

Actuator Fault Diagnosis and Accommodation in Interconnected Nonlinear Dynamical Systems

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Outline

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Actuator Fault Detection and Isolation

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- Actuator Fault Detection
- Actuator Fault Propagation
- Actuator Fault Isolation

Actuator and Sensor Fault Isolation

- Problem Formulation
- Sensor Fault Isolation
- Actuator Fault Isolation

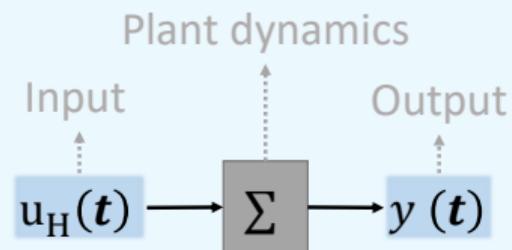
Actuator Fault Accommodation

- Problem formulation
- System Normalization
- Adaptive approximation
- Nominal Controller Design
- Control Reconfiguration

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Dynamical Systems

System Structure



Dynamical Systems

System Structure

A class of nonlinear dynamical systems: (state-space representation)

$$\Sigma : \begin{cases} \dot{x}(t) = \underbrace{Ax(t) + B(u_H(t) + u_F(t))}_{\text{Linear term}} + \underbrace{g(x(t))}_{\text{Nonlinearity}} + \underbrace{\eta(x(t), t)}_{\text{Modelling uncertainty and disturbance}} + f_p(x(t), t) \\ y(t) = h(x(t)) + \underbrace{\varepsilon_y(t)}_{\text{Measurement noise}} + f_y(x(t), u(t), t) \end{cases}$$

Dynamical Systems

System Structure

A class of nonlinear dynamical systems: (state-space representation)

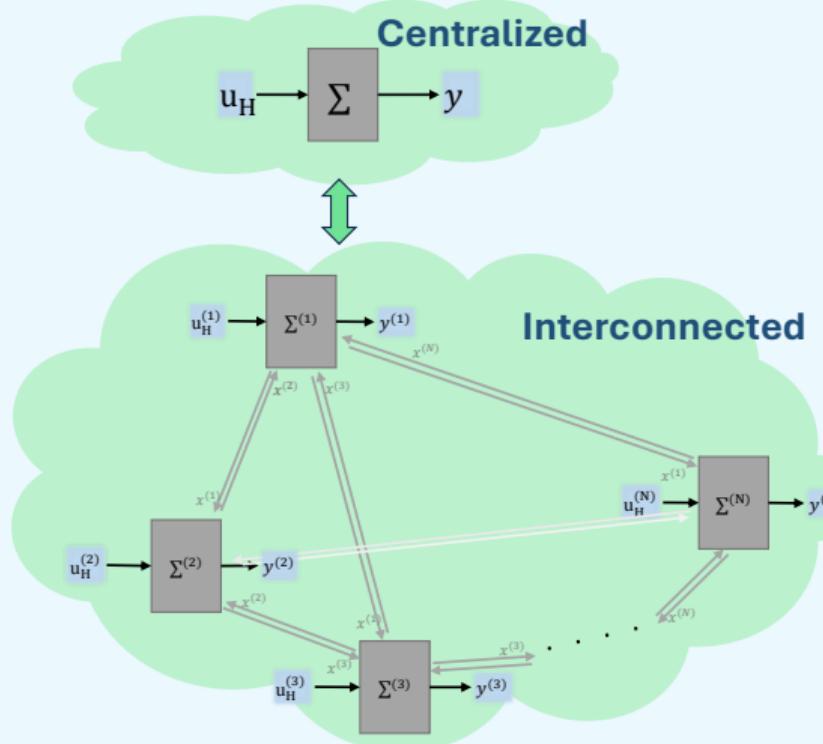
Fault types

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + B(u_H(t) + \underbrace{u_F(t)}_{\text{Actuator fault(s)}}) + g(x(t)) + \eta(x(t), t) + \underbrace{f_p(x(t), t)}_{\text{Process fault(s)}} \\ y(t) = h(x(t)) + \varepsilon_y(t) + \underbrace{f_y(x(t), u(t), t)}_{\text{Sensor fault(s)}} \end{cases}$$

Motivation

Dynamical Systems

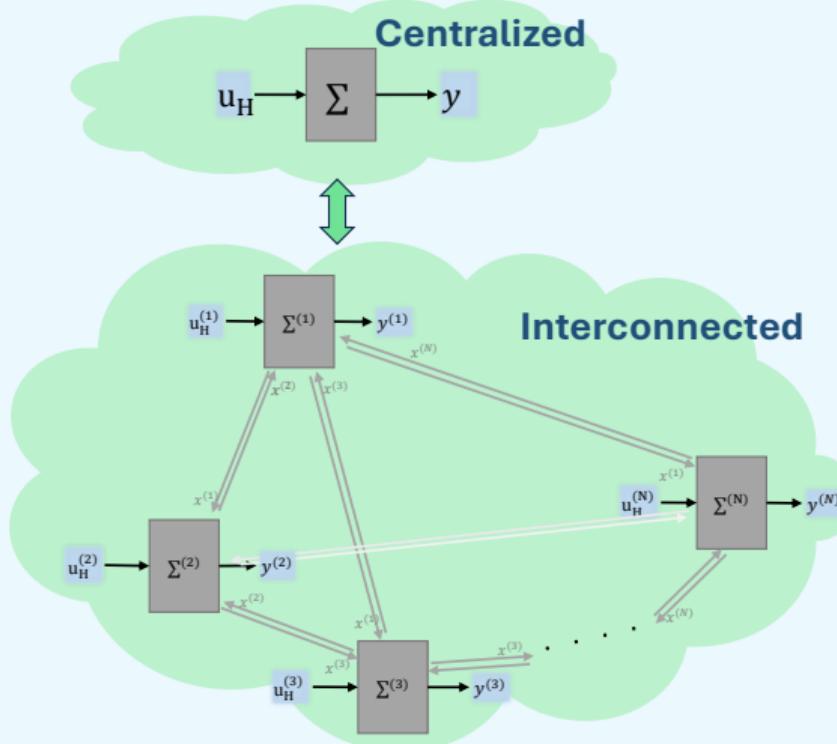
Centralized vs. Interconnected



Motivation

Dynamical Systems

Centralized vs. Interconnected



The interconnected representation:

- ▶ Provides Simplicity in Monitoring & Control
- ▶ Improves fault isolability by addressing fault propagation

► Objectives

- Actuator Fault Detection and Isolation
- Actuator and Sensor Fault Isolation
- Actuator Fault Accommodation

Actuator Fault Detection and Isolation in a Class of Nonlinear Interconnected Systems

Objectives:

- ▶ **Actuator Fault Detection**
- ▶ **Actuator Fault Isolation**

Literature Review: Actuator Fault Detection

Existing Research:

[1]. Keliris et al., 2013,

A distributed fault detection **filtering** approach for a class of interconnected continuous-time nonlinear systems, *IEEE Trans. Autom. Control*.

[2]. Reppa et al., 2014,

Adaptive approximation for multiple sensor fault detection and isolation of nonlinear uncertain systems, *IEEE Trans. Neural Netw. Learn. Syst.*

[3]. Keliris et al., 2015,

A robust nonlinear observer-based approach for distributed fault detection of input–output interconnected systems, *Automatica*.

[4]. Keliris et al., 2017,

An **integrated learning and filtering approach** for fault diagnosis of a class of nonlinear dynamical systems, *IEEE Trans. Neural Netw. Learn. Syst.*

New Contribution:

We propose a **new filtering** scheme to improve fault detection in nonlinear interconnected systems.

[5]. Reppa V, et al., 2014,

Decentralized isolation of multiple sensor faults in large-scale interconnected nonlinear systems, *IEEE Trans. Autom. Control*.

- ▶ Reasoning-based decision scheme
- ▶ Fault signature matrix
- ▶ Detection decision
- ▶ Consistency test

Dynamical system

$$\Sigma^{(I)} : \quad \dot{x}^{(I)}(t) = A^{(I)}x^{(I)}(t) + \gamma^{(I)} \left(x^{(I)}(t), u^{(I)}(t) \right) \\ + h^{(I)} \left(x^{(I)}(t), u^{(I)}(t), z^{(I)}(t) \right) \\ + \eta^{(I)} \left(x^{(I)}(t), u^{(I)}(t), z^{(I)}(t), t \right)$$

$$\mathcal{S}^{(I)} : \quad y^{(I)}(t) = C^{(I)}x^{(I)}(t) + d^{(I)}(t) + f^{(I)}(t)$$

Literature Review: Actuator Fault Isolation

[6]. Kościelny JM, Bartyś M., 2023,

A new method of diagnostic row reasoning based on trivalent residuals, *Expert Syst. Appl.*

► Reasoning-based decision scheme

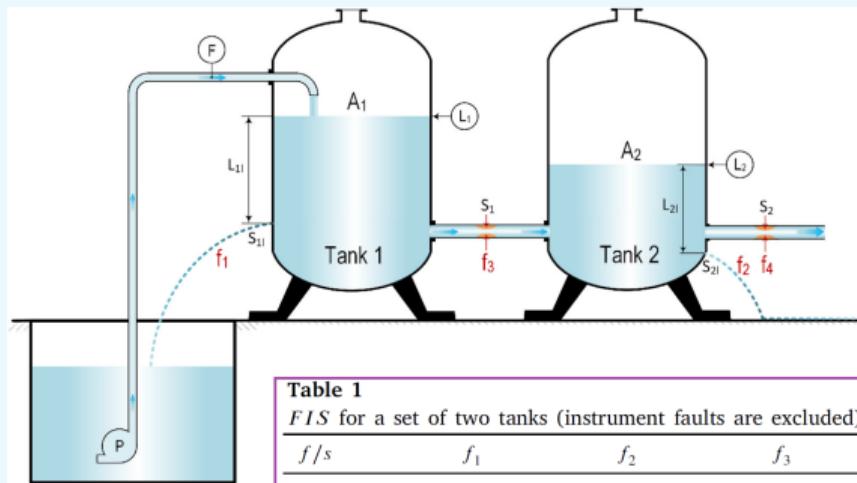


Table 1
FIS for a set of two tanks (instrument faults are excluded).

f/s	f_1	f_2	f_3	f_4
s_1	+1	0	-1	0
s_2	0	+1	+1	-1
s_3	+1	+1	0	-1

Table 2

Conflict sets $C(s_j = v_p)$ in the case of bi-valued diagnostic signals.

