N_0	h	T	N_T	# Cells	$hs_T + \varepsilon$ ($\times 10^{-3}$)	Time(s)	% Opt	True % Opt
1000	0.6260	5.66 $\pm$ 0.74	1017.42 $\pm$ 4.19	5.95 $\pm$ 0.98	7.9 $\pm$ 1.1	13.49	1.35	0.11
2000	0.5971	4.91 $\pm$ 0.52	2027.04 $\pm$ 5.00	5.02 $\pm$ 0.62	7.5 $\pm$ 0.9	18.53	1.28	0.10
3000	0.5844	4.46 $\pm$ 0.29	3023.94 $\pm$ 3.19	4.46 $\pm$ 0.29	7.0 $\pm$ 0.6	19.93	1.19	0.10
4000	0.5767	4.47 $\pm$ 0.36	4044.33 $\pm$ 5.76	4.47 $\pm$ 0.36	7.2 $\pm$ 0.6	23.47	1.23	0.12
5000	0.5715	4.21 $\pm$ 0.21	5050.10 $\pm$ 3.55	4.21 $\pm$ 0.21	6.7 $\pm$ 0.5	24.18	1.14	0.09

Table 5.2: Empirical results on the asset allocation problem, where $h' = 0.5275$.

5.5 Performance of SSAM as d_ξ increases

In this section, we examine how the performance of SSAM varies with the dimension of the random vector. To this end, we created various instances of the test problem `20TERM` by fixing some selected components of the random vector to their expected values. Recall that the original `20TERM` model (Mak et al., 1999) corresponds to a freight transportation problem where demands to be satisfied and shipped in both directions between consolidation centers (CCs) and local terminals (LTs) are random. This model is built with the convention that if a demand from a given CC to a given LT (symbolically, CC$\rightarrow$LT, where the arrow denotes the direction of shipment) is random, so is the demand in the reverse direction (i.e., from that given LT to the CC or symbolically CC$\leftarrow$LT). With that convention, there are 20 CC$\rightarrow$LT shipments with random demands (hence the name `20TERM` for the model) which results in a total of 40 individual random demands. We generate instances of `20TERM` with a smaller number of random parameters by modifying the data from the original model as follows. To generate instances with an even number of random parameters (such as 10, 20, 30, and 36), we set a fixed number of pairs of CC$\rightarrow$LT and CC$\leftarrow$LT demands to their expected values. For example, to obtain a problem instance with 30 random demand parameters we can keep stochastic every fourth CC$\rightarrow$LT demand (from the original 20) as well as their counterparts CC$\leftarrow$LT. Proceeding this way, we also manage to preserve the symmetry of the original `20TERM` model. To generate instances with an odd number of random parameters, such as instances with 15 and 25 random demands, we treat the demands CC$\rightarrow$LT and CC$\leftarrow$LT separately. For example, when we fix every second CC$\rightarrow$LT demand to their expected values while keeping stochastic every fourth CC$\leftarrow$LT demand, we obtain problem instances with 15 random demands. Proceeding this way, we created six new problem instances derived from `20TERM` plus the original 20TERM problem instance with the following dimensions of the random vector: 10, 15, 20, 25, 30, 36, and 40. Characteristics of these instances are summarized in Table 5.3. We label each instance so as to reflect the number of random demands from each type of CC→LT and CC←LT shipments. For example, the instance n-m:TERM has n random demands for shipments of the type CC→LT and m random demands for shipments of the type CC←LT. Each random demand has 2 realizations. With this notation, 20-20:TERM corresponds to the original 20TERM model.

Instance name	# of stochastic parameters	# of 1^{st} stage variables	# of 2^{nd} stage variables	# of scenarios
5-5:TERM	10	63	7.82×10^5	1.02×10^3
10-5:TERM	15	63	2.50×10^7	3.28×10^4
10-10:TERM	20	63	8.01×10^8	1.05×10^6
15-10:TERM	25	63	2.56×10^{10}	3.36×10^7
15-15:TERM	30	63	8.20×10^{11}	1.07×10^9
18-18:TERM	36	63	5.25×10^{13}	6.87×10^{10}
20-20:TERM	40	63	8.40×10^{14}	1.10×10^{12}

Table 5.3: Characteristics of the derived instances from the 20TERM model

Instance	T	N_T	# Cells	$hs_T+\varepsilon$	Time(s)	% Opt
5-5:TERM	9.10 $\pm$ 0.17	1032.68 $\pm$ 0.86	10.06 $\pm$ 0.28	2273.88 $\pm$ 106.04	126.35	0.93
10-5:TERM	9.82 $\pm$ 0.52	1035.01 $\pm$ 2.60	11.30 $\pm$ 0.72	3259.89 $\pm$ 116.17	173.78	1.34
10-10:TERM	21.28 $\pm$ 0.46	1126.72 $\pm$ 4.95	48.35 $\pm$ 1.68	3761.63 $\pm$ 79.45	539.76	1.53
15-10:TERM	25.05 $\pm$ 0.54	1179.31 $\pm$ 8.38	61.71 $\pm$ 2.11	4064.30 $\pm$ 65.32	782.24	1.65
15-15:TERM	29.97 $\pm$ 0.51	1385.87 $\pm$ 25.06	124.09 $\pm$ 4.11	4989.32 $\pm$ 85.62	1324.56	2.01
18-18:TERM	35.10 $\pm$ 0.34	2714.49 $\pm$ 86.71	271.35 $\pm$ 8.69	5342.75 $\pm$ 61.31	2659.05	2.14
20-20:TERM	40.02 $\pm$ 0.33	4601.10 $\pm$ 126.84	460.11 $\pm$ 12.68	5674.84 $\pm$ 66.58	4343.62	2.27

Table 5.4: Performance of SSAM as d_ξ increases. $N_0 = 1000$, $h' = 1.500$.

In Table 5.4 , we report the empirical performance of SSAM on the seven problem instances described in Table 5.3. We study the performance of SSAM as the dimension d_ξ increases by comparing the empirical performance results across the seven instances of 20TERM over 100 independent runs for an initial sample size $N_0 = 1000$ and $h = 1.5985$, $h' = 1.5000$. The performance metrics of Table 5.4 are computed in the same fashion as those of Table 5.2 except for the last column. For these results, as z^* is unknown for all these 20TERM instances but one, we estimated z^* of each instance with the largest lower