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### *1.1.2 Power Generation Planning: PGP2*

Next, we consider the task of selecting power generators that minimize the total cost of supplying enough electricity to satisfy regional demand. Deterministic power generation planning models have a long history dating back to Masse and Gibrat [1957]. More recently, two stage stochastic programming formulations have been proposed in Murphy, Sen and Soyster [1982] and a multi-stage model appears in Louveaux and Smeers [1988].

To begin, suppose that we wish to select the optimal sizes of three different types of generators: gas fired, coal fired, and nuclear. The annualized capital cost ($/kw) for the acquisition of a generator of type $j$ is given by $c_j$, while the cost of producing a unit of energy ($/kw-hr) is given by $f_j$. Note that while decisions regarding the generation of electricity may be postponed until regional demand is known, decisions regarding the acquisition of generation capacity cannot be postponed. Thus, we see a natural two stage progression of the decisions being undertaken. That is, the first stage decision identifies the generating capacity to be acquired, while the second stage decision identifies a more detailed operating plan undertaken in response to the imminent regional demand.

Regional demand for electricity is usually cyclic and the chronological load may be represented as shown in Figure 1.1. For the purposes of capacity planning, the chronology is not as important as the duration of the load. A load duration curve (LDC, see Figure 1.2) is a reordering of the chronological load curve. For a given load level $h$, the horizontal axis provides the number of hours during the year

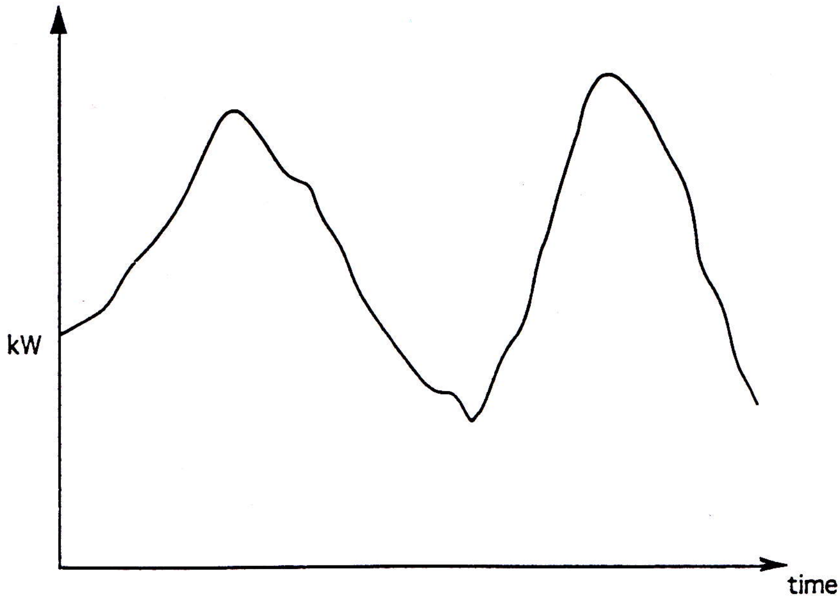


**Figure 1.1: Chronological load curve**

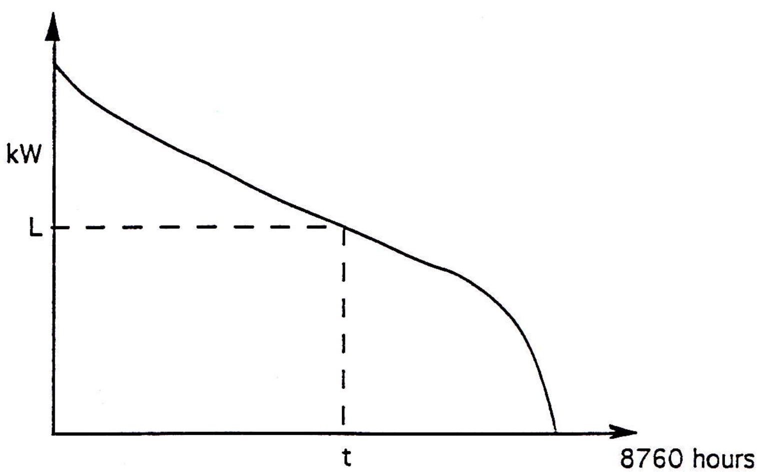


**Figure 1.2: Load duration curve**

that the load exceeds $h$. The area under the LDC gives the total energy consumption during the year. For planning purposes, the LDC is discretized into several steps, as shown in Figure 1.3.

