h_{c,new}$ = 20972 [W/m^2-K]

$h_{h,new}$ = 172.7 [W/m^2-K]

k_h = 0.6307 [W/m-K]

LMTD = 9.897 [K]

$L_{req,new}$ = 0.7647 [m]

$\dot{m}_h$ = 0.5 [kg/s]

$Nusselt_c$ = 187

$A_{cross,i}$ = 0.0002011 [m^2]

$A_{cross,i,new,1}$ = 0.00001963 [m^2]

A_i = 0.05027 [m]

A_o = 0.05969 [m]

cp_c = 4180 [J/kg-K]

c_{st} = 4181 [J/·K-kg]

$\delta_{P,c,new}$ = 238182 [Pa]

$\delta_{P,h,new}$ = 0.008698

$del_{T,2}$ = 13.5 [K]

$D_{h,new}$ = 0.1493 [m]

$D_{i,new}$ = 0.005 [m]

$D_{o,new}$ = 0.008 [m]

eps = 0.000002 [m]

f = 0.02442

$f_{h,new}$ = 0.03115

h_c = 7207 [W/m^2-K]

h_h = 5024 [W/K-m^2]

k_c = 0.6166 [W/K-m]

k_{wall} = 398 [W/m-K]

L_{req} = 8.46 [m]

$\dot{m}_c$ = 0.25 [kg/s]

m_{st} = 2244 [kg]

$Nusselt_{c,new}$ = 544.2

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$Nusselt_h$ = 1840

n_{tubes} = 20

Pr_h = 4.188

Q_c = 13584 [W]

Q_{conv} = 6270 [W]

Q_{rad} = 1663 [W]

Re_{cL} = 2141

$Re_{c,new}$ = 82405

$Re_{h,new}$ = 6745

ρ_h = 991.5 [kg/m^3]

$R_{total,new}$ = 0.0005572 [K/W]

$Time_h$ = 2.96 [hours]

$T_{c,out}$ = 311.2 [K]

$T_{h,out}$ = 311.7 [K]

$T_{mean,h}$ = 314.9 [K]

T_{set} = 328.2 [K]

$U_{o,new}$ = 3570 [W/K]

$visc_h$ = 0.0006321 [kg/m-s]

$v_{c,new}$ = 12.79 [m/s]

$v_{h,new}$ = 0.01049 [m/s]

$Nusselt_{h,new}$ = 40.88

Pr_c = 5.236

P_{wetted} = 0.322 [m]

Q_{cond} = 870 [W]

Q_h = 13584 [W]

Re_c = 25752

Re_{cU} = 2988

Re_h = 452746

ρ_c = 995.2 [kg/m^3]

R_{total} = 0.006164 [K/W]

time = 10657 [sec]

$T_{c,in}$ = 298.2 [K]

$T_{h,in}$ = 318.2 [K]

$T_{mean,c}$ = 304.7 [K]

T_{old} = 318.2 [K]

U_o = 2718 [W/K-m^2]

$visc_c$ = 0.0007725 [kg/m-s]

v_c = 1.249 [m/s]

v_h = 0.01033 [m/s]

Appendix V: Results of Primary and Third Design Sets (note: any parameter with subscript '_new' refers to the third design set)

SOLUTION

Unit Settings: SI K Pa J mass rad

$A_{cross,h} = 0.0488$ [m^2]
$A_{cross,i,new} = 0.001005$ [m^2]
$A_{flow} = 0.04766$ [m^2]
$A_{i,new} = 0.02513$ [m]
$A_{o,new} = 0.02991$ [m]
$cp_h = 4180$ [J/kg-K]
$\delta_{P,c} = 10031$ [Pa]
$\delta_{P,h} = 0.02522$ [Pa]
$del_{T,1} = 7$ [K]
$D_h = 0.231$ [m]
$D_i = 0.016$ [m]
$D_o = 0.019$ [m]
$D_{o,o} = 0.25$ [m]
$E_{st} = -9.381E+07$ [W]
$f_h = 0.01301$
$f_{new} = 0.02098$
$h_{c,new} = 13628$ [W/m^2-K]
$h_{h,new} = 199.5$ [W/m^2-K]
$k_h = 0.6307$ [W/m-K]
$LMTD = 9.897$ [K]
$L_{req,new} = 0.6619$ [m]
$\dot{m}_h = 0.5$ [kg/s]
$Nusselt_c = 187$