S	V	$C(s_j = v_p)$
s_1	1	$\{f_1, f_3\}$
s_2	1	$\{f_2, f_3, f_4\}$
s_3	1	$\{f_1, f_2, f_4\}$

Table 3

Conflict sets $C(s_j = v_p)$ in the case of three-valued diagnostic signals.

S	V	$C(s_j = v_p)$
s_1	-1	$\{f_3\}$
	+1	$\{f_1\}$
s_2	-1	$\{f_4\}$
	+1	$\{f_2, f_3\}$

Problem Formulation

System Description

The I -th subsystem considered in our study

$$\Sigma^{(I)} : \begin{cases} \dot{x}^{(I)} = A^{(I)}x^{(I)} + g^{(I)}(x^{(I)}, u^{(I)}) + h^{(I)}(x^{(I)}, z^{(I)}, u^{(I)}) + \eta^{(I)}(x^{(I)}, z^{(I)}, t) \\ y^{(I)} = x^{(I)} + \xi_y^{(I)}, \end{cases} \quad I = 1, \dots, N$$

Variables:

$$x^{(I)}, y^{(I)} \in \mathbb{R}^{n_I}, \quad z^{(I)} \in \mathbb{R}^{\bar{n}_I}, \quad u^{(I)} \in \mathbb{R}^{m_I}, \quad y^{(I)} \in \mathbb{R}^{n_I}$$

(Multiple Input Multiple Output per Subsystem)

Functions:

$$A^{(I)}x^{(I)}$$

(Known linear term)

$$g^{(I)} : \mathbb{R}^{n_I} \times \mathbb{R}^{m_I} \rightarrow \mathbb{R}^{n_I}$$

(Known nonlinear term)

$$h^{(I)} : \mathbb{R}^{n_I} \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^{m_I} \rightarrow \mathbb{R}^{n_I}$$

(Known interconnection term)

$$\eta^{(I)} : \mathbb{R}^{n_I} \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^+ \rightarrow \mathbb{R}^{n_I}$$

(Unknown uncertainty term)

$$\xi_y^{(I)}(t) \in \mathbb{R}^{n_I}$$

(Sensor noise)

Problem Formulation

System Description

The I -th subsystem considered in our study

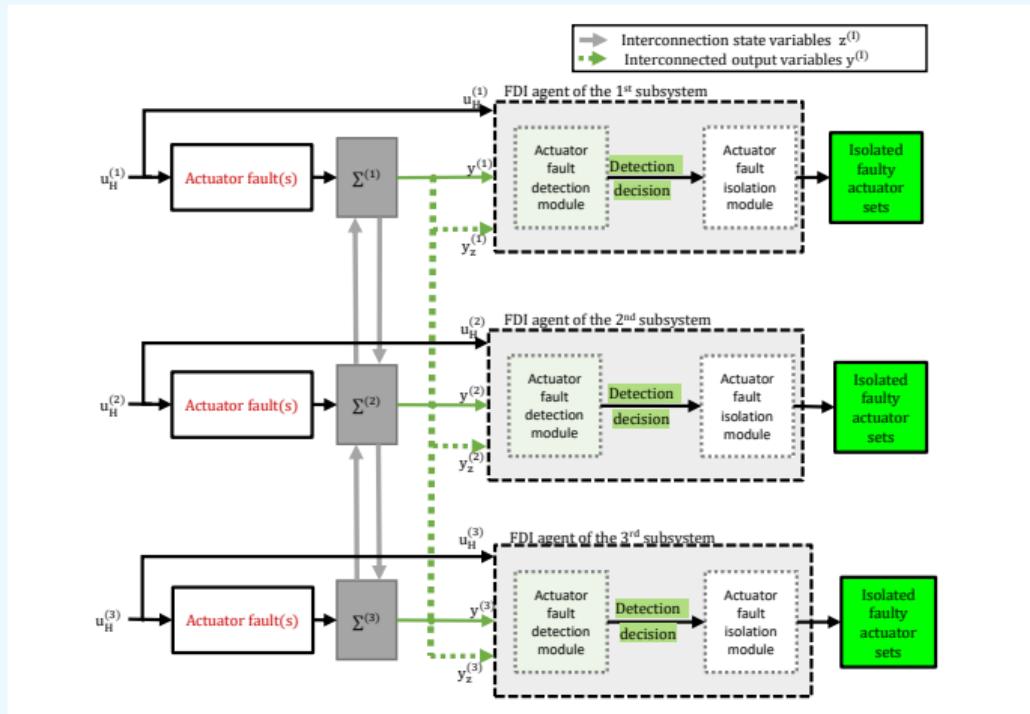
$$\Sigma^{(I)} : \begin{cases} \dot{x}^{(I)} = A^{(I)}x^{(I)} + g^{(I)}(x^{(I)}, u^{(I)}) + h^{(I)}(x^{(I)}, z^{(I)}, u^{(I)}) + \eta^{(I)}(x^{(I)}, z^{(I)}, t) \\ y^{(I)} = x^{(I)} + \xi_y^{(I)}, \end{cases} \quad I = 1, \dots, N$$

Assumptions:

- ▶ The state $x^{(I)}$ remains bounded before and after the occurrence of any actuator fault,
- ▶ $g^{(I)}$ Lipschitz in $x^{(I)}$,
- ▶ $h^{(I)}$ Lipschitz in both $x^{(I)}$ and $z^{(I)}$,
- ▶ $\eta^{(I)}$ Bounded, i.e., $|\eta^{(I)}| \leq \bar{\eta}^{(I)}$ where $\bar{\eta}^{(I)}$ is a Lipschitz in $x^{(I)}$,
- ▶ $\xi_y^{(I)}$ Bounded, i.e., $|\xi_y^{(I)}| \leq \bar{\xi}_y^{(I)}$.

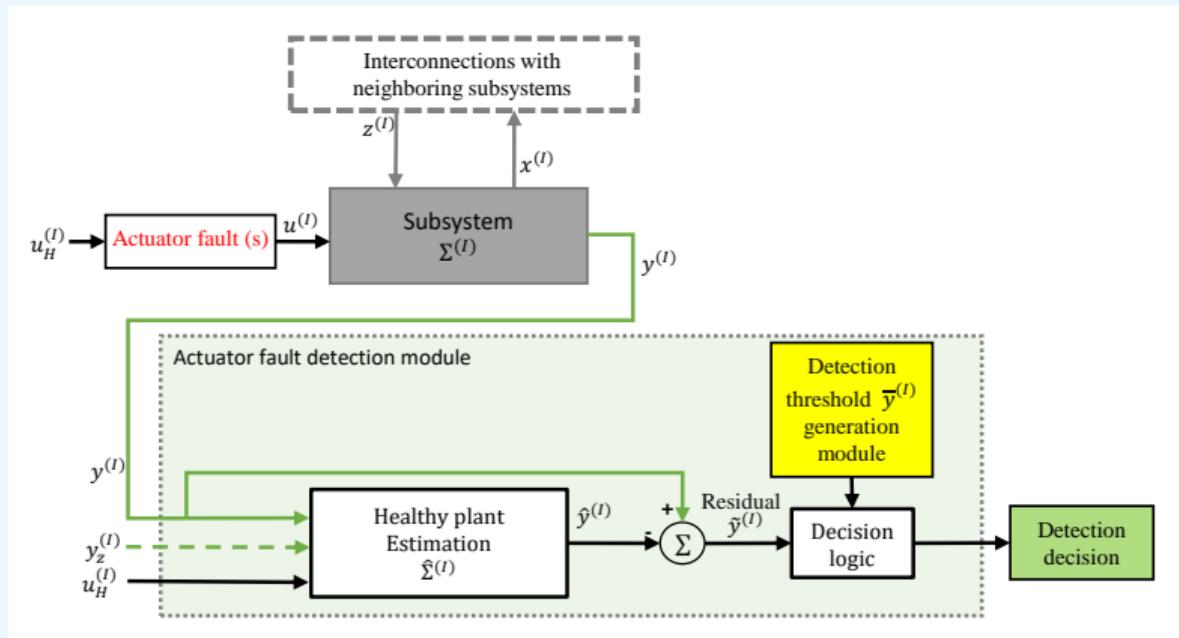
Problem formulation

The architecture of the proposed actuator FDI scheme:



Actuator Fault Detection

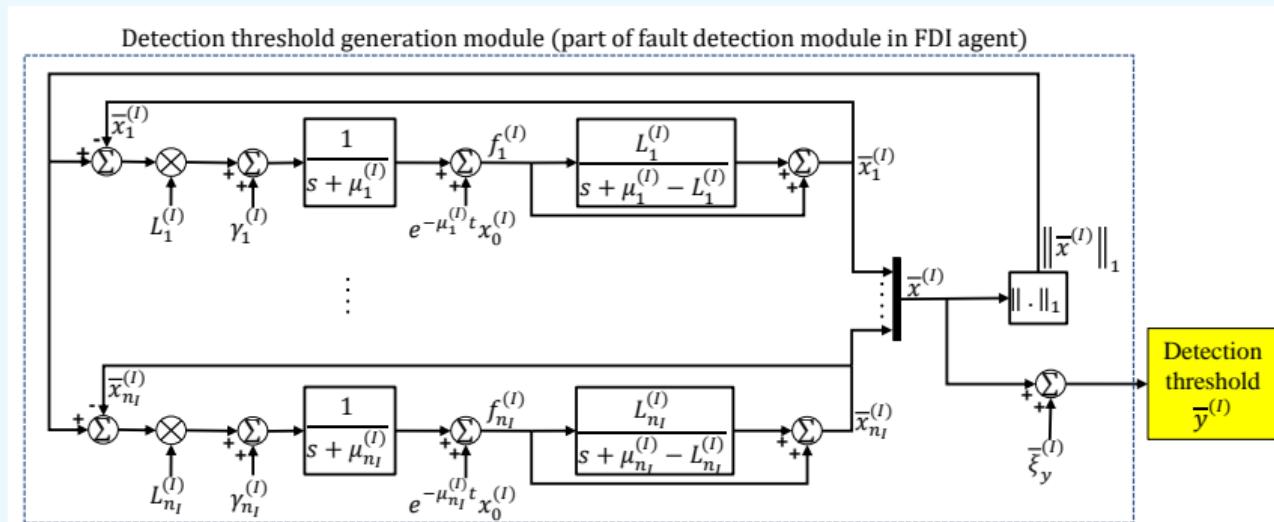
Actuator fault detection agent of the I -th subsystem:



Actuator Fault Detection

Adaptive Threshold

– Generated through a new filtering technique.



