Convection Heat Transfer: Internal Flows

Hamidreza Najafi, Ph.D.
Associate Professor
Department of Mechanical and Civil Engineering
Florida Institute of Technology
hnajafi@fit.edu

Reminders/Announcements

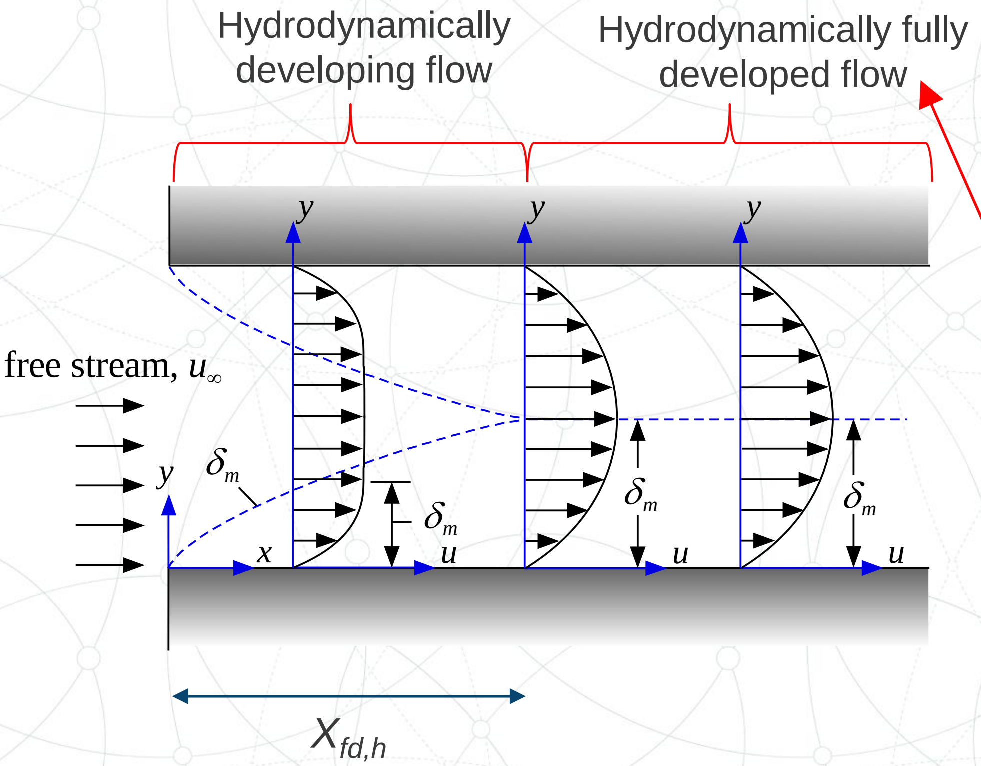
- *Comments on the Midterm exam*
- *Course Projects-Reports are Due on* **Mon, Nov 27th 2023**
- Where we go from here:
  - ***Week 12 (Nov 7th and 9th): Internal Flows***
  - ***Week 13 (Nov 14th and 16th): Internal Flows***
  - *Week 14 (Nov 21st): Internal Flows/Free Convection*
  - *Week 15 (Nov 28th and Nov 30th): Natural Convection*
  - *Week 16 (Dec 5th and 7th): Project Presentations*

Internal Forced Convection

- Conceptual Model and Basic Principles
- The Energy Balance
- Internal Flow Correlations
- Analytical and Numerical Solutions for Internal Flows

Quick Review: Internal Flow

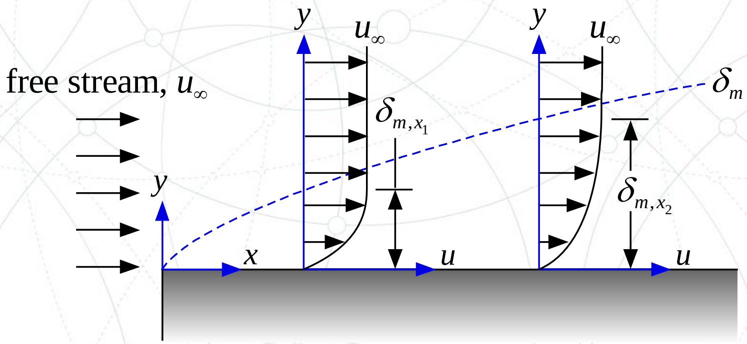
Hydrodynamic BL:



Velocity distribution does not change if $x > x_{fd,h}$

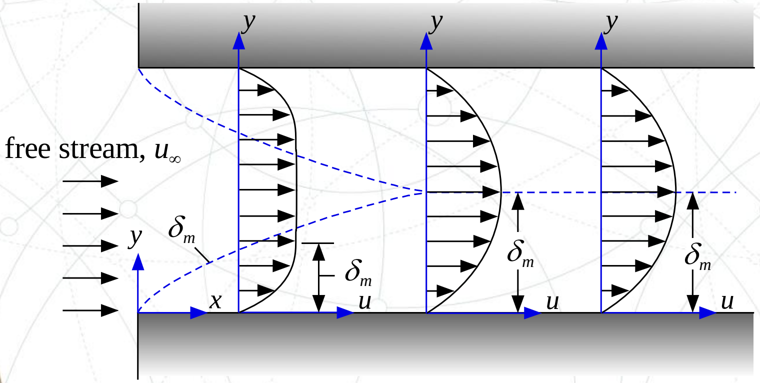
External flow:

the boundary layers could grow without bound

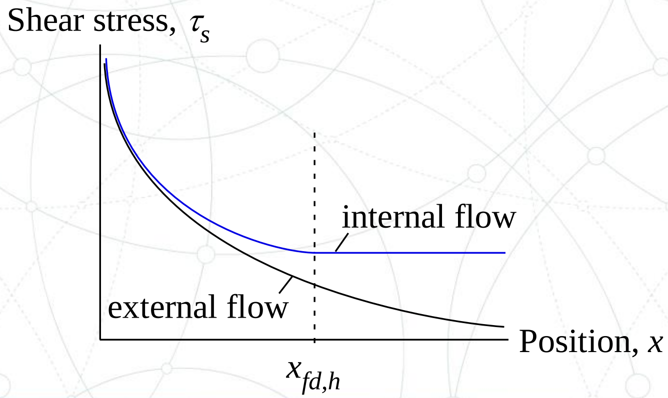


Internal flow:

the boundary layers are bounded



$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad \tau_s \approx \mu \frac{u_\infty}{\delta_m}$$



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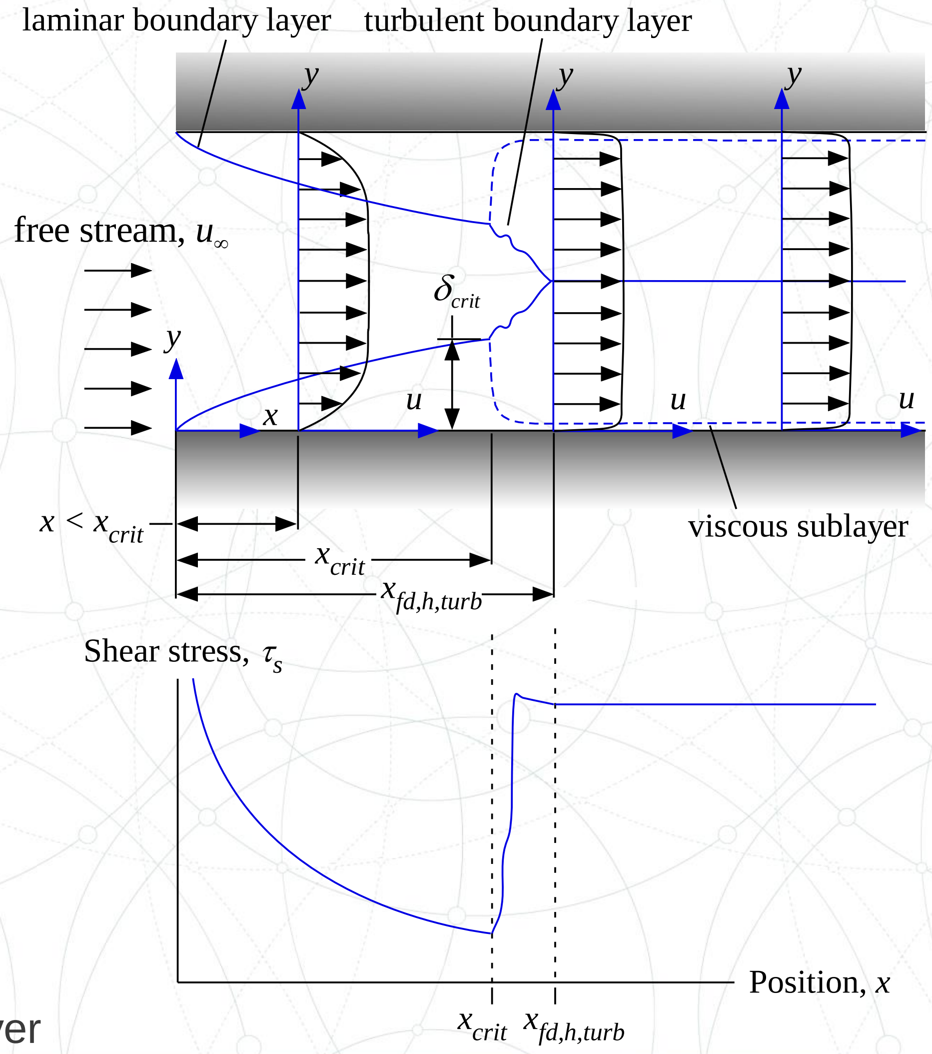
Quick Review-Momentum Considerations

- **Mean velocity** is the characteristic velocity for an internal flow

$$u_m = \frac{\boxed{\dot{m}}}{\rho \boxed{A_c}}$$

Mass flow rate

Cross-sectional area for flow



- **Laminar Hydrodynamic Entry Length**

$$\boxed{\frac{x_{fd,h,lam}}{D_h} \approx 0.06\,Re_{D_h}} \qquad \boxed{\frac{x_{fd,h,lam}}{D_h} = 0.05\,Re_{D_h}}$$

