3.6 Cargo network scheduling

Due to Mulvey and Ruszczyński [20]
(Two-stage, mixed integer linear or nonlinear stochastic problem)

$$\texttt{/cargo/4node.cor},\ \texttt{/4node.tim},\ \begin{cases}\texttt{/4node\_det.sto}\\ \texttt{/4node.sto.16}\\ \texttt{/4node.sto.32}\\ \texttt{/4node.sto.64}\end{cases}$$

3.6.1 Description

Mulvey and Ruszczyński [20] provide a two stage network problem for scheduling cargo transportation. The flight schedule is completely determined in stage one, and the amounts of cargo to be shipped are uncertain. The recourse actions are to determine which cargo to place on which flights. Transshipment, getting cargo from node m to node n by more than one flight on more than one route, is allowed. When a transshipment is made, cargo must be unloaded at some intermediate node, so that it may be loaded onto a different route going through the same node. Such nodes are called transshipment nodes. Any undelivered cargo costs a penalty.

The notation is introduced in Table 18. A route $\pi \in \mathcal{P}$ is a finite sequence of nodes $(n_1, n_2, \ldots, n_l)$ to be visited in the course of flying the route.

Table 18: Notation

$\mathcal{N}$	=the set of nodes
$\mathcal{P}$	=the set of routes
$\mathcal{A}$	=the set of aircraft types
$\boldsymbol{b}(m,n)$	=the amount of cargo to be shipped from node m to node n
$c(a)$	=the cost of an hour of flight time for aircraft type a
$h(\pi,a)$	=flight hours required for aircraft type a to complete route π
q	=the unit cargo cost for loading and unloading an aircraft
ρ	=the unit penalty for undelivered cargo
$v(\pi,j)$	=function which returns the jth node in route π
$l(\pi)$	=function which returns the number of nodes in route π
$\sigma(n)$	=the maximum number of landings allowed in node n
$d(a)$	=the maximum payload of an aircraft of type a
	(continued on the next page)

Notation (continued)	
$f(m,n)$	= the minimum number of flights from node m to node n
$h^{\max}(a)$	= the maximum flying hours for aircraft of type a
$h^{\min}(a)$	= the minimum flying hours for aircraft of type a
$x(\pi,a)$	= the number of aircraft of type a assigned to fly route π
$d(\pi,m,n)$	= the amount of cargo delivered directly from m to n on route π
$t(\pi,m,k,n)$	= the amount of cargo moving from m to n which is moved to transshipment node k on route π
$s(\pi,k,n)$	= the amount of transshipment cargo which is moved from transshipment node k to node n on route π
$y(m,n)$	= the amount of cargo moving from m to n which is undelivered
$z(\pi,j)$	= the unused capacity of leg j on route π
$U(m,n)$	= $\{\pi \in \mathcal{P} : m = v(\pi,j_1), n = v(\pi,j_2), j_1 < j_2\}$
$V_1(n)$	= $\{\pi \in \mathcal{P} : n = v(\pi,1)\}$
$V_l(n)$	= $\{\pi \in \mathcal{P} : n = v(\pi,l(\pi))\}$
$W(n)$	= $\{\pi \in \mathcal{P} : n = v(\pi,j)$ for some $j\}$

The first stage constraints include minimum flight requirements

$$\sum_{a\in\mathcal{A}} \sum_{\pi\in U(m,n)} x(\pi,a) \geq f(m,n), \quad \forall m,n \in \mathcal{N},$$

and maximum landings limits

$$\sum_{a\in\mathcal{A}} \sum_{\pi\in W(n)} x(\pi,a) \leq \sigma(n), \quad \forall n \in \mathcal{N}.$$

Assuming the operation is cyclic, we must end the round in the same state as that in which we began the round. That is,

$$\sum_{\pi\in V_1(n)} x(\pi,a) = \sum_{\pi\in V_l(n)} x(\pi,a), \quad \forall a \in \mathcal{A}, n \in \mathcal{N}.$$

Flying hours are limited by

$$h^{\min}(a) \leq \sum_{\pi\in\mathcal{P}} x(\pi,a)h(\pi,a) \leq h^{\max}(a), \quad \forall a \in \mathcal{A}.$$

For recourse constraints, a cargo material balance yields

$$\sum_{\pi \in \mathcal{P}} \left(d(\pi, m, n) + \sum_{k \in \mathcal{N}} t(\pi, m, k, n) \right) + y(m, n) \geq \boldsymbol{b(m, n)}, \quad \forall m, n \in \mathcal{N}.$$

A balance of all transshipments which go through k and wind up at n gives

$$\sum_{\pi \in \mathcal{P}} \sum_{m \in \mathcal{N}} t(\pi, m, k, n) = \sum_{\pi \in \mathcal{P}} s(\pi, k, n), \quad \forall k, n \in \mathcal{N}.$$

Finally, consider the loading and unloading which must occur throughout the course of a single route. At the initial node, we have

$$\sum_{k \in \mathcal{N}} \left(d(\pi, v(\pi, 1), k) + s(\pi, v(\pi, 1), k) + \sum_{n \in \mathcal{N}} t(\pi, v(\pi, 1), k, n) \right)$$
$$= \sum_{a \in \mathcal{A}} d(a) x(\pi, a) - z(\pi, 1), \quad \forall \pi \in \mathcal{P}.$$

