# 1. Introduction

## 1.1 Background and Motivation

## Economic growth and development are fundamental objectives for nations worldwide. A critical aspect of this growth is the concept of income convergence, which suggests that poorer economies' per capita incomes will tend to grow at faster rates than those of richer economies. Consequently, over time, all economies should converge in terms of per capita income, leading to reduced global income disparities (Barro & Sala-i-Martin, 1992).

## However, empirical evidence often contradicts the hypothesis of absolute convergence, indicating that factors beyond initial income levels significantly influence economic growth rates (Sala-i-Martin, 1996). This realization has shifted the focus toward conditional convergence, where economies converge to their own steady-state levels of per capita income, determined by country-specific factors such as savings rates, human capital, and technology levels (Mankiw, Romer, & Weil, 1992).

## Financial development emerges as a pivotal factor facilitating convergence. Well-developed financial systems enhance the efficiency of capital allocation, promote technological innovation, and facilitate risk management (Levine, 1997). By mobilizing savings and providing access to external finance, financial development enables firms and entrepreneurs in less developed countries to invest in productive projects, fostering higher economic growth rates and promoting convergence with advanced economies (Aghion, Howitt, & Mayer-Foulkes, 2005).

## In this context, introducing the concept of using Functional Data Analysis (FDA) in economic analysis offers a novel approach to understanding the dynamic relationship between financial development and income convergence. FDA allows for the analysis of data providing information about curves, surfaces, or anything else varying over a continuum (Ramsay & Silverman, 2005). Applying FDA can capture the continuous trajectories of financial development and economic growth over time, providing deeper insights into the convergence process.

## 1.2 Research Problem and Questions

## Despite extensive research, gaps remain in understanding how financial development specifically influences income convergence, particularly considering the heterogeneity across countries and over time. Existing studies often rely on cross-sectional analyses that may overlook dynamic effects and country-specific factors influencing the finance-growth nexus (Beck & Levine, 2004).

Research Problem:

The study aims to investigate the dynamic impact of financial development on income convergence among countries at different stages of development, utilizing Functional Data Analysis to capture the temporal evolution of this relationship.

Research Questions:

How does financial development influence income convergence among countries with varying initial conditions and structural characteristics?

In what ways can Functional Data Analysis enhance the understanding of the dynamic relationship between financial development and economic convergence?

## 1.3 Objectives of the Study

## The primary objectives of this study are:

## To analyze the impact of financial development on income convergence across countries, considering heterogeneity in institutional quality and economic structures.

## To employ Functional Data Analysis to model the dynamic trajectories of financial development and economic growth, capturing temporal patterns and transitions.

## To identify the mechanisms through which financial development facilitates or hinders convergence, focusing on technological adoption and innovation.

## 1.4 Significance of the Study

## This study contributes to the existing literature by addressing identified gaps:

## Theoretical Contribution: By integrating FDA into the analysis, the study offers a novel methodological approach to examining the finance-convergence relationship, capturing dynamic and continuous changes over time (Ramsay & Silverman, 2005).

## Empirical Contribution: Focusing on country-specific analyses and utilizing dynamic panel data methods addresses heterogeneity and temporal dynamics often overlooked in previous research (Arellano & Bover, 1995; Blundell & Bond, 1998).

## Policy Implications: Understanding the specific channels through which financial development affects convergence can inform policymakers in designing targeted financial policies and institutional reforms. Enhancing financial inclusion and developing robust financial systems can promote equitable growth and reduce global income disparities (Demirgüç-Kunt & Levine, 2008).

## Discuss potential policy implications.

# **The thesis is structured as follows:**

# **Chapter 1: Introduces the background, research problem, objectives, and significance of the study.**

# **Chapter 2: Provides a comprehensive literature review on income convergence theories, the role of financial development in economic growth, and gaps in previous research.**

# **Chapter 3: Outlines the methodology, including the application of Functional Data Analysis and dynamic panel data methods.**

# **Chapter 4: Presents the empirical analysis, results, and discussion of findings in relation to the research questions.**

# **Chapter 5: Concludes the study, highlighting key insights, policy recommendations, and suggestions for future research.**

**References**

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# 2. Literature Review

## 2.1 Theoretical Framework of Income Convergence

## The concept of convergence in economic growth refers to the hypothesis that poorer economies' per capita incomes will tend to grow at faster rates than richer economies. Thus, all economies should eventually converge in terms of per capita income. This concept bifurcates into two main theories: absolute convergence and conditional convergence.