- **Turbulent Internal Flow**

•Shear stress increases dramatically

•Boundary layer grows quickly and flow becomes fully developed very quickly

•Velocity gradient is concentrated across the viscous sublayer

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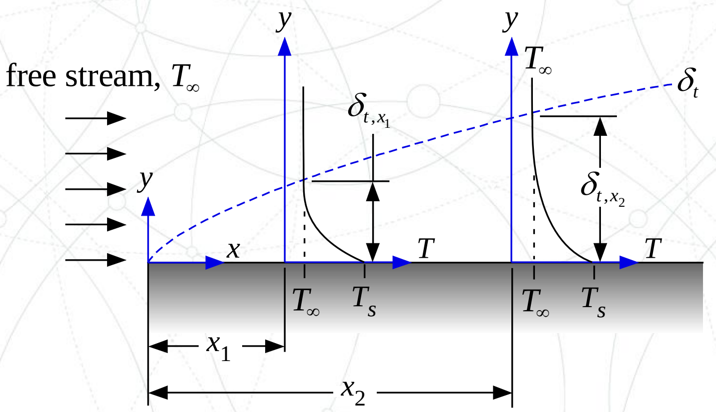
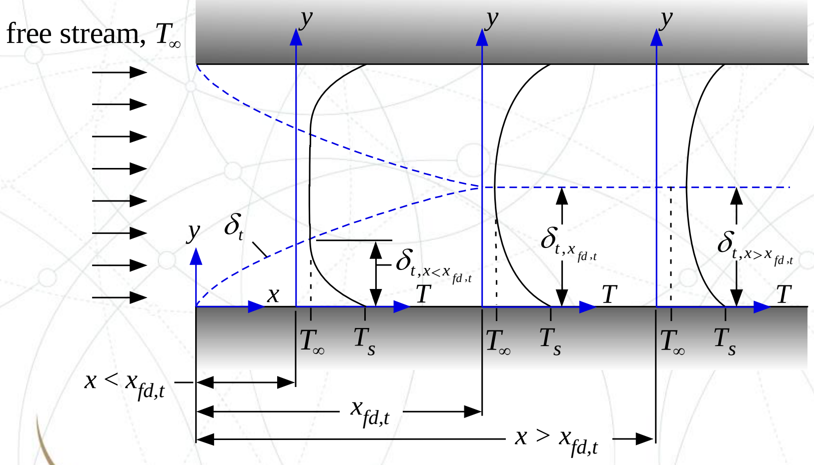
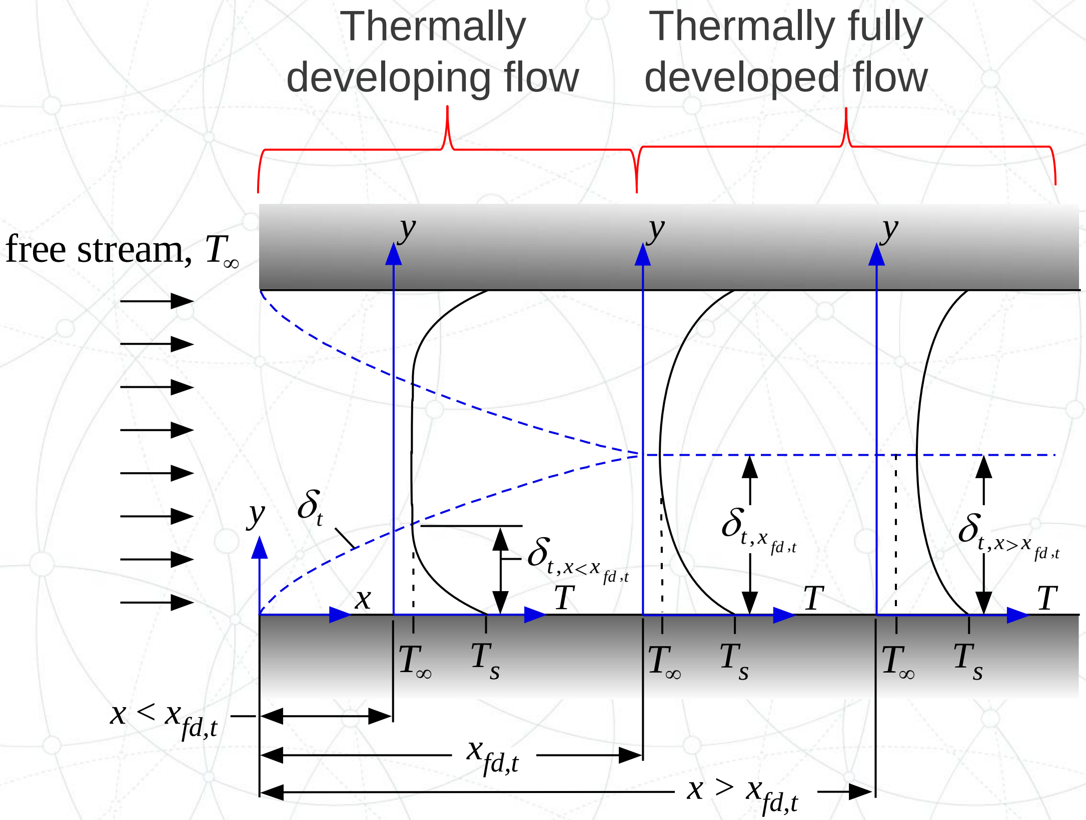
Quick Review-Thermal Considerations

External flow



Internal flow





Although the temp. profile T(r) continues to change with x, the relative shape of the profile does not change! Instead of $dT_m/dx=0$ or $dT/dx=0$, we have:

$$\frac{d}{dx}\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]=0$$

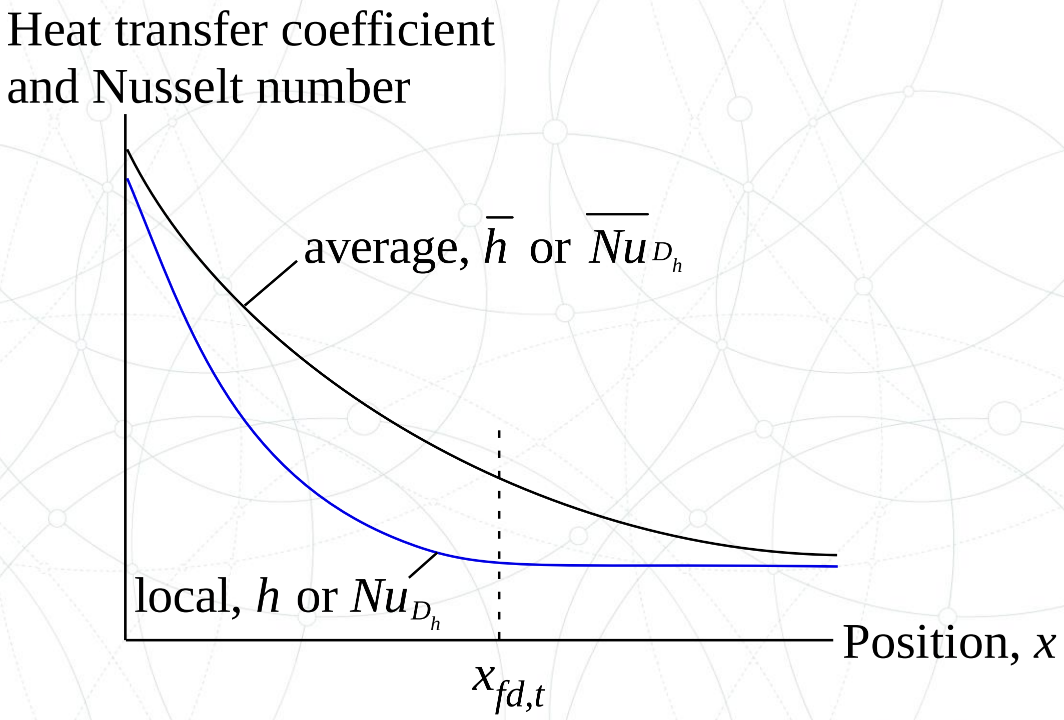
In other words, $(T_s\text{-}T)$ changes similar to $(T_s\text{-}T_m)$

Quick Review-Thermal Considerations

- **Mean temperature** $$T_m = \frac{\rho}{\dot{m}} \int_{A_c} T\, u\, dA_c$$

- **Heat transfer coefficient for laminar flow**

$$h \approx \frac{k}{\delta_t}$$



- **Laminar thermal entry length**

$$\boxed{\frac{x_{fd,t,lam}}{D_h} \approx 0.06\, Re_{D_h}\, Pr}$$

$$\boxed{\frac{x_{fd,t,lam}}{D_h} = 0.05\, Re_{D_h}\, Pr}$$

$$\frac{x_{fd,t,lam}}{x_{fd,h,lam}} = Pr$$

Quick Review-Thermal Considerations

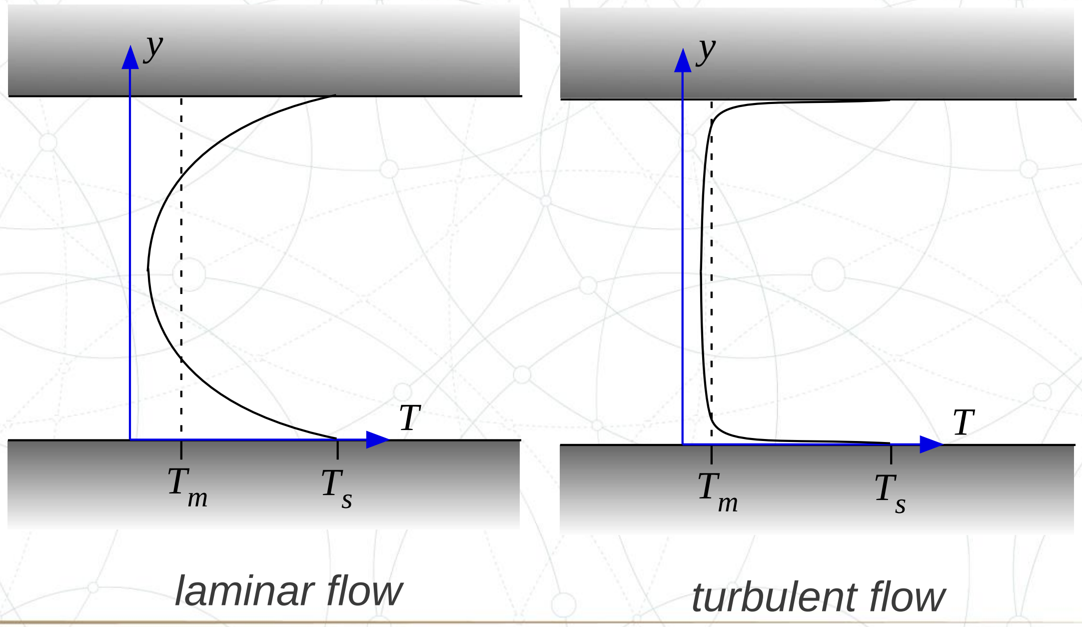
The conceptual understanding of external turbulent flows applies to internal turbulent flows

