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Increasing manufacturing flexibility is a key strategy for efficiently improving market responsiveness in the face of uncertain future product demand. Process flexibility results from being able to build different types of products in the same plant or production facility at the same time. In Part I of this paper, we develop several principles on the benefits of process flexibility. These principles are that 1) limited flexibility (i.e., each plant builds only a few products), configured in the right way, yields most of the benefits of total flexibility (i.e., each plant builds all products) and 2) limited flexibility has the greatest benefits when configured to chain products and plants together to the greatest extent possible. In Part II, we provide analytic support and justification for these principles. Based on a planning model for assigning production to plants, we demonstrate that, for realistic assumptions on demand uncertainty, limited flexibility configurations (i.e., how products are assigned to plants) have sales benefits that are approximately equivalent to those for total flexibility.

Furthermore, from this analysis we develop a simple measure for the flexibility in a given product-plant configuration. Such a measure is desirable because of the complexity of computing expected sales for a given configuration. The measure is $\Pi(M^*)$, the maximal probability over all groupings or sets of products (M) that there will be unfilled demand for a set of products while simultaneously there is excess capacity at plants building other products. This measure is easily computed and can be used to guide the search for good limited flexibility configurations.
(*Manufacturing Flexibility; Capacity Planning; Product Allocation*)

Introduction

Increasing manufacturing flexibility is a key strategy for improving market responsiveness in the face of uncertain future product demand. One type of flexibility results from being able to build different types of products in the same manufacturing plant or on the same production line at the same time. The literature on manufacturing flexibility often refers to this type of flexibility as "process flexibility" (see Browne, et al 1984 and Sethi and Sethi 1990). The purpose of this research is to help understand process flexibility, to show how much of it is needed, and to show where it should be added to most effectively respond to changes in product demand. While the focus of this research is the automotive industry, the conclusions are broadly applicable.

Process flexibility is determined by product assignment decisions, i.e., decisions on which products are to be built at which plants or on which lines. In the automotive industry, the products are vehicles defined at the nameplate level (e.g., Chevrolet Camaro, Pontiac Grand Prix, Buick Riviera). A vehicle assembly plant typically has a single assembly line, so we focus here on flexibility in the assignment of products to plants. These assignments are determined by tooling and capacity investment decisions that must be made between 1 and 3 years before production begins. Historically, the average difference between vehicle sales forecasts made in this period and actual annual vehicle sales is about 40% (both + and −). Having process flexibility is one strategy for dealing with this uncertainty. The question is how much process flexibility is needed?

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Currently in the U.S. auto industry, assignments of products to assembly plants show some degree of flexibility. Most plants build more than one product and a few products are built in more than one plant. Is this level of flexibility enough to deal adequately with uncertain product demands? If not, how should they be changed to increase flexibility? This paper will provide guidelines for planning flexibility through product assignment and capacity decisions.

This research was done to affect actual decisions on investing in flexibility. It addresses several problems that have resulted in manufacturing operations being relatively inflexible. First, in capacity and flexibility planning, investment costs for flexible operations are typically quantified; however, it is less common to quantify the benefits because demand uncertainty is not explicitly considered by the planners. Since flexibility is expensive, this normally results in decisions not to invest in it. Second, many people (in both industry and academia) think of flexibility in discrete terms—you either have no flexibility or you have total flexibility (the ability to build every product on every production line). This compounds the first problem because total flexibility is very expensive. Hence, this research focuses on quantifying the benefits from flexibility and demonstrating how they accrue as the amount of flexibility increases.

The desire to impact real decisions has influenced the approach taken in this work. We have not developed an optimization model to make product assignment decisions that minimize the cost of achieving a given level of flexibility. Rather, through a fairly simple model and analysis, we develop insights about how much and what configuration of flexibility is needed to cope with uncertainty in future demand, regardless of cost—insights that have been missing in both management and research discussions of flexibility and that managers can incorporate into their thinking in making decisions on flexibility.

We focus on flexibility's benefits exclusive of cost considerations. Of course, in making investment decisions, costs must be weighed against these benefits. However, a better understanding of the benefits is needed not only to make better investment decisions, but also to guide R&D aimed at making manufacturing processes more flexible and lowering the cost of flexibility. We develop principles on the benefits of flexibility which must be combined with cost considerations in order to make investment decisions.

The literature on manufacturing flexibility has grown dramatically in the 1980's. Sethi and Sethi (1990) provide a very comprehensive survey of this literature. Much of this literature is directed at issues related to flexible manufacturing systems (FMS) which are "computer-controlled complexes of automated material handling devices and numerically controlled machine tools that can simultaneously process a variety of products" (Stecke 1983). The focus on FMS means that deciding on how much flexibility to have is often modeled as whether to invest in 1) dedicated capacity with no flexibility or 2) an FMS which can build all products, i.e., total flexibility (Fine and Freund 1990). This approach may be appropriate in many cases, especially those involving machining operations; however, for assembly lines, the question is how much flexibility to invest in between the extremes of none and total. Andreou (1990) and Triantis and Hodder (1989) also present methods for computing the value of process flexibility. While providing insight on the factors that affect this value, the analytical approaches used are limited to only two products. Different methods are needed to show how much flexibility is needed for systems of many products and plants.

This paper is written in two parts. Part one describes principles related to:

- how much process flexibility is needed, and
- how to add it to an existing set of plants to cope most effectively with uncertain demand.

It develops these principles intuitively using examples. Part two provides an analytic demonstration supporting these findings. Additional detail on this research is reported in Jordan and Graves (1991) and Graves and Jordan (1991).

Part I. Flexibility Principles

Process flexibility provides the ability to change volumes of products produced in response to demand changes; thus, the benefits of flexibility can be measured in increased expected sales and capacity utilization. We illustrate this here through a simple two-plant, two-product example.

Consider two plants each with an annual capacity of 100 units that produce two products. The two products have random, but independent, demands with 50, 100, and 150 units being equally likely for each. Assume production levels can be set after observing demand, and that excess demand is lost. Consider the two cases of total flexibility (both plants can produce both products) and no flexibility (each plant produces one product).

If demands for the two products are 50 and 150, there are no lost sales in the flexible case, but 25% of demand is lost if there is no flexibility. Overall, with total flexibility, expected sales equals 178 units, lost sales equals 22 units, and capacity utilization is 89%. With no flexibility, expected sales are 167 units, lost sales are 33 units, and capacity utilization is 83%.

How Much Flexibility is Enough?

If a manufacturing firm only has two products and two plants, making decisions about flexibility is easy. However, with many products and many plants, these decisions become complex for several reasons. First, the sales and utilization benefits of flexibility are very difficult to calculate. Second, the number of possible product assignment configurations grows exponentially with the number of products and plants. To help sort through these, this section and the one that follows develop principles that can be used as guidelines for identifying reasonable product assignment configurations and for thus limiting the search.

To understand how to create good assignments, we will develop our intuition for how benefits accrue as we add more and more flexibility. To do this, we consider a 10-product, 10-plant example. Assume that each plant has a capacity of 100 units and that the expected demand for each product is also 100 units. The demand for each product is from a truncated normal distribution with a standard deviation of 40 units, and minimum and maximum possible demands of 20 and 180 units. This level of uncertainty is comparable to uncertainty historically seen in demand forecasts used for planning purposes in the auto industry. Again, we assume that product demands are independent.

Since computing the expected sales and capacity utilization for different product assignment configurations using analytical methods is extremely difficult, we use a simulation model. That is, we 1) randomly sample the demand for each product, 2) allocate this demand to plants to maximize the demand filled (i.e., sales) subject to product assignment decisions and capacity constraints, and 3) collect statistics on sales, lost sales, and capacity utilization for this realization of demand. This procedure is repeated to produce reliable estimates.

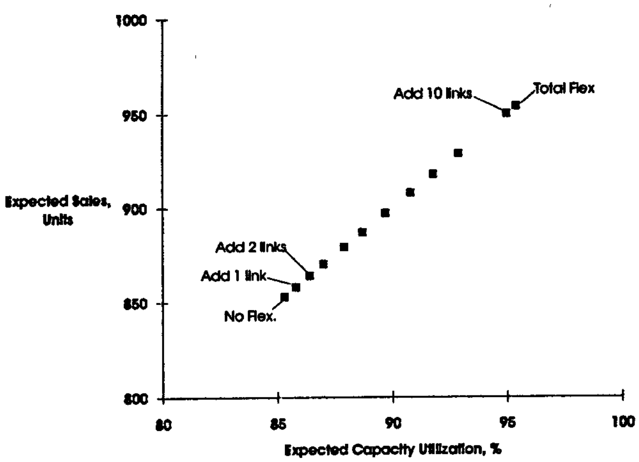
First consider the case of no flexibility—each plant builds only one product (that is, product 1 built in plant 1, product 2 in plant 2, etc.). For this configuration, expected sales are 853 units and expected capacity utilization is 85.3%. Having total flexibility (each plant can build all products) increases both expected sales and expected capacity utilization to 954 units and 95.4%, respectively. Although total flexibility may be unrealistic from an investment and manufacturing cost viewpoint, it provides a good benchmark to measure the effectiveness of other, more reasonable configurations.

