

Complete solution for right rarefaction

$$u_R - u_* = a\sqrt{\gamma} \left[\ln \left(\frac{1}{P_*} - \frac{1-\gamma}{P_e} \right) \left(\frac{P_R}{1 - \frac{1-\gamma}{P_e} P_R} \right) \right]$$

$$u_* = u_R - a\sqrt{\gamma} \left[\ln \left(\frac{1}{P_*} - \frac{1-\gamma}{P_e} \right) \left(\frac{P_R}{1 - \frac{1-\gamma}{P_e} P_R} \right) \right]$$

$$\lambda_3 = u + c \quad \frac{dx}{dt} = \frac{x}{t} = u + c$$

$$u = \frac{x}{t} - c = \frac{x}{t} - \frac{P_e \cdot a\sqrt{\gamma}}{[P_e - P(1-\gamma)]}$$

$$P_e - P(1-\gamma) = \frac{-P_e \cdot a\sqrt{\gamma}}{u - \frac{x}{t}} \quad P(1-\gamma) = P_e + \frac{P_e \cdot a\sqrt{\gamma}}{u - \frac{x}{t}}$$

$$P = \left(P_e + \frac{P_e \cdot a\sqrt{\gamma}}{u - \frac{x}{t}} \right) \left(\frac{1}{1-\gamma} \right)$$

$$u = u_R - a\sqrt{\gamma} \left[\ln \left(\frac{1}{\left(P_e + \frac{P_e \cdot a\sqrt{\gamma}}{u - \frac{x}{t}} \right) \left(\frac{1}{1-\gamma} \right)} - \frac{1-\gamma}{P_e} \right) \left(\frac{P_R}{1 - \frac{1-\gamma}{P_e} P_R} \right) \right]$$

u should be calculated by ~~newton~~ iterative method.

$$P = \left(P_e + \frac{P_e \cdot a\sqrt{\gamma}}{u - \frac{x}{t}} \right) \left(\frac{1}{1-\gamma} \right)$$

inside
the fan

$$P = \frac{P_e a^2 \gamma}{P_e - P(1-\gamma)}$$

$$SR = \frac{P_R u_R - P_* u_*}{P_R - P_*}$$

complete solution for left rarefaction
invariants

$$u_L - u_* = a \sqrt{\gamma} \left[\ln \left(\frac{1}{P_*} - \frac{1-\gamma}{P_L} \right) \left(\frac{P_L}{1 - \frac{(1-\gamma) \cdot P_L}{P_*}} \right) \right]$$

$$\lambda_1 = u - c \quad \lambda_1 = \frac{dx}{dt} = \frac{x}{t} = u - c$$

$$u = \frac{x}{t} + c$$

$$u = \frac{x}{t} + \frac{P_L \cdot a \sqrt{\gamma}}{[P_L - P(1-\gamma)]} \Rightarrow P = \left(P_L - \frac{P_L \cdot a \sqrt{\gamma}}{u - \frac{x}{t}} \right) \left(\frac{1}{1-\gamma} \right)$$

$$u = u_L + a \sqrt{\gamma} \left[\ln \left(\frac{1}{\left(P_L - \frac{P_L \cdot a \sqrt{\gamma}}{u - \frac{x}{t}} \right) \left(\frac{1}{1-\gamma} \right)} - \frac{1-\gamma}{P_L} \right) \left(\frac{P_L}{1 - \frac{(1-\gamma) \cdot P_L}{\left(P_L - \frac{P_L \cdot a \sqrt{\gamma}}{u - \frac{x}{t}} \right) \left(\frac{1}{1-\gamma} \right)}} \right) \right]$$

u should be calculated iteratively

$$P = \left(P_L - \frac{P_L \cdot a \sqrt{\gamma}}{u - \frac{x}{t}} \right) \left(\frac{1}{1-\gamma} \right)$$

inside the
fan

$$P = \frac{P_L a^2 \gamma}{P_L - P(1-\gamma)}$$

Left shock wave

$$SL = \frac{P_L u_L - P_* u_*}{P_L - P_*}$$