The heat transfer coefficient will be much larger:

$$h_{lam} \approx \frac{k}{\delta_{t,lam}} \qquad h_{turb} \approx \frac{k}{\delta_{vs}}$$

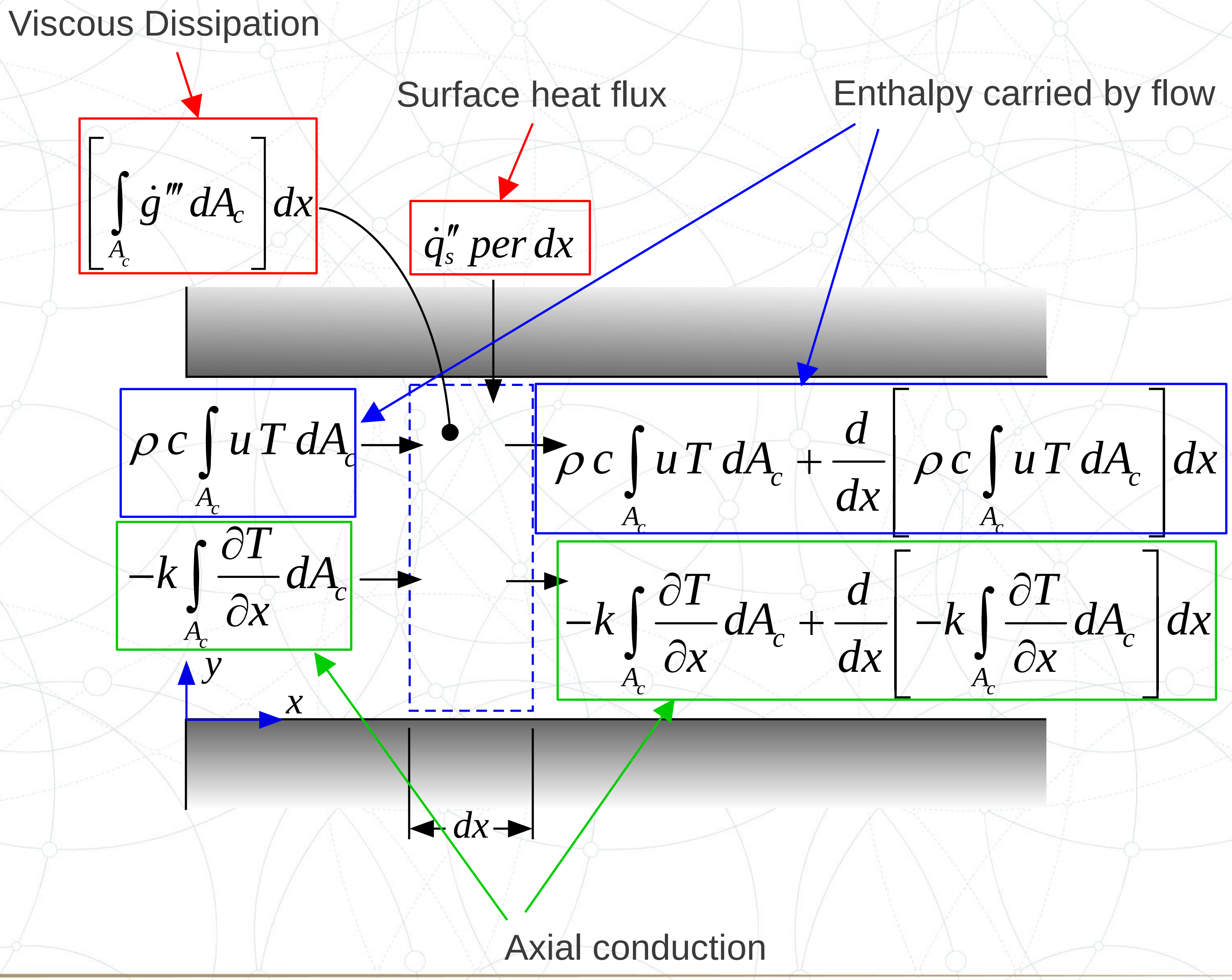
$$\delta_{t,lam} >> \delta_{vs} \quad \text{and therefore } h_{turb} >> h_{lam}$$

and the temperature gradient will be concentrated across the viscous sublayer



Internal Flows: Energy Balance

Differential Energy Balance



Energy Balance

The differential energy balance leads to:

$$\cancel{\rho c \int_{A_c} u T \, dA_c} \cancel{- k \int_{A_c} \frac{\partial T}{\partial x} dA_c} + \left[\int_{A_c} \dot{g}_v''' A_c \right] dx + \left[\int_{A_c} \dot{g}''' A_c \right] dx + \dot{q}_s'' \, per \, dx =$$

$$\cancel{\rho c \int_{A_c} u T \, dA_c} + \rho c \frac{d}{dx} \left[\int_{A_c} u T \, dA_c \right] dx \cancel{- k \int_{A_c} \frac{\partial T}{\partial x} dA_c} - k \frac{d}{dx} \left[\int_{A_c} \frac{\partial T}{\partial x} dA_c \right] dx$$

Or

$$\boxed{\underbrace{\left[\int_{A_c} \dot{g}_v''' A_c \right]}_{\text{viscous dissipation}}} + \boxed{\underbrace{\dot{g}''' A_c}_{\text{imposed volumetric generation}}} + \underbrace{\dot{q}_s'' \, per}_{\text{heat transfer from duct surface}} = \underbrace{\dot{m} c \frac{dT_m}{dx}}_{\text{enthalpy carried by flow}} + \boxed{\underbrace{\frac{d}{dx} \left[\int_{A_c} -k \frac{\partial T}{\partial x} dA_c \right]}_{\text{axial conduction}}}$$

$$\rightarrow \begin{cases} \dot{q}_s'' \, p = \dot{m} c \dfrac{dT_m}{dx} \\ \dot{q}_s'' = h\left(T_s - T_m\right) \end{cases}$$

Typically our problems do not include any imposed volumetric generation and we neglect viscous dissipation and axial conduction

Peclet number >>1

Pe=Re.Pr the ratio of the thermal energy convected to the fluid to the thermal energy conducted within the fluid

Dr. Hamidreza Najafi, Florida Institute of Technology, hnajafi@fit.edu

Energy Balance

$$\dot{q}_s'' \, p = \dot{m} c \frac{dT_m}{dx} \qquad \dot{q}_s'' = h\left(T_s - T_m\right)$$

- In general, these two equations can be solved numerically (the state variable is T_m)
- Analytical solution is available for specific cases:
  (I) Constant wall heat flux (II) Constant wall temperature (III) constant external temp.

I. Prescribed Surface Heat Flux

Mean temp. can be found as: $T_m = T_{in} + \dfrac{p}{\dot{m} c} \dot{q}_s'' \, x$

Surface temp. can be found as: $T_s = T_m + \dfrac{\dot{q}_s''}{h}$

II. Prescribed Wall Temperature

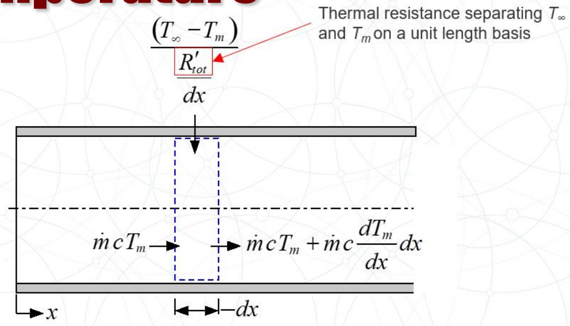
$$\frac{dT_m}{dx} = \frac{p}{\dot{m} c} h\left(T_s - T_m\right) \qquad T_m = T_s - \left(T_s - T_{in}\right)\exp\left(-\frac{p\, x\, \bar{h}}{\dot{m} c}\right)$$

$$T_{out} = T_s - \left(T_s - T_{in}\right)\exp\left(-\frac{p\, L\, \bar{h}}{\dot{m} c}\right)$$

$$q_s'' = \bar{h} A_s \Delta T_{LM} \qquad \Delta T_{LM} = \frac{\left(T_s - T_{m,o}\right) - \left(T_s - T_{m,i}\right)}{\ln\left[\left(\dfrac{T_s - T_{m,o}}{T_s - T_{m,i}}\right)\right]}$$

III. Prescribed External Temperature

$$R'_{tot} = \underbrace{\frac{1}{h_{in}\, \pi\, D_{in}}}_{\text{internal convection}} + \underbrace{\frac{\ln\left(\dfrac{D_{out}}{D_{in}}\right)}{2\, \pi\, k_{tube}}}_{\text{conduction through tube}} + \underbrace{\frac{1}{h_{out}\, \pi\, D_{out}}}_{\text{external convection}}$$



$$\frac{\left(T_\infty - T_m\right)}{R'_{tot}} = \dot{m} c \frac{dT_m}{dx}$$

$$T_{out} = T_\infty - \left(T_\infty - T_{in}\right)\exp\left(-\frac{1}{R_{tot}\, \dot{m} c}\right)$$

$$UA = \frac{1}{R_{tot}} \qquad T_{out} = T_\infty - \left(T_\infty - T_{in}\right)\exp\left(-\frac{UA}{\dot{m} c}\right)$$

Internal Forced Convection: Correlations

Convection Correlations: Internal Flows

Friction Factor

Laminar flow

Turbulent flow

Nusselt Number

Laminar flow

Turbulent flow

Friction Factor

Definitions:

Darcy Friction Factor:
$$f = \frac{-\left(\frac{dp}{dx}\right) D}{\rho u_m^2 / 2}$$

