apl1p

Our third problem is a model for electric power capacity expansion first described by Infanger (1992). The problem consists of two electrical generators that can operate at three levels. The first stage problem is to determine capacity for the two generators. The second stage decision is to determine how much to produce at each of the three operation levels on the two generators subject to (1) the availability of the two generators and (2) customer demand for each of the three operation levels. Thus, there are nine second stage variables and five second stage constraints. The availabilities of the generators and the customer demands for each level are all random giving a total of five random variables. Notation for the problem is as follows:

$$
\begin{aligned}
\text{Sets: } & I = \{1,2\} := \text{ Set of generators} \\
& J = \{1,2,3\} := \text{ Set of operation levels} \\
\text{Variables: } & x_i := \text{ Capacity of generator } i \\
& y_{ij} := \text{ Production at operation level } j \text{ on generator i} \\
& s_j := \text{ Unserved demand at operation level } j \\
\text{Parameters: } & c_i := \text{ Investment cost per unit capacity for generator } i \\
& f_{ij} := \text{ Operating cost per unit at operation level } j \text{ on generator } i \\
& u_j := \text{ Cost per unit of unserved demand at operation level } j \\
& K_i := \text{ Minimum required capacity for generator } i \\
& a_i := \text{ Availability of generator } i \text{ (random)} \\
& d_j := \text{ Customer demand for operation level } j \text{ (random)}
\end{aligned}
\tag{5.29}
$$

The problem can then be written:

$$
\begin{aligned}
\text{Minimize } & \sum_{i=1}^{4} c_i x_i + \sum_{i=1}^{2} \sum_{j=1}^{3} f_{ij} y_{ij} + \sum_{j=1}^{3} u_j s_j \\
\text{subject to: } & x_i \geq K_i, \quad \forall i \in I \\
& -a_i x_i + \sum_{j=1}^{3} y_{ij} \leq 0, \quad \forall i \in I \\
& \sum_{i=1}^{2} y_{ij} + s_j \geq d_j, \quad \forall j \in J \\
& x_i, y_{ij}, s_j \geq 0 \quad \forall i \in I, \forall j \in J.
\end{aligned}
$$

The second stage primal problem is

$$\begin{aligned}
\text{Minimize } & \sum_{i=1}^{2}\sum_{j=1}^{3} f_{ij}y_{ij} + \sum_{j=1}^{3} u_j s_j \\
\text{subject to: } & \sum_{j=1}^{3} y_{ij} \leq a_i x_i, \quad \forall i \in I \\
& \sum_{i=1}^{2} y_{ij} + s_j \geq d_j, \quad \forall j \in J \\
& y_{ij}, s_j \geq 0 \quad \forall i \in I, \forall j \in J.
\end{aligned}$$

If we again associate dual multipliers σ_i to the capacity constraints and π_j to the demand constraints, the second stage dual problem is

$$\begin{aligned}
\text{Maximize } & \sum_{j=1}^{3} d_j \pi_j - \sum_{i=1}^{2} a_i x_i \sigma_i \\
\text{subject to: } & \pi_j - \sigma_i \leq f_{ij} \quad \forall i \in I, \forall j \in J \\
& \pi_j \leq u_j \quad \forall j \in J \\
& \sigma_i, \pi_j \geq 0 \quad \forall i \in I, \forall j \in J.
\end{aligned}$$

Again, the terms in the dual objective function are correlated due to the structure of the dual constraints. Using the probability distributions for the availabilities and demands from Infanger (1992) the problem has 1280 random scenarios. Given the small number of scenarios, we can write out and solve the deterministic equivalent of this stochastic program and also evaluate the second stage objective function at every scenario given the optimal stage 1 solution. This enables us to perform some of the sensitivity analysis calculations from Section 5.3 that are too cumbersome for large problems. Table 5.2 contains the calculations of the correlation ratio (Equation 5.16) and the Pearson correlation coefficients, or PEAR,

Table 5.2: Estimating important variables - apl1p

Random Variable	Corr. Ratio	PEAR
d_1	0.0340	0.1844
d_2	0.0255	0.1597
d_3	0.0132	0.1146
a_1	0.5154	0.7153
a_2	0.3976	0.6244

(Equation 5.15) from the total enumeration of the stochastic program at the optimal stage 1 solution. We can see that the availability random variables (a_1 and a_2) are the most important given the first stage optimal solution.