An important question is how much flexibility is needed to have significant sales and utilization benefits? Put another way, can the benefits of total flexibility be achieved with something less than total flexibility? To answer this, consider adding flexibility incrementally to the no flexibility configuration. That is, we add one "link" (e.g., first assign one product to a second plant) at a time to the configuration and measure the impact on sales and utilization using the simulation model. We first add product 1 to plant 2, then product 2 to plant 3, then product 3 to plant 4, and so on. The tenth link added assigns product 10 to plant 1. This pattern for adding flexibility is not chosen arbitrarily; it is one that appears to yield the greatest benefits and is chosen based on some generic principles that will be discussed below.

Figure 1 shows the expected sales and utilization impacts of adding these links. Adding each increment of flexibility yields increasingly greater sales and utilization benefit until, after adding the tenth link, over 95% of the sales and utilization benefits of total flexibility have been achieved. Thus, adding 10 links has about the same benefits as adding the 90 links needed to achieve total flexibility. Figure 2 depicts this result.

We have said nothing to this point about the costs of achieving flexibility. Figure 1 does not identify how much flexibility to invest in for this specific example, but does show that investing in 10 new "links" would yield the benefits of total flexibility and that adding flexibility beyond this point is probably not worthwhile.

Figure 1 Impact of Incrementally Adding Flexibility on Expected Sales and Capacity Utilization



To determine the best configuration requires that sales and utilization benefits be compared to the investment and manufacturing costs associated with each level of flexibility. We will talk more about manufacturing costs in the Discussion section of this paper.

Keep in mind that Figure 2 does *not* show that building two products in every plant and having each product built in two plants is always the right level of flexibility to have. The number of products per plant, plants per product, and the product assignments required to achieve the sales and utilization benefits of total flexibility depend on the specific number of products and plants, plant capacities, product expected demands, and demand uncertainty levels. The point here is that to obtain the benefits of total flexibility for the system does not require that each plant be totally flexible; rather, a little flexibility can achieve almost all the benefits of total flexibility.

Where Should Flexibility Be Added?

To decide where to add flexibility, we have discovered that the concept of "chaining" is important. A "chain" is a group of products and plants which are all connected, directly or indirectly, by product assignment decisions. In terms of graph theory, a chain is a connected graph. Within a chain, a path can be traced from any product or plant to any other product or plant via the product assignment links. No product in a chain is built by a plant from outside that chain; no plant in a chain builds a product from outside that chain.

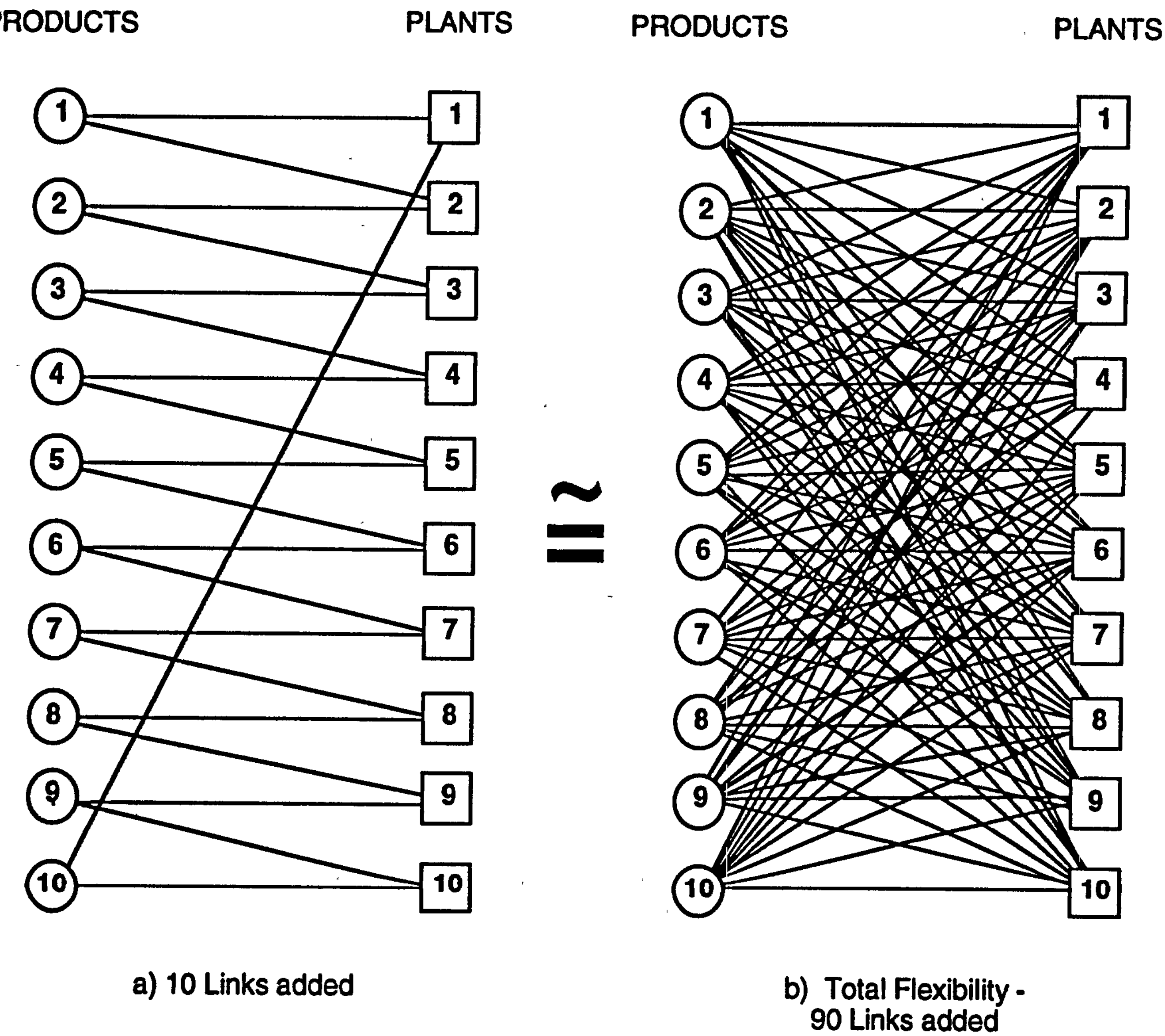
The impact that chaining has on expected sales is demonstrated by comparing the two configurations in Figure 3 for the example discussed in the previous section. By some measures, both configurations have the same level of flexibility: both have 20 product-plant "links" with two products built in each plant and each product built in two plants. However, the configuration in Figure 3a has one chain, while that in Figure 3b has five. As a consequence, these two configurations are far from equal in their sales and capacity utilization performance, as Table 1 shows. The one chain configuration yields over 95% of the increase in expected sales from total flexibility. However, having the same number of products per plant configured in five chains yields less than half of these benefits. The benefits of flexibility for responding to unforeseen changes in demand does not come only from having more products assigned per plant, but also from creating longer chains.

The intuition behind this concept is easy to grasp. The longer the chain of products and plants, the greater the opportunities are for shifting capacity from building products with lower than expected demand to those with higher than expected demand. In Figure 2a, if product 1's demand is greater than expected and product 7's is less than expected, with one long chain the volume of products built in each plant can be shifted to accommodate increased production of 1 and decreased production of 7. With 5 chains such a production shift could not be accomplished. In Part II, we develop an analytical model which demonstrates generally the benefits of chaining.

To this point, we have one clear guideline for adding flexibility: to be most effective for increasing sales, flexibility should be added to create fewer and longer chains for products and plants. This does not imply, however, that only enough flexibility should be added to create one chain. In our example, removing product 10 from plant 1 in Figure 3a still results in a single chain. Expected sales and utilization for this chain (without the product 10-plant 1 link) are 929 units and 92.9%, respectively, down from 950 and 95.0%. The reasons for this are clear: without the 10-1 link, there are no options for accommodating higher than expected demand for product 10 or for using the capacity of plant 1 if product 1's demand is lower than expected.

