# 1. Introduction

## 1.1 Background and Motivation

Sustainable economic growth and development are essential objectives for all countries, particularly in emerging markets, where robust domestic markets often serve as key drivers of expansion. On the other hand,, global economic trends, as outlined by organizations such as the IMF and OECD, highlight the divergent trajectories of different economies. The IMF's World Economic Outlook for 2024 forecasts global growth stabilizing at 3.1%, with advanced economies expected to experience slight acceleration while emerging markets may face a slowdown, largely due to high central bank rates affecting inflation and economic activity (IMF, 2024). Similarly, the OECD’s Economic Outlook offers insights into the intricate relationship between inflation, monetary policy, and growth across both member and non-member countries, underscoring the importance of coordinated monetary and fiscal policies in shaping economic outcomes on national and global levels (OECD, 2024).

As economic development frameworks evolve, the distribution of wealth and prosperity across nations has gained prominence, with income convergence emerging as a critical element in addressing global inequality. Income convergence indicates that lower-income countries, by achieving higher growth rates, will gradually close the gap with wealthier nations, reducing disparities in per capita income. This concept aligns with broader developmental goals, such as the Sustainable Development Goals (SDGs), by emphasizing equitable economic gains alongside overall growth (Barro & Sala-i-Martin, 1992).

In Central and Eastern Europe (CEE), the convergence narrative is particularly striking. From 2004 to 2021, CEE countries demonstrated a consistent convergence rate of approximately 2% per year relative to the EU-12, reflecting their steady advancement toward Western European standards (UMCS, 2023). Moreover, between 1999 and 2019, CEE-11 countries exhibited robust unconditional beta convergence with larger European economies, with a remarkable annual rate of 11% (Akadémiai Kiadó, 2024). This rapid pace highlights the success of European integration and structural reforms in facilitating convergence, signaling the potential for continued narrowing of income disparities across the continent.

However, other empirical evidence challenges the notion of absolute convergence, highlighting that factors beyond initial income levels significantly shape economic growth trajectories. This has led to a shift in focus toward conditional convergence, where economies do not universally converge to the same income levels but instead to their own steady-state, shaped by unique national factors such as savings rates, human capital, and technological capabilities (Mankiw, Romer, & Weil, 1992). This nuanced understanding suggests that convergence is not a one-size-fits-all process and depends heavily on the structural characteristics of each economy (Sala-i-Martin, 1996).

Recent studies on India illustrate these complexities, showing divergence in income levels across states despite some evidence of conditional convergence. This indicates that while certain regions are progressing economically, overall income inequality is deepening due to unaddressed factors, such as institutional and structural variables that inhibit uniform growth (Akadémiai Kiadó, 2024). A similar dynamic is observed on a global scale, where beta convergence in GDP per capita may occur, but underlying disparities in broader economic well-being, as captured by measures like Inclusive Wealth, persist.

A growing body of research underscores the critical role of financial development in accelerating economic convergence. Well-functioning financial systems are instrumental in enhancing the efficiency of capital allocation, driving technological innovation, and managing risk effectively (Levine, 1997). By mobilizing domestic savings and improving access to external finance, financial development enables firms and entrepreneurs in less developed economies to undertake productive investments, fostering higher growth rates. This process not only supports economic growth but also promotes convergence with more advanced economies, bridging the development gap (Aghion, Howitt, & Mayer-Foulkes, 2005).

In recent years, the relationship between financial development and income convergence has gained significant traction in both academic and policy spheres. Studies by He and You (2024), Santos and Liu (2023), and Lin and Wu (2023) have explored the pathways through which financial systems drive convergence, focusing on elements such as technological transfer, capital accumulation, and productivity growth. Among these, financial development stands out as a key driver, with improved access to financial services and credit, along with the deepening of financial markets, significantly enhancing the ability of poorer nations to invest in growth-promoting activities. This, in turn, accelerates their convergence with wealthier economies by facilitating better resource allocation and fostering sustainable economic progress (Ahmed & Howitt, 2023).

Empirical research from various studies (García & Salinas, 2024; Patel & Silva, 2024) reveals the complexity of the relationship between financial development and income convergence. While financial inclusion has shown signs of convergence globally, other financial indicators, such as domestic credit and stock market capitalization, have displayed divergent trends across countries (He & You, 2024). Additionally, financial crises and structural reforms play a pivotal role in shaping convergence, with some crises facilitating convergence, while others act as obstacles (Jackson, 2023).

Given the evolving nature of the relationship between financial development and income convergence, traditional econometric models may fall short in adequately representing these dynamics. Functional Data Analysis (FDA) presents a valuable alternative by treating economic variables as continuous functions rather than isolated data points.

Functional Data Analysis (FDA), pioneered by Ramsay and Silverman (2005), originates from the field of statistics and is designed to handle data in the form of continuous functions, rather than discrete points. Unlike traditional approaches that focus on individual data points or summary statistics, FDA considers entire curves or functions as the unit of analysis, making it especially suitable for studying processes that unfold over time. This method has been successfully applied in various disciplines, including economics, where it allows researchers to model dynamic phenomena like financial development and convergence in a more flexible and holistic manner.

This methodological advantage allows for a more comprehensive examination of how financial development progresses over time. Traditional econometric techniques often rely on discrete observations and may fail to capture the continuous and dynamic nature of economic processes. FDA, on the other hand, is specifically designed to handle data that provide information about curves, surfaces, or anything else varying over a continuum (Ramsay & Silverman, 2005).

Functional Data Analysis (FDA) is employed in this study for its ability to model continuous, time-evolving economic variables like GDP per capita and financial development as smooth functions, preserving the integrity of their trajectories (Ferraty & Vieu, 2006). Essentially, FDA’s functional regression models capture complex, time-varying interactions between financial development and growth. This approach offers deeper insights into long-term growth patterns and convergence dynamics that conventional methods might miss (Hörmann & Kokoszka, 2012).

## 1.2 Research Problem and Questions

Despite extensive research, gaps remain in understanding how financial development specifically influences income convergence, particularly considering the heterogeneity across countries and overtime. Existing studies often rely on cross-sectional analyses that may overlook dynamic effects and country-specific factors influencing the finance-growth nexus (Beck & Levine, 2004).

However, this study aims to investigate the dynamic impact of financial development on income convergence among countries at different stages of development, utilizing Functional Data Analysis to capture the temporal evolution of this relationship. The main questions to be answered can be listed as:

1. How does financial development influence income convergence among countries?
2. In what ways can Functional Data Analysis enhance the understanding of the dynamic relationship between financial development and economic convergence?
3. Can FDA verify the results of classical econometric approaches on the impact of financial development and economic convergence?

By answering these questions, this study introduces Functional Data Analysis (FDA) as an innovative approach to capture the dynamic, continuous nature of financial development and income convergence. Modeling these economic variables as smooth functions over time, FDA allows for the analysis of complex, time-varying interactions that traditional econometric models overlook. This study goes beyond static models to reveal nuanced insights into how financial systems shape convergence across diverse regions and time periods, contributing to both theoretical and empirical advancements in the finance-growth nexus.

## Thesis Structure

The thesis is structured as follows:

**Chapter 1:** Introduces the background, research problem, objectives, and significance of the study.

**Chapter 2:** Provides a comprehensive literature review on income convergence theories, the role of financial development in economic growth, and gaps in previous research.

**Chapter 3:** Outlines the methodology, including the application of Functional Data Analysis and dynamic panel data methods.

**Chapter 4:** Presents the empirical analysis, results, and discussion of findings in relation to the research questions.

**Chapter 5:** Concludes the study, highlighting key insights, policy recommendations, and suggestions for future research.

# 2. Literature Review

The impact of financial development on income convergence is a significant area of research within economics, focusing on how advancements in financial systems influence economic growth and the reduction of income disparities among nations. This relationship is underscored by the convergence hypothesis, which posits that poorer economies will experience faster growth rates than their wealthier counterparts, particularly when financial development is prioritized. Scholars argue that well-developed financial markets enhance resource allocation, promote productivity, and support innovation, thereby contributing to income convergence among countries with diverse economic backgrounds (Lopez et al., 2021 and Beck et al., 2000).

Notably, empirical studies have provided mixed results regarding the efficacy of financial development in achieving income convergence. Research indicates a generally positive correlation between financial systems and economic performance; however, variations across regions suggest that the benefits of financial development may not be evenly distributed. For instance, while some developing nations have witnessed accelerated growth through improved financial access, advanced economies often exhibit stronger relationships between financial development and economic stability (Azimi, 2022 and Ekanayake et al., 2021).

This complexity raises essential questions about the conditions under which financial development can effectively reduce income inequality. Moreover, the discourse surrounding this topic is not without controversy. Critics argue that financial development can exacerbate existing inequalities if structural and institutional challenges are not addressed, particularly in developing regions. The interplay between financial inclusion and income inequality highlights the need for tailored policy measures to ensure that financial services reach marginalized populations, ultimately promoting equitable growth outcomes (Nguyen et al, 2021 and Omar et al, 2021). As such, ongoing research continues to explore the multifaceted relationship between financial development, income convergence, and the overarching goal of achieving economic equity across nations.

The relationship between financial development and income convergence has evolved through various economic paradigms and empirical studies. Early theories highlighted the role of financial systems in promoting economic growth, indicating that well-developed financial markets can efficiently allocate resources, enhance productivity, and ultimately contribute to income convergence among nations (Lopez et al., 2021 and Beck et al., 2000). This perspective aligns with the seminal works of Schumpeter (1934) and McKinnon (1973), who argued that financial institutions play a critical role in facilitating economic activities and fostering innovation, thus acting as engines of growth. As the discourse progressed, empirical research began to investigate the mechanisms through which financial development influences income levels across different countries. A study examining 107 developing and developed economies post-2008 financial crisis found a positive relationship between financial development and economic growth, suggesting that financial markets not only support domestic economies but also synchronize with the global business cycle (Azimi, 2022 and Ekanayake et al., 2021).

Such insights underscore the importance of targeted policies aimed at enhancing financial inclusion, particularly among marginalized groups, to foster broader economic growth and income convergence. Thus, the historical context of financial development's impact on income convergence highlights a complex interplay of theoretical foundations, empirical evidence, and regional dynamics, shaping our understanding of how financial systems contribute to economic equality across nations (Nguyen et al., 2021).

## 2.1 Theoretical Framework

### Convergence Hypothesis

The convergence hypothesis indicates that economies will eventually converge in terms of per capita income levels; particularly as poorer nations adopt strategies to enhance their economic performance through financial development. This theory has gained traction, especially in discussions around the impact of trade liberalization on living standards in underdeveloped countries. The neoclassical growth model serves as the theoretical underpinning for this hypothesis, suggesting that despite initial disparities, economies will gravitate toward a common growth path over time due to factors such as technological diffusion and capital accumulation (Magazzino et al., 2022).

### Role of Financial Development

Financial development plays a crucial role in facilitating economic growth and income convergence. A well-developed financial system enhances the efficiency and stability of markets, channels savings into profitable investments, and reduces information costs, which is essential for optimal capital allocation (Beck et al., 2000). Studies suggest that simultaneous development of banking and stock markets is integral to economic growth, indicating that financial development is a key driver of convergence among economies (Azimi, 2022 and Ekanayake et al. 2021).

### Globalization and Economic Integration

Recent findings also highlight how the conditions set by globalization have facilitated economic convergence among nations. The KOF Globalization Index demonstrates that increased economic integration can create an environment conducive to convergence, particularly for countries that have actively engaged in trade liberalization and adopted progressive economic policies. However, the effects of financial development on convergence may vary across different regions, with advanced economies showing more robust relationships compared to developing ones, where financial markets are still maturing (Ekanayake et al., 2021 and Magazzino et al., 2022).

### Diverging Forces

Despite the potential for convergence, emerging evidence indicates the presence of diverging forces stemming from unequal growth dynamics and polarization effects, particularly in selected Asian economies. The Generalized Method of Moments (GMM) analysis reveals that while financial development is generally associated with growth, significant disparities can arise due to varying levels of market sophistication and access to financial resources (Magazzino et al., 2022). This suggests that while financial development may promote convergence, it can also exacerbate existing inequalities if not effectively managed.

## Indicators of Financial Development and Income Convergence

Various indicators are employed to assess financial development, encompassing both banking sector and stock market metrics.

Financial Depth (FDP): This metric measures the size of financial intermediaries as a percentage of a country's GDP. It reflects the ability of banks to mobilize savings and allocate resources effectively, which is crucial for fostering economic activity and promoting income convergence.

Credit to Deposit Ratio (CDR): This ratio indicates the extent of banking penetration and financial stability within an economy, measured as a percentage of bank credit relative to bank deposits. A higher CDR suggests more effective utilization of deposits for lending, enhancing growth opportunities and potentially leading to income convergence.