Actuator Fault Propagation

It is guaranteed that the occurrence of an actuator fault can only be detected by the local detection agent of the faulty subsystem;

$$\Sigma^{(I)} : \begin{cases} \dot{x}^{(I)} = f^{(I)}(x^{(I)}, z^{(I)}, \underbrace{u_f^{(I)}}_{\text{Faulty}}) \\ y^{(I)} = x^{(I)} + \xi_{y_i}^{(I)} \end{cases}$$

$$\Sigma^{(J)} : \begin{cases} \dot{x}^{(J)} = f^{(J)}(x^{(J)}, z^{(J)}, \underbrace{u_H^{(J)}}_{\text{Healthy}}) \\ y^{(J)} = x^{(I)} + \xi_{y_i}^{(I)} \end{cases} \Rightarrow |\tilde{y}_j^{(J)}| < \bar{y}_j^{(J)} \quad \forall j = 1, \dots, n_j$$

Actuator Fault Isolation

Actuator Fault Isolation

- **Signature Matrix:** $F = [F_{ij}]_{n_I \times |\mathcal{F}|}$,

$$F_{ij} = \begin{cases} 0 & \text{Fault } \mathcal{F}_{c_j} \text{ has not any affect on residual } \tilde{y}_i^{(I)} \\ -1 & \text{Fault } \mathcal{F}_{c_j} \text{ has negative affect on residual } \tilde{y}_i^{(I)} \\ +1 & \text{Fault } \mathcal{F}_{c_j} \text{ has positive affect on residual } \tilde{y}_i^{(I)} \\ * & \text{Fault } \mathcal{F}_{c_j} \text{ has unknown affect on residual } \tilde{y}_i^{(I)}, \end{cases}$$

- **Detection Decision:** $\mathcal{E}^{(I)}(t) = [\epsilon_1^{(I)}, \dots, \epsilon_{n_I}^{(I)}]$

$$\epsilon_i^{(I)}(t) = \begin{cases} +1 & \tilde{y}_i^{(I)}(t) > \bar{y}_i^{(I)}(t) \\ -1 & \tilde{y}_i^{(I)}(t) < -\bar{y}_i^{(I)}(t) \\ 0 & |\tilde{y}_i^{(I)}(t)| < \bar{y}_i^{(I)}(t) \end{cases} \quad i = 1, \dots, n_I.$$

Actuator Fault Isolation

Consistent set: (Can result in false alarms in AFI)

$$\mathcal{D}_s(t) = \left\{ \mathcal{F}_{c_k}^{(I)} \middle| \begin{cases} \varepsilon_i(t) = F_{ik} \text{ or } F_{ik} = * & \forall i \in \mathcal{J} \\ F_{ik} = 0 & \forall i \notin \mathcal{J}. \end{cases} \right\}, \quad \mathcal{J} = \left\{ j \middle| |\tilde{y}_j^{(I)}| > \bar{y}_j^{(I)} \right\}$$
$$k \in \{1, \dots, 2^{m_I} - 1\}$$

Weakly consistent set: (Contain the accurate faulty actuator set)

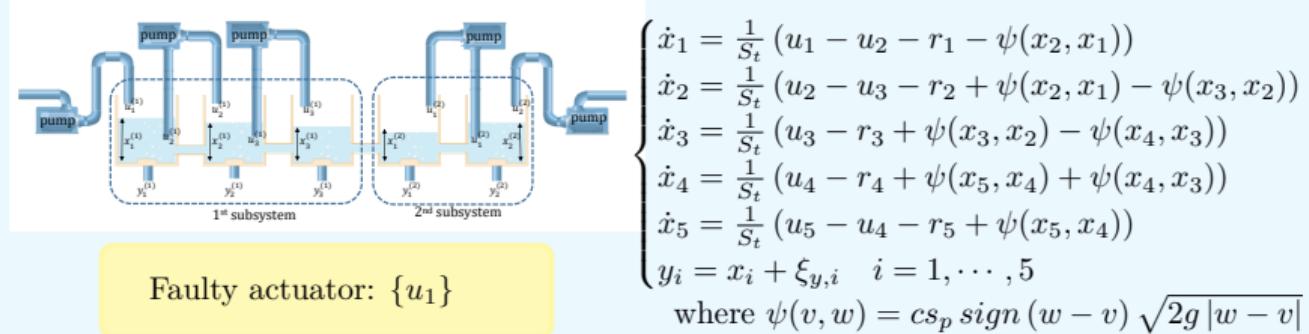
$$\mathcal{D}_w^{(I)}(t) = \left\{ \mathcal{F}_{c_k}^{(I)} \middle| \varepsilon_j^{(I)}(t) = F_{jk}^{(I)} \text{ or } F_{jk}^{(I)} = *, \quad \forall j \in \mathcal{J}^{(I)}, \quad k \in \{1, \dots, 2^{m_I} - 1\} \right\}$$

Diagnosed fault set(s): (Accurate isolated faulty actuator set)

$$\mathcal{F}_w^{(I)} \triangleq \bigcap_{i=1}^{2^{m_I}-1} \left\{ \mathcal{F}_{c_i}^{(I)} \middle| \mathcal{F}_{c_i}^{(I)} \in \mathcal{D}_w^{(I)}(t) \right\}.$$

Simulation results

Five-Tank System



Time interval	$[0, T_0^{(1)}) = [0, 5] \text{ sec}$	$[T_0^{(1)}, T_{d,1}^{(1)}) = [5, 5.3] \text{ sec}$	$[T_{d,1}^{(1)}, 35.1) = [5.3, 35.1] \text{ sec}$	$[35.1, 60] \text{ sec}$
Actual faulty actuators	—	$\mathcal{F}_{c_1}^{(1)} = \{\mathbf{u}_1^{(1)}\}$	$\mathcal{F}_{c_1}^{(1)} = \{\mathbf{u}_1^{(1)}\}$	—
Observed pattern $\mathcal{E}^{(1)}(t)$		$[0, 0, 0]^T$	$[-1, 0, 0]^T$	$[0, 0, 0]^T$
Diagnosis sets $\mathcal{D}_s^{(1)}(t)$ (consistency)		—	$\{\mathcal{F}_{c_1}^{(1)}\}$	—
Diagnosis sets $\mathcal{D}_w^{(1)}(t)$ (weak consistency)		—	$\{\mathcal{F}_{c_1}^{(1)}, \mathcal{F}_{c_4}^{(1)}, \mathcal{F}_{c_5}^{(1)}, \mathcal{F}_{c_7}^{(1)}\}$	—
Faulty identified set $\mathcal{F}_w^{(1)}(t)$	—	—	$\mathcal{F}_{c_1}^{(1)} = \{\mathbf{u}_1^{(1)}\}$	—

Actuator and Sensor Fault Isolation in a Class of Nonlinear Dynamical Systems

Objectives:

- ▶ Identification of Fault Type: Sensor(s) or Actuator(s)
- ▶ Isolation of Faulty Component(s)

A dynamical system affected by sensor and actuator faults:

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + \textcolor{red}{Bu_F(t)} + g(x(t)) + \eta(x, t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0)f_y(x, t), \end{cases}$$

Literature Review

[1]. Trunov A.B., Polycarpou M. M., 2000,

Automated fault diagnosis in nonlinear multivariable systems using a learning methodology, *IEEE Trans. Neural Netw.*

Dynamical system:

$$\begin{aligned}\dot{z} &= Az + \alpha(y, u) + \eta_z(z, u, t) + \underline{\mathcal{B}_z(t - T_z)} f_z(y, u) \\ y &= Cz + \eta_y(z, u, t) + \underline{\mathcal{B}_y(t - T_y)} f_y(u)\end{aligned}\quad (1)$$

A. Nonlinear Estimation Model

$$\begin{aligned}\dot{\hat{z}} &= A\hat{z} + \alpha(y, u) + K(y - \hat{y}) + \underline{\hat{f}_z(y, u; \hat{\theta}_z)} \\ &\quad + \Omega_z \dot{\hat{\theta}}_z + \Omega_y \dot{\hat{\theta}}_y, \quad \hat{z}(0) = z_0\end{aligned}\quad (3)$$

$$\dot{\Omega}_z = A_0 \Omega_z + Z_z(y, u; \hat{\theta}_z), \quad \Omega_z(0) = 0 \quad (4)$$

$$\dot{\Omega}_y = A_0 \Omega_y - K Z_y(u; \hat{\theta}_y), \quad \Omega_y(0) = 0 \quad (5)$$

$$\hat{y} = C\hat{z} + \underline{\hat{f}_y(u; \hat{\theta}_y)} \quad (6)$$

[2]. Talebi HA, et al., 2008,

A recurrent neural-network-based sensor and actuator fault detection and isolation for nonlinear systems with application to the satellite's attitude control subsystem, *IEEE Trans. Neural Netw.*

– Neural-Network-Based Scheme

Dynamical system:

$$\begin{aligned}\dot{x} &= Ax + g(x, u) + \eta_x(x, u, t) + T_A(x, u) \\ y &= Cx + \eta_y(x, u, t) + T_S(x, u)\end{aligned}\quad (1)$$

Estimation model:

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + g(\hat{x}, u) + \hat{T}_A(\hat{x}, u, W_1) \\ \hat{y} &= C\hat{x} + \hat{T}_S(\hat{x}, u, W_2)\end{aligned}\quad (6)$$

Literature Review

[3]. Zhang X, et al.,2008,

Design and analysis of a fault isolation scheme for a class of uncertain nonlinear systems, *Annual Reviews in Control*.

– Generalized observer scheme (FIEs are sensitive to all but one fault)

Dynamical system:

$$\begin{aligned}\dot{x} &= Ax + \gamma(y, u) + \eta(x, u, t) + \beta_x(t - T_x)\phi(y, u) \\ y &= Cx + d(x, u, t) + \beta_y(t - T_y)F\theta(t)\end{aligned}\quad (1)$$

Fault set:

(2)

$$\mathcal{F}_P \triangleq \{\phi^1(y, u), \dots, \phi^N(y, u)\}.$$

Fault Isolation Decision Scheme: If, for each $r \in \{1, \dots, N+l\} \setminus \{s\}$, there exist some finite time $t^r > T_d$ and some $j \in \{1, \dots, l\}$, such that $|\epsilon_{y_j}^r(t^r)| > \mu_j^r(t^r)$, then the occurrence of fault s is concluded.