For the remaining nodes in the route, a payload balance yields

$$\sum_{k \in \mathcal{N}} \left(d(\pi, v(\pi, j), k) + s(\pi, v(\pi, j), k) + \sum_{n \in \mathcal{N}} t(\pi, v(\pi, j), k, n) \right)$$
$$- \sum_{k \in \mathcal{N}} \left(d(\pi, k, v(\pi, j)) + s(\pi, k, v(\pi, j)) + \sum_{n \in \mathcal{N}} t(\pi, k, v(\pi, j), n) \right)$$
$$= z(\pi, j-1) - z(\pi, j), \quad \forall \pi \in \mathcal{P}, j = 2, \ldots (l(\pi) - 1).$$

The objective is to minimize the costs and penalties. Mulvey and Ruszczyński [20] provide both a linear objective function

$$\text{minimize } Z_1 = \sum_{\pi \in \mathcal{P}} \sum_{a \in \mathcal{A}} c(a) h(\pi, a) x(\pi, a) +$$
$$\underset{\boldsymbol{b(m,n)}}{E} \left\{ q \sum_{\pi \in \mathcal{P}} \sum_{(m,n) \in \pi} \left[d(\pi, m, n) + s(\pi, m, n) + \sum_{k \in \mathcal{N}} t(\pi, m, n, k) \right] \right.$$
$$\left. + \rho \sum_{m \in \mathcal{N}} \sum_{n \in \mathcal{N}} y(m, n) \right\},$$

and a nonlinear objective function

$$\text{minimize } Z_2 = \sum_{\pi \in \mathcal{P}} \sum_{a \in \mathcal{A}} c(a) h(\pi, a) x(\pi, a) +$$
$$\underset{\boldsymbol{b(m,n)}}{E} \left\{ \Phi \left(q \sum_{\pi \in \mathcal{P}} \sum_{(m,n) \in \pi} \left[d(\pi, m, n) + s(\pi, m, n) + \sum_{k \in \mathcal{N}} t(\pi, m, n, k) \right] \right.\right.$$
$$\left.\left. + \rho \sum_{m \in \mathcal{N}} \sum_{n \in \mathcal{N}} y(m, n) \right) \right\},$$

where

$$\Phi(x) = \alpha \exp(\beta x). \tag{42}$$

3.6.2 Problem statement

Given Φ as either the identity function or as in (42), the problem is to

$$\text{minimize } Z = \sum_{\pi \in \mathcal{P}} \sum_{a \in \mathcal{A}} c(a) h(\pi, a) x(\pi, a) +$$
$$\underset{\boldsymbol{b(m,n)}}{E} \left\{ \Phi \left(q \sum_{\pi \in \mathcal{P}} \sum_{(m,n) \in \pi} \left[d(\pi, m, n) + s(\pi, m, n) + \sum_{k \in \mathcal{N}} t(\pi, m, n, k) \right] \right.\right.$$
$$\left.\left. + \rho \sum_{m \in \mathcal{N}} \sum_{n \in \mathcal{N}} y(m, n) \right) \right\},$$

subject to

$$\sum_{a \in \mathcal{A}} \sum_{\pi \in U(m,n)} x(\pi, a) \geq f(m, n), \quad \forall m, n \in \mathcal{N},$$
$$\sum_{a \in \mathcal{A}} \sum_{\pi \in W(n)} x(\pi, a) \leq \sigma(n), \quad \forall n \in \mathcal{N},$$
$$\sum_{\pi \in V_1(n)} x(\pi, a) = \sum_{\pi \in V_l(n)} x(\pi, a), \quad \forall a \in \mathcal{A}, n \in \mathcal{N},$$
$$h^{\min}(a) \leq \sum_{\pi \in \mathcal{P}} x(\pi, a) h(\pi, a) \leq h^{\max}(a), \quad \forall a \in \mathcal{A},$$
$$\sum_{\pi \in \mathcal{P}} \left(d(\pi, m, n) + \sum_{k \in \mathcal{N}} t(\pi, m, k, n) \right) + y(m, n) \geq \boldsymbol{b(m,n)}, \quad \forall m, n \in \mathcal{N},$$

$$\sum_{\pi\in\mathcal{P}}\sum_{m\in\mathcal{N}} t(\pi,m,k,n) = \sum_{\pi\in\mathcal{P}} s(\pi,k,n), \quad \forall k,n\in\mathcal{N},$$

$$\sum_{k\in\mathcal{N}}\left(d(\pi,v(\pi,1),k) + s(\pi,v(\pi,1),k) + \sum_{n\in\mathcal{N}} t(\pi,v(\pi,1),k,n)\right) = \sum_{a\in\mathcal{A}} d(a)x(\pi,a) - z(\pi,1), \quad \forall \pi\in\mathcal{P},$$

$$\sum_{k\in\mathcal{N}}\left(d(\pi,v(\pi,j),k) + s(\pi,v(\pi,j),k) + \sum_{n\in\mathcal{N}} t(\pi,v(\pi,j),k,n)\right) - \sum_{k\in\mathcal{N}}\left(d(\pi,k,v(\pi,j)) + s(\pi,k,v(\pi,j)) + \sum_{n\in\mathcal{N}} t(\pi,k,v(\pi,j),n)\right) = z(\pi,j-1) - z(\pi,j), \quad \forall \pi\in\mathcal{P}, j = 2,\ldots(l(\pi)-1),$$

$$x(\pi,a), d(\pi,m,n), t(\pi,m,k,n), s(\pi,k,n), y(m,n), z(\pi,j) \geq 0$$
$$x(\pi,a)\in\mathbb{Z}.$$