The implication of this is that once enough flexibility

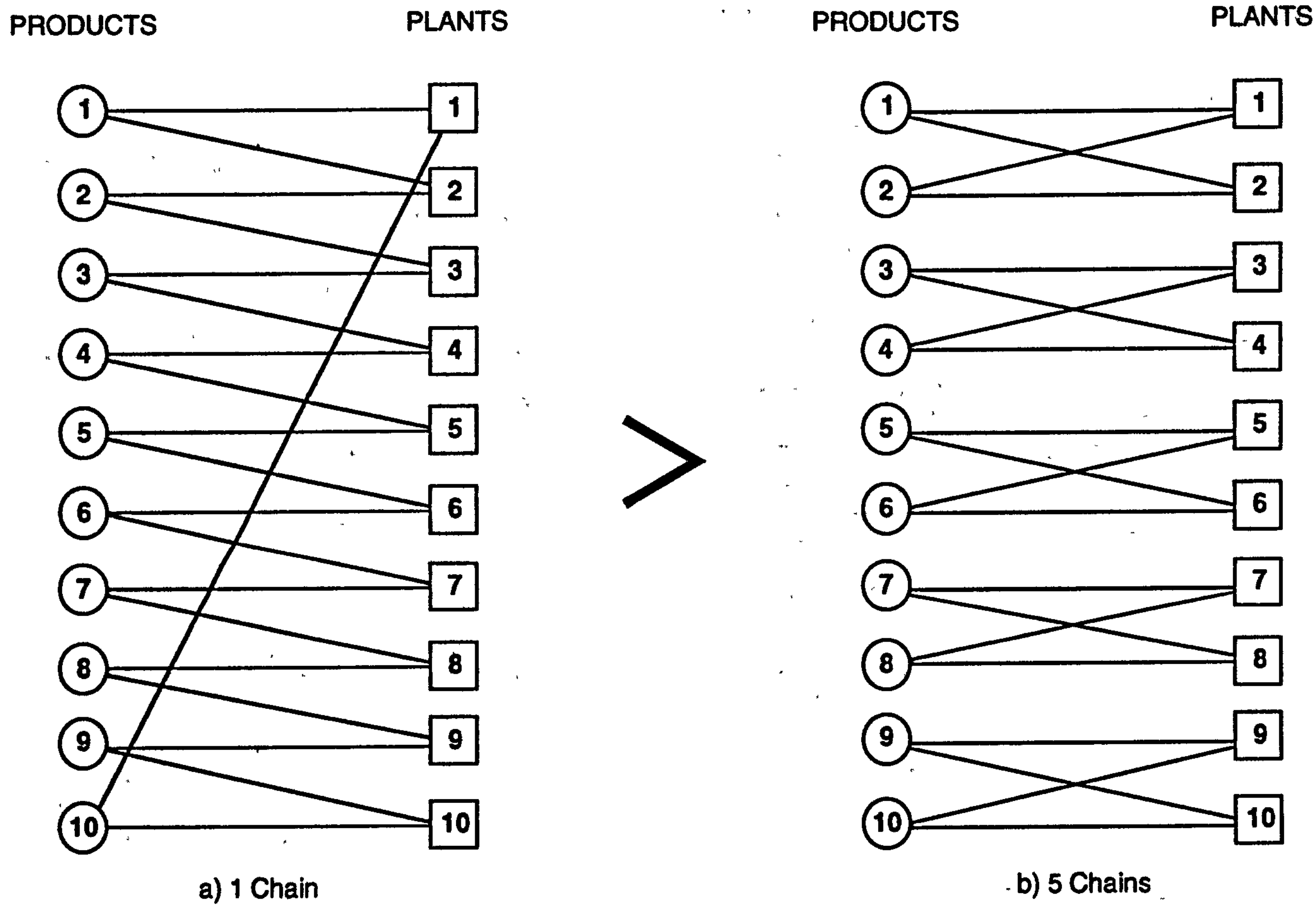
Figure 2 Flexibility Configurations with Approximately Equal Ability to Respond to Demand Uncertainty



has been added to create a chain, a little more flexibility may be needed to reach the benefits of total flexibility. However, it is difficult to prescribe general guidelines for how flexibility should be added within a chain. For the idealized example in Figures 1–3 where the number of plants and products are equal and expected demands and capacities are identical, there is a specific way in which this flexibility must be added: to be most effective it must "close the chain" (in terms of graph theory, we want to create a cycle). That is, adding the link between product 10 and plant 1 creates a complete circuit around which production can be varied in response to demand changes. Further, it balances the number of products/plant and plants/product throughout the chain.

For more realistic cases, the idea of adding flexibility to create a complete circuit does not necessarily apply and we have no firm guidelines for adding flexibility within a chain. However, based on this example and

Figure 3 Flexibility Configurations with Fewer Chains Have Greater Benefits



on tests with real products and plants, the following guidelines are helpful for identifying the best way to add flexibility once a single chain has been formed:

- try to equalize the number of plants (measured in total units of capacity) to which each product in the chain is directly connected;
- try to equalize the number of products (measured in total units of expected demand) to which each plant in the chain is directly connected; and
- try to create a circuit(s) that encompasses as many plants and products as possible.

We will illustrate these guidelines in a more realistic example later.

Table 1 Expected Sales for Different Flexibility Configurations

Configuration	E[Sales]
no flexibility	853 units
20 links in 5 chains (Fig 3b)	896
20 links in 1 chain (Fig. 3a)	950
total flexibility	954

Factors Affecting the Benefits of Flexibility

Two factors that affect the benefits of flexibility are 1) correlations in product demand and 2) total capacity relative to expected total demand. We now examine these factors *in the context* of the 10-product, 10-plant example.

Product Demand Correlation. If product demands are positively correlated, then opportunities for changing this product mix, and therefore the benefits of process flexibility, are reduced. Conversely, negative correlations increase the benefits of flexibility. We demonstrate these relationships in Part II.

The real issue for capacity planning is whether product demand correlations should be a major factor in deciding assignments of products to plants. That is, if you could identify negatively correlated products is it important to build them in the same plant? The intuition behind chaining, described above, suggests that this is not important (at least not for realistic levels of demand correlation and uncertainty). It is only important that negatively correlated products be included in the same chain. The key behind chaining is that all products in the chain effectively share all the plant capacity in the chain.

We illustrate this with the current example. Assume that in Figure 2a that product pairs (1, 3), (3, 5), (5, 7), (7, 9), and (9, 1) all have correlation coefficients of −0.5. All other pairs have correlations of zero. This is a case where no negatively correlated product pair shares a plant. We compare this to a second case where the product pairs (1, 2), (3, 4), (5, 6), (7, 8), and (9, 10) have correlation coefficients of −0.5, with all remaining pairs having coefficients of zero. In this case, each negatively correlated product pair shares a plant. We apply the simulation model to evaluate these cases. Both yield identical total expected sales of 966 units. This is to be compared to 969 for total flexibility and 853 units for no flexibility. Producing the negatively correlated products in the same plants is of secondary importance to producing them in the same chain for this example. We have consistently found this to be true for a wide range of both realistic and idealized examples.

Total Production Capacity. To see the basic relationship between flexibility and capacity consider some extreme cases. If each plant's capacity is less than the minimum possible demand for each product, then each plant is fully utilized under any possible demand. Adding flexibility has no value. At the other extreme, if each plant's capacity exceeds the maximum possible product demand, then all demand would always be filled and again flexibility would have no value.

To see how the value of flexibility changes when plant capacities are varied within these extremes, we varied the total capacity for the 10 plants between 500 and 1500 units (capacity is always split equally among the plants). Expected sales and capacity utilization are plotted in Figure 4 for each capacity level for no flexibility and the one-chain configuration (Figure 2a). The curve for total flexibility is essentially identical to that for one-chain flexibility. The dashed lines show the impact of changing flexibility at a given capacity level; the solid lines show the impact of changing capacity at a given flexibility level. For example, point D plots expected sales and utilization when total capacity equals 1000 units and there is no flexibility. Point E shows the impact of creating one chain with the same capacity.

This graph shows that the benefits of adding flexibility are substantial for a wide range of total capacity. When capacity equals expected demand going from no flexibility to one-chain flexibility increases expected sales by 11.8%. Certainly, if total capacity deviates far enough from expected demand, then flexibility has little or no value. However, even when capacity is 25% above or below expected demand, expected sales increase by more than 5%. While these numbers are specific to this example, the general result applies to more realistic situations generally. This is an important result because expected product demand is quite dynamic. This result says that we only need capacity roughly in balance with expected demand to get sales and utilization benefits from adding flexibility.

Figure 4 also shows that adding flexibility can be substituted for changing capacity to reach sales and capacity utilization goals. For example, with no flexibility and a total capacity of 880 units (point A), expected sales are 786 units. Expected sales could be increased to about 855 units either by adding total flexibility (point B) or by adding 12 units of capacity to each plant (point D). Similarly, capacity utilization goals could either be reached by changing flexibility or changing capacity.