$p = 1, \dots, N$, the p th process fault,

$q = N+1, \dots, N+l$, the q th sensor fault,

Literature Review

[4]. Keliris C, et al., 2016,

An integrated learning and filtering approach for fault diagnosis of a class of nonlinear dynamical systems, *IEEE Trans. Neural Netw. Learn. Syst.*

– Dedicated observer scheme (FIEs are sensitive to single fault)

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + f(x(t), u(t)) + \eta(x(t), u(t)) \\ \quad + \beta^x(t - T_0^x)\phi(x(t), u(t)) \end{cases} \quad (1)$$
$$y(t) = x(t) + \xi(t) + \beta^y(t - T_0^y)\sigma(t) \quad (2)$$

V. FAULT TYPE IDENTIFICATION

A. Process Fault

$$\dot{\hat{x}}_p(t) = g(y(t), u(t)) + \hat{\eta}(y(t), u(t), \hat{\theta}(t)) \quad (28)$$

$$\epsilon_p(t) \triangleq H_1(s)[y(t) - \hat{x}_p(t)] \quad (29)$$

$$r_p(t) \triangleq H_2(s)[\epsilon_p(t)] \quad (30)$$

B. Sensor Faults

$$\dot{\hat{x}}_s(t) = A\hat{x}_s(t) + f(\hat{x}_s(t), u(t)) + \hat{\eta}_L(y(t), u(t)) + \Lambda\epsilon_s^y(t) \quad (33)$$

$$\hat{y}_s(t) = \hat{x}_s(t) + \hat{\sigma}(t) \quad (34)$$

$$\dot{\hat{\sigma}}(t) = \mathcal{P}_\sigma(\Gamma^y\epsilon_s^y(t)) \quad (35)$$

Fault Identification Logic:

$$|r_p^{(k)}(t)| \leq \bar{r}_p^{(k)}(t) \text{ for all } k = 1, \dots, n$$

for all $t > T_d + T_{L,2}$.

Fault Identification Logic:

$$|\epsilon_s^y(t)| \leq \bar{\epsilon}_s^y(t) \text{ and } |r_s^{(k)}(t)| \leq \bar{r}_s^{(k)}(t) \text{ for all } k = 1, \dots, n$$

for all $t > T_d$.

Literature Review

[5]. Reppa V, et al., 2014,

Decentralized isolation of multiple sensor faults in large-scale interconnected nonlinear systems, *IEEE Trans. Autom. Control*.

Dynamical system:

$$\begin{aligned}\Sigma^{(I)} : \quad \dot{x}^{(I)}(t) = & A^{(I)}x^{(I)}(t) + \gamma^{(I)}\left(x^{(I)}(t), u^{(I)}(t)\right) \\ & + h^{(I)}\left(x^{(I)}(t), u^{(I)}(t), z^{(I)}(t)\right) \\ & + \eta^{(I)}\left(x^{(I)}(t), u^{(I)}(t), z^{(I)}(t), t\right) \quad (1)\end{aligned}$$

$$\mathcal{S}^{(I)} : \quad y^{(I)}(t) = C^{(I)}x^{(I)}(t) + d^{(I)}(t) + f^{(I)}(t) \quad (4)$$

Assumption 1:

the corresponding reference signals of the interconnection variables $z^{(I)}(t)$, denoted by $z_r^{(I)}(t)$, are known.

$$|z^{(I)}(t) - z_r^{(I)}(t)| \leq \bar{z}^{(I)}, \forall t \geq 0.$$

[1] Keliris C, et al., 2015,

Distributed fault diagnosis for process and sensor faults in a class of interconnected input–output nonlinear discrete-time systems, *Int. J. Control.*

Literature Review

[6]. Reppa V, et al., 2017,

Optimization of observer design for sensor fault detection of nonlinear systems, *Proc. IEEE Conf. Decis. Control.*

- Bias (and abrupt) faults
- Sensor fault detection
- Single sensor fault
- Detectability

Dynamical system:

$$\dot{x}(t) = Ax(t) + \gamma(x(t), u(t)) + \eta(t) \quad (1)$$

$$y(t) = x(t) + d(t) + f(t), \quad (2)$$

Abrupt & Bias Faults:

$$f(t) = f_0 + \tilde{f}(t), \quad (34)$$

$f_0 \in \mathbb{R}^n$ is the (unknown) non-zero offset

Corollary 3.4: Every single sensor fault $f_i = f_{0i} + \tilde{f}_i$ with $|f_{0i}| \geq f_{0i}^{\min}$ and $|\tilde{f}_i| \leq \tilde{f}_i^*$ is strongly detectable, if $|V_j^{-1}A_i| > |V_j^{-1}|\Gamma$.

$$f_{0i}^{\min} = \min\{f_i : (|\lambda_j^{-1}V_j^{-1}A_i| - |\lambda_j^{-1}V_j^{-1}|\Gamma)|f_i| > c\} \quad (53)$$

$$c = |\lambda_j^{-1}V_j^{-1}|(2\bar{\eta} + \Gamma(2\|\bar{d}\| + \tilde{f}_i^*)) + (|V_j^{-1} - \lambda_j^{-1}V_j^{-1}A_i| + |V_j^{-1}|)(2\bar{d} + [0, \dots, \tilde{f}_i^*, \dots, 0]^\top) \quad (54)$$

[7]. Chen W, Saif M., 2007,

Adaptive actuator fault detection, isolation and accommodation in uncertain systems, *Int. J. Control.*

- Actuator fault detection and isolation
- Known and distinguishable fault functions

$$\dot{x} = Ax - ay + b_1 u_1 + \cdots + b_m u_m$$
$$y = x_1,$$

$$u_1(t) = u(t), u_2(t) = f_2(t) + u(t), \dots, u_m(t) = f_m(t) + u(t),$$
$$u_j(t) = \bar{u}_j, \quad t \geq t_j, \quad j \in 1, 2, \dots, m \quad (29)$$

$$f_j(t) = d f_j \sin(\omega_j t), \quad 2 \leq j \leq m$$

$C_m^1 + \cdots + C_m^{m-1}$ adaptive estimates need to isolate the faults.

Literature Review

[8]. Talebi HA, Khorasani K., 2012,

A neural network-based multiplicative actuator fault detection and isolation of nonlinear systems, *IEEE Trans. Control Syst. Technol.*

- Actuator fault detection and isolation
- Neural Network-Based Scheme

Dynamical system:

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t)) + \eta_x(x, u, t) \\ y(t) &= Cx(t) + \eta_y(x, u, t)\end{aligned}$$

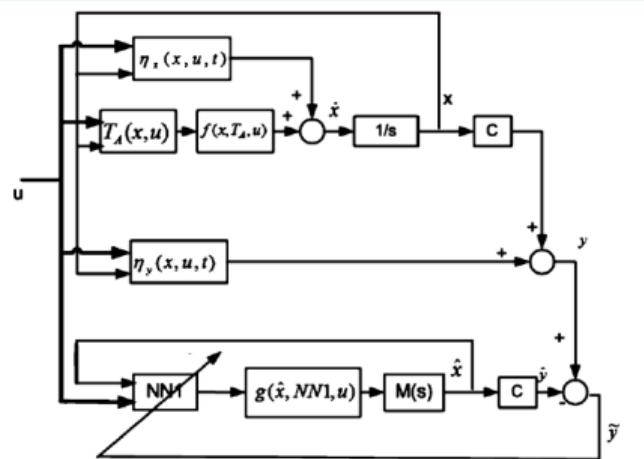
Estimation model:

$$\hat{x} = A\hat{x} + g(\hat{x}, \hat{u}_f)$$

$$\hat{y} = C\hat{x}$$

$$\hat{u}_f = \hat{T}_A(\hat{x}, u, W)u,$$

$$\hat{T}_A(\hat{x}, \hat{W}) = \hat{W}\sigma(\hat{V}\hat{x})$$



- ▶ **Address Sensor Fault Propagation**

Key Contribution

- ▶ Address Sensor Fault Propagation
- ▶ Identification of Fault Type (Sensor/Actuator)

Key Contribution

- ▶ Address Sensor Fault Propagation
- ▶ Identification of Fault Type (Sensor/Actuator)
 - No Need for Persistency of Excitation

Definition (Persistence of excitation):

A signal $y(t)$ is persistently exciting if there are positive scalars $\eta_1, \eta_2, \mathcal{T} \in \mathbb{R}^+$ such that

$$\eta_1 I \leq \int_t^{t+\mathcal{T}} y(\tau) y^T(\tau) d\tau \leq \eta_2 I \quad \forall t \in \mathbb{R}^+.$$

Persistently of exciting \iff sufficient variability over time

Key Contribution

- ▶ Address Sensor Fault Propagation
- ▶ Identification of Fault Type (Sensor/Actuator)
- ▶ **Accurate Isolation of Multiple Faulty Sensors: a Decentralized scheme**

Key Contribution

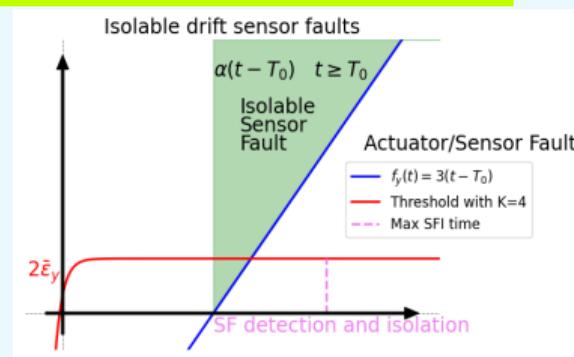
- ▶ Address Sensor Fault Propagation
- ▶ Identification of Fault Type (Sensor/Actuator)
- ▶ Accurate Isolation of Multiple Faulty Sensors: a Decentralized scheme

$$\text{Sensor fault isolability} \iff |f_{y,i}| > 4\bar{\xi}_{y,i}$$

Key Contribution

- ▶ Address Sensor Fault Propagation
- ▶ Identification of Fault Type (Sensor/Actuator)
- ▶ Accurate Isolation of Multiple Faulty Sensors: a Decentralized scheme

KIOS Testbed simulation results



WaterSafe state vector
 $[\mathcal{L}_1, \mathcal{Q}_1, \mathcal{L}_2, \mathcal{Q}_2, \mathcal{L}_3, \mathcal{Q}_3, \mathcal{L}_4, \mathcal{Q}_4]$

It can easily show that drift faults greater than $3(t - T_0)$ for all $T_0 = 5$ can be detectable for states (tank levels):
 $[\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4]$

$$\tilde{y}_{si}^{F_y}(T_d) - \tilde{y}_{si}^{f_y}(T_d) = F_{y_i}(T_d) - f_{y_i}(T_d) - \int_{T_0}^{T_d} e^{A_{k_i}(T_d - \tau)} \left[g(\hat{x}_{s_i}^{F_y}, \underline{y}_i) - g(\hat{x}_{s_i}^{f_y}, \underline{y}_i) + K_i(F_{y_i}(\tau) - f_{y_i}(\tau)) \right] d\tau$$