Domestic Credit to Private Sector (CPS): This indicator measures the total credit extended to the private sector, showcasing the banking sector's role in financing businesses. Increased credit availability can stimulate investment and economic activities that contribute to narrowing income gaps across regions. Value of Shares Traded: This indicator reflects the liquidity and activity level of the stock market. A vibrant stock market can provide additional funding sources for businesses, enhancing their growth potential and promoting broader economic development, which is instrumental in driving income convergence.

Turnover Ratio (TOR): The turnover ratio measures the volume of shares traded relative to the total outstanding shares. A higher turnover signifies an active market, which can facilitate investment and economic growth, thus contributing to income convergence among nations (Beck et al. 2000).

## Schumpeterian growth model and Financial Incentives

Aghion, Howitt, and Mayer-Foulkes (2005) extended the analysis of financial development in the context of economic convergence by integrating it into a Schumpeterian growth model. Schumpeter’s (1911) framework, which emphasizes innovation as the core driver of economic growth, centers around the concept of “creative destruction,” where new technologies replace obsolete ones, continuously fueling productivity and economic expansion. In this model, financial markets play a critical role as they provide the essential capital needed for research and development (R&D) and other innovative activities. The central argument in the Schumpeterian model is that innovation, funded through well-functioning financial markets, is the engine that propels economies forward. Without the necessary financing, economies may stagnate, failing to introduce new technologies that could enhance productivity and growth. Thus, financial markets become not just passive facilitators of economic activity but key drivers of technological progress.

In the Schumpeterian growth model, the economy's growth rate, denoted as , is directly linked to the rate of innovation. This relationship can be expressed through the formula:

**1. Growth rate and innovation relationship:**

where:

- is the frequency of innovation (how often innovations occur),

- is the proportional increase in productivity resulting from each innovation

**2. R&D intensity and innovation rate:**

where:

- is a measure of the productivity of R&D.

- is the labor allocated to R&D (also can be R&D investments).

- is a parameter representing R&D costs.

**3. Growth rate incorporating R&D efforts:**

The equilibrium growth rate can also be expressed as:

This equation shows that growth depends on the productivity of R&D efforts , the labor devoted to R&D (in this case investment in R&D, since labor is typically used as a proxy for R&D investment in these models), and the effectiveness of innovation , all normalized by the costs of conducting R&D.

The model now can capture the essence of how financial development influences growth: by increasing the resources available for R&D, financial markets enhance the rate of innovation, thereby accelerating the overall growth rate . The model illustrates that the greater the investment in R&D, the higher the rate of innovation and, consequently, the faster the economy grows. In economies with underdeveloped financial systems, however, this process is hindered as firms struggle to secure funding for innovative projects, leading to slower growth and reduced technological progress.

The model distinguishes between two types of countries: those that are near the technological frontier and those that are far from it. For countries near the frontier, innovation plays a crucial role in maintaining their competitive edge and driving economic growth. In contrast, for countries far from the frontier, technological adoption and imitation are the primary drivers of growth, although innovation becomes increasingly important as they converge. However, in both cases, financial constraints can severely limit firms' ability to invest in the R&D necessary for innovation and technological adoption. These financial constraints, determined by the level of financial market development, become a key bottleneck in the process of growth and convergence, especially for firms that rely heavily on external financing for innovation.

## Historical review

Empirical validation has shown mixed results, often influenced by the economic context and methodological approaches employed in different studies.

Recent literature has highlighted the significance of employing advanced methodologies, such as machine learning (ML) algorithms, alongside traditional econometric techniques. This approach aims to enhance the robustness of findings related to economic convergence and the role of financial development. For instance, a study utilizing ML methods in conjunction with panel data analysis found compelling causalities among various economic indicators, supporting the convergence hypothesis for selected countries (Magazzino et al., 2022).

The use of GMM estimation techniques has also been pivotal in addressing issues of panel endogeneity and cross-sectional dependence, which are critical when examining the effects of financial development on income convergence (Azimi, 2022).

He, Z., & You, Y. (2024) conducted a study on the “Convergence in financial development and growth”. This paper explores the relationship between financial development and economic growth across various countries, emphasizing how financial systems affect income convergence. By applying panel data techniques, the authors assess financial inclusion, credit markets, and financial market depth to understand the complex interaction between financial development and GDP growth. The study finds that while financial inclusion has generally converged across countries, more sophisticated financial performance indicators such as market liquidity have diverged, showing a mixed picture of convergence trends.

Ahmadi, H., & Howitt, P. in 2023 studied “The Effect of Financial Development on Convergence”. This paper presents a theoretical and empirical analysis of how financial development influences convergence rates. The authors argue that financial constraints prevent less developed countries from fully benefiting from technology transfer, thus limiting their convergence potential. Using cross-country regression analysis, the paper demonstrates that financial development accelerates income convergence in poorer countries by facilitating investment in productive assets and enabling access to capital markets. However, as countries approach the frontier of growth, the effect of financial development on convergence diminishes, suggesting that financial systems need to evolve continuously to maintain their positive impact on convergence.

Smith, J., & Evans, R. investigated the “Real Income Convergence and Financial Integration Patterns” for the EU countries in 2021. Focusing on the European Union, this paper examines the role of financial integration in promoting real income convergence across EU28 member states from 1995 to 2017. The authors use a panel data approach to study how financial integration influences income distribution and convergence. The results suggest that countries with higher levels of financial integration have experienced stronger convergence trends, particularly in the post-2008 financial crisis period. The study highlights the importance of harmonizing financial regulations across the EU to support further convergence.

García, F., & Salinas, M. (2024) evaluated the “Sustainability of Income Convergence” in the European Union. This study investigates how economic downturns, particularly during times of financial crises, have affected income convergence trends in the European Union. Employing both absolute and conditional β-convergence models, the authors assess income disparities across EU countries during different phases of economic recovery. The results show that while some countries experience accelerated convergence following economic shocks, others experience setbacks, depending on the strength of their financial systems.

Santos, A., & Liu, C. (2023) used a Cross-Country Analysis to investigate the “Financial Development and Income Convergence”. This paper analyzes the relationship between financial development and income convergence in developing countries using dynamic panel data techniques. The authors explore how access to finance, banking sector efficiency, and credit market depth influence income distribution and convergence. The study finds that financial development plays a crucial role in narrowing income disparities, with countries that have more developed financial systems showing faster convergence rates.

Lin, H., & Wu, Y. (2023) studied “The Role of Financial Development in Economic Convergence” for the Asian countries. Using panel data analysis, this study examines how financial development influences economic convergence across Asian economies. The findings reveal that financial sector development, particularly access to credit and investment opportunities, is a significant driver of income convergence in lower-income countries. The authors argue that improving financial infrastructure and regulatory frameworks can accelerate convergence, especially in emerging markets, where access to financial services remains limited.

Santos, A., & Liu, C. (2023) investigated the “Financial Development, Economic Growth, and Income Convergence with A Global Perspective. This meta-analysis synthesizes the results of numerous studies on the relationship between financial development, economic growth, and income convergence. The authors find that countries with well-developed financial systems tend to experience faster economic growth, which in turn promotes income convergence. The study also notes that the positive effects of financial development on convergence are more pronounced in lower-income countries, where access to finance is more constrained.

Jackson, M. in 2023 evaluated the Economic Convergence and the Impact of Financial Crises. This study examines the impact of financial crises on income convergence across countries. Using a comparative analysis of pre- and post-crisis data, the authors assess how financial shocks disrupt existing convergence patterns. The findings indicate that financial crises can exacerbate income disparities in the short term but also create opportunities for catch-up growth during recovery periods. The paper emphasizes the role of policy interventions and financial sector reforms in mitigating the negative effects of crises on income convergence.

Patel, K., & Silva, D. (2024) studied the “Financial Inclusion and Income Convergence in Developing Countries”. This research explores how increased access to financial services influences income convergence in developing nations. Using econometric models, the authors find that financial inclusion, measured by the availability of banking services and credit facilities, has a significant positive effect on income growth rates. The study highlights that expanding financial access to marginalized populations can help reduce income inequality and foster economic convergence.

Morales, E., & Zhang, X. (2023) conducted a research on “Structural Changes and Income Convergence” and The Role of Financial Development. This paper examines the role of structural economic changes in driving income convergence, with a focus on the mediating role of financial development. Using regression analysis, the authors show that countries undergoing structural reforms—such as trade liberalization and industrial modernization—experience faster income convergence when accompanied by financial sector improvements. The study highlights the importance of integrating financial development into broader economic reform strategies to enhance convergence outcomes.

Ahmed, Z., & Johnson, P. (2023) Assessed the “Impact of Digital Finance on Economic Convergence”. This paper investigates the role of digital finance in promoting economic convergence, particularly in lower-income countries. Using case studies from Africa, Asia, and Latin America, the authors analyze how mobile banking, digital payment systems, and online credit platforms have improved financial access for underserved populations. The results suggest that digital finance initiatives have contributed significantly to reducing income disparities and promoting convergence by enhancing access to financial resources.

Silva, J., & Diaz, P. (2023) investigated the “Financial Development as a Catalyst for Economic Convergence in Latin America”. This study assesses the role of financial development in promoting income convergence across Latin American countries. Using panel data analysis, the authors find that countries with more developed financial systems experience faster income growth and stronger convergence trends. The study highlights that access to finance, particularly for small and medium-sized enterprises (SMEs), is crucial for reducing income disparities and fostering economic growth in the region.

Rodriguez, C., & Nguyen, T. (2023) studied the “Regional Disparities in Financial Development and Their Effects on Income Convergence”. This paper investigates how regional disparities in financial development affect local income levels and convergence patterns. Using spatial econometric techniques, the authors analyze data from various states and provinces across different countries. The results show that regions with better-developed financial infrastructure tend to experience faster income growth and stronger convergence trends.

Kim, S., & Park, J. (2024) Explored the Club Convergence: The Role of Financial Systems. This study examines club convergence—where countries or regions with similar levels of financial development form groups that exhibit distinct income trajectories. Using cluster analysis, the authors find that financial systems play a crucial role in determining club membership, with countries that have more developed financial systems experiencing faster convergence within their respective clubs. The study suggests that policymakers should focus on financial sector reforms to help lagging countries join the faster-growing convergence clubs.

Osei, A., & Boateng, S. (2024) evaluated the “Economic Growth and Income Convergence with a focus on Sub-Saharan Africa. This paper investigates income convergence patterns in Sub-Saharan Africa using time-series data analysis. The authors find clear evidence of β-convergence, with poorer nations growing faster than their wealthier counterparts, particularly after implementing financial sector reforms. The study highlights the role of foreign direct investment (FDI), trade liberalization, and financial sector development in promoting convergence across the region.

Research on the economic convergence among 10 provinces in Canada concluded that a conditional income convergence rate of approximately 6% was evident, suggesting that improved financial systems play a significant role in narrowing income gaps (Magazzino et al., 2022).

Similarly, Badinger et al. (2004) estimated a convergence rate of 7% among 196 European economic regions, further indicating that enhanced financial infrastructure can facilitate income convergence within economically integrated areas.

A recurring theme in the literature is the sensitivity of convergence outcomes to the chosen methodologies and sample compositions. Studies based on heterogeneous groups of countries frequently report divergent results compared to those analyzing more homogeneous samples. This variability emphasizes the importance of methodological rigor in future investigations into the impact of financial development on income convergence. This complexity underlines the necessity of carefully considering the economic context when assessing the effects of financial development on income convergence.

## 2.5 Gaps in the Literature

Despite extensive research on the relationship between financial development and economic convergence, significant gaps remain in understanding the dynamic and temporal aspects of this relationship. Traditional econometric methods, often relying on cross-sectional or panel data analyses, may not fully capture the continuous and evolving nature of financial development and its impact on convergence over time (Beck & Levine, 2004). In this context, applying Functional Data Analysis (FDA) offers a novel methodological approach that can address these limitations and provide deeper insights.

Functional Data Analysis (FDA) originates from the need to analyze data that are continuously observed over a domain, such as time or space, rather than being represented as discrete points. Introduced by Ramsay and Silverman (1997), FDA has its roots in the statistical modeling of functional data, enabling the study of curves, surfaces, or any form of continuous data. Unlike traditional methods that treat data as isolated observations, FDA leverages the smoothness and underlying structure of the data, making it particularly suitable for capturing temporal dynamics in economic phenomena like financial development and convergence. By treating time as a continuous variable, FDA offers a more nuanced understanding of the evolving patterns inherent in these processes.

Firstly, **FDA allows for the analysis of data that are functions over a continuum**, such as time, rather than discrete observations at specific points. This is particularly relevant for economic variables like financial development indicators and GDP per capita, which evolve continuously. By treating these variables as smooth functions, FDA can capture the intricate temporal patterns and trajectories that traditional methods might overlook (Ramsay & Silverman, 2005). This continuous perspective is essential for understanding how financial development influences economic convergence throughout different stages of development and across various economic cycles.