## Absolute Convergence

## Absolute convergence posits that all economies will converge to the same steady-state level of per capita income regardless of their initial conditions or structural characteristics. This theory assumes that all countries have access to the same technology, preferences, and savings rates, leading to similar long-term growth paths. Under the neoclassical Solow-Swan growth model, diminishing returns to capital imply that poorer countries with less capital per worker should grow faster than richer countries (Sala-i-Martin, 1996).

## However, empirical evidence often contradicts the absolute convergence hypothesis. For instance, when examining a broad cross-section of countries, the expected negative relationship between initial income levels and subsequent growth rates is weak or nonexistent (Barro & Sala-i-Martin, 1992). This suggests that factors other than initial income levels play significant roles in determining economic growth rates.

## Conditional Convergence

## Conditional convergence, on the other hand, suggests that economies converge to their own steady-state levels of per capita income, which are determined by country-specific factors such as savings rates, population growth, human capital, and technology levels. When these factors are held constant, poorer countries are expected to grow faster than richer ones until they reach their unique steady states (Mankiw, Romer, & Weil, 1992).

## Sala-i-Martin (1996) provided extensive empirical analysis supporting the conditional convergence hypothesis. By incorporating additional variables that account for differences in countries' steady-state determinants, such as human capital and population growth, the negative relationship between initial income and growth becomes statistically significant. This implies that convergence is conditional upon countries sharing similar structural characteristics and policies.

## Moreover, the concept of "club convergence" has been introduced to explain why convergence might occur among groups of countries with similar characteristics but not globally. Countries within a "convergence club" share similar structural features, institutional frameworks, and levels of technological advancement, leading them to converge among themselves but not necessarily with countries outside the club (Galor, 1996).

## Mechanisms Through Which Financial Development Affects Convergence

## Financial development plays a pivotal role in influencing economic convergence by enhancing the efficiency of capital allocation, promoting technological innovation, and facilitating risk management. The mechanisms through which financial development affects convergence can be understood through both theoretical models and empirical evidence.

## Facilitation of Capital Accumulation and Allocation

## Financial systems mobilize savings and allocate capital to its most productive uses. In economies with well-developed financial markets, firms and entrepreneurs have better access to external finance, enabling them to invest in capital-intensive and high-yield projects (Levine, 1997). This efficient allocation of resources fosters higher economic growth rates in less developed countries, promoting convergence with more advanced economies.

## Greenwood and Jovanovic (1990) developed a model illustrating how financial intermediaries facilitate growth by pooling savings and directing them toward profitable investment opportunities. By reducing information asymmetries and transaction costs, financial institutions enable investors to fund projects that they otherwise could not, leading to increased capital accumulation and growth.

## Promotion of Technological Innovation and Diffusion

## Financial development affects convergence by influencing the rate of technological innovation and the adoption of existing technologies. Aghion, Howitt, and Mayer-Foulkes (2005) proposed that financial constraints can hinder a country's ability to innovate or adopt frontier technologies, slowing down the convergence process. In their model, economies with underdeveloped financial systems face barriers in funding research and development (R&D) activities, leading to persistent productivity gaps with technologically advanced countries.

## Empirical evidence supports this view. Industries that are more dependent on external finance tend to grow faster in countries with more developed financial systems (Rajan & Zingales, 1998). This suggests that financial development is crucial for industries that require significant upfront investment in innovation and technology adoption.

## Risk Diversification and Management

## Developed financial markets provide instruments and institutions that help in diversifying and managing risks associated with investment projects. By allowing for the pooling and sharing of risks, financial development encourages investment in higher-return but riskier projects, which can lead to faster economic growth (King & Levine, 1993). This risk mitigation is particularly important for entrepreneurs in developing countries who might otherwise be deterred from investing due to uncertainty.

## Enhancement of Human Capital Investment

## Financial development also facilitates investments in human capital by providing credit for education and training. Access to educational loans enables individuals from poorer backgrounds to acquire skills and knowledge, contributing to a more productive workforce (Galor & Zeira, 1993). This human capital accumulation is essential for technological adoption and innovation, further promoting convergence.

## Reduction of Poverty Traps and Income Inequality

## Underdeveloped financial systems can contribute to poverty traps by limiting access to credit for the poor, preventing them from investing in education or entrepreneurial activities (Banerjee & Newman, 1993). By improving financial inclusion, countries can reduce income inequality and promote broader-based economic growth, aiding the convergence process.