Fanning Friction coefficient: $C_f = f / 4$

- **For Laminar flow-circular tubes:**

Local friction factor (fully developed): $f_{fd,h} = \frac{64}{\mathrm{Re}_{D_h}} \rightarrow$ obtained by solving Momentum equation

Average (apparent) friction factor that includes the developing region (Shah and London):

$$\bar{f} = \frac{4}{\mathrm{Re}_{D_h}} \left[\frac{3.44}{\sqrt{L^+}} + \frac{\frac{1.25}{4L^+} + \frac{64}{4} - \frac{3.44}{\sqrt{L^+}}}{1 + \frac{0.00021}{(L^+)^2}} \right] \quad \text{and} \quad L^+ = \frac{L}{D_h \mathrm{Re}_{D_h}}$$

Dimensionless length for hydrodynamically developing internal flow

Friction Factor

- **For Laminar flow- rectangular ducts:**

Local friction factor (fully developed):

$$f_{fd,h} = \frac{96}{\mathrm{Re}_{D_h}}\left(1 - 1.3553AR + 1.9467AR^2 - 1.7012AR^3 + 0.9564AR^4 - 0.2537AR^5\right)$$

$AR \rightarrow$ Aspect Ration: $\mathrm{L_{min}/L_{max}}$

Note that: if $AR \rightarrow 0$ then $f_{fd,h} = \dfrac{96}{\mathrm{Re}_{D_h}}$

Average (apparent) friction factor that includes the developing region:

$$\bar{f} = \frac{4}{\mathrm{Re}_{D_h}}\left[\frac{3.44}{\sqrt{L^+}} + \frac{\dfrac{1.25}{4L^+} + \dfrac{f_{fd,h}\mathrm{Re}_{D_h}}{4} - \dfrac{3.44}{\sqrt{L^+}}}{1 + \dfrac{0.00021}{(L^+)^2}}\right] \quad \text{and} \quad L^+ = \frac{L}{D_h \mathrm{Re}_{D_h}}$$

Friction Factor

- **For Laminar flow- Annular ducts:**

Local friction factor (fully developed):

$$f_{fd,h} = \frac{64}{\mathrm{Re}_{D_h}} \sqrt{\frac{\left(1-RR^2\right)}{1+RR^2-\left(\frac{1-RR^2}{\ln(RR^{-1})}\right)}} \quad \text{and} \quad RR = R_i / R_o$$

Note that: if $RR \to 0$ then $f_{fd,h} = \dfrac{64}{\mathrm{Re}_{D_h}}$

Average (apparent) friction factor that includes the developing region:

$$\bar{f} = \frac{4}{\mathrm{Re}_{D_h}} \left[\frac{3.44}{\sqrt{L^+}} + \frac{\frac{1.25}{4L^+} + \frac{f_{fd,h}\,\mathrm{Re}_{D_h}}{4} - \frac{3.44}{\sqrt{L^+}}}{1+\frac{0.00021}{(L^+)^2}} \right] \quad \text{and} \quad L^+ = \frac{L}{D_h\,\mathrm{Re}_{D_h}}$$

Friction Factor

Dr. Hamidreza Najafi, Florida Institute of Technology, hnajafi@fit.edu

- **For Turbulent flow:**

Local friction factor (fully developed) in a smooth duct (Petukhov 1970):

$$f_{fd,h,e=0} = \frac{1}{[0.79\ln(\mathrm{Re}_{D_h}) - 1.64]^2} \quad \text{for} \quad 3000 < \mathrm{Re}_{D,h} < 5\times10^6$$

Local friction factor (fully developed) in a duct with given roughness:

Colebrook 1939: $$\frac{2}{\sqrt{f_{fd,h}}} = 3.48 - 1.7373\ln\left(\frac{2e}{D_h} + \frac{2(9.35)}{\mathrm{Re}_{D_h}\sqrt{f_{fd,h}}}\right)$$

Zigrang and Sylvester 1982: $$f_{fd,h} = \left\{-2\log_{10}\left[\frac{2e}{7.54D_h} - \frac{5.02}{\mathrm{Re}_{D_h}}\log_{10}\left(\frac{2e}{7.54D_h} + \frac{13}{\mathrm{Re}_{D_h}}\right)\right]\right\}^{-2}$$

Since the entry region is so short, the Average (apparent) friction factor and local value are almost equal:

$$\overline{f} \approx f_{fd,h}\left(1 + \left(\frac{D_h}{L}\right)^{0.7}\right)$$

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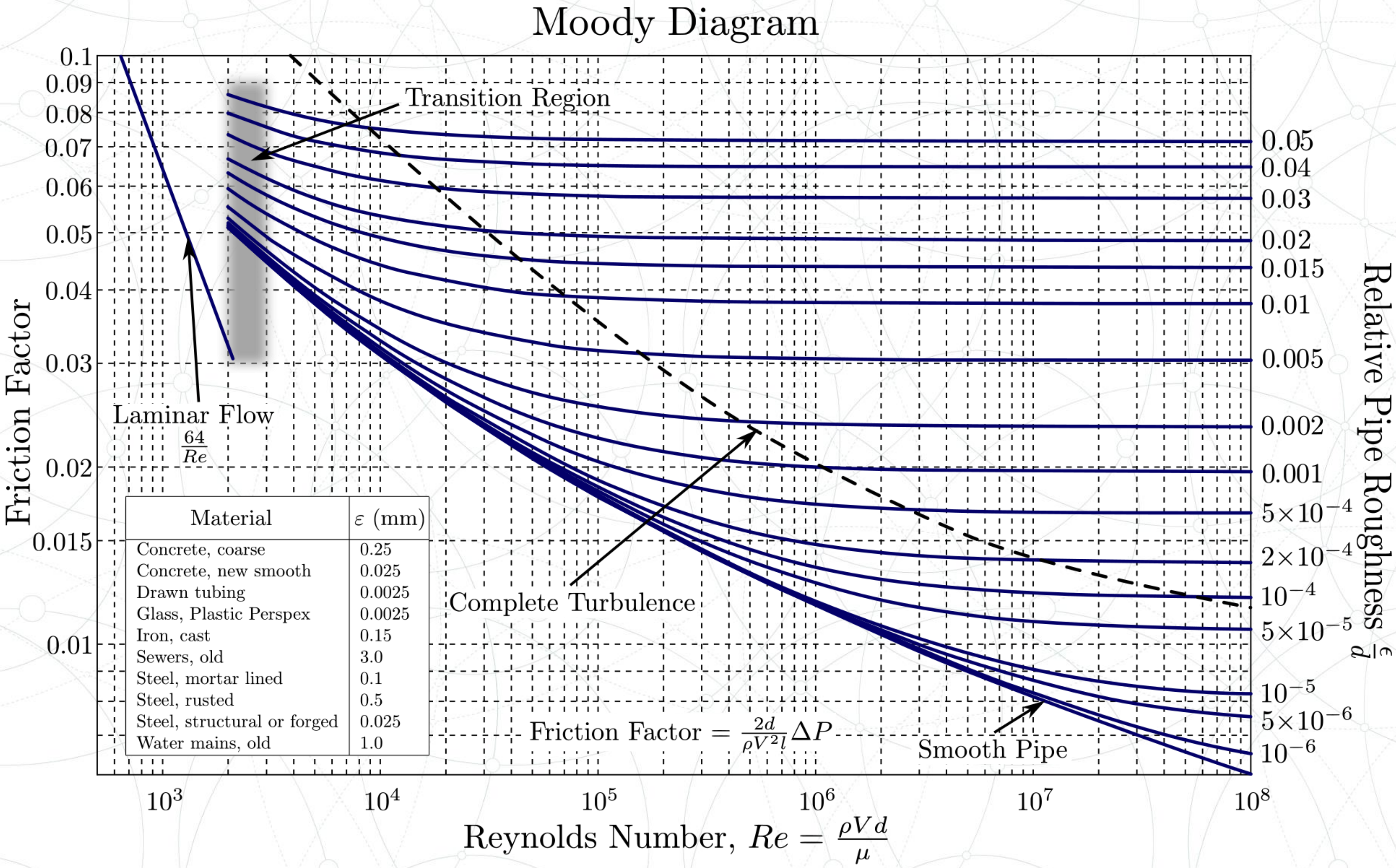
Friction Factor

- **For Turbulent flow:**



Nusselt Number

- **For Laminar hydrodynamically and thermally fully developed flow-circular tubes:**

Constant surface heat flux: $Nu_{D_h,H,fd} = 4.36$

Constant surface temperature: $Nu_{D_h,T,fd} = 3.66$

- **Average Nusselt number for laminar flow in circular tubes:**

To find average Nusselt number, the inlet conditions must be considered.

Let's define the Graetz number: $$Gz = \frac{D_h \mathrm{Re}_{D_h} \mathrm{Pr}}{L}$$

For simultaneously developing flow (combined entry length problem) with constant wall temperature:

$$\overline{Nu}_{D_h,T} = 3.66 + \frac{\left[0.049 + 0.02/\mathrm{Pr}\right] Gz^{1.12}}{\left[1 + 0.065 Gz^{0.7}\right]}$$

For simultaneously developing flow (combined entry length problem) with uniform wall heat flux:

$$\overline{Nu}_{D_h,H} = 4.36 + \frac{\left[0.1156 + 0.08569/\mathrm{Pr}^{0.4}\right] Gz}{\left[1 + 0.1158 Gz^{0.6}\right]}$$

*For thermal entry length problems (hydrodynamically fully developed and thermally developing): Use the average Nusselt correlations with Pr approaching to infinity.