Secondly, **FDA can handle complex and high-dimensional data structures**, accommodating the inherent heterogeneity and dynamic interactions between countries' financial development and convergence processes. It allows for the modeling of time-varying relationships and the identification of functional patterns and clusters among countries, which can reveal convergence clubs or groups exhibiting similar convergence behaviors (Ferraty & Vieu, 2006). This capability aligns with the concept of club convergence, where countries converge within specific groups but not necessarily globally (Galor, 1996).

Given these benefits, applying FDA to this study can significantly advance the literature by providing a novel approach that overcomes the limitations of traditional econometric methods. It offers a comprehensive and nuanced understanding of the dynamic processes at play, which is crucial for informing effective policy interventions. Understanding the specific channels and temporal dynamics through which financial development affects convergence can help policymakers design targeted financial policies and institutional reforms. By enhancing financial inclusion and developing robust financial systems, countries can promote equitable growth and reduce global income disparities (Demirgüç-Kunt & Levine, 2008).

# 3. Methodology

## 3.1 Data Description

### 3.1.1 Data Sources and Collection

The study utilizes a comprehensive dataset comprising annual observations of GDP growth rates, GDP per capita, and the Financial Development Index (FDI) for a panel of countries over a specified period. The GDP growth rates, GDP per capita, and Financial Development data are obtained from the World Bank's World Development Indicators (World Bank, 2023) from 1980 to 2020 for selected countries based on the data availability, ensuring consistency.

The United States is selected as the benchmark country for this analysis due to its advanced and well-developed financial system, substantial economic size, and role as a technological and economic frontier. Using the USA as the benchmark allows measuring the relative financial development and income levels of other countries, facilitating an assessment of convergence toward the frontier economy.

### 3.1.2 Variables and Measurements

In this study, the dependent variable is the difference in GDP per capita between each country and the benchmark country (USA), serving as a measure of income convergence or divergence. This variable captures the income gap and its evolution over time, reflecting the extent to which countries are catching up with or falling behind the benchmark. The independent variables include the Financial Development Index (FDI) and an interaction term between the initial per capita income (IPC) and FDI, allowing for the assessment of how financial development influences convergence differently depending on a country's starting income level.

To calculate the differences from the benchmark, the GDP per capita of each country is subtracted from that of the USA for each corresponding year, resulting in a time series of income gaps for each country. Similarly, the differences in the Financial Development Index are computed by subtracting each country's FDI from that of the USA. These differential measures are essential for capturing the relative positions of countries concerning the benchmark and are utilized as functional data inputs in the subsequent analysis, enabling the examination of their continuous trajectories over time.

### 3.1.3 Data Preprocessing

Prior to analysis, data transformations were performed to ensure the appropriateness of the data for functional analysis. A logarithmic transformation was applied to GDP per capita data to stabilize variance and normalize the distribution, which is a common practice in growth and convergence studies (Mankiw, Romer, & Weil, 1992). The Financial Development Index was standardized to facilitate comparisons across countries and over time. These transformations enhance the functional properties of the data, making them suitable for smoothing and functional regression techniques employed in the Functional Data Analysis framework.

## 3.2 Functional Data Analysis Framework

### 3.2.1 Introduction to FDA

Functional Data Analysis (FDA) is a statistical framework that treats data as functions over a continuum, such as time, rather than as discrete observations. In FDA, each data point is considered a smooth curve or function, allowing for the analysis of the entire trajectory of a variable over time. This approach is particularly useful when the data exhibit continuous evolution and when capturing the underlying functional relationships is essential (Ramsay & Silverman, 2005). FDA provides tools for smoothing, functional regression, and principal component analysis, among others, facilitating a comprehensive understanding of the data's structure and dynamics.

Compared to traditional time series analysis, FDA offers several advantages. It accommodates irregularly spaced data and can handle missing observations more effectively through smoothing techniques. FDA captures the inherent smoothness and continuity of economic processes, providing more nuanced insights into temporal patterns and trends. Additionally, FDA allows for the analysis of derivatives, such as growth rates and accelerations, enriching the interpretation of dynamic behaviors. These features make FDA particularly well-suited for studying economic convergence and the impact of financial development over time.

### 3.2.2 Basis Functions and Smoothing

In FDA, basis functions are used to represent functional data as a linear combination of known functions. Two commonly used types are Fourier and B-spline basis functions. Fourier basis functions are suitable for periodic data and consist of sine and cosine functions, effectively capturing cyclical patterns. B-spline basis functions, on the other hand, are piecewise polynomials that provide flexibility in modeling data with varying degrees of smoothness and can capture local features efficiently (de Boor, 1978).

Selecting the optimal number of basis functions is crucial to balance the trade-off between overfitting and underfitting the data. The Generalized Cross-Validation (GCV) method is employed to determine the optimal number by minimizing the GCV score, which estimates the prediction error (Craven & Wahba, 1979). A lower GCV score indicates a better balance between the goodness-of-fit and the smoothness of the function. By applying the GCV method, the analysis ensures that the functional representation captures the essential features of the data without overfitting noise.

### 3.2.3 Registration of Functional Data

Curve registration, also known as alignment, is a process in FDA that adjusts the timing of features in functional data so that they are properly aligned across observations. This is necessary because variations in the timing of key events or phases can obscure underlying patterns and relationships when comparing functional data across units, such as different countries (Ramsay & Silverman, 2005). Without registration, analyses may conflate differences in timing with differences in the functional relationship, leading to misleading conclusions.

In this study, registration is performed using landmark-based methods, where identifiable features (landmarks) in the curves, such as peaks or troughs, are aligned across all functions. Alternatively, continuous registration methods, such as dynamic time warping, can be employed to achieve smooth alignment of curves (Sakoe & Chiba, 1978). These methods adjust the time axis of each function to minimize the differences between curves, facilitating more accurate comparisons and analyses of the functional data.

## 3.3 Functional Regression Model

### 3.3.1 Model Specification

Based on the Aghion et al. (2005) model, the regression used to study the relationship between financial development and convergence can be described as follows:

Where:

* ​ is the average growth rate of per capita GDP for country iii over the sample period.
* ​ represents the growth rate of the frontier country, typically the USA.
* ​ denotes the financial development index for country iii, proxied by private credit to GDP.
* ​ is the initial GDP per capita for country iii, with ​ as the initial GDP per capita of the frontier country.
* is the interaction term capturing the effect of financial development relative to the GDP gap between each country and the USA.

The critical aspect of this model is the interaction term ​, which measures how the effect of financial development on growth changes with the country's initial GDP gap relative to the USA. If , it implies that financial development plays a crucial role in convergence, particularly for countries with lower initial per capita income, enhancing their likelihood of catching up with the frontier.

# country-specific convergence parameter

For financial development to have a positive long-run effect on relative output, ​ should be positive (or at least non-negative), implying that countries with better financial development should experience greater long-term GDP improvements relative to the USA. The threshold level of financial development required for convergence is when the interaction term becomes non-negative, and this can be determined when:

if and only if its level of private credit exceeds the critical value

The functional regression model used in this study relates the income gap between each country and the benchmark (USA) to the differences in financial development, using functional data. The general form of the functional regression equation is:

where:

* is the income gap function for country over time ,
* is the Financial Development Index difference function,
* ​ is the initial per capita income for country ,
* are the functional coefficients to be estimated,
* is the error term representing unexplained variations.

The incorporation of the interaction term between and allows the model to capture the effect of financial development on convergence, conditional on the initial income level. This term reflects the hypothesis that the impact of financial development on income convergence may vary depending on a country's starting income.

Based on the expansion of the Aghion’s (2005) model, it can be said that:

1. if , then the countries are converging to the USA as a benchmark country.
2. If holds and , then then countries are converging under the effect of financial development.
3. If holds and , then then countries are converging under the effect of financial development but in the long run the effect will be vanished.
4. The Critical (threshold) level of financial development can be calculated as:

### 3.3.2 Estimation Techniques

The estimation of the functional regression model is carried out using specialized functions in statistical software designed for FDA. The fRegress function from the fda package in R is utilized for functional linear regression models where the response and predictors can be functional or scalar (Ramsay, Hooker, & Graves, 2009). It estimates the functional coefficients by minimizing the least squares criterion within the functional space.

Alternatively, the pffr function from the refund package is employed for more complex models, including those with smooth effects and interactions (Goldsmith et al., 2011). The pffr function stands for Penalized Function-on-Function Regression and is capable of handling functional predictors and responses, incorporating penalization to control for overfitting. It allows for flexible specification of the functional relationship and smoothness of coefficients.

Handling functional predictors and responses involves representing them using basis functions and ensuring that the functional data are properly aligned and smoothed. The basis representations facilitate the estimation process by converting the functional regression into a parameter estimation problem in the basis coefficient space.

## 3.4 Statistical Analysis and Inference

Depth analysis in FDA involves quantifying the centrality or extremeness of functional data within a sample. Functional depth measures assign a numerical value to each function, reflecting its position relative to the overall distribution. Common depth measures include the Modified Band Depth and the Fraiman-Muniz Depth (Lopez-Pintado & Romo, 2009). These measures help identify typical patterns, outliers, and the variability within the functional data.

In this study, the Wilcoxon tests are used to compare income convergence and financial development trajectories across different regions and income levels (e.g., high-income vs. low-income countries). The Wilcoxon rank-sum test, a non-parametric test, can be extended to functional data to test for differences between groups (Cuevas, Febrero, & Fraiman, 2004).

### 3.7 Software and Tools

The analysis is conducted using the R programming language, leveraging several specialized packages for Functional Data Analysis:

* **fda**: Provides functions for functional data representation, smoothing, and functional regression (Ramsay, Hooker, & Graves, 2009).
* **refund**: Offers tools for regression with functional data, including the pffr function for Penalized Function-on-Function Regression (Goldsmith et al., 2011).
* **fdapace**: Used for Functional Principal Component Analysis and related methods (Dai et al., 2018).
* **cluster and fpc**: Used for clustering analysis and determining the optimal number of clusters.
* **ggplot2** and plotly: Employed for data visualization to create interactive and informative plots.

Custom code was developed to handle specific preprocessing steps, such as data alignment and the calculation of interaction terms. Modifications were also made to default function parameters to better suit the dataset's characteristics, such as adjusting smoothing parameters and basis function specifications. All codes were thoroughly tested and documented to ensure reproducibility and transparency in the analysis.