Example

This section presents an example based on a real set of vehicles and assembly plants. The specific numbers used in this study have been adjusted to protect proprietary data, but the magnitudes of product demands and uncertainty levels are typical of those for automobiles at the nameplate level (e.g., Chevrolet Camaro). Also, the plant capacities used are representative of automobile assembly plants.

Figure 4 Impact of Capacity Changes on Benefits of Flexibility

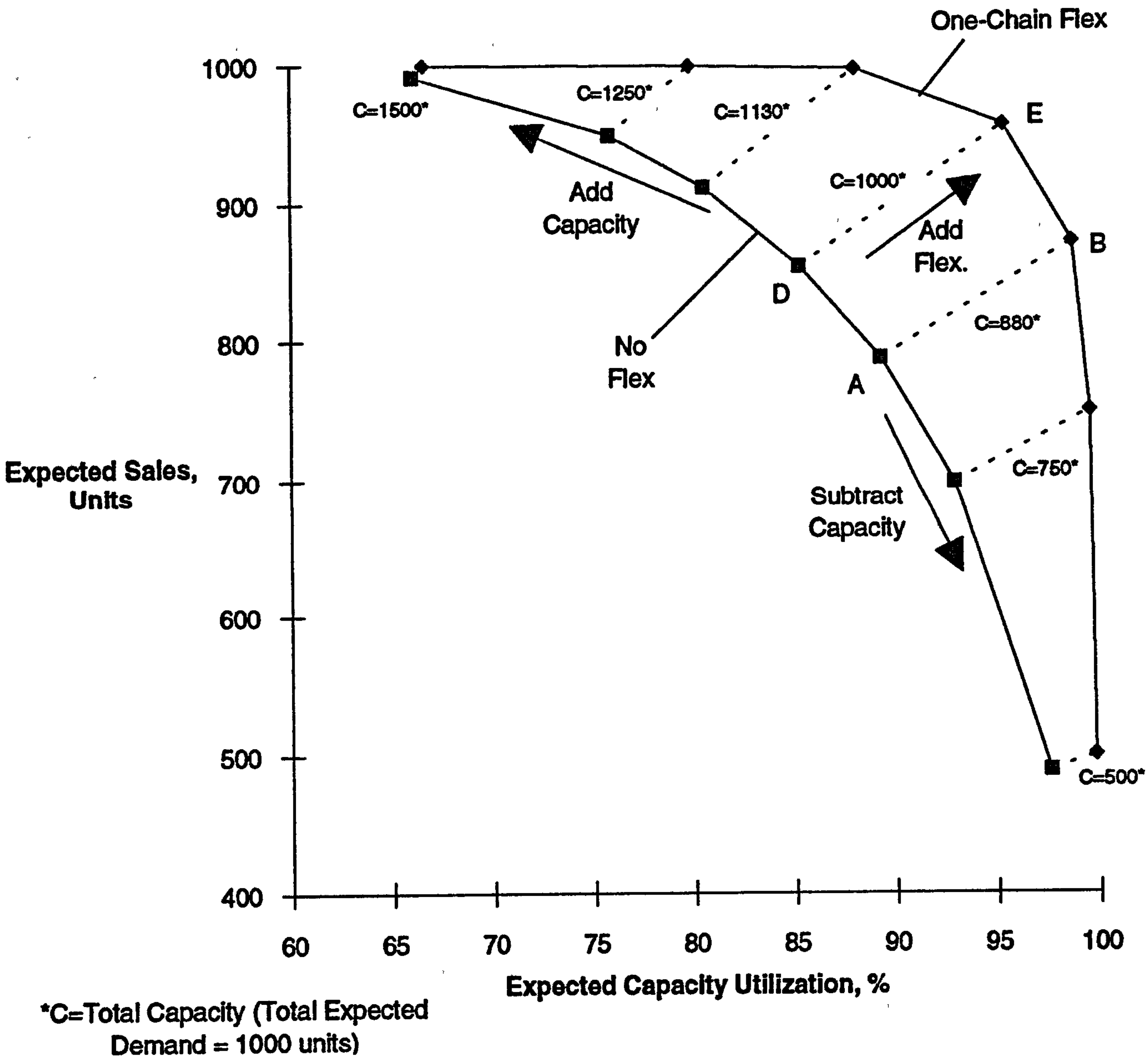
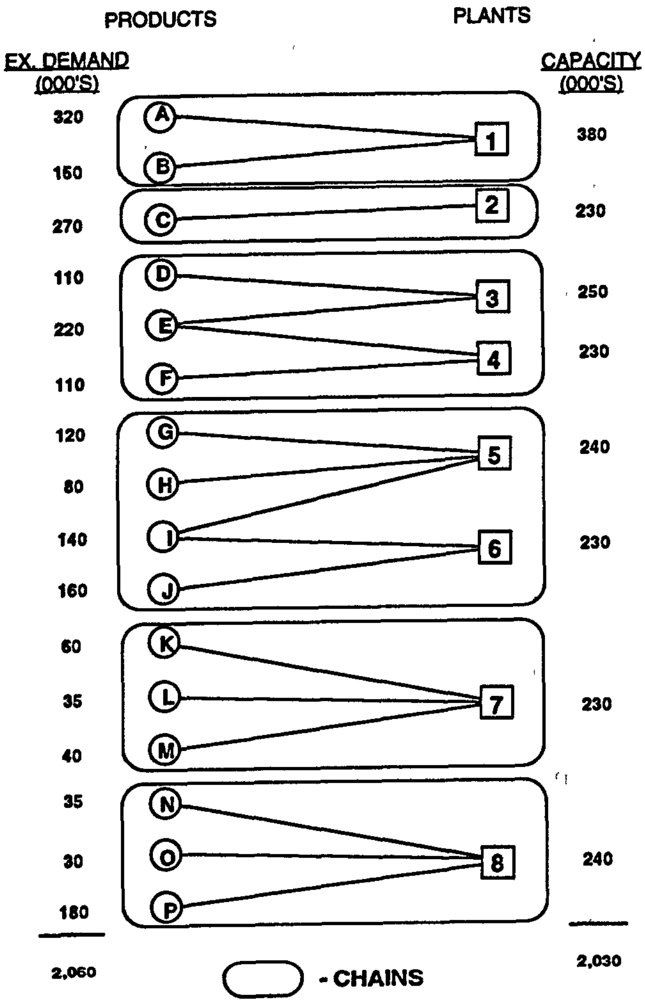


Figure 5 shows the plants and products used in this example. It also shows expected demand for each product, plant capacities, planned product assignments, and the resulting product-plant chains. We assume that product demands are truncated ($\pm 2\sigma$) normally distributed random variables with a standard deviation, σ, equal to 40% of expected demand. In this example, the sixteen vehicles considered fall into three groups: products A to F (compact cars), products G to M (full-sized cars), and products N to P (luxury cars). We assume that product demands for pairs of vehicles within the same group are positively correlated with a coefficient of 0.3; all intergroup pairs of products have coefficients of 0.0. For plants building multiple products, we assume that each product's tooling capacity (i.e., the capacity for each individual product within the plant) is 80% of the plant capacity.

With these data, the simulation model was used to

Figure 5 Example Products, Plants, Chains, Base Assignments, Expected Demands, and Capacities



calculate how these product assignments, and changes to them, impact expected sales and capacity utilization. For the base assignments, expected sales and capacity utilization are 1.76 million units and 86.6%. Having total flexibility at each plant would increase expected sales by 9.1% to 1.92 million units and capacity utilization to 94.7%. To find out how much flexibility would have to be added to achieve the benefits of total flexibility, new product assignment links were added one-by-one to those in Figure 5. These added links chain the products and plants together, as shown in Figure 6. They were added based on the principles described above, and because, based on output from the simulation model, they connected products with high expected lost sales with under-utilized plants. The number by each new link shows the order in which they were added.

Table 2 shows how adding each new link affects expected sales and capacity utilization. After adding six links, 94% of the sales and utilization benefits of total flexibility (i.e., 94% of the difference between total flexibility and the base assignments) have been achieved.