Key Contribution

- ▶ Address Sensor Fault Propagation
- ▶ Identification of Fault Type (Sensor/Actuator)
- ▶ Accurate Isolation of Multiple Faulty Sensors: a Decentralized scheme
- ▶ **Isolation of Multiple Faulty Actuators: an Adaptive Approximation Scheme**

Problem Formulation

System Description

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + Bu_F(t) + g(x(t)) + \eta(x, t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0)f_y(x, t), \end{cases}$$

Variables:

$x, y \in \mathbb{R}^n, u \in \mathbb{R}^m$. (Actuators and sensors can be faulty)

Functions:

Ax

(known linear term)

$g : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$

(known nonlinear term)

$\eta : \mathbb{R}^n \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^+ \rightarrow \mathbb{R}^n$

(unknown uncertainty term)

$\xi_y(t) \in \mathbb{R}^n$

(sensor noise)

$f_y(x, t) \in \mathbb{R}^n$

(sensor fault)

Problem Formulation

System Description

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + Bu_F(t) + g(x(t)) + \eta(x, t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0)f_y(x, t), \end{cases}$$

Assumptions:

- ▶ g Differentiable and hence Lipschitz in x
- ▶ Modeling uncertainty and measurement noise boundedness:
 - $|\eta_i(x, t)| \leq \bar{\eta}_i \quad i = 1, \dots, n, \quad \bar{\eta}_i$ known constant!
 - $|\xi_{y,i}(t)| \leq \bar{\xi}_{y,i} \quad i = 1, \dots, n, \quad \bar{\xi}_{y,i}$ known constant!
- ▶ Sensor and actuator fault boundedness:
 - $|f_{y,i}| < \bar{f}_{y,i} \quad i = 1, \dots, n \quad \bar{f}_{y,i}$ known constant!
 - $|u_{F,i}(t)| < \bar{u}_{F,i} \quad i = 1, \dots, n \quad \bar{u}_{F,i}$ known constant!
- ▶ Sensor fault isolability: Sensor fault $f_{y,i}$ will be isolated if we have
 - $|f_{y,i}| > 4\bar{\xi}_{y,i} \quad i \in \{1, \dots, n\}.$

Problem Formulation

System Description

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + Bu_F(t) + g(x(t)) + \eta(x, t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0)f_y(x, t), \end{cases}$$

► Actuator fault structure:

$$u_F(t) \triangleq -\Upsilon(t - T_0)\Lambda u_H(t),$$

$$\Upsilon_i(t - T_{0,i}) = \begin{cases} 0 & \text{for all } t < T_{0,i} \\ 1 - \exp(-\delta_i(t - T_{0,i})) & \text{for all } t \geq T_{0,i}, \end{cases}$$

► Sensor fault structure (abrupt & bias faults)

$$\beta_i(t - \mathcal{T}_{0,i}) = \begin{cases} 0 & \text{if } t < \mathcal{T}_{0,i} \\ 1 & \text{if } t \geq \mathcal{T}_{0,i} \end{cases}$$

Sensor Fault Isolation Module:

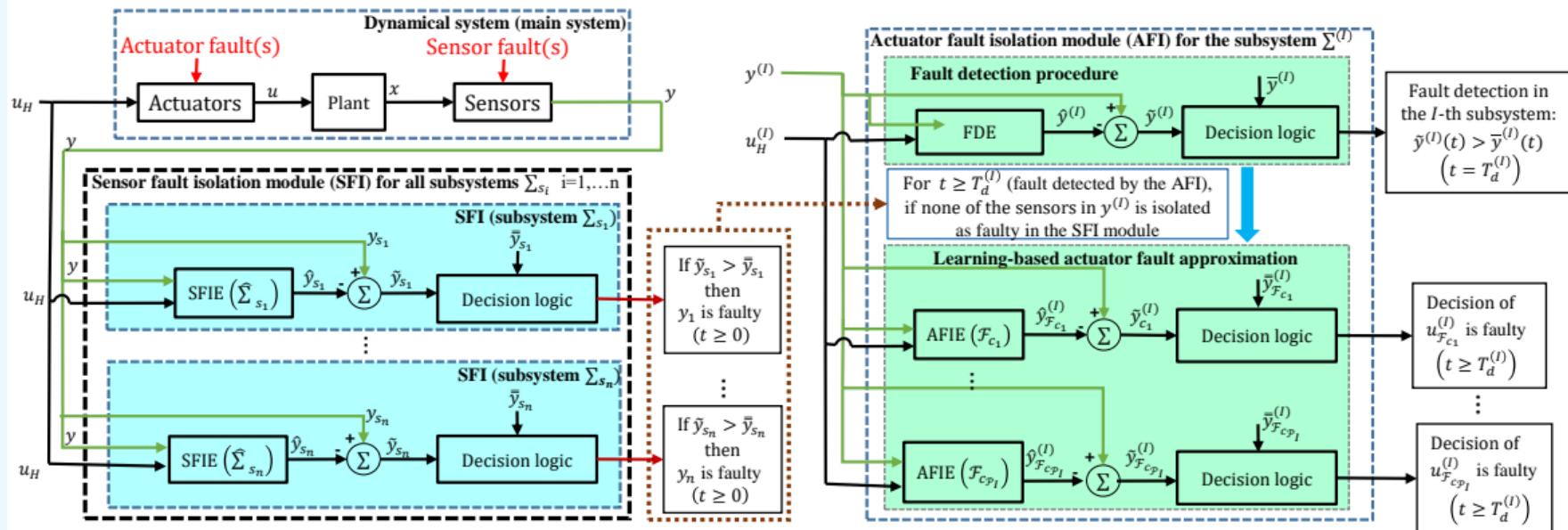
- ▶ SFI Decomposition Procedure
- ▶ A Modified Luenberger estimator design
- ▶ Reasoning-based sensor fault isolation

Actuator Fault Isolation Module:

- ▶ AFI Decomposition Procedure
- ▶ Fault Detection Procedure (designed in our first work)
- ▶ Adaptive Approximation-Based Actuator Fault Estimation and Isolation

Actuator/Sensor Fault Isolation Modules

The architecture of actuator and sensor fault isolation modules



Sensor Fault Isolation

SFI Decomposition Procedure

Decompose the main system into n subsystems:

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + Bu_F(t) + g(x(t)) + \eta(x, t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0)f_y(x, t), \end{cases} \quad \Downarrow$$

$$\Sigma_{s_i} : \begin{cases} \dot{x}_i = a_{ii}x_i + A_i\underline{x}_i + B_iu_H + B_iu_F + g_i(x) + \eta_i(x, t) \\ y_i = x_i + \xi_{y,i} + \beta_i(t - \mathcal{T}_{0,i})f_{y,i}(x, t), \end{cases} \quad i = 1, \dots, n$$

where variables:

$x_i \in lR$ (*i*-th state signal)

$y_i \in lR$ (*i*-th output signal)

$\underline{x}_i \triangleq [x_1, \dots, x_{i-1}, 0, x_{i+1}, \dots, x_n]^T$

$A_i = [a_{i1}, \dots, a_{in}]$ (*i*-th row of A)

$B_i = [b_{i1}, \dots, b_{im}]$ (*i*-th row of B)

A Modified Luenberger Estimator Design

By utilizing the Hurwitz stability in [2], we have

$$A - \bar{\mathcal{K}} = \Pi \triangleq [\pi_{ij}]_{n \times n} = \begin{bmatrix} \boxed{\pi_{11}} & \pi_{12} & \cdots & \pi_{1i} & \cdots & \pi_{1n} \\ \pi_{21} & \boxed{\pi_{22}} & \cdots & \pi_{2i} & \cdots & \pi_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \pi_{i1} & \pi_{i2} & \cdots & \boxed{\pi_{ii}} & \cdots & \pi_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{ni} & \cdots & \boxed{\pi_{nn}} \end{bmatrix}$$

Sensor Fault Isolation

A Modified Luenberger estimator design

$$\Sigma_{s_i} : \begin{cases} \dot{x}_i = a_{ii}x_i + A_i\underline{x}_i + B_iu_H + B_iu_F + g_i(x) + \eta_i(x,t) \\ y_i = x_i + \xi_{y,i} + \beta_i(t - \mathcal{T}_{0,i})f_{y,i}(x, t), \end{cases}$$



$$\hat{\Sigma}_{s_i} : \begin{cases} \dot{\hat{x}}_{s_i} = a_{ii}\hat{x}_{s_i} + A_i\underline{y}_i + B_iu_H + g_i(\hat{x}_{s_i}, \underline{y}_i) + \mathcal{K}_{ii}(y_i - \hat{y}_{s_i}) \\ \hat{y}_{s_i} = \hat{x}_{s_i}, \end{cases}$$

where the estimator gain \mathcal{K}_{ii} has to be selected such that:

$$\mathcal{K}_{ii} > a_{ii} + L_{g_i},$$

$$|a_{ii} - \mathcal{K}_{ii}| > \sum_{j=1, j \neq i}^n |a_{ij}|.$$

Sensor Fault Isolation

Reasoning-based sensor fault isolation

Theorem 3.1 (SFI Threshold for the i -th Sensor)

If the i -th sensor is healthy, then we have

$$|\tilde{y}_{s_i}(t)| \leq \bar{y}_{s_i}(t) \quad \forall t \geq 0,$$

where

$$\bar{y}_{s_i}(t) \triangleq \bar{x}_{s_i}(t) + \bar{\xi}_{y,i}(t),$$

$$\bar{x}_{s_i}(t) \triangleq \mathcal{C}_i(t) + \int_0^t e^{(L_{g_i} + \mathcal{A}_{ii})(t-\tau)} \mathcal{C}_i(\tau) d\tau,$$

$$\mathcal{C}_i(t) \triangleq e^{\mathcal{A}_{ii}t} \bar{x}_{i,0} + \int_0^t e^{\mathcal{A}_{ii}(t-\tau)} [\gamma_i - \mathcal{A}_{ii}\bar{\xi}_{y,i}] d\tau,$$

$$\gamma_i \triangleq |B_i| \bar{u}_F + \sum_{j=1, j \neq i}^n |a_{ij}| (\bar{\xi}_{y,j} + \bar{f}_{y,j}) + L_{g_i} \sum_{j \in \mathcal{Z}_i} (\bar{\xi}_{y,j} + \bar{f}_{y,j}) + \bar{\eta}_i + a_{ii}\bar{\xi}_{y,i},$$

Sensor Fault Isolation

Reasoning-based sensor fault isolation

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$$\mathcal{C}_i(t) \triangleq e^{\mathcal{A}_{ii}t} \bar{x}_{i,0} + \int_0^t e^{\mathcal{A}_{ii}(t-\tau)} [\gamma_i - \mathcal{A}_{ii}\bar{\xi}_{y,i}] d\tau,$$

$$\gamma_i \triangleq |B_i| \bar{u}_F + \sum_{i=1, i \neq i}^n |a_{ij}| (\bar{\xi}_{y,j} + \bar{f}_{y,j}) + L_{g_i} \sum_{j \in \mathcal{F}_i} (\bar{\xi}_{y,j} + \bar{f}_{y,j}) + \bar{\eta}_i + a_{ii}\bar{\xi}_{y,i},$$

If the i -th sensor is healthy, then

$\forall t_{0,i} > 0$, & $\forall \epsilon_i > 0$ $\exists \mathcal{M}_{ii} > 0$, such that:

$$\forall \mathcal{K}_{ii} > \mathcal{M}_{ii}, \Rightarrow |\tilde{y}_{s_i}(t)| < 2\bar{\xi}_{y,i} + \epsilon_i \quad \forall t > t_{0,i}$$

Assumption

Sensor fault isolability:

$$|f_{y,i}| > 4\bar{\xi}_{y,i}$$

AFI decomposition procedure

Actuator faults can only be detected in their corresponding subsystems.



