$P = a^2 \rho$ $$\begin{bmatrix} \rho \\ \rho u \end{bmatrix}_t + \begin{bmatrix} \rho u \\ \rho u^2 + a^2 \rho \end{bmatrix} = 0$$

$$q_{(x,t)} = \begin{bmatrix} \rho \\ \rho u \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$f(q) = \begin{bmatrix} \rho u \\ \rho u^2 + a^2 \rho \end{bmatrix} = \begin{bmatrix} q_2 \\ q_2^2/q_1 + a^2 q_1 \end{bmatrix}$$

$$f'(q) = \begin{bmatrix} 0 & 1 \\ -(q_2/q_1)^2 + a^2 & \frac{2q_2}{q_1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -u^2 + a^2 & 2u \end{bmatrix}$$

$\lambda_1 = u - a$ $\lambda_2 = u + a$

$r^1 = \begin{bmatrix} 1 \\ u - a \end{bmatrix}$ $r^2 = \begin{bmatrix} 1 \\ u + a \end{bmatrix}$

For two shock waves:

Hugoniot Loci

$$s(q_* - q) = f(q_*) - f(q)$$

$$s(\rho_* - \rho) = \rho_* u_* - \rho u$$

$$s(\rho_* u_* - \rho u) = \rho_* u_*^2 - \rho u^2 + a^2(\rho_* - \rho)$$

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$$\cancel{-2uu_*\nu + u_*^2\nu = 0}$$

$$S = \frac{\rho_* u_* - \rho u}{\rho_* - \rho}$$

$$(\rho_* u_* - \rho u)\left(\frac{\rho_* u_* - \rho u}{\rho_* - \rho}\right) = \rho_* u_*^2 - \rho u^2 + a^2\rho_* - a^2\rho$$

$$\rho_*^2 \cancel{u_*^2} + \rho^2 \cancel{u^2} - 2\rho u \rho_* u_* = \rho_*^2 \cancel{u_*^2} - \rho\rho_* u^2 + a^2\rho_*^2 - a^2\rho\rho_*$$

$$-\rho\rho_* u_*^2 + \rho^2 \cancel{u^2} - a^2\rho\rho_* + a^2\rho^2$$

$$-2\underline{\rho u}\rho_* u_* + \rho\rho_* u^2 - a^2\rho_*^2 + \underline{a^2\rho\rho_*} + \rho\rho_* u_*^2 + \underline{a^2\rho\rho_*} - a^2\rho^2 = 0$$

$$\rho\rho_* u^2 - 2\rho u\rho_* u_* \underbrace{- a^2\frac{\rho_*}{\rho} + 2a^2 \cancel{\rho\rho_*} - \frac{a^2\rho}{\rho_*} + \rho\rho_* u_*^2}_{} = 0$$

$$u^2 - 2uu_* - a^2\frac{\rho_*}{\rho} + 2a^2 - \frac{a^2\rho}{\rho^*} + u_*^2 = 0$$

$$\Delta = 4u_*^2 + 4a^2\frac{\rho_*}{\rho} - 8a^2 + 4\frac{a^2\rho}{\rho^*} - 4u_*^2$$

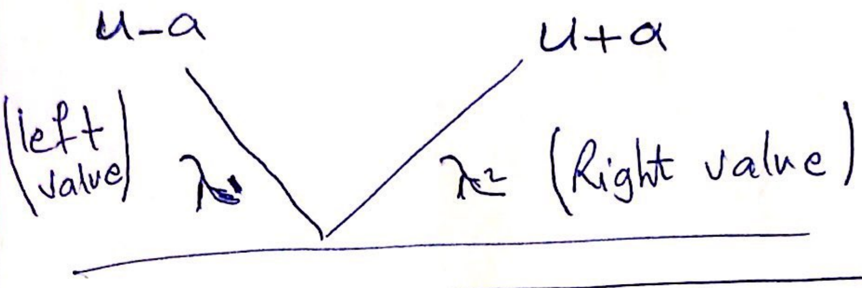
$$\frac{2u_* - \sqrt{\cancel{4u_*^2} + 4a^2\frac{\rho_*}{\rho} - 8a^2 + \frac{4a^2\rho}{\rho^*} - \cancel{4u_*^2}}}{2}$$

$$u(\rho) = u_* \pm \sqrt{\cancel{u_*^2} + a^2\frac{\rho_*}{\rho} - 2a^2 + \frac{a^2\rho}{\rho^*} - \cancel{u_*^2}}$$

$$u(\rho) = u_* \pm \sqrt{\cancel{u^2} + a^2\frac{\rho_*}{\rho} + a^2\frac{\rho}{\rho^*} - 2a^2 - \cancel{u_*^2}}$$

$-$ sign give locus of 1-shocks

$+$ sign " " " 2-shocks



$$u_m = u_r + a\sqrt{\frac{\rho_r}{\rho_m} + \frac{\rho_m}{\rho_r} - 2}$$ from right side

$$u_m = u_l - a\sqrt{\frac{\rho_l}{\rho_m} + \frac{\rho_m}{\rho_l} - 2}$$ from left side

if we put them equal, we can solve for ρ_m, u_m

$$f = u_r - u_l + a\sqrt{\frac{\rho_r}{\rho_m} + \frac{\rho_m}{\rho_r} - 2} + a\sqrt{\frac{\rho_l}{\rho_m} + \frac{\rho_m}{\rho_l} - 2} =$$

Newton method for non linear equation :

$$0 = f(x) = f(x_0) + \frac{df}{dx}(x - x_0)$$

$$x = x_0 - \frac{f(x_0)}{df/dx}$$

$$\frac{df}{d\rho} = a.\tfrac{1}{2}\left(\frac{\rho_r}{\rho_m} + \frac{\rho_m}{\rho_r} - 2\right)^{-1/2} \cdot \left[-\frac{\rho_r}{\rho_m^2} + \frac{1}{\rho_r}\right] +$$

$$a.\tfrac{1}{2}\left(\frac{\rho_l}{\rho_m} + \frac{\rho_m}{\rho_l} - 2\right)^{-1/2} \cdot \left[-\frac{\rho_l}{\rho_m^2} + \frac{1}{\rho_l}\right]$$

u_m and ρ_m is calculated by iterative method.

$$S_1 = \frac{\cancel{\rho_m} * u_m - \rho_L . * u_L}{\rho_m - \rho_L}$$

$$S_2 = \frac{\rho_m * u_m - \rho_R * u_R}{\rho_m - \rho_R}$$