There are many ways to add six links to the initial assignments in Figure 5 to create one chain. Not all have equal benefits. However, there are several plans that do have nearly all the benefits of total flexibility. Figure 6 shows one plan. An alternative plan, for instance, adding links (A-2), (A-3), (C-7), (D-6), (J-8), and (P-1) to Figure 5 yields virtually identical sales and

Figure 6 Product Assignment Links Added to Create One Chain

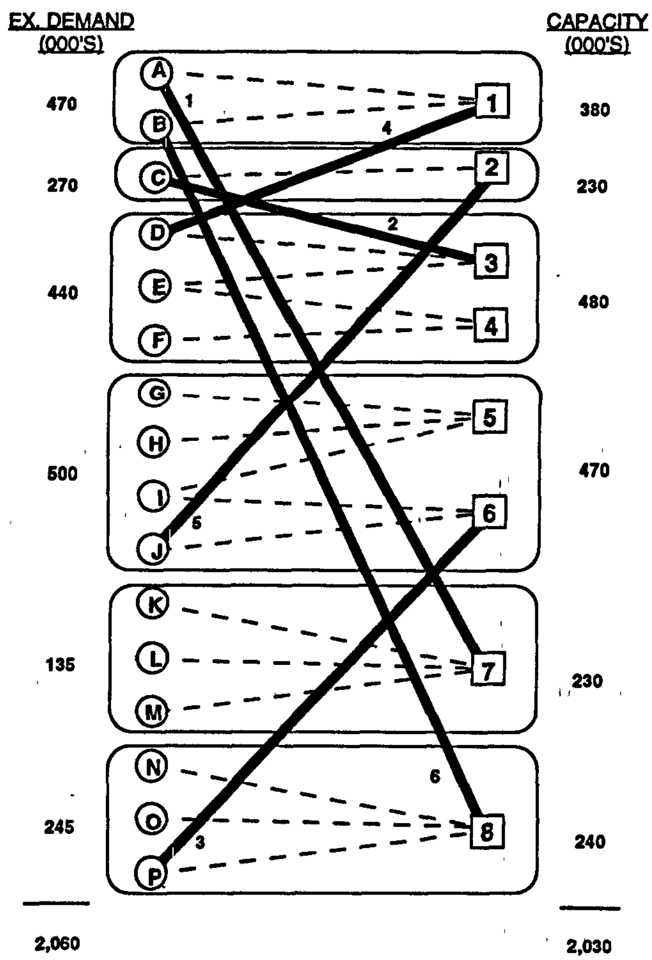


Table 2 **Impact of Adding Flexibility in Example**

Configuration	E[Sales]
Base Assignments	1.766 million units
Add 1 link	1.822
Add 2 links	1.842
Add 3 links	1.850
Add 4 links	1.862
Add 5 links	1.891
Add 6 links	1.914
total flexibility	1 923

utilization benefits. The key point is that there is not one "optimal" flexibility plan; rather there are many that are near-optimal.

Discussion

Part I has developed principles for how much flexibility is needed and where it should be added to respond most effectively to uncertain demand. These flexibility principles are:

(1) A small amount of flexibility added in the right way can have virtually all the benefits of total flexibility.

(2) The right way to add flexibility is to create fewer, longer plant-product chains.

(3) Once a plant-product chain has been created, a little more flexibility may be appropriate. This flexibility should be added in a way that better balances the assignment of products to plants, and/or that creates circuits. However, there are rapidly diminishing benefits to adding more flexibility within the chain.

(4) There is not a single flexibility plan which optimizes the benefits of flexibility.

Although developed and presented in terms of automobile assembly plants, these principles apply to all manufacturing operations in which there are parallel facilities producing multiple products.

In the introduction we stated that this research was intended to affect industry practice. In fact, it has had a substantial impact on the way capacity planning is done at GM. These principles are widely disseminated within GM and have had an impact for several reasons: 1) they contradict conventional thinking on flexibility, namely that you need to be able to build many or all products in the same plant with very expensive automated machinery to be flexible and 2) they are simple and graphic enough to be easily taught and remembered. For instance, "chaining" has become entrenched in the lexicon of GM manufacturing planning. We conjecture that a more complex model would have had nowhere near the impact. Complex models have their place, especially for guiding specific decisions. However, simple models—if focused on the right questions—can often reveal new principles that can greatly improve management decision-making. We feel this work illustrates this.

This paper is on the benefits of flexibility. However, its results imply something about the cost of flexibility. First, many people faced with investing in flexibility fear excessive costs because they assume that to be beneficial flexibility must mean that each plant will have to build many or all products. We have demonstrated here that this is not the case. Automotive manufacturers can get the benefits of total flexibility in vehicle assembly without increasing significantly the number of products each plant builds. For the example, building only two to four products in each plant (see Figure 6) provides almost all the benefits of total flexibility at a fraction of the cost.

Second, people may fear that adding flexibility will be expensive because it means that each plant will have to build extremely diverse products. Again, the results demonstrated here show that this is not necessary. The principle of chaining implies that the benefits of total flexibility can be realized by building fairly similar products in most plants. That is, becoming flexible does not require that all plants build, for example, cars and trucks. Rather, the benefits of total flexibility can be achieved by chaining with most plants building either cars or trucks and only one or two plants building both. It is creating longer product-plant chains that results in flexibility benefits, not building diverse products in a single plant.

Third, costs may be lower than anticipated because there is not one optimal product assignment that must be implemented to get the benefits of flexibility. This means that there are opportunities for choosing among "good" flexibility plans to minimize investment and manufacturing costs or satisfy other objectives.

These flexibility principles also imply how flexibility and capacity planning should be performed. Often flexibility is planned by focusing exclusively on products with negatively correlated demand. Even if we could identify such products, this approach will miss most of the benefits that flexibility has to offer. To obtain the benefits of flexibility, we only need negatively correlated products in the same chain, not in the same plant. This does not mean that a manufacturer should necessarily create one single chain encompassing all its plants and products. Deciding how much manufacturing flexibility to invest in requires an analysis of both the benefits and costs. However, since the benefits of flexibility come from creating long chains of products and plants, capacity planning should encompass as many plants and products as possible.

This paper looks at flexibility in assembly manufacturing networks. To take full advantage of this assembly flexibility, you may need to add flexibility (or extra capacity) in sub-assembly or component (e.g., engines, sheet metal) production. Further research is addressing flexibility in more complex production networks.

Part II. Analytical Justification

In Part II, we provide some analytical justification for the findings of Part I. We first develop a model and use it to show that limited flexibility (i.e., chaining) can provide the benefits of total flexibility. Then we demonstrate the benefits of combining chains.

Model for Demonstrating the Benefits of Limited Flexibility

Suppose we have n plants available to produce m products. Assume that we are given the annual production capacity of each plant, and we need to decide how to configure or tool each plant; that is, on an annual basis we determine which plants have the capability to produce which products over the coming year. The annual demand for each product is a random variable. For a given plant configuration (i.e., a specification of which products are produced at which plants), we want to project how the demand will be met over the course of the year as it is realized. We assume that demand that cannot be met due to capacity constraints is lost. The basis for comparison between different plant configurations is the expected amount of demand that can be met by the set of production facilities. Presumably, greater levels of flexibility will result in a larger percentage of product demand being satisfied.

We first state the general model for evaluating a given configuration. We then develop the argument to show that limited flexibility can provide the same level of benefits as total flexibility.

We denote a configuration by a set of ordered pairs, call it A, where $(i, j) \in A$ indicates that plant j can produce product i. We find it useful to think of a configuration as a bipartite graph with $m + n$ nodes, one node for each product and one node for each plant, and with arc set given by A. We assume that we can express the demands and capacities in common units so that one unit of capacity at plant j is required to produce one unit of product i, for any product i such that $(i, j) \in A$.

We evaluate a configuration A in terms of the expected amount of demand that can be met by the plants, or equivalently in terms of the expected demand shortfall that cannot be covered by the plants. For evaluating the expected demand shortfall for configuration A, we assume that the actual volume that each plant builds of each product assigned to it is such that the total unmet demand is minimized. In effect, we assume that we first observe the realizations of the demand random variables and then we determine how much of each product is produced by each plant. (This is an approximation since in practice one must allocate production capacity to the products in real time as the demand is realized.) If we know the demand realization for product i, call it d_i, for all products, then we find the minimum shortfall for A, $V(A)$, by solving the following linear program:

$$V(A) = \min \sum_{i=1}^{m} s_i$$

s.t.

$$\sum_{(i,j)\in A} x_{ij} + s_i \geq d_i \quad \forall i = 1, 2, \cdots m,$$

$$\sum_{(i,j)\in A} x_{ij} \leq c_j \quad \forall j = 1, 2, \cdots n. \qquad [P1]$$

The decision variables x_{ij} denote the amount of demand for product i supplied by plant j, and are defined only if plant j is configured to produce product i; s_i is the amount of demand for product i that cannot be satisfied, i.e., its shortfall; and c_j is the capacity at plant j. There are two sets of constraints, the first represents the demand requirements and defines the shortfalls, and the second enforces the plant capacities. The objective is to minimize the sum of the shortfalls. Implicitly we have assumed that all shortfalls are equally costly, but that can be easily changed by weighting the shortfalls.