Each actuator has to appear only in one subsystem:

$$u_i \in \mathcal{U}^{(I)} \text{ iff } u_i \notin \mathcal{U}^{(J)}, \text{ for all } I, J \in \{1, \dots, N\}, I \neq J.$$

Fault detection procedure

The fault detection scheme in our previous work is utilized for actuator fault detection.

Actuator Fault Isolation

Adaptive Approximation-Based Actuator Fault Estimation and Isolation

The r_I -th adaptive estimator $\hat{\Sigma}_{\mathcal{F}_{r_I}}^{(I)}$, $r_I = 1, \dots, \mathcal{P}_I$ are given by:

$$\hat{\Sigma}_{\mathcal{F}_{r_I}}^{(I)} : \begin{cases} \dot{\hat{x}}_{\mathcal{F}_{r_I}}^{(I)} = A^{(I)}\hat{x}_{\mathcal{F}_{r_I}}^{(I)} + B^{(I)}(\hat{u}_{\mathcal{F}_{r_I}}^{(I)} + \hat{u}_{\mathcal{F}_{r_I}}^{(I)}) + p(\hat{x}_{\mathcal{F}_{r_I}}^{(I)}) \\ \quad + q^{(I)}(\hat{x}_{\mathcal{F}_{r_I}}^{(I)}, y_z^{(I)}) + K^{(I)}(y^{(I)} - \hat{y}_{\mathcal{F}_{r_I}}^{(I)}) + \Omega_{\mathcal{F}_{r_I}}^{(I)}\dot{\hat{u}}_{\mathcal{F}_{r_I}}^{(I)} \\ \hat{y}_{\mathcal{F}_{r_I}}^{(I)}(t) = \hat{x}_{\mathcal{F}_{r_I}}^{(I)}(t) \\ \dot{\Omega}_{\mathcal{F}_{r_I}}^{(I)} = A_k^{(I)}\Omega_{\mathcal{F}_{r_I}}^{(I)} + B_{\mathcal{F}_{r_I}}^{(I)} \\ \dot{\hat{u}}_{\mathcal{F}_{r_I}}^{(I)} = \mathcal{P}\{\Gamma_{\mathcal{F}_{r_I}}^{(I)}(\Omega_{\mathcal{F}_{r_I}}^{(I)})^T D^{(I)}[\tilde{y}_{\mathcal{F}_{r_I}}^{(I)}]\}, \end{cases}$$

where

$$\dot{\hat{u}}_{\mathcal{F}_{r_I}}^{(I)} = \Gamma_{\mathcal{F}_{r_I}}^{(I)}(\Omega_{\mathcal{F}_{r_I}}^{(I)})^T D[\tilde{y}_{\mathcal{F}_{r_I}}^{(I)}] - \mathcal{X}_{\mathcal{F}_{r_I}}^{(I)}\Gamma_{\mathcal{F}_{r_I}}^{(I)} \frac{\hat{u}_{\mathcal{F}_{r_I}}^{(I)} \hat{u}_{\mathcal{F}_{r_I}}^{(I)T}}{\hat{u}_{\mathcal{F}_{r_I}}^{(I)} \Gamma_{\mathcal{F}_{r_I}}^{(I)} \hat{u}_{\mathcal{F}_{r_I}}^{(I)}} \mathcal{P}^{(I)},$$

Adaptive Approximation-Based Actuator Fault Estimation and Isolation

AFI Threshold Design: Considering the accurate faulty actuator set is $\mathcal{F}_{k_I}^{(I)}$, and the fault is detected at time $T_d^{(I)}$, then, we have

$$|\tilde{y}_{\mathcal{F}_{k_I},i}^{(I)}(t)| \leq \bar{\bar{y}}_{\mathcal{F}_{k_I},i}^{(I)}(t) \quad \forall i = 1, \dots, n_I \quad \forall t \geq T_d^{(I)} + t_D^{(I)},$$

where

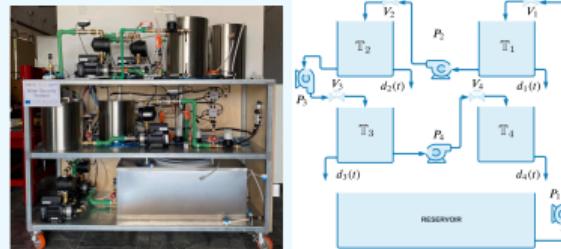
$$\bar{\bar{y}}_{\mathcal{F}_{k_I},i}^{(I)}(t) \triangleq \bar{x}_{\mathcal{F}_{k_I},i}^{(I)}(t) + \bar{\xi}_y^{(I)}(t) \quad \forall t \geq T_d^{(I)} + t_D^{(I)},$$

$$\bar{x}_{\mathcal{F}_{k_I},i}^{(I)}(t) \triangleq \mathbb{E}_{\mathcal{F}_{k_I},i}^{(I)}(t) + L_i^{(I)} \int_{T_d^{(I)}}^t e^{(a_{k,i}^{(I)} + L_i^{(I)})(t-\tau)} \mathbb{E}_{\mathcal{F}_{k_I},i}^{(I)}(\tau) d\tau,$$

$$\mathbb{E}_{\mathcal{F}_{k_I},i}^{(I)}(t) \triangleq \left| \Omega_{\mathcal{F}_{k_I},i}^{(I)} \right| \kappa_{\mathcal{F}_{k_I}}^{(I)} + \int_{T_d^{(I)}}^t e^{a_{k,i}^{(I)}(t-\tau)} \mu_{\mathcal{F}_{k_I},i}^{(I)} d\tau + e^{a_{k,i}^{(I)}(t-T_d^{(I)})} \bar{x}_{d,i}^{(I)},$$

Simulation results

The KIOS Water Systems Testbed



$$\Sigma_I: \begin{cases} \dot{\mathcal{L}}_I(t) = \frac{1}{S_I} [\mathcal{Q}_I(t) - \mathcal{Q}_{I+1}(t) - d_I(t)] \\ \dot{\mathcal{Q}}_I(t) = \Psi_A(\mathcal{Q}_I(t), u_I(t)) \end{cases} \quad I = 1, 2, 3, 4,$$

$\mathcal{Q}_{I+1}(t)$ denotes the outflow rate of tank \mathbb{T}_I at time t

Local Feedback Controller (LFC)

$$\Psi_A^{(I)}(x_2^{(I)}, u_I) \triangleq \begin{cases} \lambda_I u_I - \alpha_I x_2^{(I)} & 0.1 < u_I \leq 1 \\ -\vartheta_I x_2^{(I)} & 0 \leq u_I \leq 0.1, \end{cases}$$

[1] Vrachimis, Stelios, et al. "WaterSafe: A water network benchmark for fault diagnosis research." IFAC-PapersOnLine 55.6 (2022): 655-660.

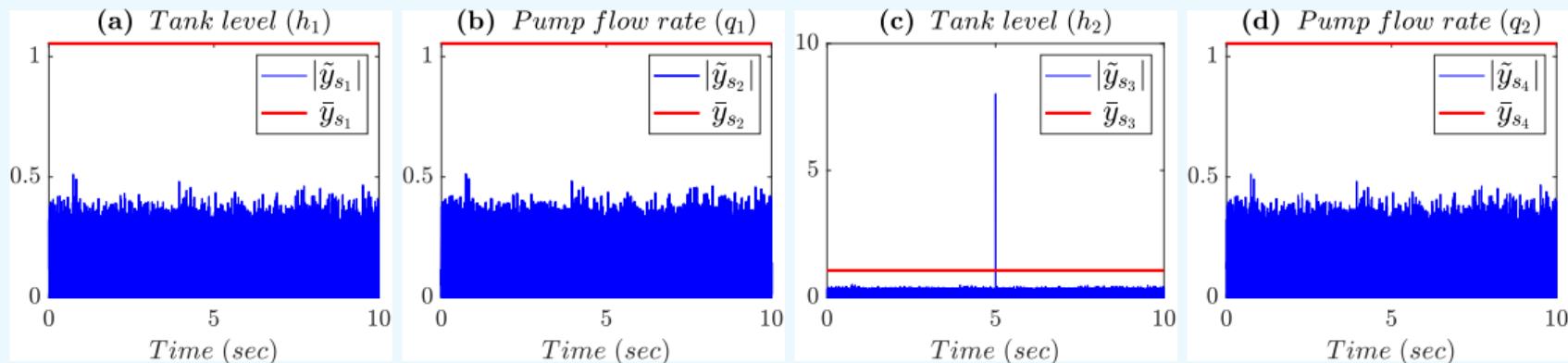
Simulation results(1)

Isolation of Faulty Sensors

The faulty sensors are y_3 , y_7 and y_8 .