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## 2.2 Financial Development and Economic Growth

## Review of Seminal Papers Linking Financial Development to Growth

## The relationship between financial development and economic growth has been a central topic in economic research. Seminal papers have explored how financial systems contribute to growth by improving the allocation of resources, facilitating investment, and promoting technological innovation.

## Early Theoretical Foundations

## The foundational work by Schumpeter (1911) emphasized the role of financial intermediaries in fostering innovation and economic development. Schumpeter argued that banks play a crucial role in identifying and funding entrepreneurs who can implement new technologies, thus driving growth.

## Goldsmith (1969) provided empirical evidence on the positive correlation between financial development and economic growth across countries. He suggested that financial institutions mobilize savings and allocate them efficiently, leading to capital accumulation and growth.

## Financial Intermediation and Growth

## McKinnon (1973) and Shaw (1973) independently developed the "financial repression" hypothesis, arguing that government interventions in the financial sector, such as interest rate ceilings and high reserve requirements, hinder financial development and thus economic growth. They advocated for financial liberalization to enhance the efficiency of financial markets.

## King and Levine (1993a, 1993b) conducted empirical studies demonstrating that financial development is a strong predictor of long-term economic growth. They used cross-country regressions to show that countries with better-developed financial systems experience faster growth, capital accumulation, and productivity improvements. Their work emphasized the importance of financial depth, measured by indicators like the ratio of liquid liabilities to GDP.

## Endogenous Growth Models and Financial Markets

## Endogenous growth theories incorporated financial development into growth models. Bencivenga and Smith (1991) showed that financial intermediation affects growth by altering the composition of savings between productive capital and unproductive liquid assets. Financial markets reduce liquidity risks and encourage investment in productive assets, enhancing growth.

## Levine (1997) provided a comprehensive survey of the finance-growth nexus, highlighting mechanisms such as mobilizing savings, facilitating transactions, improving resource allocation, and promoting technological innovation. He emphasized the role of financial markets in overcoming information asymmetries and reducing transaction costs.

## Rajan and Zingales (1998) investigated the link between financial development and industry growth. They found that industries dependent on external finance grow disproportionately faster in countries with more developed financial markets. This suggests that financial development eases financing constraints, allowing firms to invest and expand.

## Aghion et al. (2005) and Their Methodology

## Aghion, Howitt, and Mayer-Foulkes (2005) extended the analysis of financial development and convergence using a Schumpeterian growth model. They focused on how financial development affects a country's ability to catch up technologically with leading economies.

## Schumpeterian Approach

## The Schumpeterian growth model, rooted in the work of Schumpeter (1911), emphasizes innovation as the engine of economic growth. In this framework, growth results from the process of creative destruction, where new technologies replace outdated ones. Financial markets are crucial as they provide the necessary funding for R&D and innovation activities (Aghion et al., 2005).

## Financial Development as an Incentive for Technological Transfer

## Aghion et al. (2005) argued that financial constraints impede a country's ability to innovate or adopt existing technologies. In countries with underdeveloped financial systems, firms struggle to obtain financing for innovation, leading to persistent technology gaps with frontier economies.

## Their model showed that financial development facilitates technological transfer by:

## Funding Innovation and Adoption: Financial markets provide capital for firms to invest in R&D and adopt advanced technologies. This reduces the technology gap between countries (Aghion et al., 2005).

## Reducing Risks Associated with Innovation: By diversifying risks, financial intermediaries encourage firms to undertake innovative projects that they might otherwise avoid due to uncertainty.

## Enhancing Human Capital Investment: Access to finance allows individuals and firms to invest in education and training, which are essential for absorbing and implementing new technologies.

## Methodology of Aghion et al. (2005)

## Aghion et al. (2005) employed both theoretical modeling and empirical analysis:

## Theoretical Model: They developed a Schumpeterian growth model where growth is driven by technological innovation. The model incorporated financial constraints that limit firms' ability to invest in innovation. Financial development alleviates these constraints, allowing for faster technological catch-up and growth.

## Empirical Analysis: They tested the model using cross-country regression analysis. The empirical strategy involved:

## Measuring financial development using indicators like private credit to GDP.

## Assessing technological gap by comparing countries' productivity levels to the technological frontier (e.g., the United States).

## Estimating growth equations to examine the interaction between financial development and the initial technological gap.