In Graves and Jordan (1991) we show that the optimal objective value for [P1] is equal to

$$V(A) = \max_{M} \left\{ \sum_{i \in M} d_i - \sum_{j \in P(M)} c_j \right\}. \tag{1}$$

The maximization in (1) is over all subsets M (including the null set) of the index set $\{1, 2, \cdots m\}$. That is, the maximization is over all subsets of the products. For any given subset of products M, $P(M)$ is the subset of plants that can produce at least one of the products in M. Thus, $j \in P(M)$ if and only if there is at least one $i \in M$ such that $(i, j) \in A$. Each term within the maximization in (1) is the difference between the demand for some subset of products and the maximum capacity available to make that subset of products.

To understand the derivation of (1), we observe that [P1] is equivalent to a maximum flow network problem. Rather than minimize the demand shortfall, we can restate the objective as maximizing the amount of demand met, since the shortfall equals the difference between the total demand and the amount of demand met by production. Thus, the objective can be stated as maximizing the amount of flow from plants to products. Then we can develop an interpretation of (1) as being the minimum cut for the network (see Lawler 1976).

To determine the expected shortfall for a configuration, we take the expectation of (1) over the possible realizations of the demands $\{d_1, d_2, \cdots d_m\}$:

$$E[V(A)] = E\left[\max_{M} \left\{ \sum_{i \in M} D_i - \sum_{j \in P(M)} c_j \right\} \right] \tag{2}$$

where D_i denotes the random variable for demand for product i. In general, this cannot be easily evaluated. However, we will use (2) to analyze special cases and to draw general inferences, as discussed below.

General Argument. We wish to compare the expected demand shortfall for total flexibility with that for limited flexibility. We will assume that product demands are independent random variables with demand for product i being $N(\mu_i, \sigma_i^2)$. Total flexibility corresponds to each plant having the ability to manufacture all m of the products. From (2) we can easily show that the expected shortfall for total flexibility is given by

$$ES_{tf} = E\left[\max\left\{ 0, \sum_{i=1}^{m} D_i - \sum_{j=1}^{n} c_j \right\} \right]. \tag{3}$$

Limited flexibility corresponds to having a configuration A that is more restrictive than total flexibility. The expected shortfall is given in general by (2) for any given A. We wish to show that the expected shortfall for limited flexibility, as discussed in Part I, can be approximately equal to ES_{tf}. It is very difficult to make a direct comparison of $E[V(A)]$ given by (2) to ES_{tf} given by (3). Rather than do this, we will examine the probability that the shortfall for limited flexibility exceeds the shortfall for total flexibility:

$$\Pr\left[\max_{M} \left\{ \sum_{i \in M} D_i - \sum_{j \in P(M)} c_j \right\} > \max\left\{ 0, \sum_{i=1}^{m} D_i - \sum_{j=1}^{n} c_j \right\} \right]. \tag{4}$$

If this probability is small, then we contend that $E[V(A)]$ is approximately equal to ES_{tf}.

To argue that (4) is small, we focus on the probability that the shortfall for a given subset of products M can exceed the shortfall for total flexibility; that is, we will argue that for any subset M

$$\Pi(M) = \Pr\left[\left\{ \sum_{i \in M} D_i - \sum_{j \in P(M)} c_j \right\} > \max\left\{ 0, \sum_{i=1}^{m} D_i - \sum_{j=1}^{n} c_j \right\} \right] \tag{5}$$

is small. In particular, we will focus on M that maximizes this probability, call it M^*, and then show that this probability, $\Pi(M^*)$, is small. If this is true then we expect (4) to be small.

To evaluate (5), we let

$$a = \sum_{i \in M} D_i - \sum_{j \in P(M)} c_j \quad \text{and} \quad b = \sum_{i=1}^{m} D_i - \sum_{j=1}^{n} c_j - a.$$

For a given set of products M, a is the difference between demand for the products in M and the maximum capacity available to produce these products; b is the difference between the demand for the products in the complement of M and the remaining capacity.

Since we have assumed that each product demand is a normally-distributed independent random variable, we find that:

$$a \text{ is } N\left(\sum_{i \in M} \mu_i - \sum_{j \in P(M)} c_j, \sum_{i \in M} \sigma_i^2\right) \text{ and}$$

$$b \text{ is } N\left(\sum_{i \notin M} \mu_i - \sum_{j \notin P(M)} c_j, \sum_{i \notin M} \sigma_i^2\right).$$

Furthermore, a and b are independent. With these observations, we can now reexpress (5) as follows:

$$\begin{aligned} \Pi(M) &= \Pr\{a > \max(0, a + b)\} \\ &= \Pr\{0 > \max(-a, b)\} \\ &= \Pr\{0 < a\} \Pr\{0 > b\} \\ &= [1 - \Phi(z_1)]\Phi(z_2) \end{aligned} \tag{6}$$

where $z_1 = -E[a]/\sigma[a]$, $z_2 = -E[b]/\sigma[b]$, $\Phi(z)$ is the cumulative distribution function for a standard normal random variable, and $E[\ ]$ and $\sigma[\ ]$ signify the mean and standard deviation for a random variable, respectively.

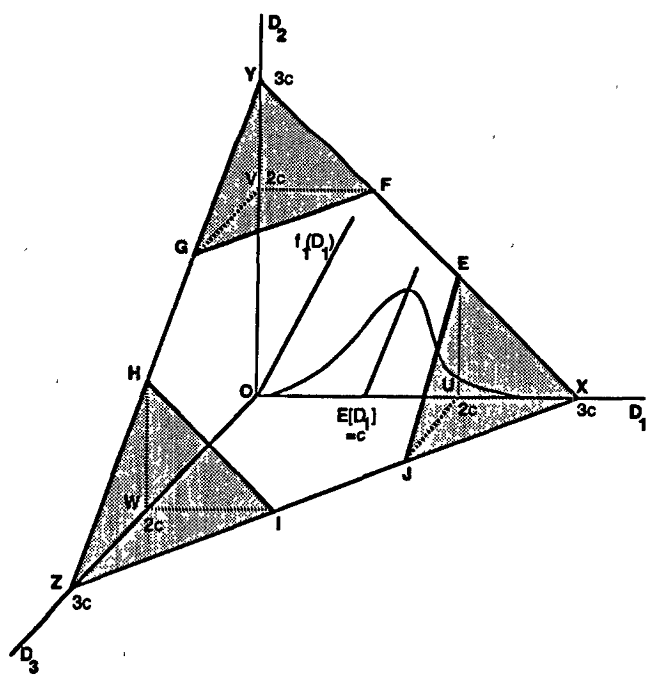
From (6) we can interpret $\Pi(M)$ as the probability of having unfilled demand for the set of products M (i.e., $\Pr\{0 < a\}$), while simultaneously having excess capacity in plants that don't build any of the products in M (i.e., $\Pr\{0 > b\}$). If $\Pi(M)$ is large, then there would seem to be an opportunity to reduce the expected demand shortfall by adding more flexibility to enable the excess capacity to be applied to products in M. If $\Pi(M)$ is small, then there is little benefit from adding additional flexibility to produce the products in M.

Three-product, Three-plant Example. Suppose that we have three products and three plants. We want to show that limited flexibility provides virtually the same expected sales benefits as total flexibility. Total flexibility is when each plant can produce all three products. Limited flexibility corresponds to each plant being able to produce two products and each product being produced by two plants; we assume that plant 1 produces products 1 and 2, plant 2 produces products 2 and 3, and plant 3 produces products 1 and 3. For this example we will also assume that each plant has the same capacity ($c_i = c$ for $i = 1, 2, 3$) and that the product demands are identically and normally distributed random variables, each with mean μ and standard deviation σ.

Our approach for this can be visualized using Figure 7, a three-dimensional graph with axes plotting the demand for each product. The graph shows the production capabilities of both total flexibility and limited flexibility. With total flexibility we can produce up to $3c$ units of any combination of products 1, 2, or 3. Hence, we can satisfy any mix of product demands whose coordinates are in the region XYZ0. With limited flexibility, again we can produce up to $3c$ units in total, but we are restricted to producing no more than $2c$ of any particular product. Limited flexibility restricts production to the smaller region EFGHIJUVW0 (i.e., all but the shaded regions). We term the regions XYZ0 and EFGHIJUVW0 to be the production possibilities region for total and limited flexibility, respectively.

Superimposed on these production possibilities regions we have a multivariate distribution of demand. For example, Figure 7 shows the probability distribution for product 1 demand. Similar distributions could be drawn for the other products. The analysis that follows shows that for reasonable demand distributions and with capacity roughly balanced with demand, the

Figure 7 Production Possibilities Graph for Limited and Total Flexibility



probability that demand falls within the production possibilities region for total flexibility, but outside the region for limited flexibility, is small.