Nusselt Number

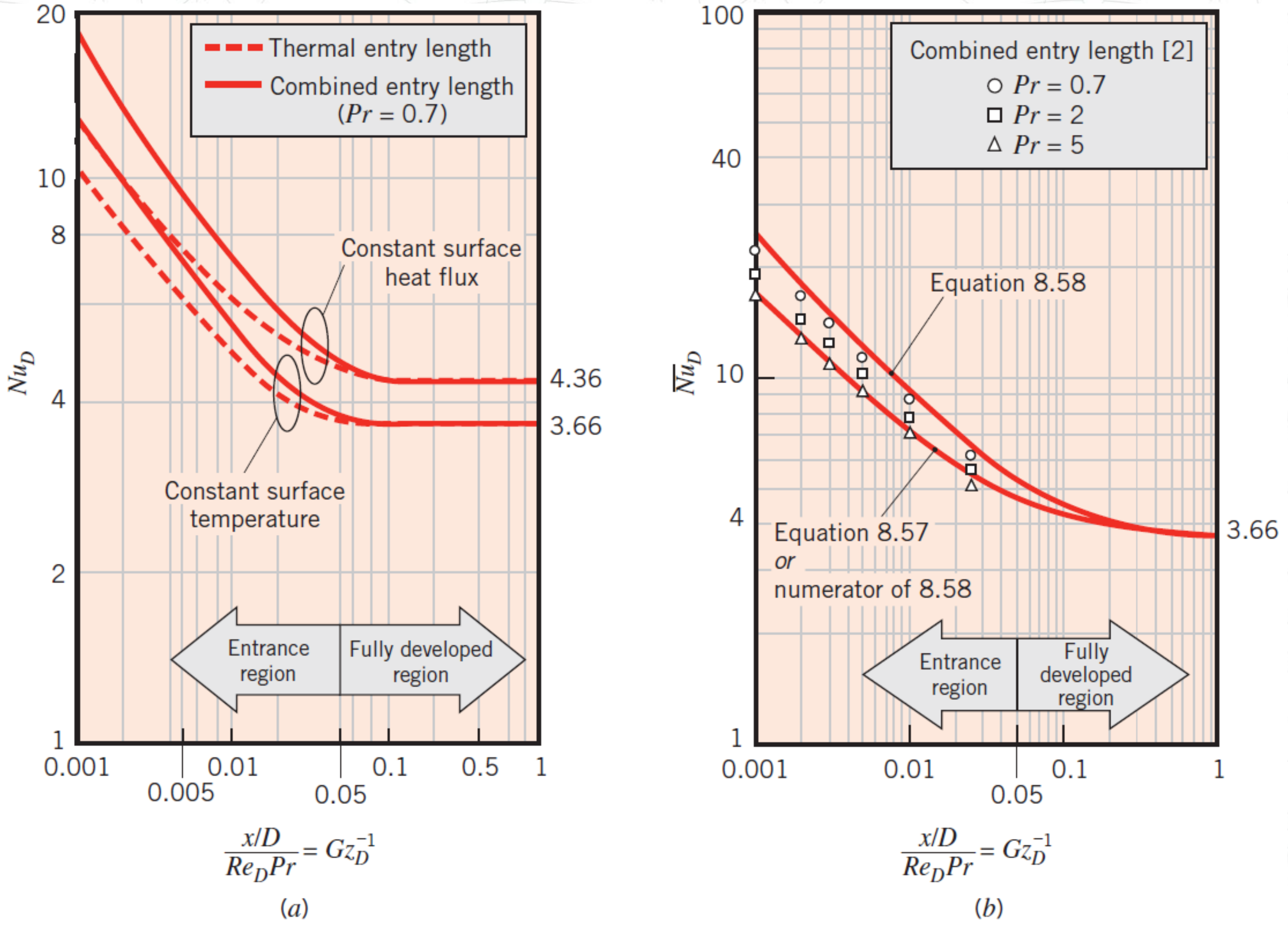


FIGURE 8.10 Results obtained from entry length solutions for laminar flow in a circular tube with constant surface temperature: (*a*) Local Nusselt numbers. (*b*) Average Nusselt numbers.

* T. L. Bergman, A. S. Lavine, F. P. Incropera, D. P. DeWitt, Fundamentals of Heat and Mass Transfer, 8th Edition, Wiley (2017).

Nusselt Number

- **Laminar flow in Rectangular ducts (Shah and London 1978):**

Constant surface temperature:

$$Nu_{D_h,T,fd} = 7.541\left(1 - 2.61AR + 4.97AR^2 - 5.119AR^3 + 2.702AR^4 - 0.548AR^5\right)$$

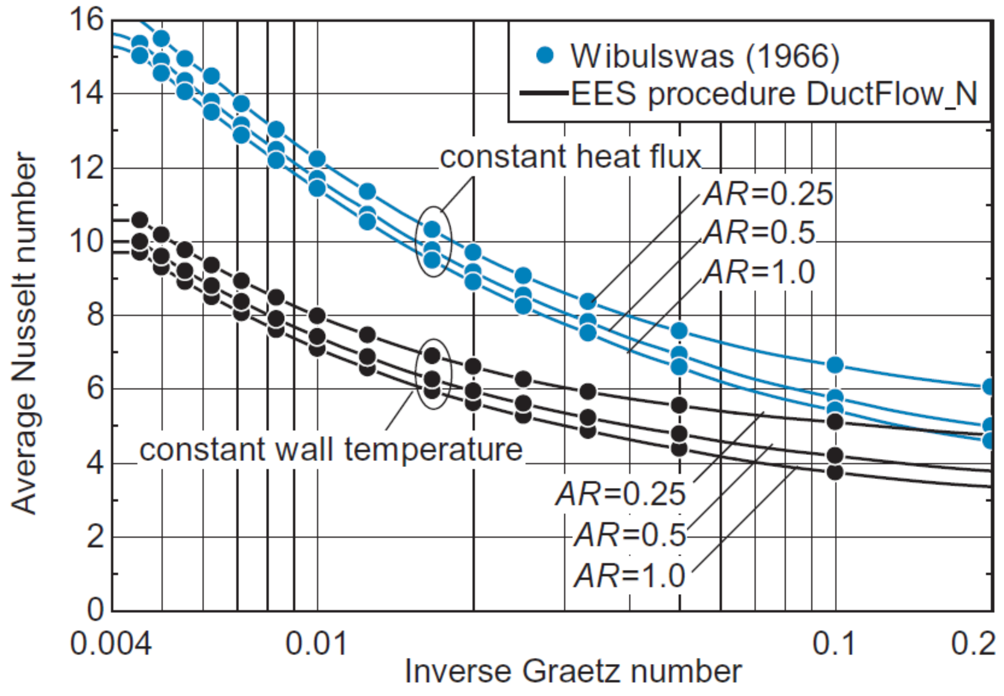
Constant surface heat flux:

$$Nu_{D_h,H,fd} = 7.541\left(1 - 2.042AR + 3.085AR^2 - 2.477AR^3 + 1.058AR^4 - 0.186AR^5\right)$$

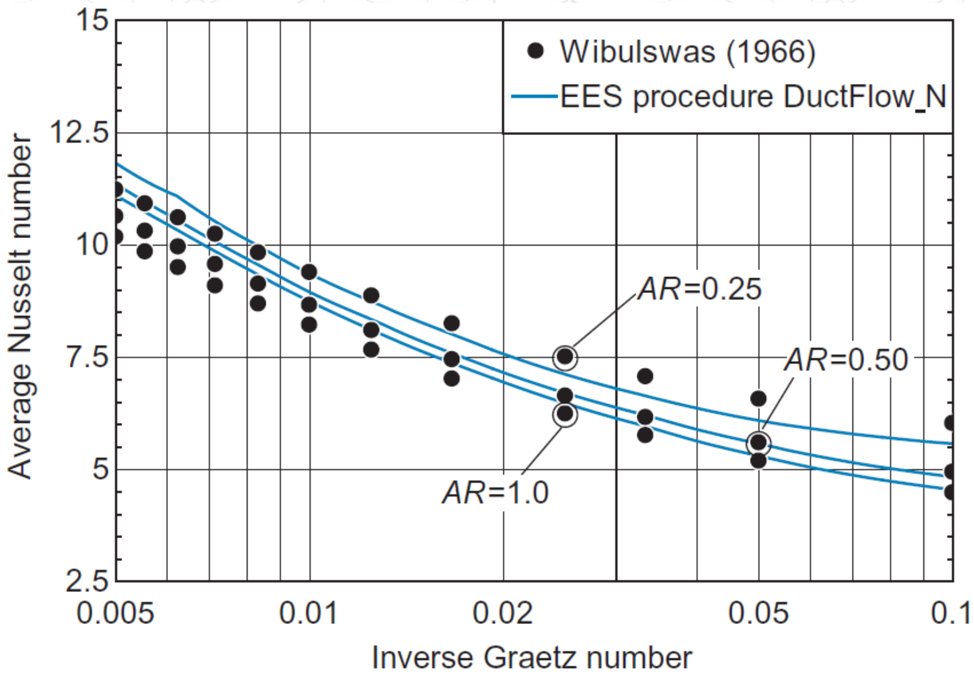
Average Nu is given in the figure on the next slide:

Nusselt Number

➤ **Laminar flow in Rectangular ducts :**



Simultaneously developing flow



Thermally developing flow for uniform surface heat flux

* G. Nellis and S. Klein, Heat Transfer, Cambridge University Press (2008).

Nusselt Number

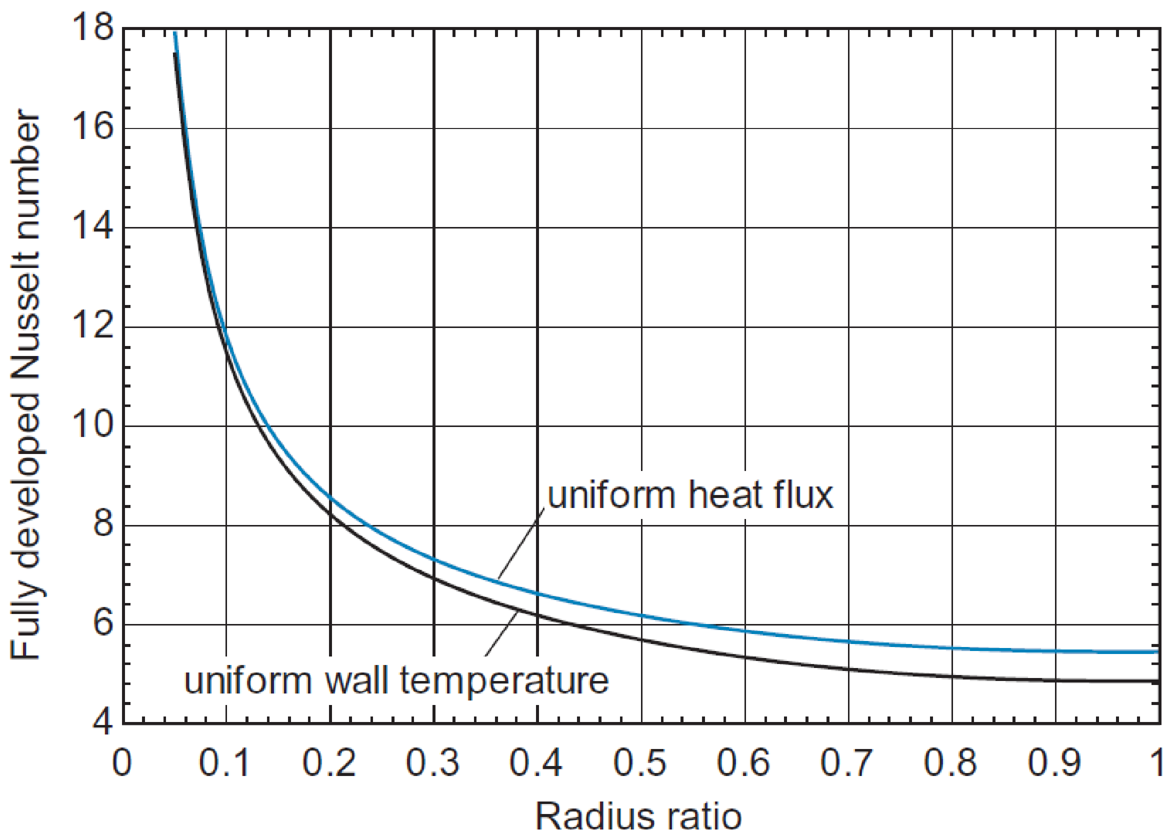
➢ **Nu and friction factor for fully developed Laminar flow in tubes of different cross sections:**