## Their findings indicated that financial development has a more significant impact on growth in countries that are closer to the technological frontier. In contrast, for countries far from the frontier, financial development alone is insufficient to spur growth without complementary factors like human capital.

## Detailed Discussion

## Aghion et al. (2005) highlighted the non-linear relationship between financial development and economic growth. They proposed that:

## In Advanced Economies: Financial development stimulates innovation by providing funds for cutting-edge R&D. Since these countries operate near the technological frontier, innovation is essential for growth.

## In Developing Economies: The primary growth driver is the adoption and imitation of existing technologies. While financial development helps, other factors like education and institutional quality are also critical.

## Their model suggested that financial development can lead to divergence if only advanced economies benefit from innovation financing, leaving behind countries with underdeveloped financial systems.

## Policy Implications

## The study implied that policies promoting financial development should be tailored to a country's stage of development. For developing countries, strengthening financial institutions must go hand-in-hand with investments in education and institutional reforms to maximize growth benefits.

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## 2.5 Gaps in the Literature

# Identifying Gaps in Previous Research

# Despite extensive research on the relationship between financial development and economic growth, several areas remain underexplored:

# Heterogeneity Across Countries: Much of the existing literature, including seminal works by King and Levine (1993) and Aghion et al. (2005), tends to use cross-country analyses that may overlook country-specific factors. There is a need for studies that account for heterogeneity in institutional quality, regulatory frameworks, and cultural factors that influence the effectiveness of financial development on growth.

# Dynamic Effects Over Time: Previous research often provides a static view of the finance-growth nexus. Limited attention has been paid to how the relationship evolves over different stages of economic development or during periods of financial crises and reforms (Beck & Levine, 2004).

# Role of Financial Inclusion: While financial development is generally measured using aggregate indicators like private credit to GDP, these metrics may not capture the extent of financial inclusion. The impact of access to financial services by underserved populations on growth and convergence remains insufficiently addressed (Demirgüç-Kunt & Levine, 2008).

# Microeconomic Channels: The specific micro-level mechanisms through which financial development affects firm behavior, innovation, and productivity are not fully understood. Studies focusing on firm-level data could provide deeper insights into these channels (Claessens & Laeven, 2005).

# Impact on Income Inequality and Poverty Reduction: Although some studies acknowledge that financial development can influence income distribution (Galor & Zeira, 1993), there is a gap in understanding how this, in turn, affects overall economic growth and convergence.

# Technological Adoption in Developing Countries: Aghion et al. (2005) emphasize financial development's role in technological innovation near the frontier but less is known about how it affects technological adoption in countries far from the frontier. The interplay between financial development, technology transfer, and human capital in these contexts requires further exploration.

# Positioning the Study to Fill These Gaps

# The proposed study aims to address these gaps by:

# Conducting Country-Specific Analyses: Focusing on individual countries or regions allows for a more nuanced understanding of how local factors influence the finance-growth relationship. This approach can account for institutional, cultural, and policy differences that affect financial development's impact on growth.

# Employing Dynamic Panel Data Methods: Utilizing methodologies like the System GMM estimator (Arellano & Bover, 1995; Blundell & Bond, 1998) enables the study to capture the dynamic effects of financial development over time, including during periods of economic upheaval or policy change.

# Measuring Financial Inclusion: Incorporating indicators of financial inclusion, such as the proportion of the population with access to banking services, allows the study to assess how inclusive financial development affects economic growth and convergence.

# Utilizing Firm-Level Data: By analyzing micro-level data from firms across different industries and countries, the study can investigate the specific channels through which financial development influences innovation, productivity, and growth (Beck, Demirgüç-Kunt, & Maksimovic, 2005).

# Exploring the Role of Income Distribution: Examining how financial development affects income inequality and how this, in turn, impacts economic growth can provide insights into the broader social implications of financial policies (Banerjee & Duflo, 2003).

# Focusing on Technological Adoption in Less Developed Countries: The study will investigate how financial development facilitates the adoption of existing technologies in countries far from the technological frontier. It will consider the roles of human capital and institutional quality in this process, building on and extending the framework of Aghion et al. (2005).

# Policy Analysis and Recommendations: By identifying the conditions under which financial development most effectively promotes growth and convergence, the study can offer policy recommendations tailored to countries at different development stages.

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# 3. Methodology

## 3.1 Data Description

### 3.1.1 Data Sources and Collection

* + - Describe datasets for GDP growth, GDP per capita, and Financial Development Index.
    - Explain the selection of the benchmark country (USA).