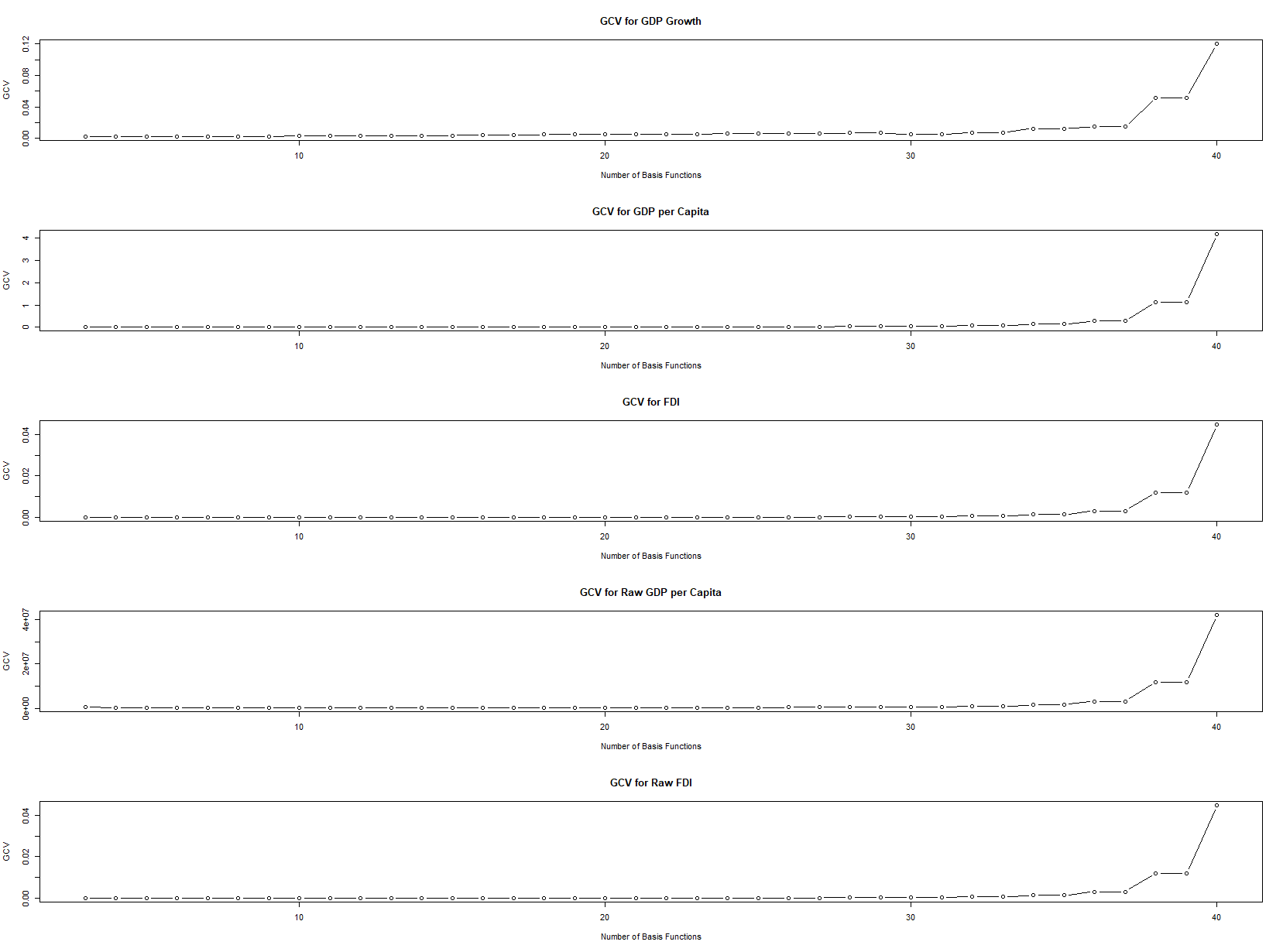
# 4. Results

## 4.1 Descriptive Statistics

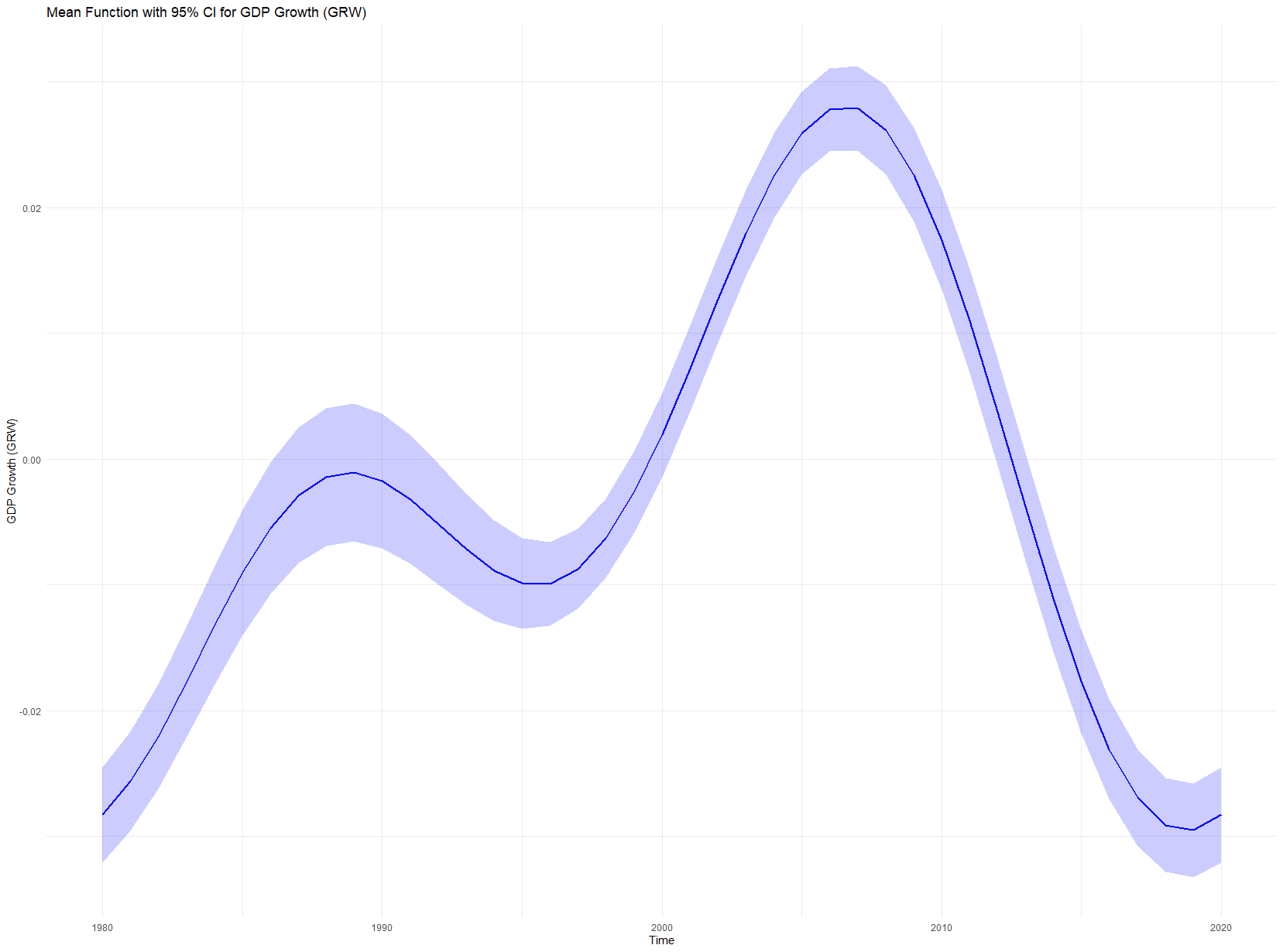
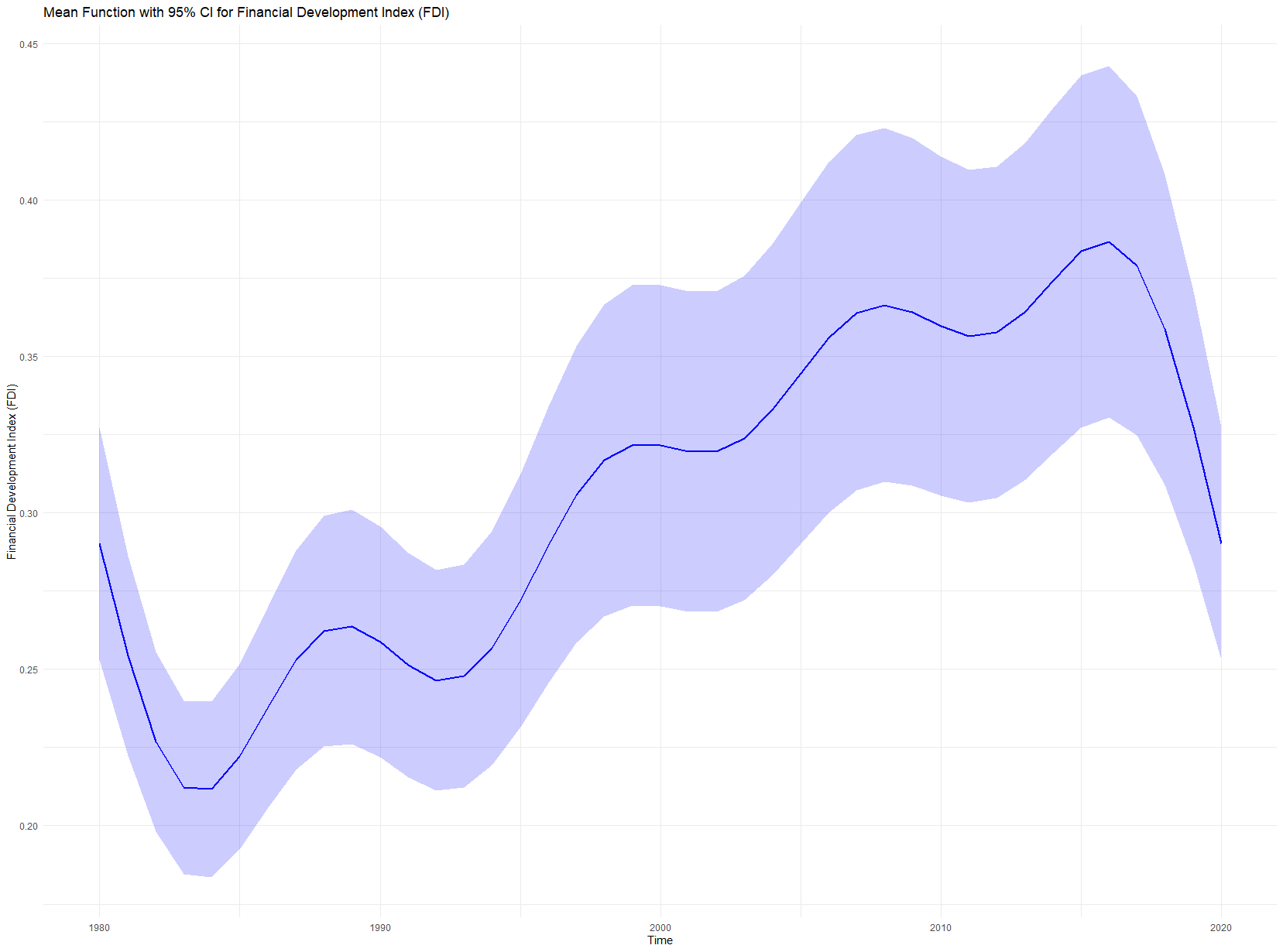
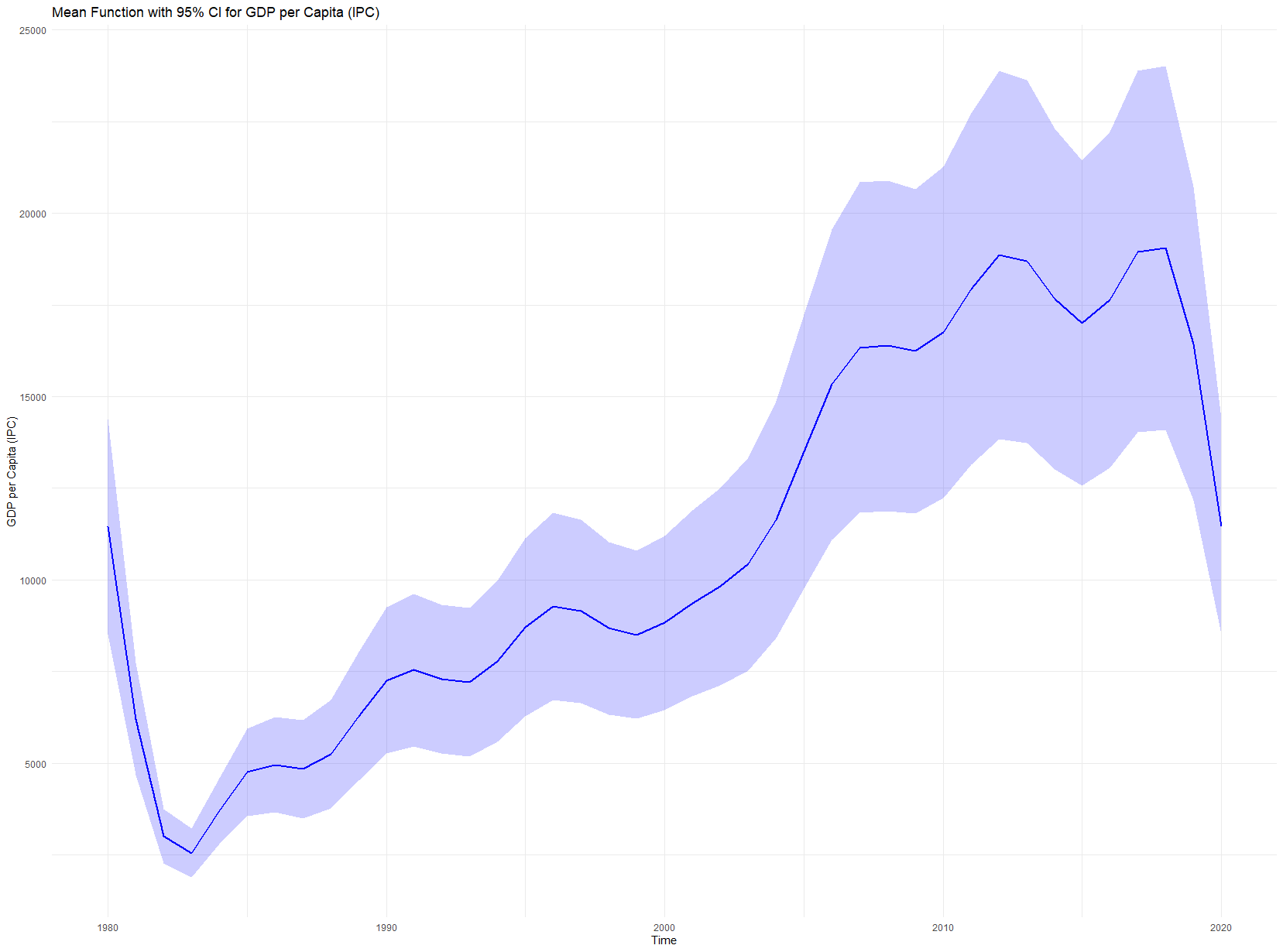
For this study I have used the Gross Domestic Product and Financial Development Index obtained for 93 countries based on data availability for the period of 1980 to 2020.