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform T_s)	$f.Re_{Dh}$
(circle)	—	4.36	3.66	64
a, b (rectangle)	1.0	3.61	2.98	57
a, b (rectangle)	1.43	3.73	3.08	59
a, b (rectangle)	2.0	4.12	3.39	62
a, b (rectangle)	3.0	4.79	3.96	69
a, b (rectangle)	4.0	5.33	4.44	73
a, b (rectangle)	8.0	6.49	5.60	82
(parallel plates)	∞	8.23	7.54	96
Heated / Insulated	∞	5.39	4.86	96
(triangle)	—	3.11	2.49	53

* T. L. Bergman, A. S. Lavine, F. P. Incropera, D. P. DeWitt, Fundamentals of Heat and Mass Transfer, 8th Edition, Wiley (2017).

Nusselt Number

- Laminar flow in Annular ducts



* G. Nellis and S. Klein, Heat Transfer, Cambridge University Press (2008).

Nusselt Number

- **Turbulent flow**

  - Not affected by the shape of the cross section or the boundary condition
  - Sensitive to surface roughness

• Gnieliski (1976):

$$Nu_{D_h,fd} = \frac{\left(\frac{f_{fd}}{8}\right)\left(\mathrm{Re}_{D_h} - 1000\right)\mathrm{Pr}}{1 + 12.7\left(\mathrm{Pr}^{2/3} - 1\right)\sqrt{f_{fd}/8}} \qquad 0.5 < \mathrm{Pr} < 2000 \quad \text{and } 2300 < \mathrm{Re}_{D_h} < 5\times10^6$$

• Kakac (1987): $\overline{Nu}_{D_h} \approx Nu_{D_h,fd}\left[1 + C\left(\frac{x}{D_h}\right)^{-m}\right]$ $\qquad C=1$ and $m=0.7$

• For smooth tubes, Dittus-Boelter equation may be used: $Nu_{D_h,fd} = 0.023\mathrm{Re}_{D_h}^{4/5}\mathrm{Pr}^n$

$$\begin{cases} n = 0.4 & \text{if } T_s > T_m \quad \text{(heating)} \\ n = 0.3 & \text{if } T_s < T_m \quad \text{(cooling)} \end{cases} \qquad 0.6 \le \mathrm{Pr} \le 160 \quad \text{and} \quad \mathrm{Re}_{D_h} \ge 10000, \quad L/D > 10$$

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Summary of Correlations

Friction Factor-Quick Review

Dr. Hamidreza Najafi, Florida Institute of Technology, hnajafi@fit.edu

Definitions: $$f=\frac{-\left(\frac{dp}{dx}\right)D}{\rho u_m^2/2}$$

➢ For Laminar flow-circular tubes:

Local friction factor (fully developed):

$$f_{fd,h}=\frac{64}{\mathrm{Re}_{D_h}}\rightarrow \text{obtained by solving Momentum equation}$$

Average (apparent) friction factor that includes the developing region (Shah and London):

$$\bar{f}=\frac{4}{\mathrm{Re}_{D_h}}\left[\frac{3.44}{\sqrt{L^+}}+\frac{\frac{1.25}{4L^+}+\frac{64}{4}-\frac{3.44}{\sqrt{L^+}}}{1+\frac{0.00021}{(L^+)^2}}\right]\quad \text{and}\quad L^+=\frac{L}{D_h\,\mathrm{Re}_{D_h}}$$

Dimensionless length for hydrodynamically developing internal flow

➢ For Laminar flow- rectangular ducts:

Local friction factor (fully developed):

$$f_{fd,h}=\frac{96}{\mathrm{Re}_{D_h}}\left(1-1.3553AR+1.9467R^2-1.7012AR^3+0.9564AR^4-0.2537AR^5\right)$$

$AR\rightarrow$ Aspect Ration: L_{min}/L_{max} , Note that: if $AR\rightarrow 0$ then $f_{fd,h}=\frac{96}{\mathrm{Re}_{D_h}}$

Average (apparent) friction factor that includes the developing region:

$$\bar{f}=\frac{4}{\mathrm{Re}_{D_h}}\left[\frac{3.44}{\sqrt{L^+}}+\frac{\frac{1.25}{4L^+}+\frac{f_{fd,h}\mathrm{Re}_{D_h}}{4}-\frac{3.44}{\sqrt{L^+}}}{1+\frac{0.00021}{(L^+)^2}}\right]\quad \text{and}\quad L^+=\frac{L}{D_h\,\mathrm{Re}_{D_h}}$$

➢ For Laminar flow- Annular ducts:

Local friction factor (fully developed):

$$f_{fd,h}=\frac{64}{\mathrm{Re}_{D_h}}\sqrt{\frac{\left(1-RR^2\right)}{1+RR^2-\left(\frac{1-RR^2}{\ln(RR^{-1})}\right)}}\quad \text{and}\quad RR=R_i/R_o$$

Note that: if $RR\rightarrow 0$ then $f_{fd,h}=\frac{64}{\mathrm{Re}_{D_h}}$

Average (apparent) friction factor that includes the developing region:

$$\bar{f}=\frac{4}{\mathrm{Re}_{D_h}}\left[\frac{3.44}{\sqrt{L^+}}+\frac{\frac{1.25}{4L^+}+\frac{f_{fd,h}\mathrm{Re}_{D_h}}{4}-\frac{3.44}{\sqrt{L^+}}}{1+\frac{0.00021}{(L^+)^2}}\right]\quad \text{and}\quad L^+=\frac{L}{D_h\,\mathrm{Re}_{D_h}}$$

➢ For Turbulent flow:

Local friction factor (fully developed) in a smooth duct (Petukhov 1970):

$$f_{fd,h,e=0}=\frac{1}{[0.79\ln(\mathrm{Re}_{D_h})-1.64]^2}\quad \text{for}\quad 3000<\mathrm{Re}_{D,h}<5\times10^6$$

Local friction factor (fully developed) in a duct with given roughness:
Colebrook 1939:

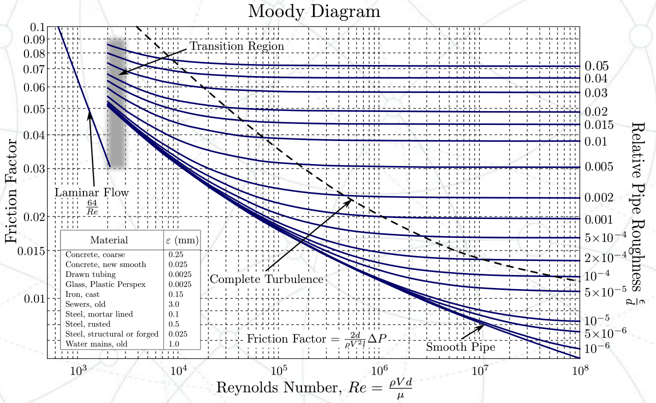
$$\frac{2}{\sqrt{f_{fd,h}}}=3.48-1.7373\ln\left(\frac{2e}{D_h}+\frac{2(9.35)}{\mathrm{Re}_{D_h}\sqrt{f_{fd,h}}}\right)$$

Zigrang and Sylvester 1982:

$$f_{fd,h}=\left\{-2\log_{10}\left[\frac{2e}{7.54D_h}-\frac{5.02}{\mathrm{Re}_{D_h}}\log_{10}\left(\frac{2e}{7.54D_h}+\frac{13}{\mathrm{Re}_{D_h}}\right)\right]\right\}^{-2}$$

Since the entry region is so short, the Average (apparent) friction factor and local value are almost equal:

$$\bar{f}\approx f_{fd,h}\left(1+\left(\frac{D_h}{L}\right)^{0.7}\right)$$



Nusselt Number: Laminar Flow- Quick Review

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Circular Tubes:

- For Laminar hydrodynamically and thermally fully developed

Constant surface heat flux: $Nu_{D_h,H,fd} = 4.36$

Constant surface temperature: $Nu_{D_h,T,fd} = 3.66$

Graetz number: $Gz = \dfrac{D_h \, \mathrm{Re}_{D_h} \, \mathrm{Pr}}{L}$

- For simultaneously developing flow (combined entry length problem) with constant wall temperature:

$$\overline{Nu}_{D_h,T} = 3.66 + \frac{[0.049 + 0.02/\mathrm{Pr}]Gz^{1.12}}{[1 + 0.065Gz^{0.7}]}$$

- For simultaneously developing flow (combined entry length problem) with uniform wall heat flux:

$$\overline{Nu}_{D_h,H} = 4.36 + \frac{[0.1156 + 0.08569/\mathrm{Pr}^{0.4}]Gz}{[1 + 0.1158Gz^{0.6}]}$$

*For thermal entry length problems (hydrodynamically FD & thermally developing): Use the Avg. Nu correlations with Pr approaching to infinity.