$A_{cross,i} = 0.0002011$ [m^2]
$A_{cross,i,new,1} = 0.00005027$ [m^2]
$A_i = 0.05027$ [m]
$A_o = 0.05969$ [m]
$cp_c = 4180$ [J/kg-K]
$c_{st} = 4181$ [J/·K-kg]
$\delta_{P,c,new} = 21569$ [Pa]
$\delta_{P,h,new} = 0.008145$
$del_{T,2} = 13.5$ [K]
$D_{h,new} = 0.1378$ [m]
$D_{i,new} = 0.008$ [m]
$D_{o,new} = 0.00952$ [m]
$eps = 0.000002$ [m]
$f = 0.02442$
$f_{h,new} = 0.03056$
$h_c = 7207$ [W/m^2-K]
$h_h = 5024$ [W/K-m^2]
$k_c = 0.6166$ [W/K-m]
$k_{wall} = 398$ [W/m-K]
$L_{req} = 8.46$ [m]
$\dot{m}_c = 0.25$ [kg/s]
$m_{st} = 2244$ [kg]
$Nusselt_{c,new} = 353.6$

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$Nusselt_h = 1840$
$n_{tubes} = 20$
$Pr_h = 4.188$
$Q_c = 13584$ [W]
$Q_{conv} = 6270$ [W]
$Q_{rad} = 1663$ [W]
$Re_{CL} = 2141$
$Re_{c,new} = 51503$
$Re_{h,new} = 7309$
$\rho_h = 991.5$ [kg/m^3]
$R_{total,new} = 0.0004822$ [K/W]
$Time_h = 2.96$ [hours]
$T_{c,out} = 311.2$ [K]
$T_{h,out} = 311.7$ [K]
$T_{mean,h} = 314.9$ [K]
$T_{set} = 328.2$ [K]
$U_{o,new} = 3467$ [W/K]
$visc_h = 0.0006321$ [kg/m-s]
$v_{c,new} = 4.998$ [m/s]
$v_{h,new} = 0.01058$ [m/s]

$Nusselt_{h,new} = 43.59$
$Pr_c = 5.236$
$P_{wetted} = 0.3459$ [m]
$Q_{cond} = 870$ [W]
$Q_h = 13584$ [W]
$Re_c = 25752$
$Re_{CU} = 2988$
$Re_h = 452746$
$\rho_c = 995.2$ [kg/m^3]
$R_{total} = 0.006164$ [K/W]
$time = 10657$ [sec]
$T_{c,in} = 298.2$ [K]
$T_{h,in} = 318.2$ [K]
$T_{mean,c} = 304.7$ [K]
$T_{old} = 318.2$ [K]
$U_o = 2718$ [W/K-m^2]
$visc_c = 0.0007725$ [kg/m-s]
$v_c = 1.249$ [m/s]
$v_h = 0.01033$ [m/s]

Appendix VI: Parametric study done to study the influence of increasing the mass flow rate of hot fluid and cold fluid (Done for Primary and Third sets)

Parametric Table: Table 3

	$\dot{m}_h$ [kg/s]	U_o [W/K-m2]	L_{req} [m]	$L_{req,new}$ [m]	$U_{o,new}$ [W/m2-K]
Run 1	0.5	2718	8.46	0.2987	3849
Run 2	0.8214	2730	7.646	0.2695	3873
Run 3	1.143	2735	7.345	0.2587	3883
Run 4	1.464	2737	7.189	0.253	3889
Run 5	1.786	2739	7.092	0.2496	3892
Run 6	2.107	2740	7.027	0.2472	3895
Run 7	2.429	2741	6.98	0.2455	3896
Run 8	2.75	2742	6.945	0.2443	3898
Run 9	3.071	2743	6.917	0.2433	3899
Run 10	3.393	2743	6.895	0.2425	3900
Run 11	3.714	2743	6.876	0.2418	3901
Run 12	4.036	2744	6.861	0.2413	3901
Run 13	4.357	2744	6.848	0.2408	3902
Run 14	4.679	2744	6.837	0.2404	3902
Run 15	5	2744	6.827	0.24	3903

1..20	1 $\dot{m}_c$ [kg/s]	2 $L_{req,new}$ [m]	3 L_{req} [m]	4 U_o [W/K-m2]	5 $U_{0,new}$ [W/K]	6 $\delta_{P,c}$ [Pa]	7 $\delta_{P,c,new}$ [Pa]
Run 1	0.005	3.29	0.1488	2480	11.21	0.3591	127
Run 2	0.05737	0.4245	5.767	765.1	1039	529.4	1029
Run 3	0.1097	0.4957	6.487	1360	1779	1819	3737
Run 4	0.1621	0.5584	7.209	1895	2446	3989	8396
Run 5	0.2145	0.6198	7.941	2394	3067	7185	15351
Run 6	0.2668	0.6838	8.715	2868	3655	11597	25049
Run 7	0.3192	0.7531	9.561	3321	4217	17481	38084
Run 8	0.3716	0.8306	10.51	3758	4757	25186	55265
Run 9	0.4239	0.92	11.61	4180	5277	35211	77735
Run 10	0.4763	1.027	12.93	4589	5782	48294	107194
Run 11	0.5287	1.159	14.57	4987	6271	65613	146339