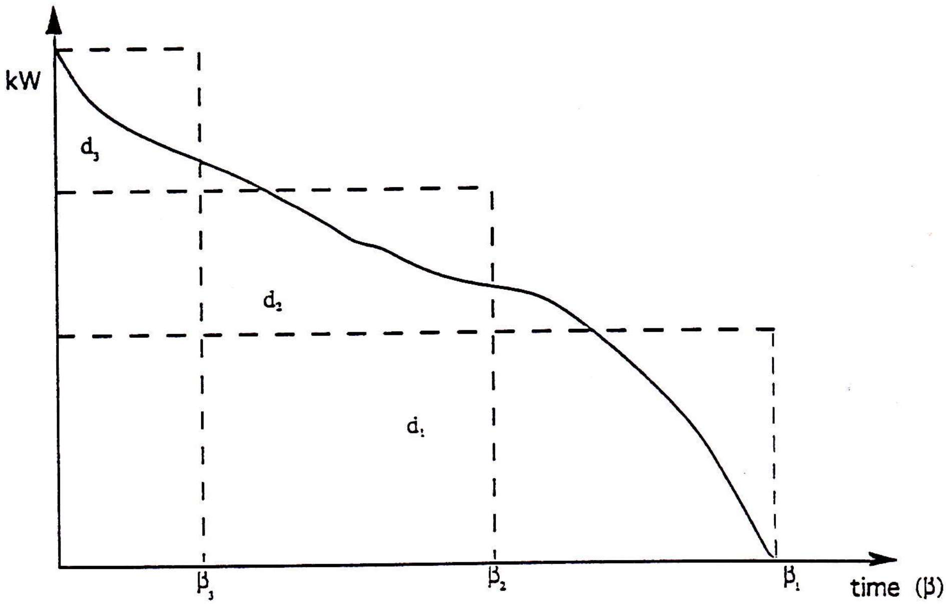


**Figure 1.3: Discretized load duration curve**

To formulate a model of the power generation planning problem, let $J = \{1, \ldots, n\}$ denote the set of possible generation equipment, and $I = \{1, \ldots, m\}$ the set of steps of the discretized LDC. For $j \in J$, we are given the costs $c_j$ and $f_j$ described above. For $i \in I$, we require the step sizes, $\omega_i$, and the load duration $\beta_i$. We assume that the durations of the steps, $\{\beta_i\}_{i \in I}$, which depend on characteristics such as the number of workdays vs. weekend days, etc. during the planning period are predictable. However, the step sizes, which correspond to the actual demand for power during the $i^{th}$ load segment will vary in response to several unpredictable factors (e.g., weather). Thus, these

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demands for electrical power are treated as random variables, $\tilde{\omega}_i$. The decision variables are identified as

$x_j$ = kw of generation capacity type $j$

$y_{ij}$ = kw of demand segment $i$ served using generator type $j$.

Finally, the model presented below includes two first stage constraints: a minimum capacity of $M$ and a budgetary restriction of $\$b$ are imposed.

Thus, the power generation planning problem may be written as

$$\begin{aligned} \text{Min} \quad & \sum_{j \in J} c_j x_j + E[h(x, \tilde{\omega})] \qquad\qquad (PGP2) \\ \text{s.t.} \quad & \sum_{j \in J} c_j x_j \leq b \\ & \sum_{j \in J} x_j \geq M \\ & x_j \geq 0 \qquad \forall j \in J \end{aligned}$$

where

$$\begin{aligned} h(x, d) = \text{Min} \quad & \sum_{i \in I} \sum_{j \in J} f_j \beta_i y_{ij} \\ \text{s.t.} \quad & -x_j + \sum_{i \in I} y_{ij} \leq 0 \qquad \forall\, j \in J \\ & \sum_{j \in J} y_{ij} = \omega_i \qquad \forall\, i \in I \\ & y_{ij} \geq 0 \qquad \forall\, i \in I,\ j \in J \end{aligned}$$

The first stage constraints require that the total cost of the acquired capacity satisfies the budget restriction as well as the minimum capacity restriction. In the second stage, the first set of constraints restricts

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the total power that can be obtained from a generator with capacity $x_j$, while the second set of constraints ensures that the power requirement of each step of the LDC is satisfied. Finally, as with CEP1, the data for PGP2 can be found in the appendix of this chapter.

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*PGP2 Data*

The problem data presented below is a slight modification of that given in Louveaux and Smeers [1988]. Our modifications of the data achieves three objectives: (a) allows relatively complete recourse, (b) includes more than one random variable and (c) provides a little more room for decision making in the first stage. Complete recourse is accomplished via the inclusion of an opportunity to subcontract additional generating capacity (of each type) whenever demand exceeds generating capabilities, at a cost of $p$ per kW of capacity. The random variables given below are discretized from the normal distributions that appear in Louveaux-Smeers data. The data is as follows.

$n = 4$, $m = 3$, $b = 220$, $M = 15$, $p = 1000$,

$c_j = (10, 7, 16, 6)$, $f_j = (40, 45, 32, 55)$

$\beta_i = (1, 0.6, 0.1)$

The demand distributions are as follows:

| $\tilde{d}_1$ | | $\tilde{d}_2$ | | $\tilde{d}_3$ | |
|---|---|---|---|---|---|
| *Value* | *Prob.* | *Value* | *Prob.* | *Value* | *Prob.* |
| 0.5 | 0.00005 | 0.0 | 0.0013 | 0.0 | 0.0013 |
| 1.0 | 0.00125 | 1.5 | 0.0215 | 0.5 | 0.0215 |
| 2.5 | 0.0215 | 2.5 | 0.2857 | 1.5 | 0.2857 |
| 3.5 | 0.2857 | 4.0 | 0.3830 | 3.0 | 0.3830 |
| 5.0 | 0.3830 | 5.5 | 0.2857 | 4.5 | 0.2857 |
| 6.5 | 0.2857 | 6.5 | 0.0215 | 5.5 | 0.0215 |
| 7.5 | 0.0215 | 8.0 | 0.00125 | 7.0 | 0.00125 |
| 9.0 | 0.00125 | 8.5 | 0.00005 | 7.5 | 0.00005 |
| 9.5 | 0.00005 | | | | |

**Table A.1: PGP2 Demand Distributions**