This latter statement equates to showing that (4) is small. Since this statement is approximate in nature, we cannot give a rigorous demonstration; rather, we try to provide a compelling and intuitive argument for the plausibility of the general finding. Our approach is to argue that $\Pi(M)$, given by (5), is small for all choices for M.

For this example, it is easy to see that $\Pi(M) = 0$ for all M except when $M = \{1\}$ or $M = \{2\}$ or $M = \{3\}$. Consider $\Pi(M)$ for $M = \{1\}$; the other two cases are similar. We can rewrite (5) as:

$$\Pi(M) = \Pr\left[\{D_1 - 2c\} > \max\left\{0, \sum_{i=1}^{3} D_i - 3c\right\}\right]. \quad (7)$$

By rearranging terms and assuming that product demands are independent, we can simplify (7) as

$$\Pi(M) = \Pr[0 > -D_1 + 2c]\Pr[0 > D_2 + D_3 - c]. \quad (8)$$

From this expression we see that $\Pi(M)$ corresponds to the probability that demand falls in the shaded region EJUX in Figure 7. Now it is easy to evaluate (8) in terms of the cumulative distribution function $\Phi(z)$ for a standard normal random variable:

$$\Pi(M) = [1 - \Phi(z_1)]\Phi(z_2) \quad (9)$$

where $z_1 = (2c - \mu)/\sigma$ and $z_2 = (c - 2\mu)/\sigma\sqrt{2}$.

To make the argument that $\Pi(M)$ is small, we evaluate (9) for typical parameter values. We expect that c, the average plant capacity, would approximately equal the mean product demand μ. With this assumption we can reexpress $z_1 = \mu/\sigma$ and $z_2 = -\mu/\sigma\sqrt{2}$. For a typical vehicle at the nameplate level, the ratio μ/σ is at least 2; for $\mu/\sigma = 2$, $\Pi(M) = (0.023)(0.079) = 0.0018$. For larger ratios of μ/σ, this probability is smaller. For average plant capacity greater than mean product demand ($c > \mu$), this probability is smaller. We thus conclude that for typical parameters the probability $\Pi(M)$ for $M = \{1\}$ is small.

A similar development can be made for the cases for $M = \{2\}$ and $M = \{3\}$, and will correspond to evaluating the probability that demand falls in the regions FGVY and HIWZ in Figure 7, respectively. Indeed, when $c = \mu$, each of these probabilities will also be equal to 0.0018. Thus, for this case, the probability that the shortfall for limited flexibility exceeds the shortfall for total flexibility, as given by (4), is 3*(0.0018) = 0.0054.

This completes the example and the demonstration that limited flexibility (two products per plant) provides virtually the same performance as total flexibility (three products per plant). To show the generality of this result, we next discuss how this analytic justification extends to cases with more than three products and plants, to cases with non-identical products and non-equal-sized plants.

n Identical Products and n Identical Plants. Suppose that we have n products and n plants, where each plant has the same capacity c and product demands are i.i.d. random variables with each being $N(\mu, \sigma^2)$. We wish to compare the expected demand shortfall for total flexibility with that for limited flexibility. The expected shortfall for total flexibility, in which each plant can produce all n products, is given by (3).

For limited flexibility, we will only consider configurations where all plants build the same number of products, call it h, and each product is built in the same number of plants, namely h. We will only consider cases where products are built in up to one-half of the plants: $2 \le h < n/2$.

We consider only configurations of the form shown in Figure 2a for $n = 10$ and $h = 2$. That is, we assume that product i can be produced by h plants: plants i to $i + h - l$ for $i = 1, 2, \cdots n - h + l$, and plants i to n and l to $i + h - l - n$ for $i = n + 2 - h, \cdots n$. After eliminating dominated terms in (4), we can restrict the maximization to subsets of consecutive integers, namely subsets of the form $M = \{i, i + l, \cdots j\}$ where we assume circular indexing if $i > j$. For such sets of consecutive products, the set of plants that can produce these products, $P(M)$, is also a consecutive set, namely $\{i, i + l, \cdots j + h - l\}$. So, for example, in Figure 2a, if we consider the subset $M = \{8, 9, 10, 1, 2\}$, then $P(M) = \{8, 9, 10, 1, 2, 3\}$. The argument will be to show that $\Pi(M)$ given in (5) is small for any (consecutive) product subset M.

At this point the argument follows that developed earlier for the general case. To argue that (4) is small we will show that $\Pi(M)$ given by (5) is small for all sets M. Let $M = \{i, \ldots, j\}$, and let $k = |M|$. Paralleling the development of (6), we define

$$a = D_i + \cdots + D_j - (c_i + \cdots + c_{j+h-l})$$
$$= D_i + \cdots + D_j - (k + h - l)c,$$
$$b = D_l + \cdots + D_n - nc - a.$$

Thus, a and b are independent random variables: a is $N(k\mu - (k + h - l)c, k\sigma^2)$ and b is

$$N((n - k)\mu - (n - k - h + l)c, (n - k)\sigma^2).$$

We can now restate (6) to evaluate $\Pi(M)$:

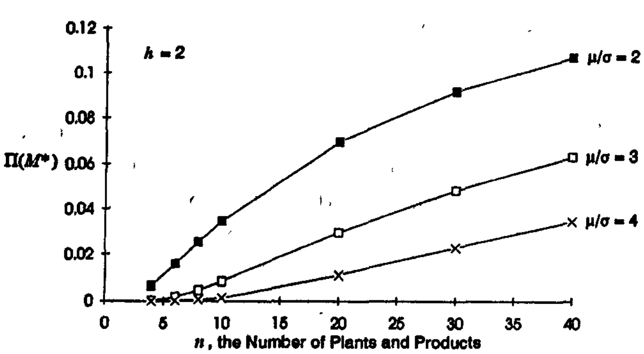
$$\Pi(M) = [1 - \Phi(z_1)]\Phi(z_2) \quad \text{where}$$
$$z_1 = -[k\mu - (k + h - 1)c]/\sigma\sqrt{k} \quad \text{and}$$
$$z_2 = -[(n - k)\mu - (n - k - h + 1)c]/\sigma\sqrt{n - k}. \quad (10)$$

To judge the strength of the argument, we need to evaluate $\Pi(M)$ for typical parameters. We assume that plant capacity equals expected product demand $c = \mu$. To show that $\Pi(M)$ is small for all M, we consider the set M^* that maximizes $\Pi(M)$. From (10) we can show that M^* is any product subset that contains one-half of the products. So, $k = |M^*| = n/2$, and $z_1 = (h - 1)\mu/\sigma\sqrt{0.5n} = -z_2$.

Consider now the case when each product is built in two plants: $h = 2$. With $\mu/\sigma = 2$ and $n = 10$, $\Pi(M^*) = 0.0346$. Though not as small as in the three product/three plant case, this value for $\Pi(M^*)$ indicates that there is little chance that adding flexibility beyond this limited amount will increase sales.

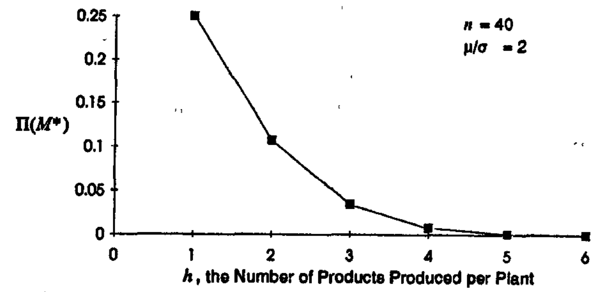
Figure 8 shows how $\Pi(M^*)$ varies with μ/σ and n for $h = 2$. We see that $\Pi(M^*)$ increases with greater demand uncertainty (lower values of μ/σ) and with more products or plants (larger n). This suggests that for high levels of demand uncertainty and many products and plants limited flexibility with only two products per plant ($h = 2$) may not provide the same ability to respond to demand changes as total flexibility.

Figure 8 Flexibility Measure vs. Number of Plants and Products



We also investigate configurations with more products per plant. Figure 9 shows how changing the number of products per plant from 1 to 6 affects $\Pi(M^*)$ for $n = 40$ and $\mu/\sigma = 2$. Going from $h = 2$ to $h = 3$ reduces $\Pi(M^*)$ from about 0.11 to 0.03; increasing h beyond this point drives $\Pi(M^*)$ to zero. Even with high demand uncertainty and many products and plants, limited flexibility with only three or four products per plant can provide nearly all of the benefits of total flexibility.

Figure 9 Flexibility Measure vs. Products per Plant



To make this point further, we compare the expected sales for limited and total flexibility using the simulation model. Table 3 shows the results for cases with 10 and 40 products and plants, and confirms that limited flexibility with very small numbers of products per plant can have the same sales benefits as total flexibility.