The illustrative example, test problem APL1P, is a model of a simple power network with one demand region. There are two generators with different investment and operating costs, and the demand is given by a load duration curve with three load levels: base, medium, and peak. We index the generators with $j = 1, 2$, and the demands with $i = 1, 2, 3$. The variables x_j, $j = 1, 2$, denote the capacities which can be built and operated to meet demands d_i, $i = 1, 2, 3$. The variable y_{ij} denotes the operating level for generator j in load level i with operating cost f_{ij}. The variable y_{is} defines the unserved demand in load level i which can be purchased with penalty cost $f_{is} > f_{ij}$. The subscript s is not an index, but denotes only an unserved demand variable. The per-unit cost to build generator j is c_j. Finally, the model is formulated with complete recourse, which means that at any given choice of x, demand is satisfied for all outcomes. In this model, building new generators competes with

Infanger 1992

Table 1

APL1P test problem data.

Generator capacity costs (10^5\$/MW, a)

$c_1 = 4.0,\ c_2 = 2.5$

Generator operating costs (10^5\$/MW, a)

$f_{11} = 4.3 \quad f_{21} = 8.7$
$f_{12} = 2.0 \quad f_{22} = 4.0$
$f_{13} = 0.5 \quad f_{23} = 1.0$

Unserved demand penalities (10^5\$/MW, a)

$f_{1s} = f_{2s} = f_{3s} = 10.0$

Minimum generator capacities (MW)

$b_1 = b_2 = 1000$

Demands (MW)

No.	1	2	3	4
Outcome	900	1000	1100	1200
Probability	0.15	0.45	0.25	0.15

Availabilities of generators

Generator 1 (β_1)

No.	1	2	3	4
Outcome	1.0	0.9	0.5	0.1
Probability	0.2	0.3	0.4	0.1

Generator 2 (β_2)

No.	1	2	3	4	5
Outcome	1.0	0.9	0.7	0.1	0.0
Probability	0.1	0.2	0.5	0.1	0.1

purchasing unserved demand through the cost function, yet there is a minimum capacity b_j which has to be built for each load level. The availabilities of the two generators $\beta_j, j = 1, 2$, and the demands in each load level $d_i, i = 1, 2, 3$, are uncertain. Generator one has four possibilities, while generator two has five, and each demand has four. All of the data values are given in table 1 and the problem can be formulated as follows:

$$\text{minimize} \quad \sum_{j=1}^{2} c_j x_j + E\left\{ \sum_{j=1}^{2} \sum_{i=1}^{3} f_{ij} y_{ij}^{\omega} + \sum_{i=1}^{2} f_{is} y_{is}^{\omega} \right\}$$

$$\text{subject to} \quad x_j \geq b_j, \qquad j = 1, 2,$$

$$-\alpha_j^{\omega} x_j + \sum_{i=1}^{3} y_{ij}^{\omega} \leq 0, \qquad j = 1, 2,$$

$$\sum_{j=1}^{2} y_{ij}^{\omega} + y_{is}^{\omega} \geq d_i^{\omega}, \qquad i = 1, 2, 3,$$

$$x_j, \ y_{ij}^{\omega}, \ y_{is}^{\omega} \qquad j = 1, 2, \ i = 1, 2, 3.$$

We will take $\omega \in \Omega$ when solving the universe problem and $\omega \in S$ when solving a problem with sampling.

The number of possible demands and availabilities results in $4*5*4^3 = 1280$ possible outcomes in Ω, and thus 1280 subproblems have to be solved in each iteration of Benders decomposition for the universe case. We compare the universe solution with solutions gained by the importance sampling algorithm. Table 2 shows the results in the case of 20 samples out of the possible 1280 combinations and

Table 2

Model APL1P, 20 samples (100 replications of the experiment).

	Correct	Mean	95% conf [%]	Bias [%]
No. univ	1280			
No. iter		7.6		
G1	1800.0	1666.5	57.0	−7.4
G2	1571.4	1732.5	52.5	10.2
θ	13513.7	13729.4	21.3	1.6
Obj	24642.3	24726.7	2.1	0.3
Est. conf [%]	left	1.5		
Est. conf [%]	right	1.9		
Coverage		0.90		

without an improvement phase. One hundred replications of the same experiment with different seeds were run to obtain statistical information about the accuracy of the solution and the estimated confidence interval. The mean over the 100 replications of the objective function value (total costs) differs from the universe solution by 0.3%. From the distribution of the optimum objective function value derived from the 100 replications of the experiment, a 95% confidence interval is computed: ±2.1%. In each replication, a 95% confidence interval of the solution is estimated. The mean over all replications of the estimated confidence interval is