### 4.1.1 Preprocessing and Summary Statistics

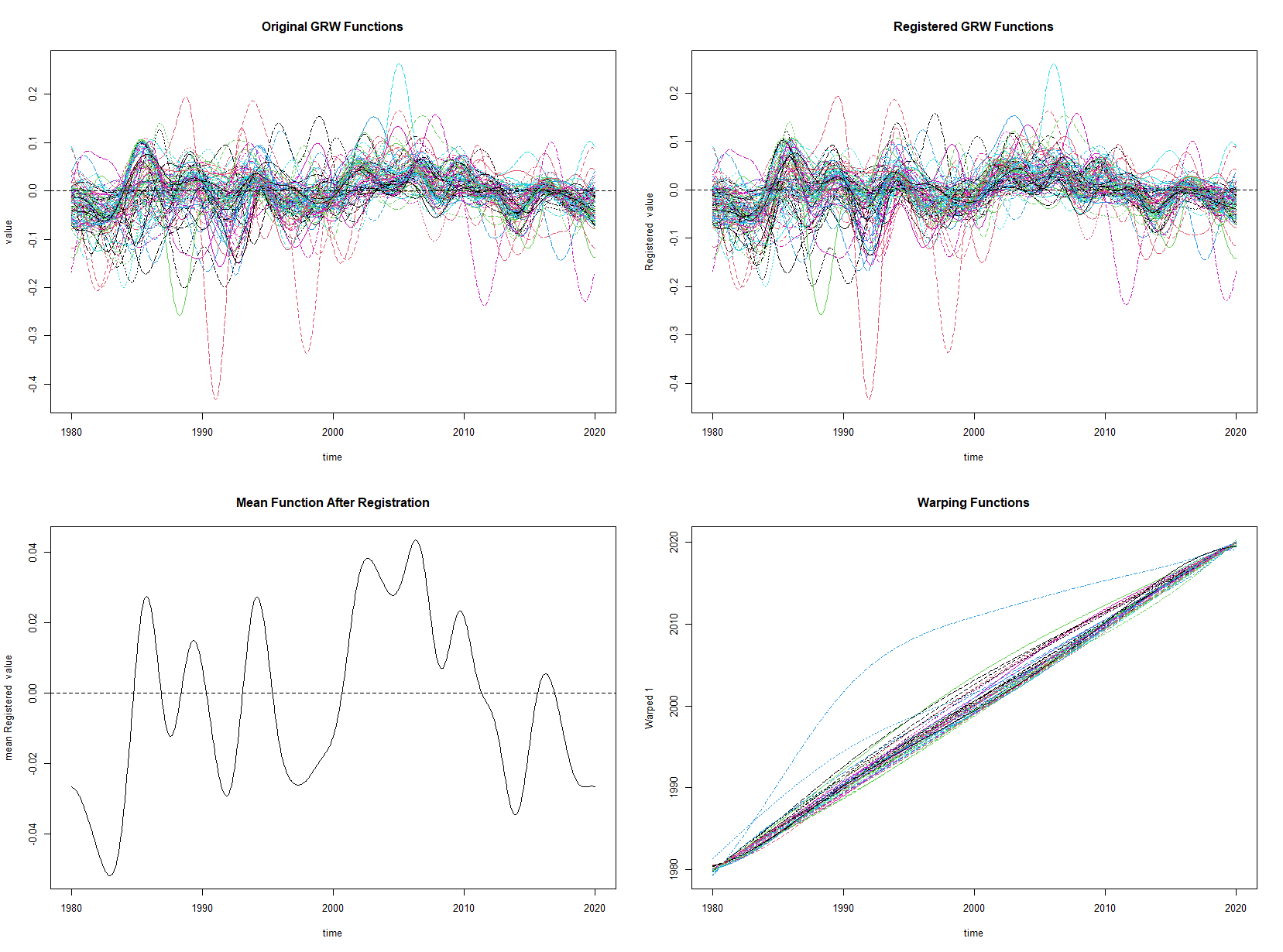
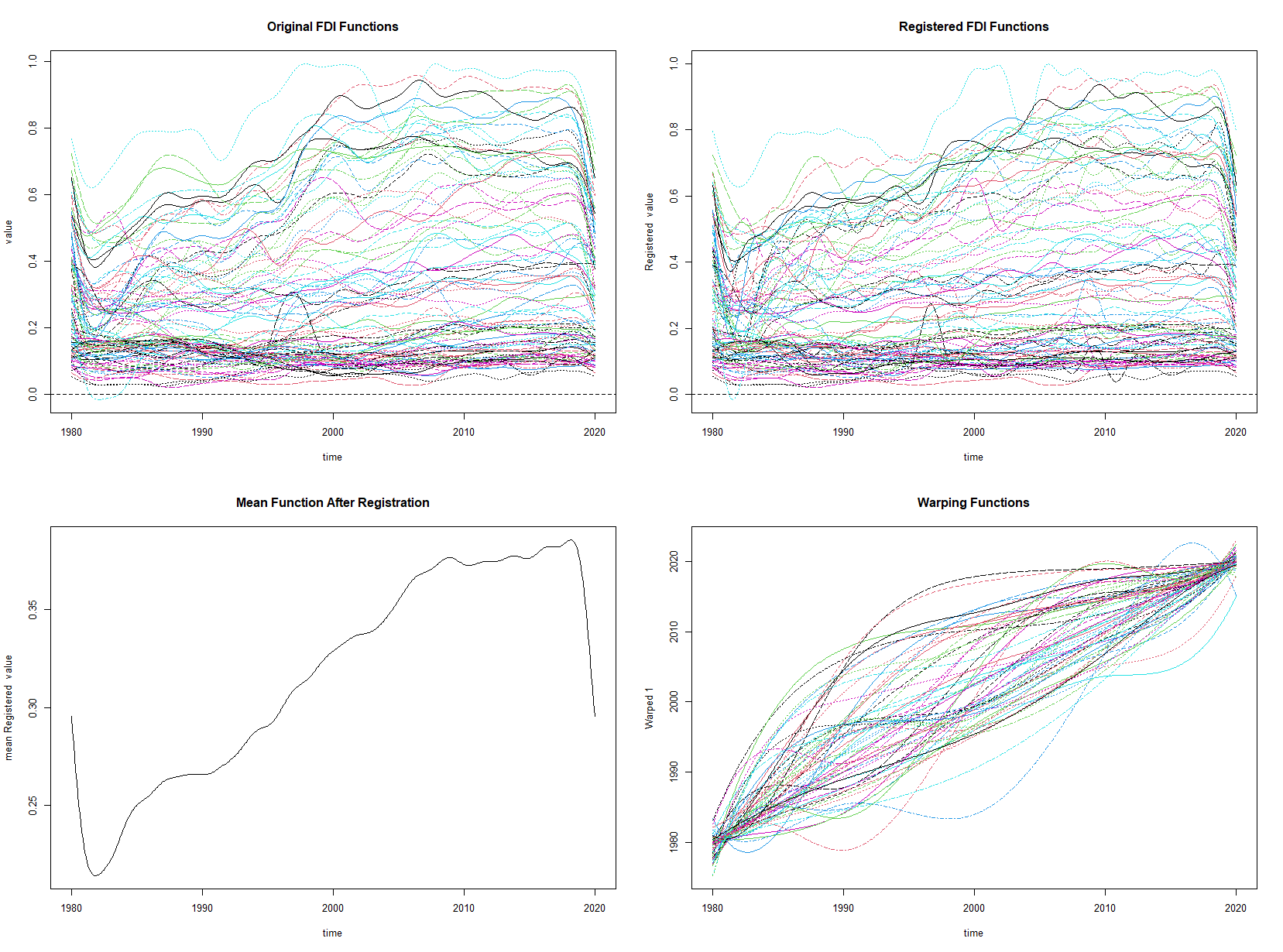
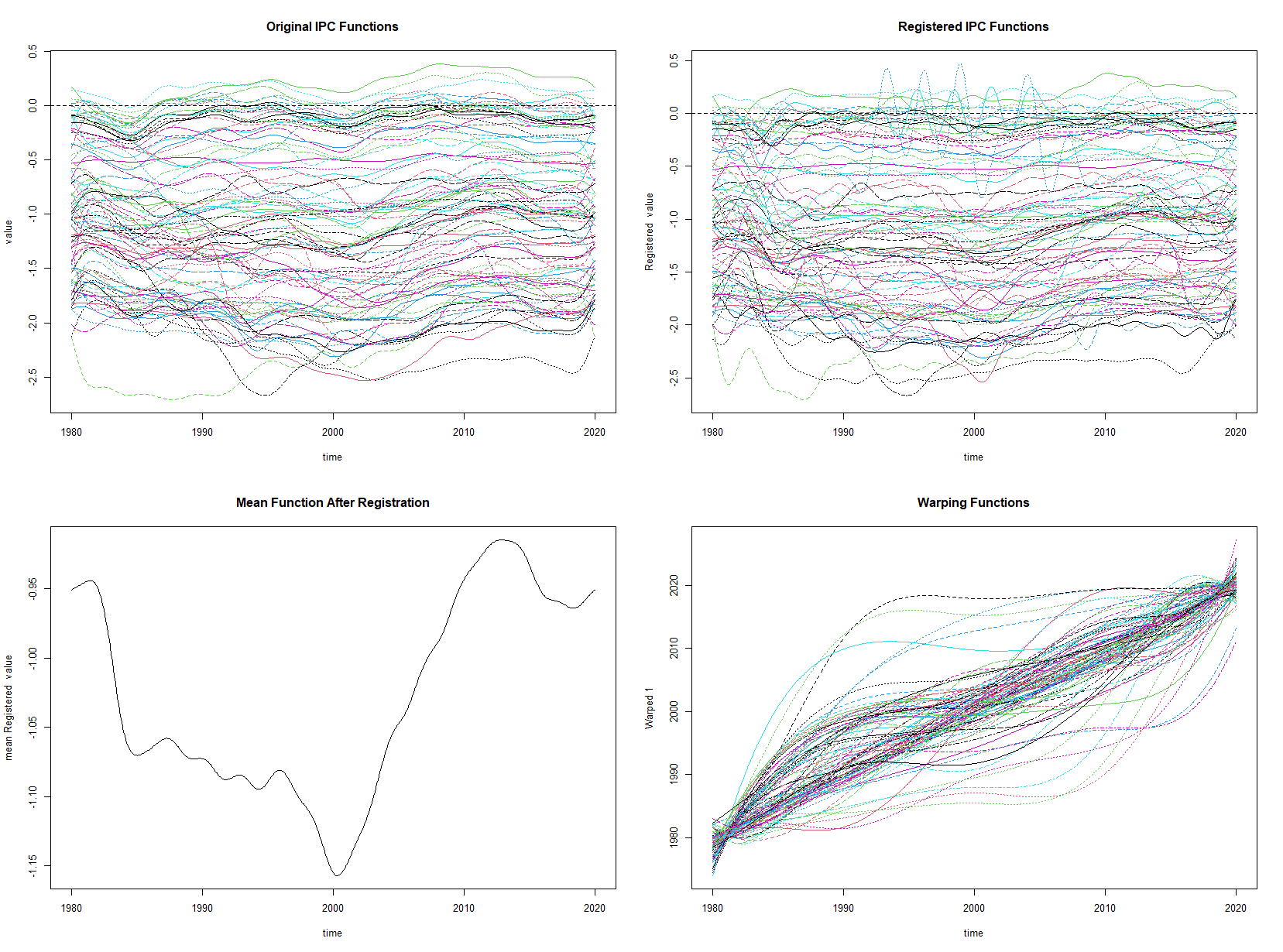
First, I used Fourier Basis to create functional data objects for smoothing purposes. To choose the best number of basis GCV method has been used with penalty parameter of 0.0001 for all functions.



Thus the optimal number of basis can be chosen from the range up to 35. I used the highest number of basis to capture even small noises on the GDP and FDI signals.