Rectangular Ducts

Constant surface temperature: $$Nu_{D_h,T,fd} = 7.541\left(1 - 2.61AR + 4.97AR^2 - 5.119AR^3 + 2.702AR^4 - 0.548AR^5\right)$$

Constant surface heat flux: $$Nu_{D_h,H,fd} = 7.541\left(1 - 2.042AR + 3.085AR^2 - 2.477AR^3 + 1.058AR^4 - 0.186AR^5\right)$$

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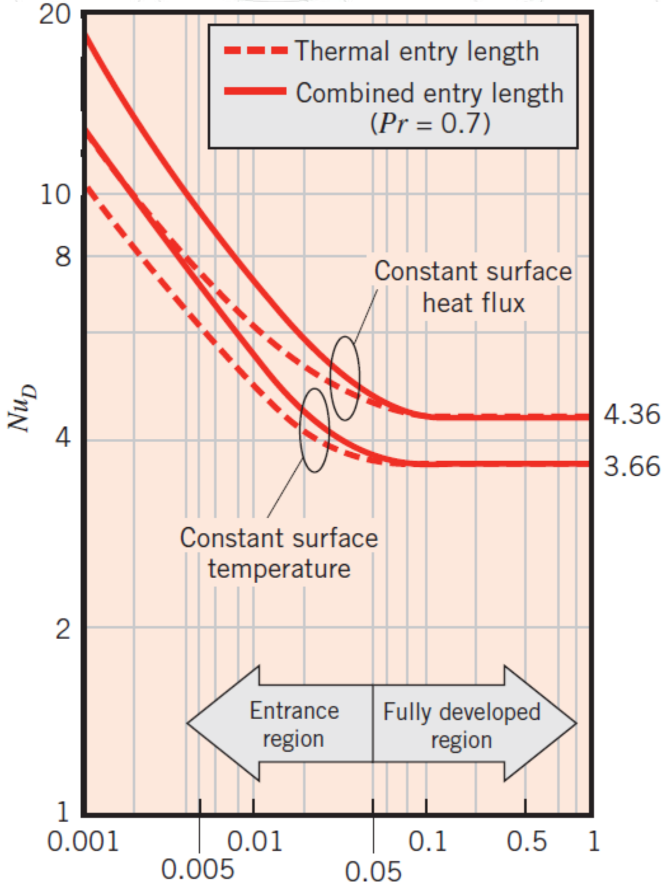
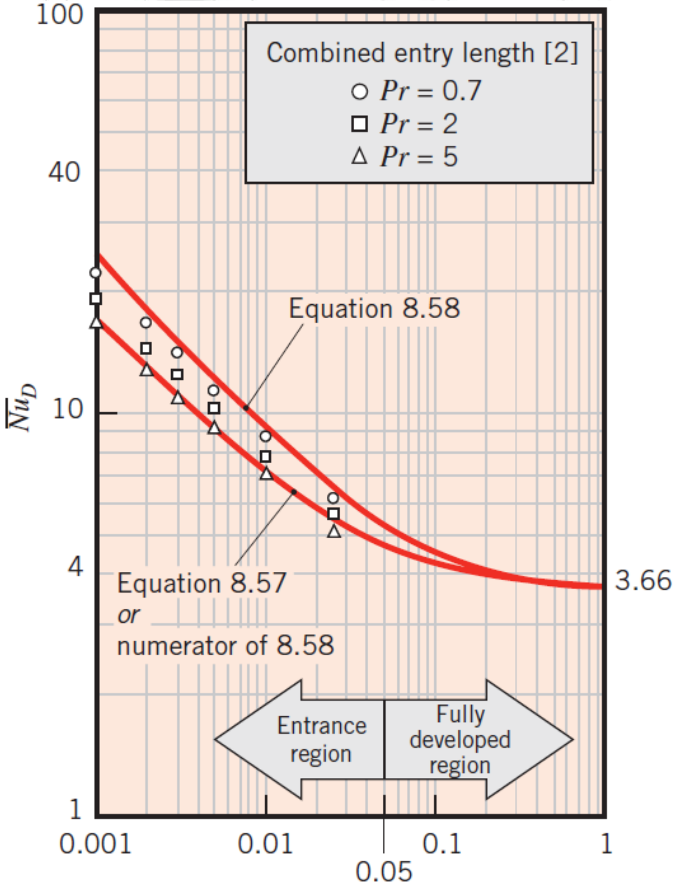
Nusselt Number-Laminar Flow- Quick Review

Laminar flow in a circular tube with constant surface temp



$$\frac{x/D}{Re_D Pr} = Gz_D^{-1}$$

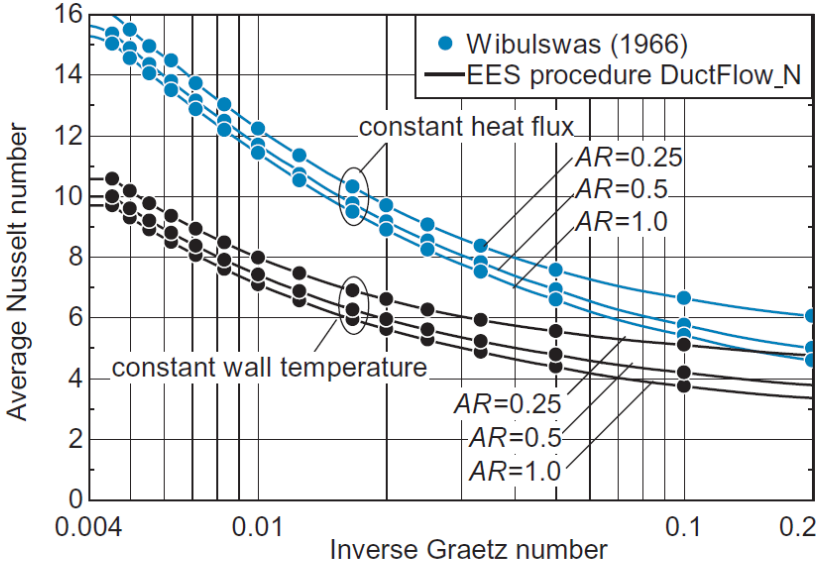
(a)



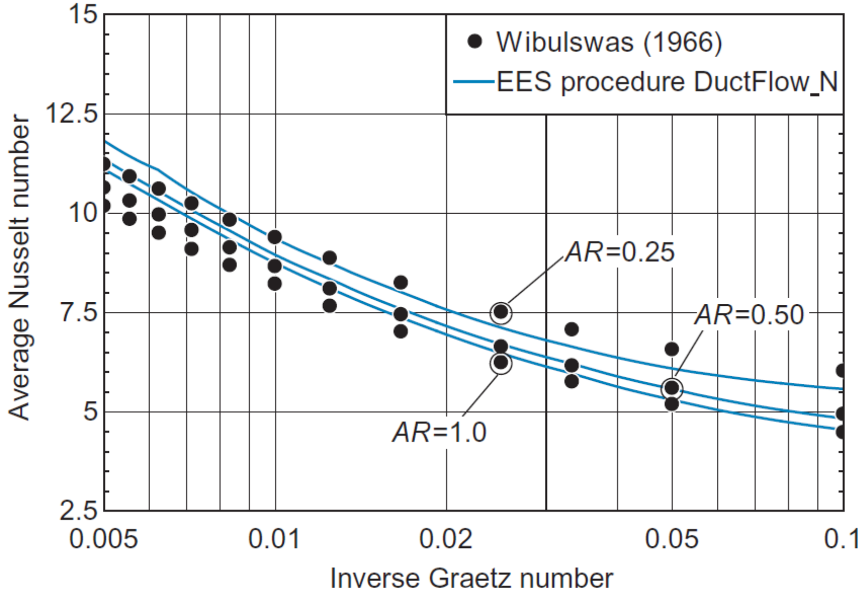
$$\frac{x/D}{Re_D Pr} = Gz_D^{-1}$$

(b)

Laminar flow in Rectangular ducts :



Simultaneously developing flow



Thermally developing flow (uniform surface heat flux)

* T. L. Bergman, A. S. Lavine, F. P. Incropera, D. P. DeWitt, Fundamentals of Heat and Mass Transfer, 8th Edition, Wiley (2017).

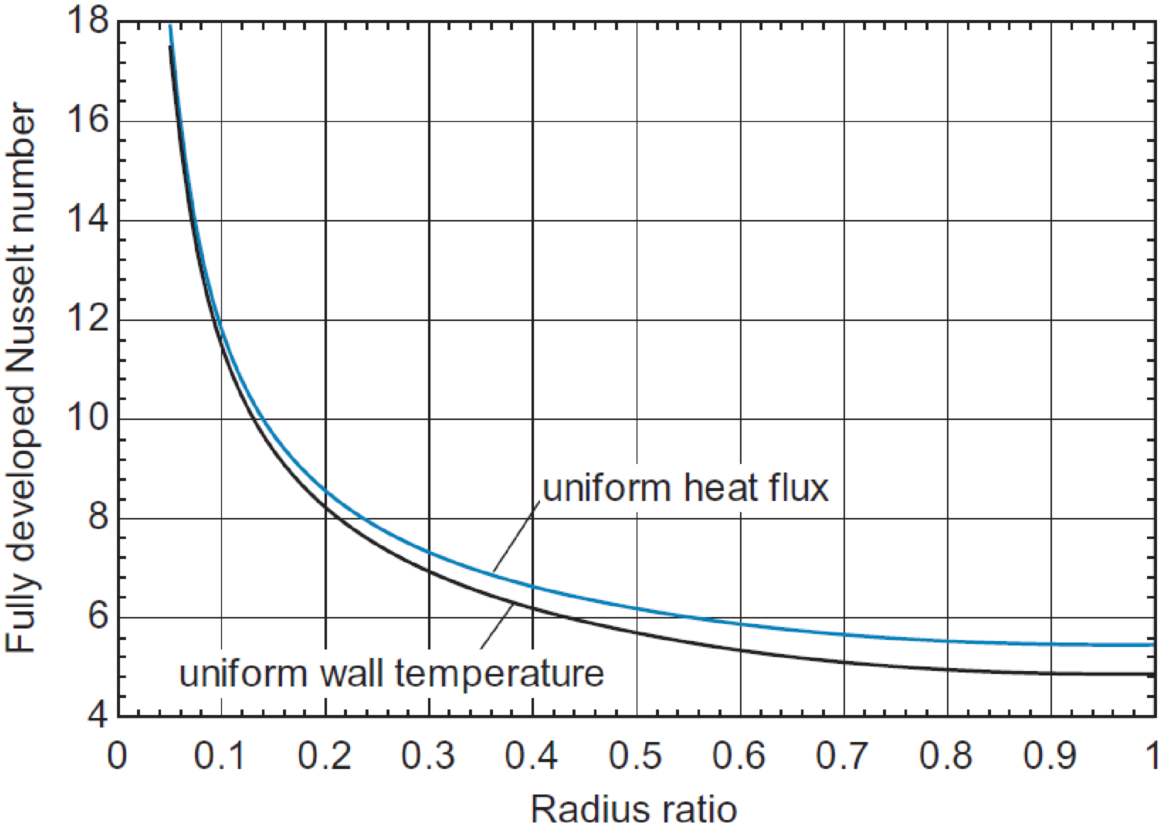
Dr. Hamidreza Najafi, Florida Institute of Technology, hnajafi@fit.edu

