QUASI MONTE CARLO SIMULATION APPROACH TO PATH-DEPENDENT OPTIONS PRICING

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Objectives

- To analyze and compare various Quasi Monte Carlo techniques for simulating path-dependent options pricing.
- For Asian options in particular, Regular Monte Carlo simulations were tested versus Quasi Monte Carlo simulations using Sobol and Halton sequences.

Introduction

- To value a path-dependent option, the whole underlying asset path is required to be considered. The simulation need to generate a sequence of underlying price at each time instant.
- Low-discrepancy sequences offer the highest probability that the value of every single asset price will be simulated in the sequence.
- High dimensional problems with a large number of time instances can not be efficiently solved by PDE or Lattice methods. $d = \frac{T}{\Delta t}$ Where T is the time interval between current price and price at maturity.
- QMS involve simulating asset prices on a fine grid (where Δt determines how fine it it) by utilizing a low discrepancy sequence that fills the grid more uniformly than uncorrelated pseudo random points.
- QMS Avoids clustering by repelling each point from each other as far as possible in a hyper-cube.

Halton Sequence

A low discreptancy sequence where:

$$n = (...d_4 d_3 d_2 d_1 d_0)_b$$

where b is a prime number. For an m dimensional sequence:

$$n = \sum_{k=0}^{m} d_k b^k$$

The n^{th} number in Halton sequence with base b is:

$$n = \sum_{k=0}^{m} d_k b^{-(k+1)}$$

Arithmetic average price for Asian option - Halton.

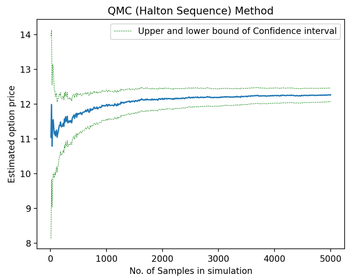

Figure 1:QMC convergence within CI bounds: Halton

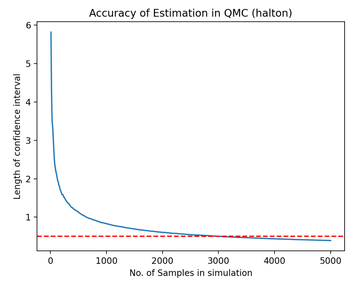

Figure 2:Testing estimation accuracy vs exact value for Halton sequence.

Sobol Sequence

Let $\{n_k\}_{k\in\mathbb{N}}$ be a representation in base 2 of any integer n. Then the n^{th} element S_n is defined as:

$$S_n = (\sum_{k=1}^{\infty} n_k \frac{V_k}{2^k}) \, mod \, 1$$

Each Sobol sequence is based of a polynomial with coefficients equal either 0 or 1.

Arithmetic average price for Asian option - Sobol.

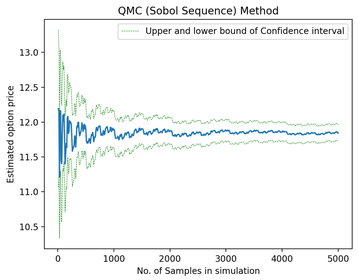

Figure 3:QMC convergence within CI for Sobol sequence.

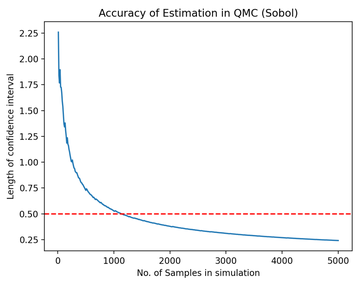

Figure 4:Testing estimation accuracy vs exact value for Sobol sequence.

MC method for Asian option:

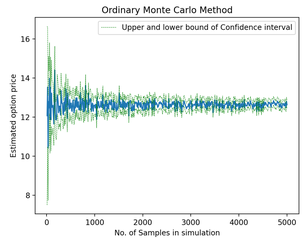

Figure 5:MC convergence within CI

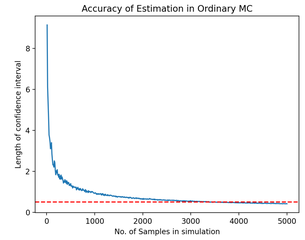

Figure 6:Testing estimation accuracy vs exact value.

Simulation Method	Estimated Price at largest sample	Convergence sample size	CI length at largest sample
Ordinary MC	$12.38	3380	0.417
QMC - Sobol	$11.84	1130	0.240
QMC - Halton	$12.27	3000	0.39

Figure 7:Table of final results

References

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Path-dependent options pricing: A quasi monte carlo simulation approach with matlab.
SSRN Electronic Journal, 07 2010.