### 4.1.2 Visualizations



## 4.2 FDA Outcomes

### 4.2.2 Functional Regression Findings

Generalized Additive Model (GAM) Results

Model Specification

Family: Gaussian

Link function: Identity

Formula: Y∼te(ipc,t,bs=c("ps","ps"),k=c(10,10))+te(fdi,t,bs=c("ps","ps"),k=c(10,10))+te(ipc\_fdi,t,bs=c("ps","ps"),k=c(10,10))Y \sim te(\text{ipc}, t, bs = c(\text{"ps"}, \text{"ps"}), k = c(10, 10)) + te(\text{fdi}, t, bs = c(\text{"ps"}, \text{"ps"}), k = c(10, 10)) + te(\text{ipc\\_fdi}, t, bs = c(\text{"ps"}, \text{"ps"}), k = c(10, 10))Y∼te(ipc,t,bs=c("ps","ps"),k=c(10,10))+te(fdi,t,bs=c("ps","ps"),k=c(10,10))+te(ipc\_fdi,t,bs=c("ps","ps"),k=c(10,10))

Parametric Coefficients

| Coefficient | Estimate | Std. Error | t-value | p-value | Significance |
| --- | --- | --- | --- | --- | --- |
| Intercept | -0.0028678 | 0.0003901 | -7.351 | 2.13e-13 | \*\*\* |

Significance Codes

\*\*\*: 0.001

\*\*: 0.01

\*: 0.05

.: 0.1

: 1

Approximate Significance of Smooth Terms

| Smooth Term | edf | Ref.df | F-value | p-value | Significance |
| --- | --- | --- | --- | --- | --- |
| te(ipc, t) | 48.15 | 56.62 | 5.980 | <2e-16 | \*\*\* |
| te(fdi, t) | 42.51 | 87.00 | 3.403 | <2e-16 | \*\*\* |
| te(ipc\_fdi, t) | 17.86 | 87.00 | 1.171 | <2e-16 | \*\*\* |