### 3.1.2 Variables and Measurements

* + - Define dependent and independent variables.
    - Discuss the calculation of differences from the benchmark.

### 3.1.3 Data Preprocessing

* + - Handling missing values and outliers.
    - Justification for data transformations.

## 3.2 Functional Data Analysis Framework

### 3.2.1 Introduction to FDA

* + - Explain the basics of FDA.
    - Advantages over traditional time series analysis.

### 3.2.2 Basis Functions and Smoothing

* + - Describe Fourier and B-spline basis functions.
    - Criteria for selecting the optimal number of basis functions (GCV method).

### 3.2.3 Registration of Functional Data

* + - Explain the need for curve registration.
    - Methods used for aligning curves.

## 3.3 Functional Regression Model

### 3.3.1 Model Specification

* + - Present the functional regression equation.
    - Incorporation of the interaction term between IPC and FDI.

### 3.3.2 Estimation Techniques

* + - Discuss the use of fRegress and pffr functions.
    - Handling of functional predictors and responses.

### 3.3.3 Interpretation of Coefficients

* + - Explain how to interpret functional beta coefficients.
    - Focus on the interaction term's significance.

## 3.4 Statistical Analysis and Inference

* + **3.4.1 Depth Analysis**
    - Describe depth measures used.
  + **3.4.2 Hypothesis Testing**
    - Outline the Wilcoxon tests for regions and income levels.

## 3.5 Principal Component Analysis (PCA)

* + Explain the application of PCA to functional data.
  + Interpretation of principal components.

### 3.6 Clustering Analysis

* + Methods for clustering countries based on interaction effects.
  + Justification for the number of clusters chosen.

### 3.7 Software and Tools

* + List R packages and functions used.
  + Any custom code or modifications made.

# 4. Results

* **4.1 Descriptive Statistics**

### 4.1.1 Summary Statistics

* + - Present tables for mean, median, variance, and standard deviation.

### 4.1.2 Visualizations

* + - Plot mean functions with confidence intervals.
    - Boxplots for GRW, IPC, and FDI.

## 4.2 FDA Outcomes

### 4.2.1 Smoothing and Registration Results

* + - Display the optimal number of basis functions selected.
    - Show plots before and after registration.

### 4.2.2 Functional Regression Findings

* + - Present estimated beta functions.
    - Discuss the significance of coefficients, especially the interaction term.
    - Interpretation in the context of convergence.

## 4.3 Statistical Tests Results

### 4.3.1 Depth Analysis

* + - Present depth measures and discuss patterns.

### 4.3.2 Wilcoxon Test Findings

* + - Report test statistics and p-values for regions and income levels.

## 4.4 PCA and Clustering Results

### 4.4.1 Principal Components

* + - Explain the variance explained by each component.
    - Show plots of the principal component functions.

### 4.4.2 Clustering Analysis

* + - Display cluster assignments.
    - Visualizations of clusters (e.g., scatter plots of PC scores).
    - Discuss characteristics of each cluster.

## 4.5 Visualization of Interaction Effects

### 4.5.1 Heatmaps and 3D Surface Plots

* + - Present heatmaps of beta surfaces.
    - Show covariance heatmaps and discuss implications.

### 4.5.2 Acceleration Analysis

* + - Plots of acceleration (second derivative) functions.
    - Interpretation in the economic context.

## 4.6 Discussion

### 4.6.1 Comparison with Previous Studies

* + - How do your findings align or contrast with Aghion et al. (2005)?

### 4.6.2 Implications of Findings

* + - What do the results suggest about the role of financial development in income convergence?

### 4.6.3 Policy Recommendations

* + - Based on the results, what policy actions could be recommended?

# 5. Conclusion

## 5.1 Summary of Key Findings

* + Recap the main results of the study.

## 5.2 Contributions to Literature

* + Highlight how your research adds to existing knowledge.

## 5.3 Limitations of the Study

* + Discuss any limitations in data, methodology, or scope.

## 5.4 Suggestions for Future Research

* + Propose areas where further investigation is needed.

# 6. References

* List all sources cited in the thesis following an appropriate citation style.

**Additional Sections (Optional):**

* **Appendices**
  + Include any supplementary material, such as code snippets, additional tables, or figures.
* **Acknowledgments**
  + Acknowledge individuals or institutions that supported your research.