Nusselt Number-Laminar Flow-Quick Review

➢ **Nu and friction factor for fully developed Laminar flow in tubes of different cross sections:**

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q''_s)	(Uniform T_s)	$f\,Re_{D_h}$
(circle)	—	4.36	3.66	64
a, b	1.0	3.61	2.98	57
a, b	1.43	3.73	3.08	59
a, b	2.0	4.12	3.39	62
a, b	3.0	4.79	3.96	69
a, b	4.0	5.33	4.44	73
a, b	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated / Insulated	∞	5.39	4.86	96
(triangle)	—	3.11	2.49	53

➢ **Laminar flow in Annular ducts**



* G. Nellis and S. Klein, Heat Transfer, Cambridge University Press (2008).

* T. L. Bergman, A. S. Lavine, F. P. Incropera, D. P. DeWitt, Fundamentals of Heat and Mass Transfer, 8th Edition, Wiley (2017).

Nusselt Number-Quick Review

- **Turbulent flow**

  - Not affected by the shape of the cross section or the boundary condition

  - Sensitive to surface roughness

    - Gnieliski (1976):

$$Nu_{D_h,fd} = \frac{\left(\frac{f_{fd}}{8}\right)\left(\mathrm{Re}_{D_h} - 1000\right)\mathrm{Pr}}{1 + 12.7\left(\mathrm{Pr}^{2/3} - 1\right)\sqrt{f_{fd}/8}} \qquad 0.5<\mathrm{Pr}<2000 \quad \text{and } 2300<\mathrm{Re}_{D_h} < 5\times10^6$$

    - Kakac (1987): $\overline{Nu}_{D_h} \approx Nu_{D_h,fd}\left[1 + C\left(\frac{x}{D_h}\right)^{-m}\right]$ $\qquad C=1$ and $m=0.7$

    - For smooth tubes, Dittus-Boelter equation may be used: $Nu_{D_h,fd} = 0.023\mathrm{Re}_{D_h}^{4/5}\mathrm{Pr}^n$

$$\begin{cases} n = 0.4 & \text{if } T_s > T_m \quad \text{(heating)} \\ n = 0.3 & \text{if } T_s < T_m \quad \text{(cooling)} \end{cases} \qquad 0.6 \le \mathrm{Pr} \le 160 \quad \text{and} \quad \mathrm{Re}_{D_h} \ge 10000$$

$$L/D > 10$$

Examples

Convection Correlations: Internal Flows

Example 1

A method to generate electric power from solar irradiation involves concentrating sunlight onto absorber tubes that are placed at the focal points of parabolic reflectors. The absorber tubes carry a liquid *concentrator flid* that is heated as it flows through the tubes. After it leaves the concentrating field, the fluid enters a heat exchanger, where it transfers thermal energy to the *working flid* of a Rankine cycle. The cooled concentrator fluid is returned to the concentrator field after it exits the heat exchanger. A power plant consists of many concentrators.

The net effect of a single concentrator-tube arrangement *may be approximated* as one of creating a constant heating condition at the surface of the tube. Consider conditions for which a concentrated heat flux of $q_s'' = 20{,}000\ \mathrm{W/m^2}$, assumed to be uniform over the tube surface, heats a concentrator fluid of density, thermal conductivity, specific heat, and viscosity of $\rho = 700\ \mathrm{kg/m^3}$, $k = 0.078\ \mathrm{W/m \cdot K}$, $c_p = 2590\ \mathrm{J/kg \cdot K}$, and $\mu = 0.15 \times 10^{-3}\ \mathrm{N \cdot s/m^2}$, respectively. The tube diameter is $D = 70$ mm, and the mass flow rate of the fluid in a single concentrator tube is $\dot{m} = 2.5$ kg/s.

1. If the concentrator fluid enters each tube at $T_{m,i} = 400°\mathrm{C}$ and exits at $T_{m,o} = 450°\mathrm{C}$, what is the required concentrator length, L? How much heat q is transferred to the concentrator fluid in a single concentrator-tube arrangement?
2. What is the surface temperature of the tube at the exit of a concentrator, $T_s(L)$?

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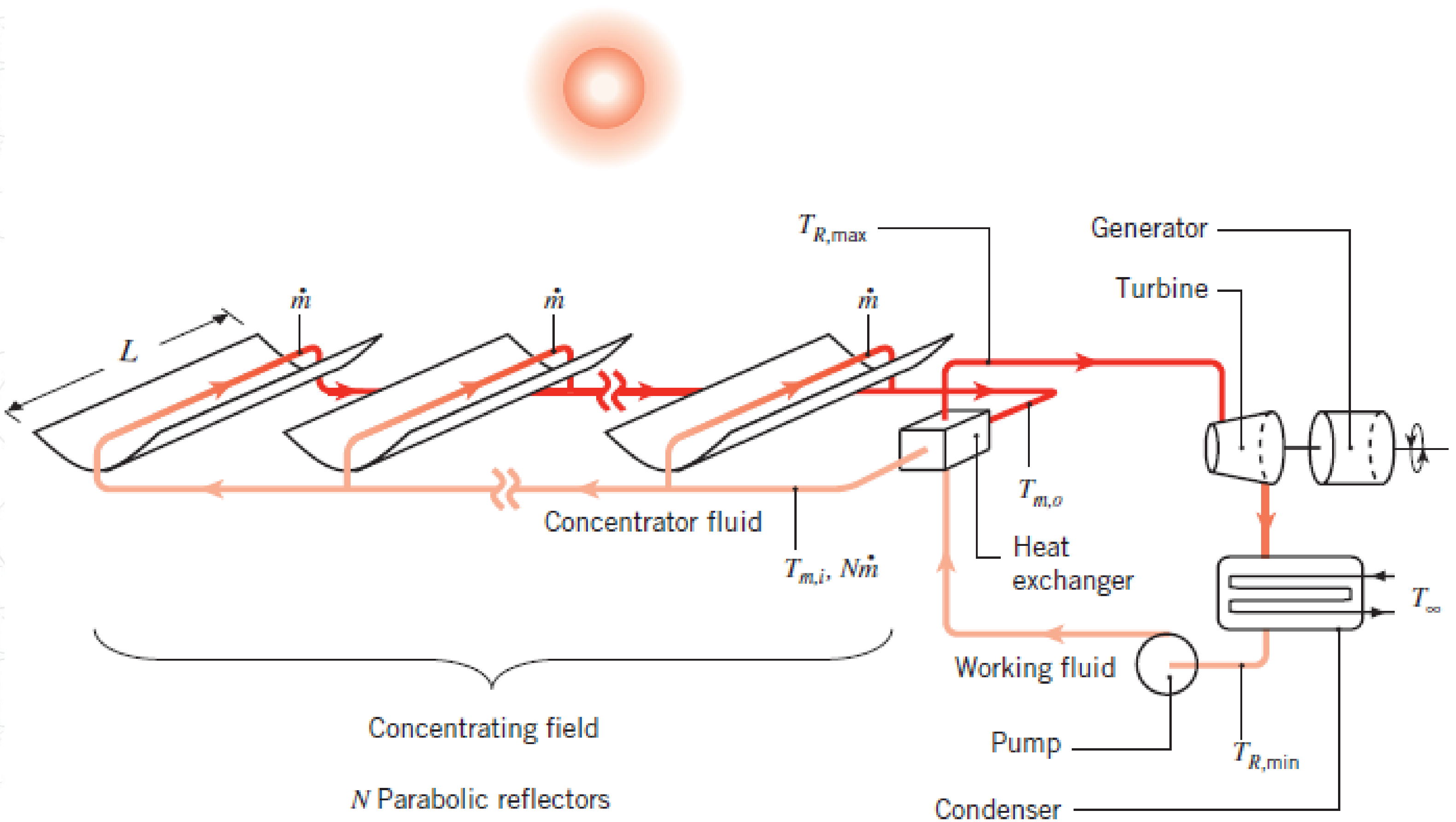
Convection Correlations: Internal Flows

Dr. Hamidreza Najafi, Florida Institute of Technology, hnajafi@fit.edu

Example 1



Convection Correlations: Internal Flows

Example 2

In the final stages of production, a pharmaceutical is sterilized by heating it from 25 to 75 C as it moves at 0.2 m/s through a straight thin-walled stainless steel tube of 12.7-mm diameter. A uniform heat flux is maintained by an electric resistance heater wrapped around the outer surface of the tube. If the tube is 10 m long, what is the required heat flux? If fluid enters the tube with a fully developed velocity profile and a uniform temperature profile, what is the surface temperature at the tube exit and at a distance of 0.5 m from the entrance? Fluid properties may be approximated as ρ=1000 kg/m3, cp=4000 J/kg K, μ= 2×10^{-3} kg/s m, k=0.8 W/m K, and Pr = 10.

Convection Correlations: Internal Flows

Dr. Hamidreza Najafi, Florida Institute of Technology, hnajafi@fit.edu

Example 3

Consider the flow of oil at 10°C in a 40-cm-diameter pipeline at an average velocity of 0.5 m/s. A 300-m-long section of the pipeline passes through icy waters of a lake at 0°C. Measurements indicate that the surface temperature of the pipe is very nearly 0°C. Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow oil in the pipe.