Model Summary

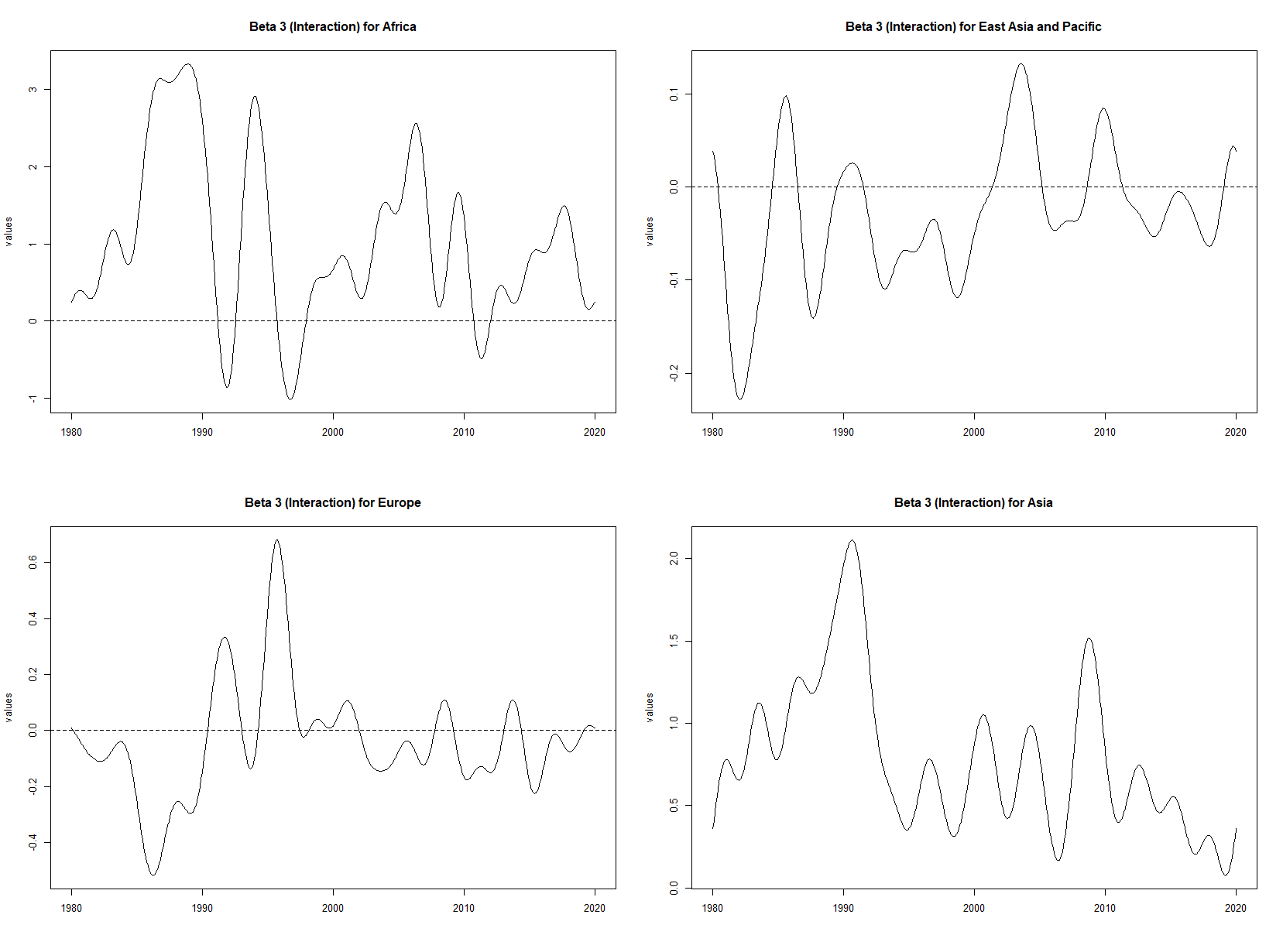
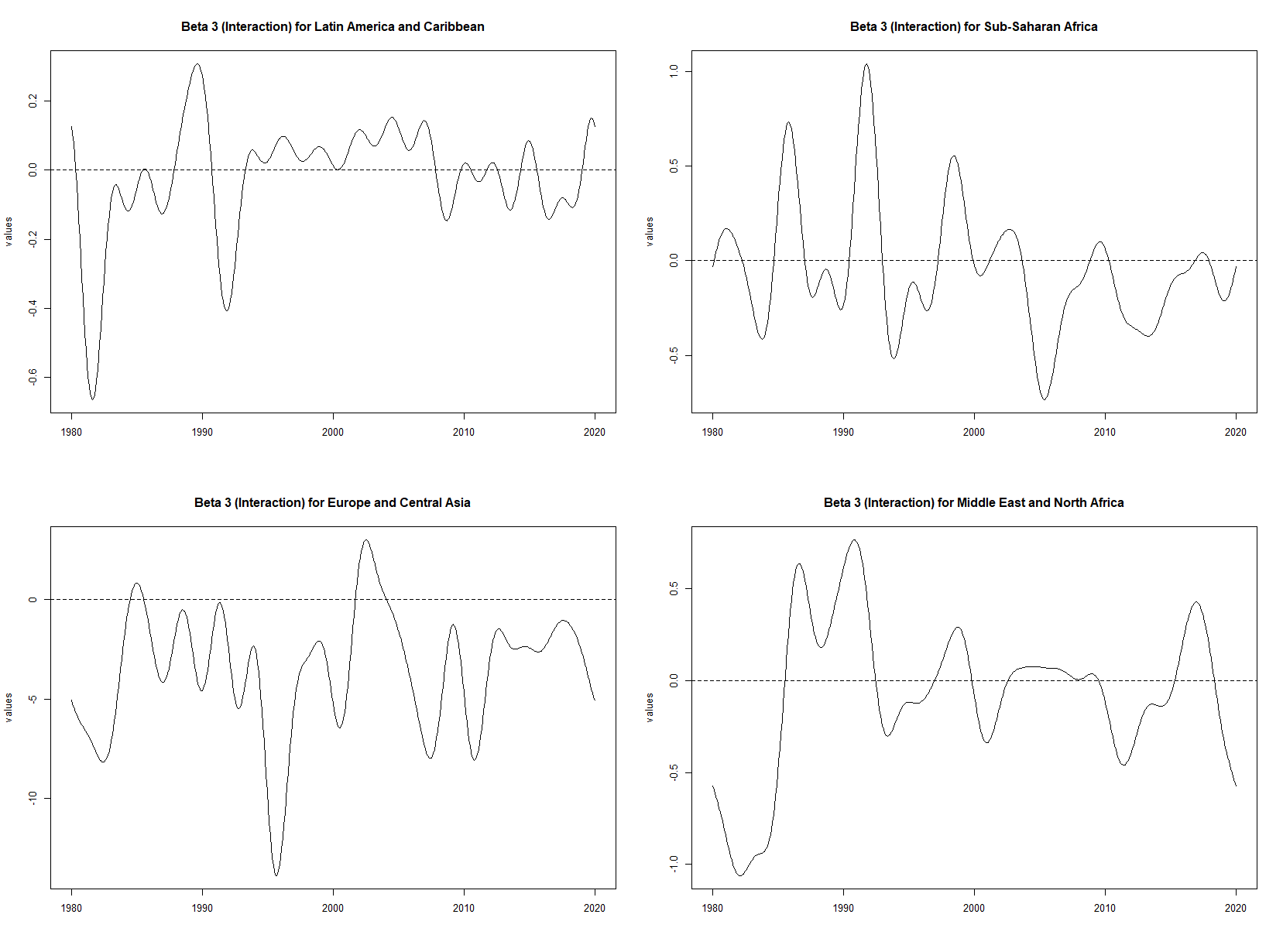
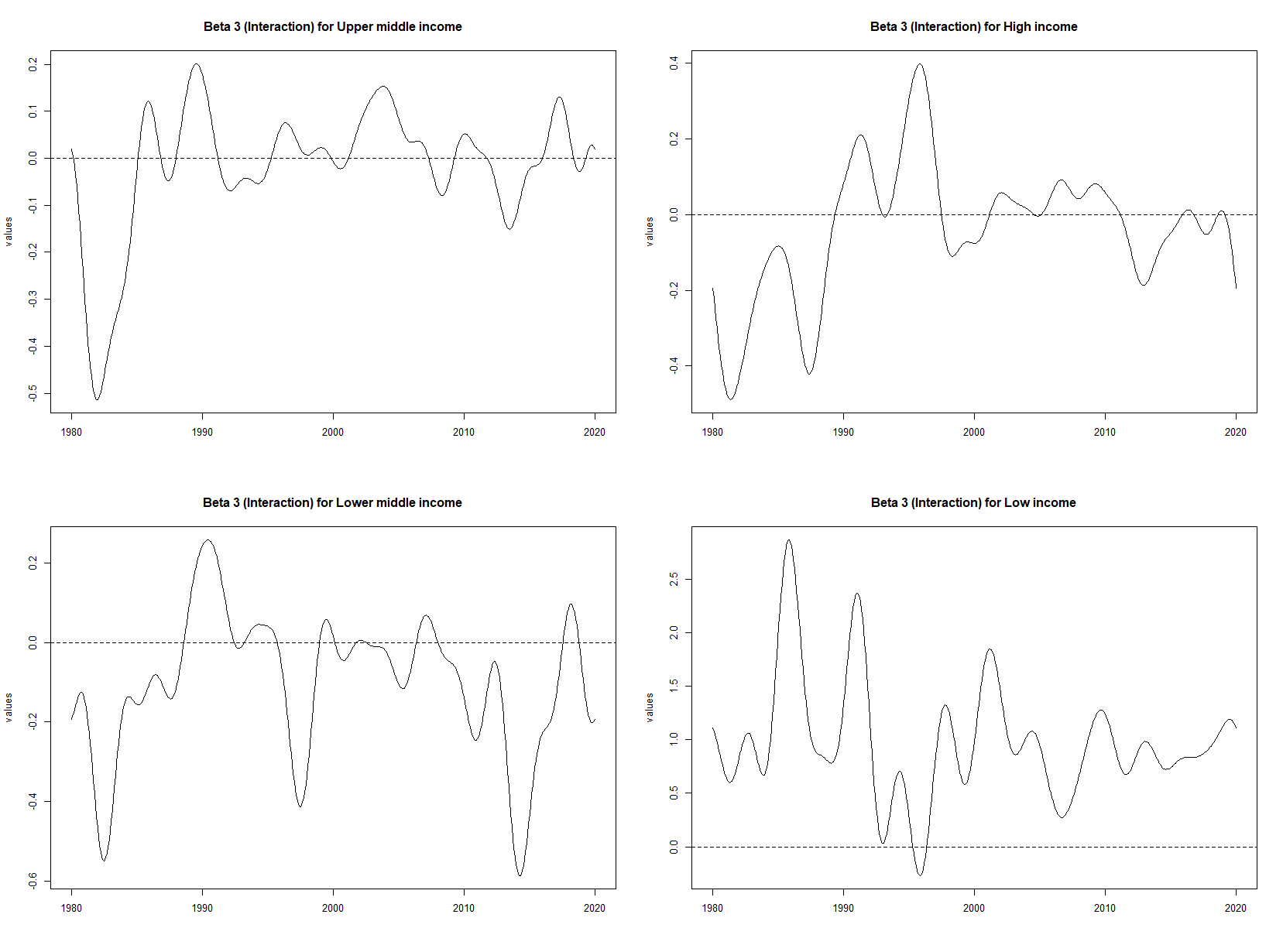
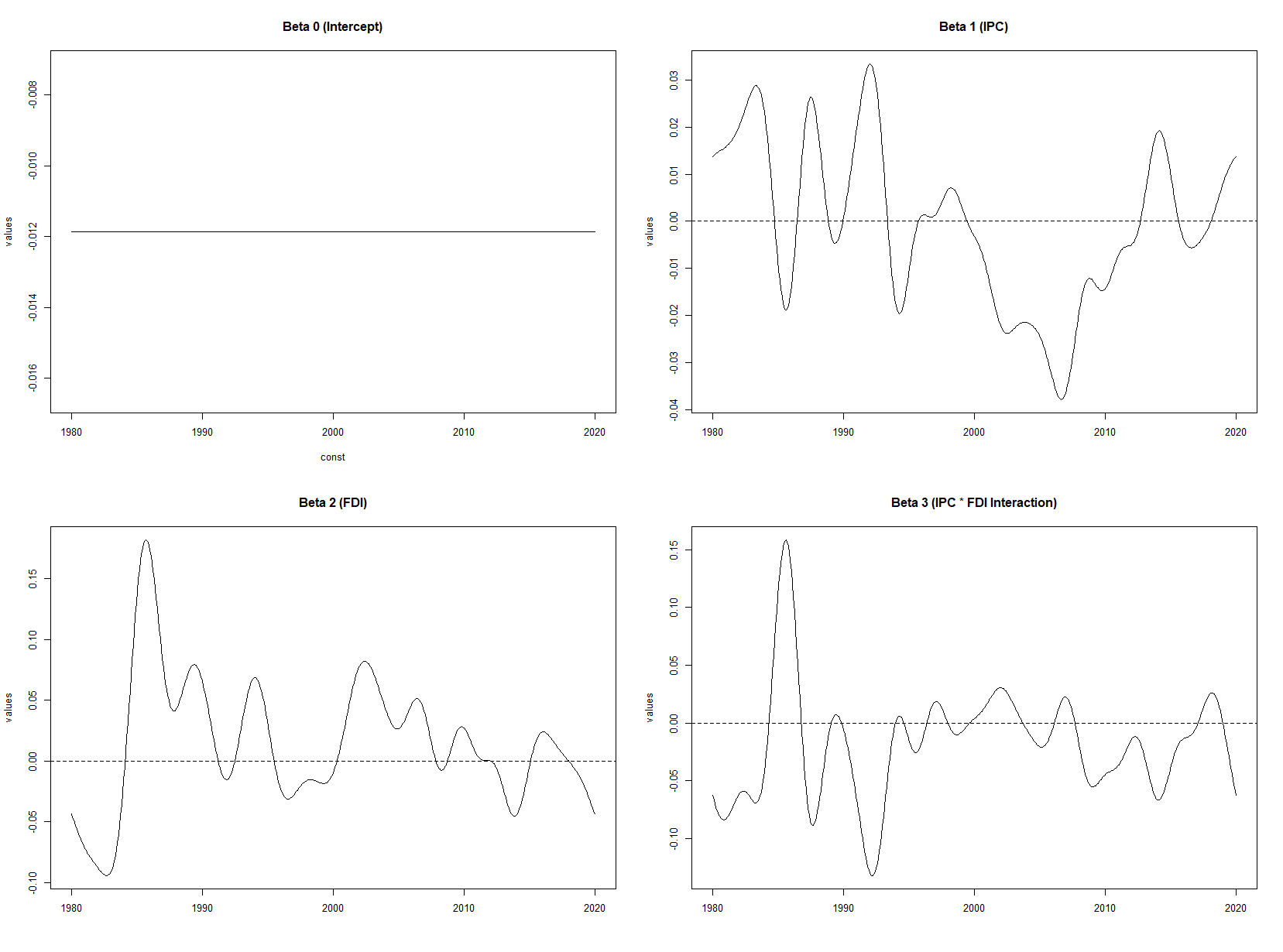
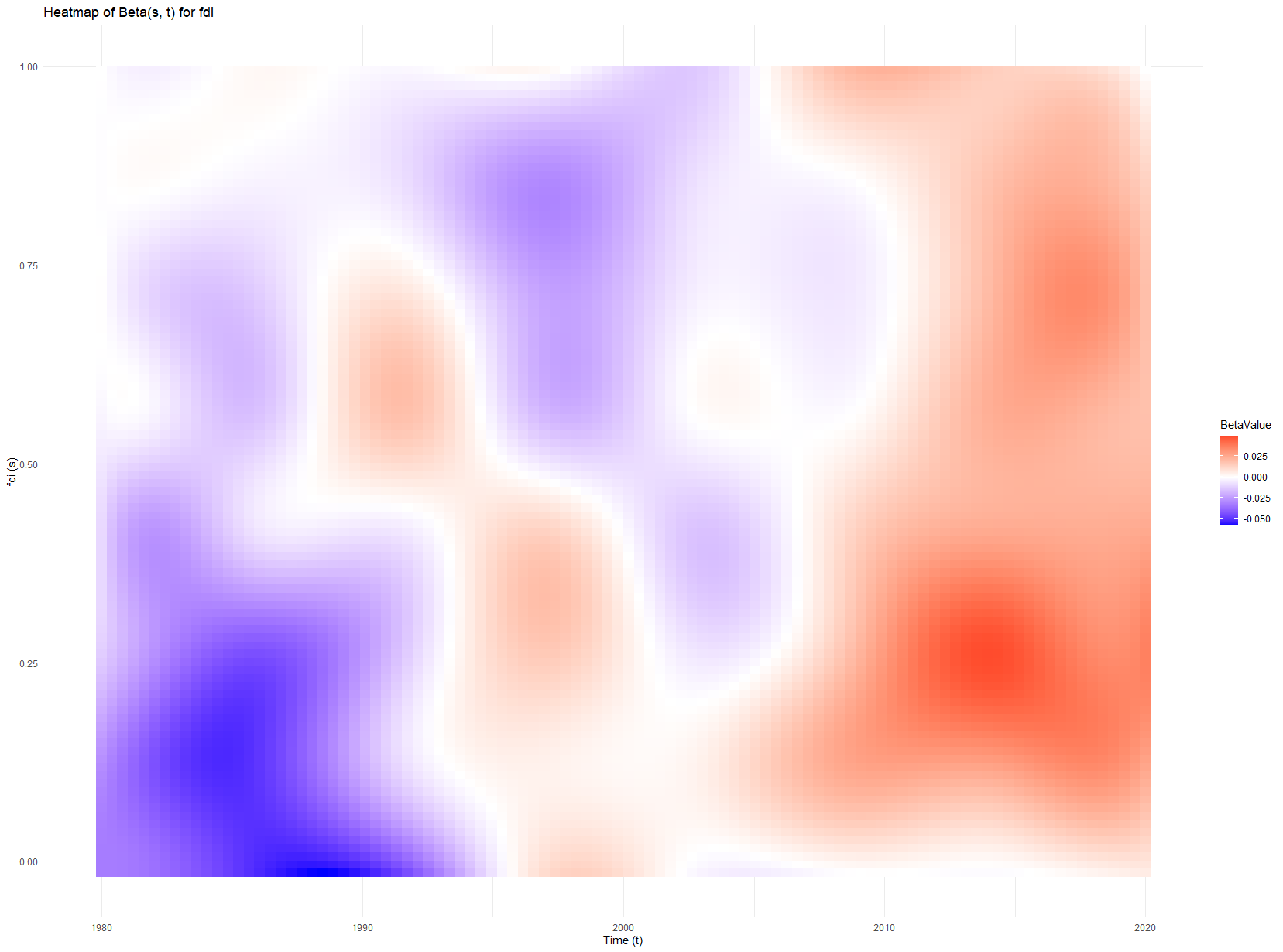
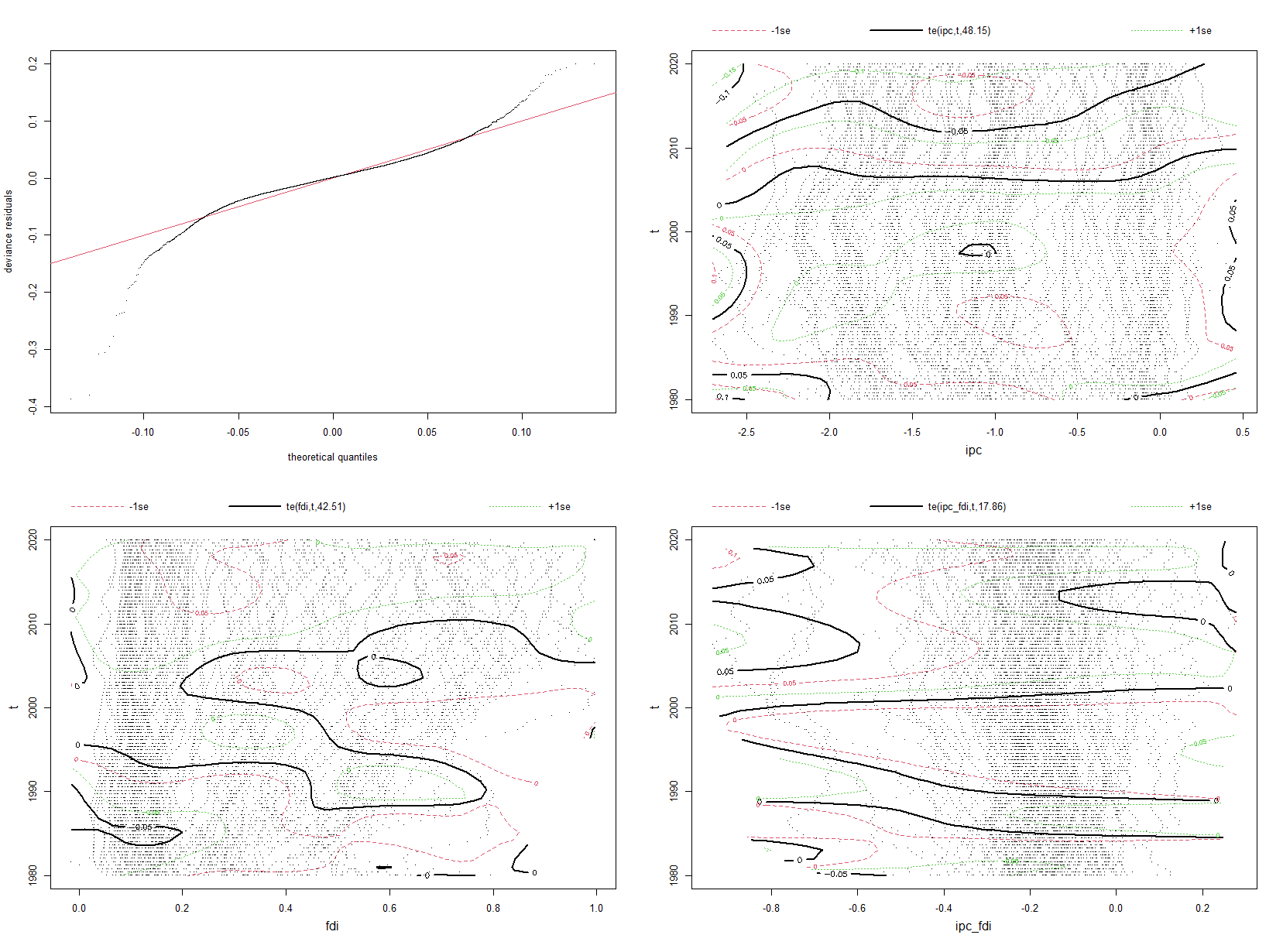
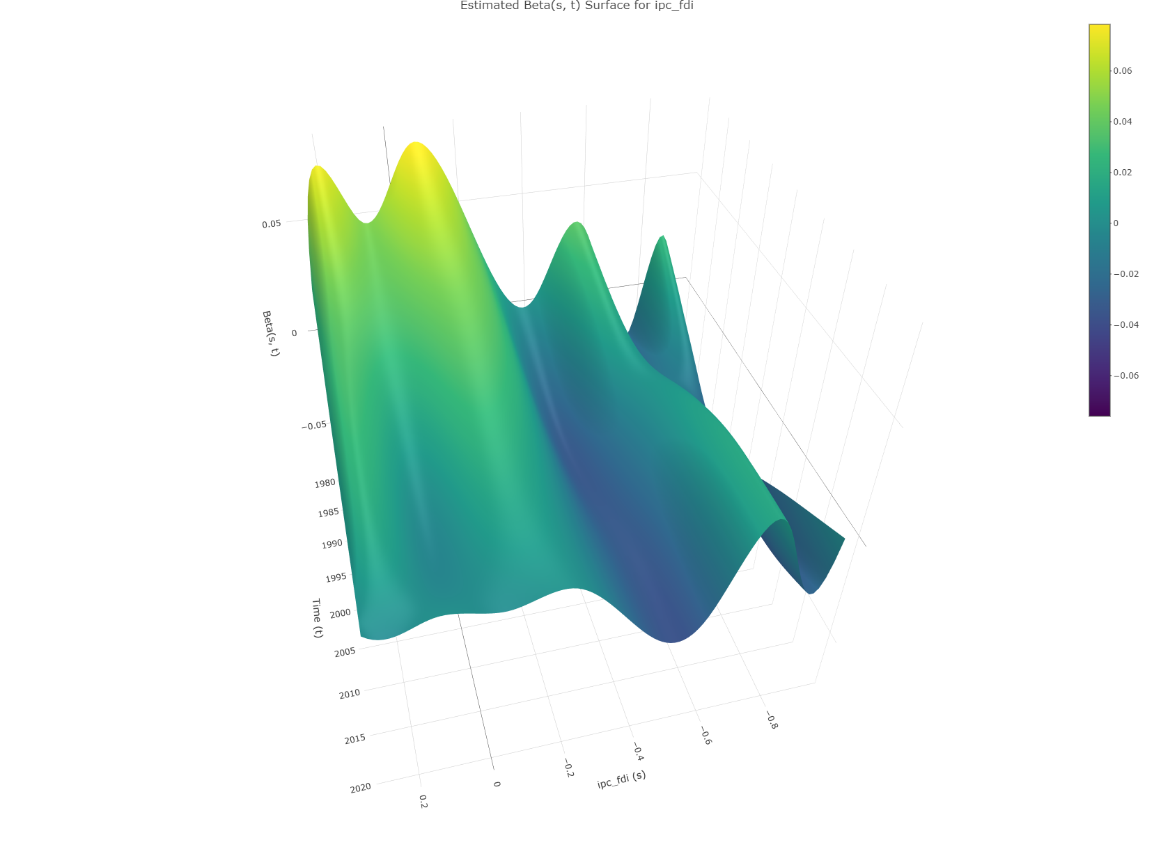
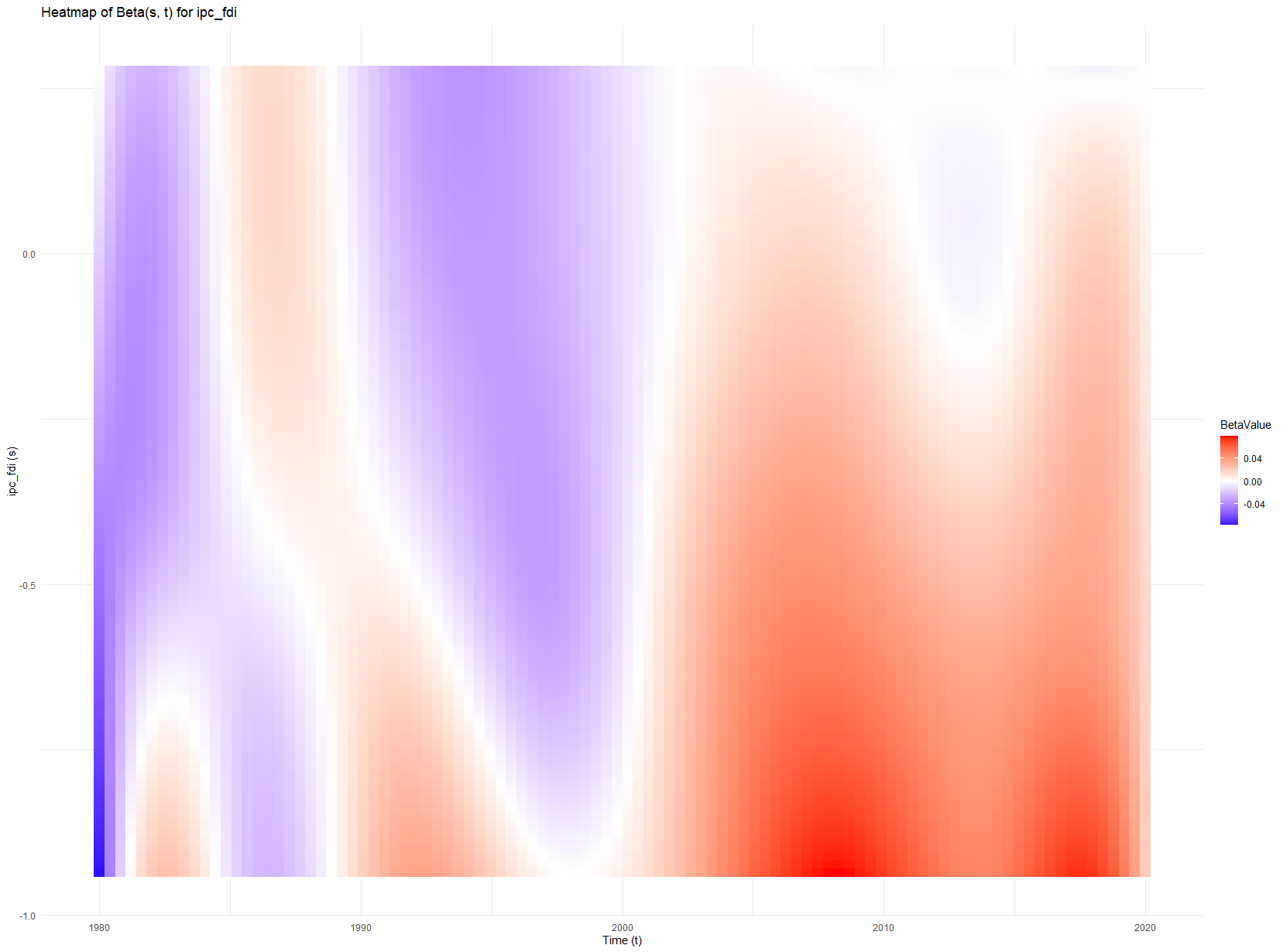
Adjusted R-squared: 0.29