Table 3 Expected Sales and $\Pi(M^*)$ for Different Levels of Flexibility

($\mu/\sigma = 2$ for all cases)

n	h	$\Pi(M^*)$	Expected Sales
40	1 (no flexibility)	0.25	3283
40	2	0.11	3767
40	3	0.034	3870
40	4	0.0081	3909
40	40 (total flex.)	–	3922
10	1 (no flex.)	0.25	822
10	2	0.034	941
10	3	0.0014	954
10	4	0.000013	955
10	10 (total flex.)	–	955

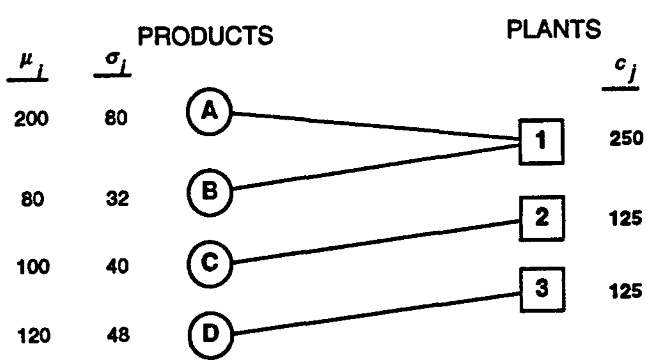
Moreover, the table shows how well $\Pi(M^*)$ reflects the expected sales for a configuration: when $\Pi(M^*)$ reaches about 0.03, expected sales for the configuration is at about 99% of the sales level with total flexibility. This is useful because $\Pi(M^*)$ is much easier to compute than expected sales, which must be computed by simulation for limited flexibility configurations. Thus, we use $\Pi(M^*)$ as a practical measure of a configuration's flexibility.

***m* Nonidentical Products and *n* Nonidentical Plants.** Suppose that we have m products and n plants, where plant j has capacity c_j and demand for product i is an independent normal random variable with mean μ_i and variance σ_i^2. We wish to compare the expected demand shortfall for total flexibility with that for limited flexibility. We will do this with a single example, since the general case does not lend itself to a more systematic analysis.

Consider the example in Figure 10 with four products and three plants. The figure provides the mean and standard deviation for demand for each product, the capacity for each plant, and an assignment of products to each plant. This assignment has little flexibility, in that each product is produced at only one plant.

We use (6) to compute $\Pi(M)$ for all subsets of products for the configuration shown in Figure 10. As shown in the figure, $M^* = \{A, B\}$ and $\Pi(M^*) = 0.436$. This large value for $\Pi(M^*)$ indicates a good chance that there will be both unfilled demand for products A and B and excess capacity in plants 2 and 3. Hence, there is likely to be substantial sales benefits from building one of the M^* products in either plant 2 or 3. Suppose we provide plant 2 with the capability to produce product A. In Table 4, we show the results for adding this product-plant link. Since $\Pi(M^*)$ is still fairly large, we add additional product-plant links as described in the table. After adding three links, $\Pi(M^*)$ has dropped to 0.0087, signifying that there is little to gain from adding more flexibility. We confirm these results using the simulation model to calculate the expected sales for each of the configurations in the table.

Figure 10 **Three-Plant, Four-Product Example**



$M^*=\{A,B\}$, $P(M)=\{1\}$, $\Pi(M^*)=0.436$

Table 4 **Simulation Analysis for 3-Plant, 4-Product Example**

Configuration	$\Pi(M^*)$	Expected Sales
Base (Figure 10)	0.436	426
Add link (*A*-2)	0 282	443
Add links (*A*-2) & (*C*-3)	0 220	451
Add links (*A*-2), (*C*-3) & (*D*-1)	0.0087	458
Total Flexibility	–	459

This example illustrates how to use $\Pi(M^*)$ to construct a good configuration with limited flexibility. If $\Pi(M^*)$ is large, then the product set M^* and the plants that build M^*, $\Pi(M^*)$, effectively are a bottleneck on improving sales. To improve the sales performance of the configuration, we need to relax the bottleneck. We do this by adding a link to the configuration that connects one of the products in M^* to a plant outside of $\Pi(M^*)$.

Furthermore, this example supports our conjecture that $\Pi(M^*)$ is a useful measure of flexibility. There is little value from adding more product-plant links to configurations with low values of $\Pi(M^*)$, e.g., less than 0.05; when $\Pi(M^*)$ is suitably small, then we expect that the expected sales for the limited flexibility configuration will nearly equal that for total flexibility. When $\Pi(M^*)$ is large, though, there remains opportunity to improve the configuration by adding more flexibility according to the rules described above.

This example and discussion also begin to provide some intuition for the value of chaining, as described in Part I. The example starts with a configuration with three chains, and then adds links one at a time to reduce the number of chains. After adding two links the configuration is connected and all of the products and plants are chained together. However, there is still improvement possible from adding one more link that effectively closes the chain. Thus, we need to use not just $\Pi(M^*)$, but also the principles of chaining to construct the best candidate configurations.

A Model of the Benefits of Chaining

Consider m products that are manufactured in n plants. The demand for any product i is a random variable and demands for all products are identically normally distributed with mean μ and standard deviation σ. Demands are assumed to be correlated with the same correlation coefficient ρ between any pair of product demands. Also assume that all plants have identical capacity, c.

Suppose the products and plants are divided into K chains. Assume that the numbers of plants and products per chain are continuous variables:

α = number of products/chain = m/K, and

β = number of plants/chain = n/K.

While unrealistic, this assumption greatly simplifies the analysis and makes little difference in the accuracy of the equations developed below. Also, let

D_j = the demand for all products in chain j, and

S_j = the sales of all products in chain j.

Assume that within each chain product assignment decisions have added enough flexibility to yield the benefits of total flexibility (we have seen that this requires only a few product-plant links beyond those needed to create the chain). With this assumption then

$$S_j = \min(\beta c, D_j). \tag{11}$$

Since D_j is the sum of α normally distributed random variables, it is also normally distributed with parameters:

$$E[D_j] = \alpha\mu, \tag{12}$$

$$V[D_j] = \alpha\sigma^2 + \alpha(\alpha - 1)\rho\sigma^2$$
$$= \sigma^2(\alpha^2\rho + \alpha(1 - \rho)). \tag{13}$$

Let $\phi(\cdot)$ and $\Phi(\cdot)$ be the standard normal density and cumulative distribution functions, respectively, for D_f.[1] Then

$$E[S_j] = \int_{-\infty}^{\beta c} x\phi((x - E[D_j])/\sqrt{V[D_j]})dx + \beta c(1 - \Phi(z)) \tag{14}$$

where

$$z = (\beta c - E[D_j])/\sqrt{V[D_j]}. \tag{15}$$

From Winkler et al. (1972), evaluating the integral yields

$$E[S_j] = \alpha\mu\Phi(z) - \sigma\sqrt{\alpha^2\rho + \alpha(1 - \rho)}\,\phi(z) + \beta c[1 - \Phi(z)]. \tag{16}$$

Let T be the total sales for all products, i.e., $T = \sum_j S_j$. Then

$$E[T] = m\mu\Phi(z) - \sigma\sqrt{m^2\rho + Km(1 - \rho)}\,\phi(z) + nc[1 - \Phi(z)] \tag{17}$$

where

$$z = (\beta c - \alpha\mu)/\sigma\sqrt{\alpha^2\rho + \alpha(1 - \rho)}$$
$$= (nc - m\mu)/\sigma\sqrt{m^2\rho + Km(1 - \rho)}. \tag{18}$$

Equations (17) and (18) show that total sales is a rather complex function of K, the number of chains. However, in specific cases the equations simplify significantly. In particular, suppose that total capacity balances total expected demand ($m\mu = nc$). In this case, $z = 0$, $\phi(z) = 0.3989$, and $\Phi(z) = 0.5$. Then (17) becomes

$$E[T] = m\mu - 0.3989\sqrt{m^2\rho + Km(1 - \rho)}. \tag{19}$$

As this equation shows, when capacity balances expected demand, $E[T]$ is convex in K. That is, there are increasing returns to expected sales from reducing the number of chains. However, this increase, as well as the overall benefits of flexibility, is reduced as product demand correlation increases.[2]

[1] Equation (11) and the equations that follow assume that the probability that $D_j < 0$ is small.

[2] We would like to thank Larry Burns, Bob Glubzinski, Al Steurer, and Rick Robbins for valuable discussions about capacity planning and manufacturing flexibility that contributed greatly to this research. We greatly appreciate technical comments on earlier versions of the paper by Bob Inman and Rakesh Vohra. Finally, we thank Joe Thomas for editorial help that has greatly improved the paper.

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