The faults $f_{y,3} = 8$, $f_{y,7} = 3$ and the intermittent fault as $f_{y,8} = 5 \quad t \in [2j, 2j + 0.5]$ for all $j = 1, 2, 3$,

SFI results of the SFI module for subsystems $\Sigma_{s_i} \quad i = 1, \dots, 8$



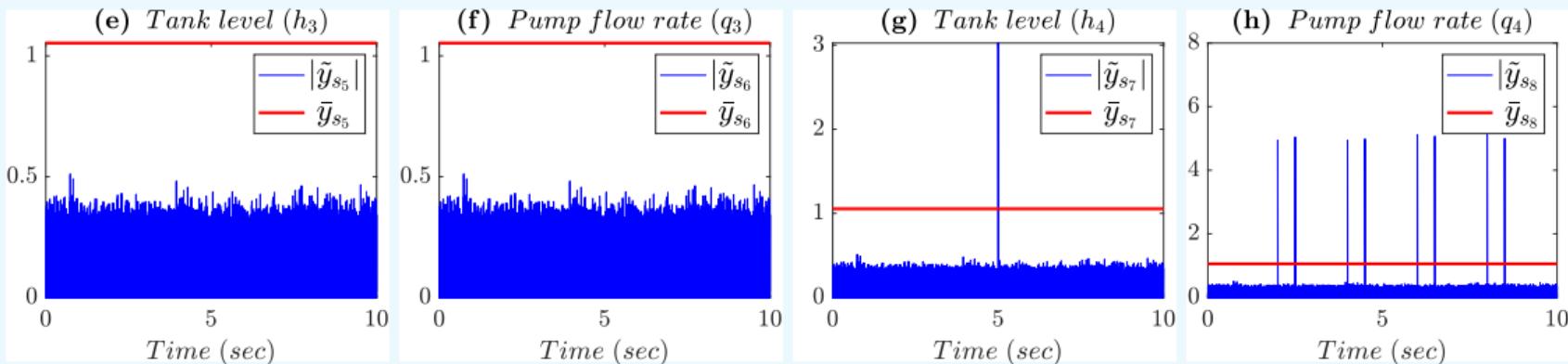
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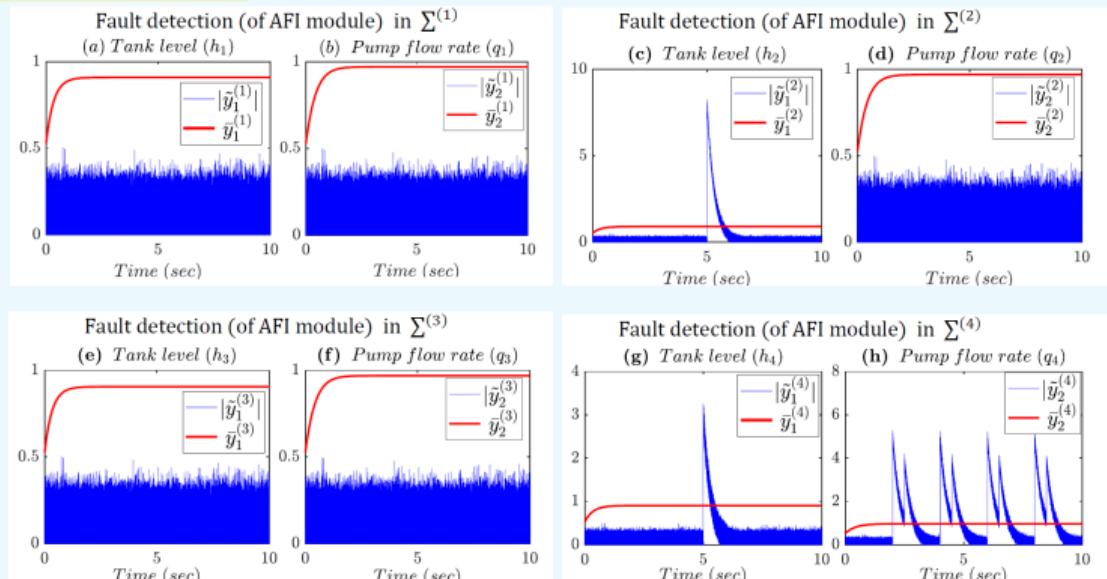
Simulation results(1)

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The faults $f_{y,3} = 8$, $f_{y,7} = 3$ and the intermittent fault as $f_{y,8} = 5 \quad t \in [2j, 2j + 0.5]$ for all $j = 1, 2, 3$,

AFI results (fault detection) of the AFI module for $\Sigma^{(i)} \quad i = 1, \dots, 4$.

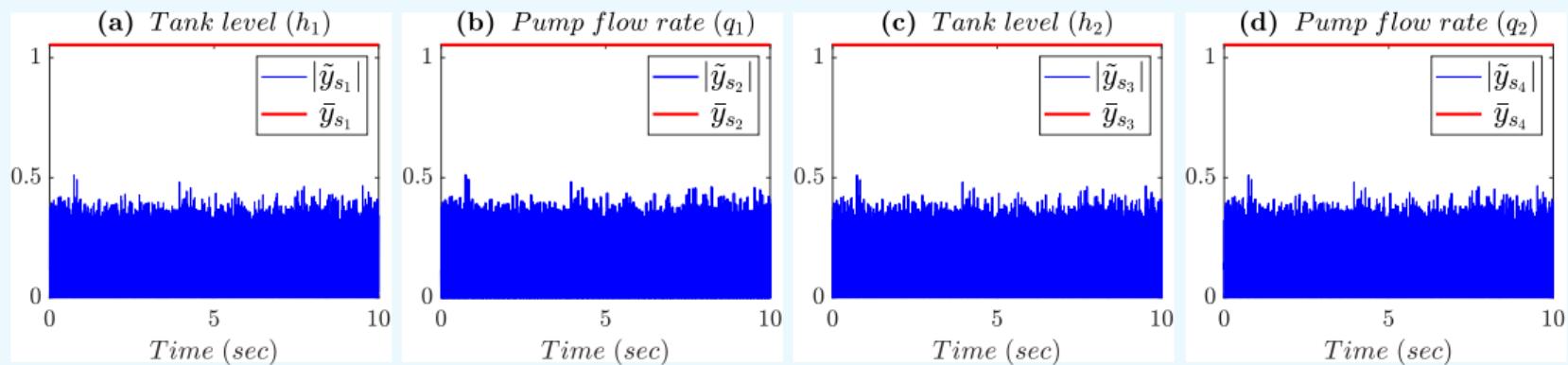


Simulation results(2)

Isolation of Faulty Actuator

The faulty actuators are u_1 and u_2 (i.e., $u_1^{(1)}$ and $u_1^{(2)}$).

- SFI results of the SFI Module for subsystems $\Sigma_{s_i} \quad i = 1, 2, 3, 4$



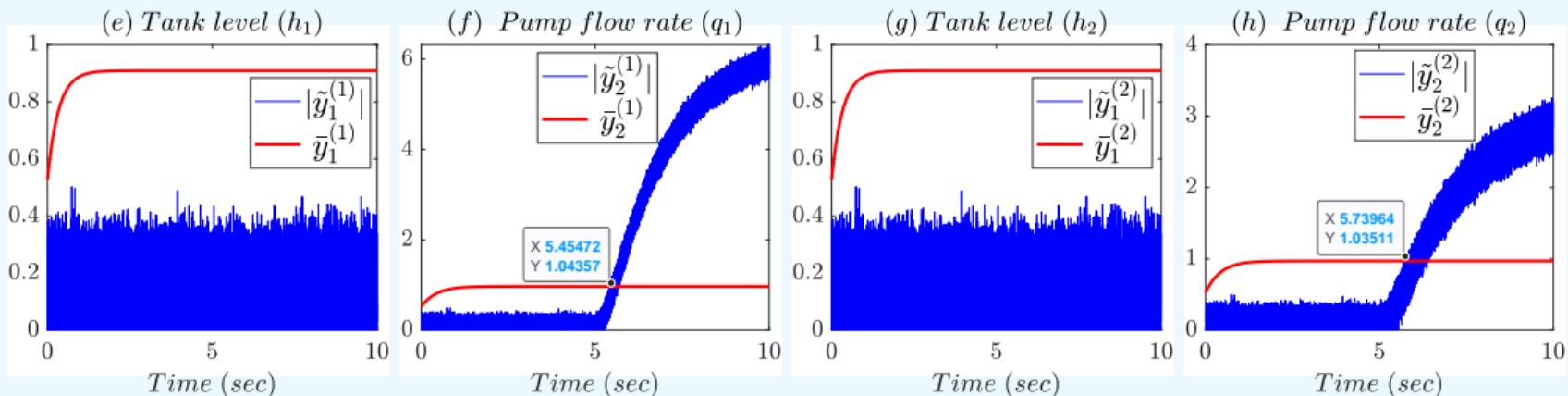
[2] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*, 2024.

Simulation results(2)

Isolation of Faulty Actuator

The faulty actuators are u_1 and u_2 (i.e., $u_1^{(1)}$ and $u_1^{(2)}$).

- Fault detection result of the AFI Module for subsystems $\Sigma^{(1)}$ and $\Sigma^{(2)}$



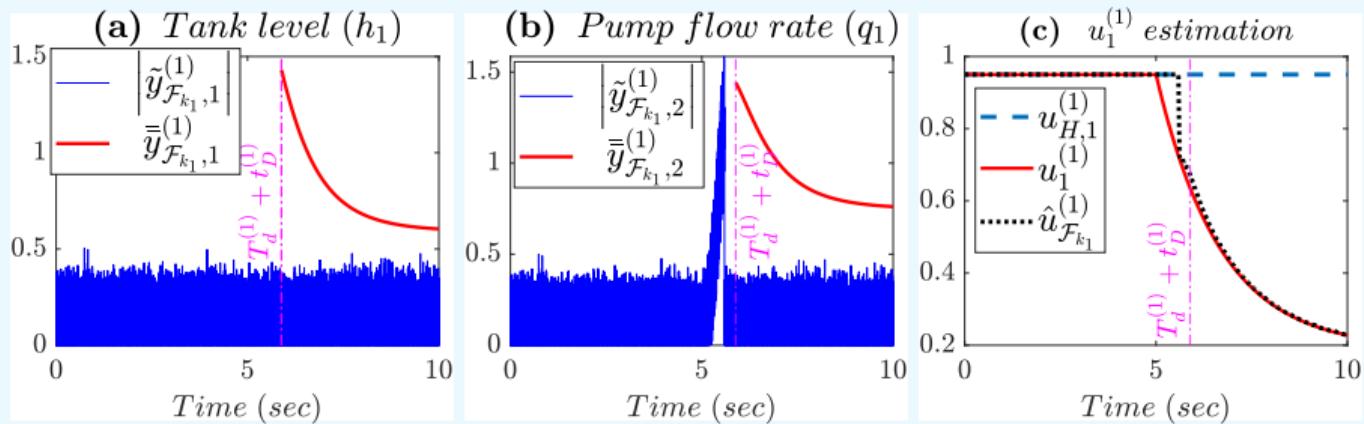
[2] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*, 2024.

Simulation results(2)

Isolation of Faulty Actuator

The faulty actuators are u_1 and u_2 (i.e., $u_1^{(1)}$ and $u_1^{(2)}$).