Deviance Explained: 29.9%

REML Score: -17119

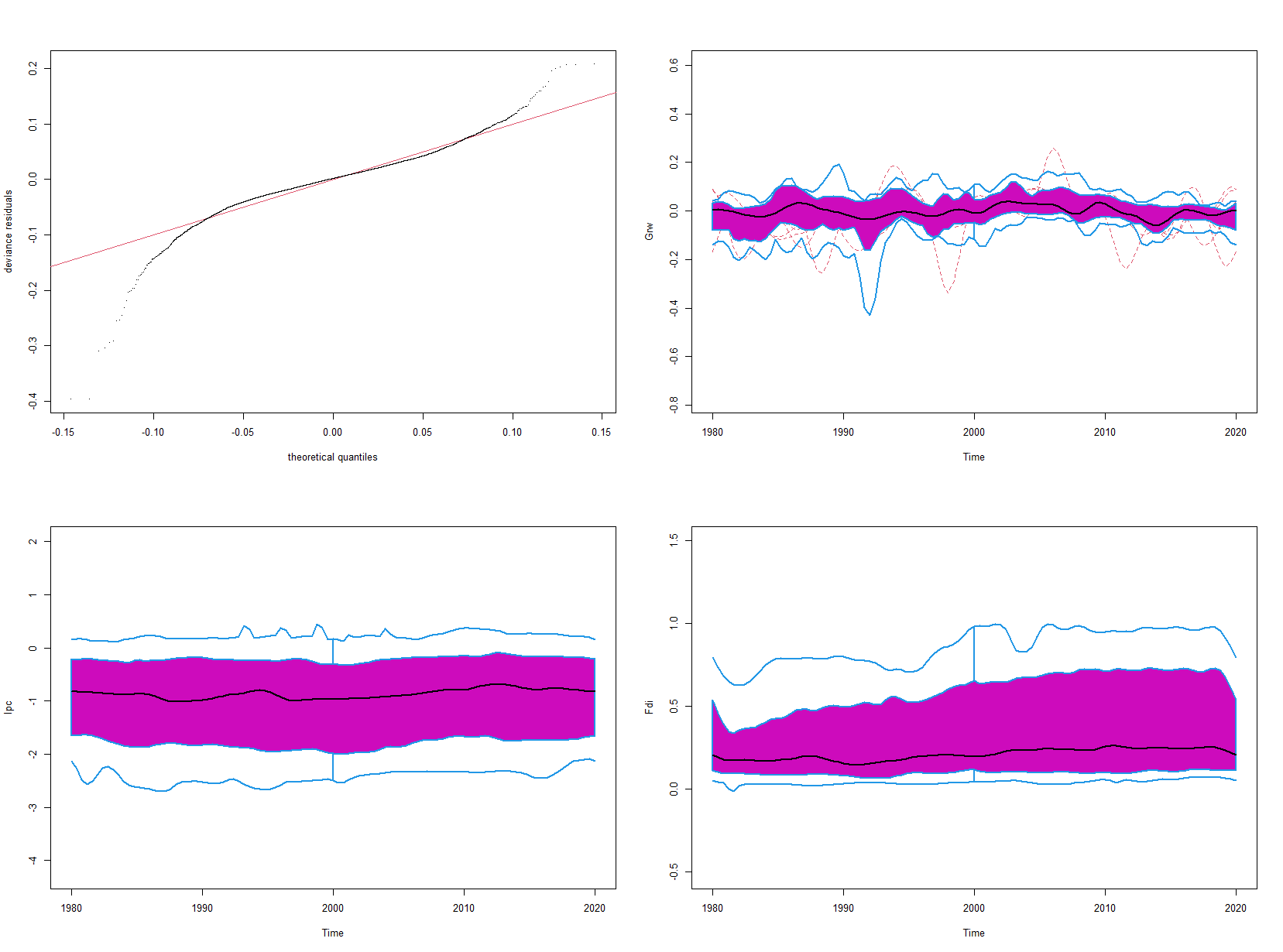
Scale Estimate: 0.0014153

Sample Size (n): 9300



## 4.3 Statistical Tests Results

### 4.3.1 Depth Analysis



### 4.3.2 Wilcoxon Test Findings

## **Wilcoxon Rank Sum Test Results**

The Wilcoxon rank sum test was performed to examine potential differences in location shift between depth distributions across various regions and income levels. The results are as follows:

| Region/Income Group | W-Statistic | p-value | Significance (α = 0.05) | Conclusion |
| --- | --- | --- | --- | --- |
| Africa | 235 | 0.2846 | Not Significant | No significant difference in location shift. |
| East Asia and Pacific | 438.5 | 0.8865 | Not Significant | No significant difference in location shift. |
| Europe | 998.5 | 0.000467 | Significant | Significant difference in location shift. |
| Asia | 294 | 0.6113 | Not Significant | No significant difference in location shift. |
| Latin America and Caribbean | 602.5 | 0.1598 | Not Significant | No significant difference in location shift. |
| Sub-Saharan Africa | 491.5 | 0.003176 | Significant | Significant difference in location shift. |
| North America | 84 | 0.1624 | Not Significant | No significant difference in location shift. |
| Europe and Central Asia | 158.5 | 0.07598 | Not Significant | No significant difference in location shift. |
| Middle East and North Africa | 207.5 | 0.4072 | Not Significant | No significant difference in location shift. |
| Middle East | 54 | 0.7799 | Not Significant | No significant difference in location shift. |

**Note**: A p-value less than 0.05 indicates a statistically significant result, suggesting a difference in location shift between groups.

| Region/Income Group | W-Statistic | p-value | Significance (α = 0.05) | Conclusion |
| --- | --- | --- | --- | --- |
| Upper middle income | 761 | 0.0828 | Not Significant | No significant difference in location shift. |
| High income | 1324 | 0.0002515 | Significant | Significant difference in location shift. |
| Lower middle income | 625 | 0.8385 | Not Significant | No significant difference in location shift. |
| Low income | 449 | 0.05079 | Marginal | Marginal difference in location shift. |

**Note**: A p-value less than 0.05 indicates a statistically significant result, suggesting a difference in location shift between groups.

## 4.5 Visualization of Interaction Effects

### 44.6 Discussion

### 4.6.1 Comparison with Previous Studies

* + - How do your findings align or contrast with Aghion et al. (2005)?

### 4.6.2 Implications of Findings

* + - What do the results suggest about the role of financial development in income convergence?

### 4.6.3 Policy Recommendations

* + - Based on the results, what policy actions could be recommended?

# 5. Conclusion

## 5.1 Summary of Key Findings

* + Recap the main results of the study.

## 5.2 Contributions to Literature

* + Highlight how your research adds to existing knowledge.

## 5.3 Limitations of the Study

* + Discuss any limitations in data, methodology, or scope.

## 5.4 Suggestions for Future Research

* + Propose areas where further investigation is needed.

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**Additional Sections (Optional):**

* **Appendices**
  + Include any supplementary material, such as code snippets, additional tables, or figures.
* **Acknowledgments**
  + Acknowledge individuals or institutions that supported your research.