- ▶ AFI results of the AFI Module for subsystems $\Sigma^{(1)}$



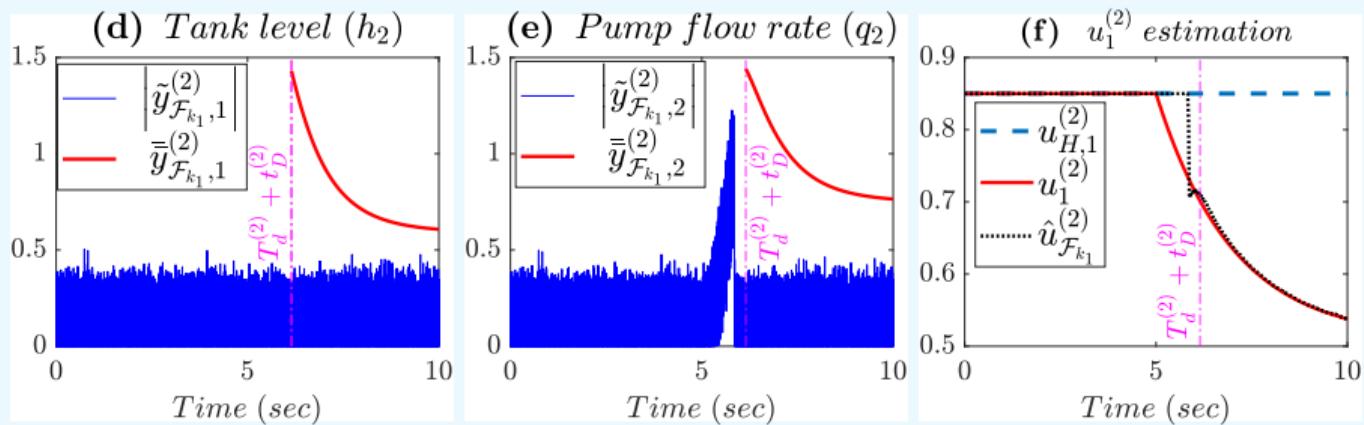
[2] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*, 2024.

Simulation results(2)

Isolation of Faulty Actuator

The faulty actuators are u_1 and u_2 (i.e., $u_1^{(1)}$ and $u_1^{(2)}$).

- ▶ AFI results of the AFI Module for subsystems $\Sigma^{(2)}$



[2] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*, 2024.

Actuator Fault Accommodation for a Class of Nonlinear Uncertain Interconnected Systems: An Adaptive Command Filtering-Based Scheme

Objectives:

- ▶ Nominal Controller Design
- ▶ Actuator Fault Accommodation

[1]. Lai G, et al., 2018,

Adaptive compensation for infinite number of actuator failures based on tuning function approach, *Automatica*.

- **Actuator fault compensation scheme**
- **Actuators with the same relative degrees**
- **No need for FDI**

Dynamical system:

$$\dot{x}_i = x_{i+1} + \theta^T \varphi_i(\bar{x}_i), \quad i = 1, \dots, n-1$$

$$\dot{x}_n = \varphi_0(\bar{x}_n, \xi) + \theta^T \varphi_n(\bar{x}_n, \xi) + \sum_{j=1}^m b_j \beta_j(\bar{x}_n, \xi) u_j$$

$$\dot{\xi} = \psi(\bar{x}_n, \xi) + \phi(\bar{x}_n, \xi) \theta$$

$$y = x_1,$$

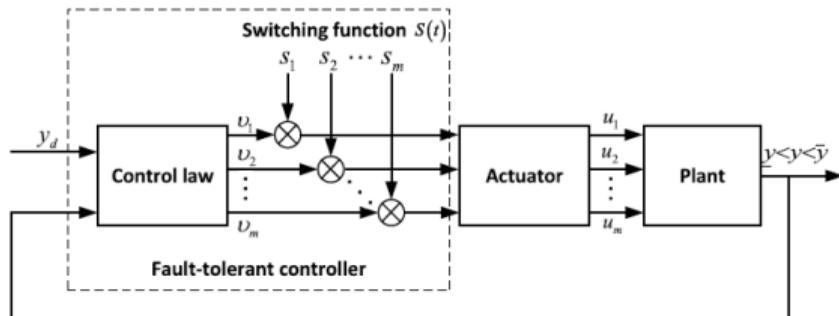
[2]. Ruan Z, et al., 2020,

Performance-guaranteed fault-tolerant control for uncertain nonlinear systems via learning-based switching scheme, *IEEE Trans. Neural Netw. Learn. Syst.*, ¹

- Actuators with the same relative degrees
- Switching-based scheme
- No need for FDI

Dynamical system:

$$\begin{aligned}\dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t) \\ \dot{x}_n &= f_n(\bar{x}_n) + g_n^T(\bar{x}_n)u + d_n(t) \\ y &= x_1\end{aligned}$$



Another switching scheme is proposed in:

Yang, Qinmin, Shuzhi Sam Ge, and Youxian Sun. "Adaptive actuator fault tolerant control for uncertain nonlinear systems with multiple actuators." *Automatica* 60 (2015): 92-99.



Literature Review

[3]. Wu C, et al., 2017,

Observer-based adaptive fault-tolerant tracking control of nonlinear nonstrict-feedback systems, *IEEE Trans. Neural Netw. Learn. Syst.*

- It supposed the fault has already been isolated
- Actuators with the same relative degrees

Dynamical system:

$$\begin{aligned}\dot{x}_i &= x_{i+1} + f_i(x), i = 1, 2, \dots, n-1 \\ \dot{x}_n &= g^T u + f_n(x) \\ y &= x_1.\end{aligned}\tag{1}$$

Literature Review

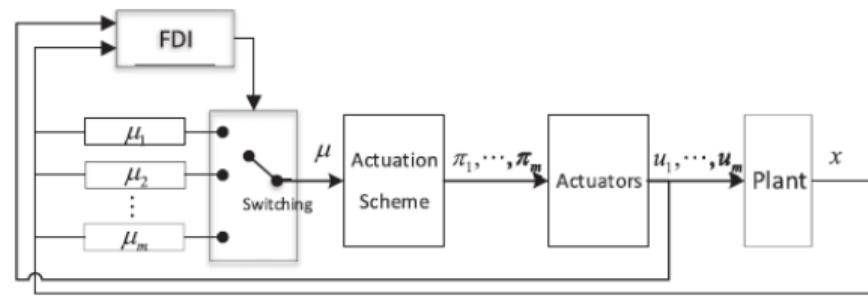
[4]. Zhang K, et al., 2021.

Fault-tolerant control for systems with unmatched actuator faults and disturbances,
IEEE Trans. Autom. Control.

- Actuators with various relative degrees
- Centralized scheme
- Sliding surface FTC scheme
- It utilizes FDI information
- No modeling uncertainty

Consider a class of nonlinear over-actuated systems

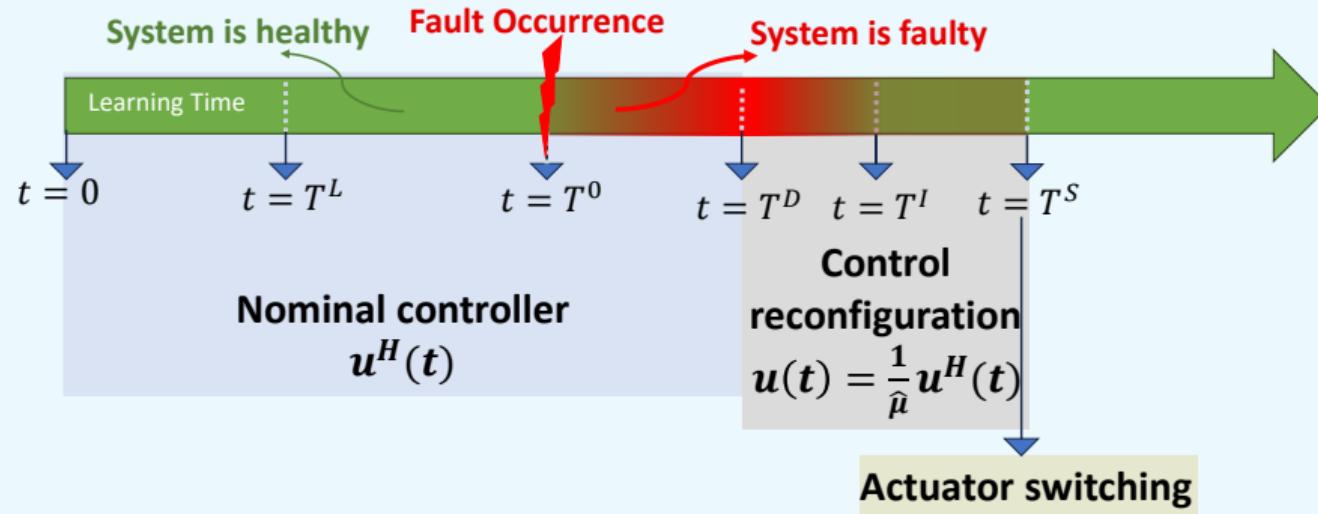
$$\begin{aligned}\dot{\zeta} &= f(\zeta) + \sum_{i=1}^m g_i(\zeta)u_i(t) + \sum_{i=1}^n D_i(\zeta)d_i(t) \\ y &= h(\zeta)\end{aligned}\quad (1)$$



Key contributions

1. **A distributed backstepping control scheme** is designed for systems with multiple fault-prone actuators, ensuring bounded signals in both healthy and faulty conditions.
2. **The "explosion of complexity"** issue associated with backstepping design is mitigated by applying a finite-time command filter technique.
3. **An adaptive approximation method** is proposed, overcoming limitations of existing uncertainty estimation techniques and accurately estimating both local and interconnected uncertainties.
4. **A unified actuator fault accommodation scheme** is introduced in this work that addresses both partial and total loss of actuator effectiveness, stabilizing the system and handling multiple fault-prone actuators, modeling uncertainty, and unknown interconnections.

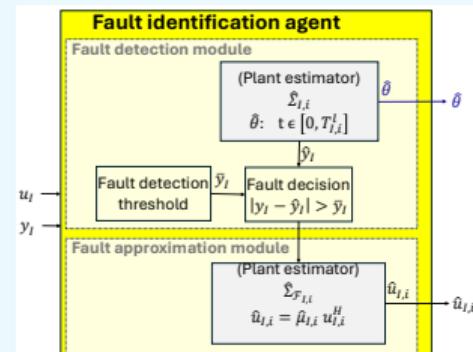
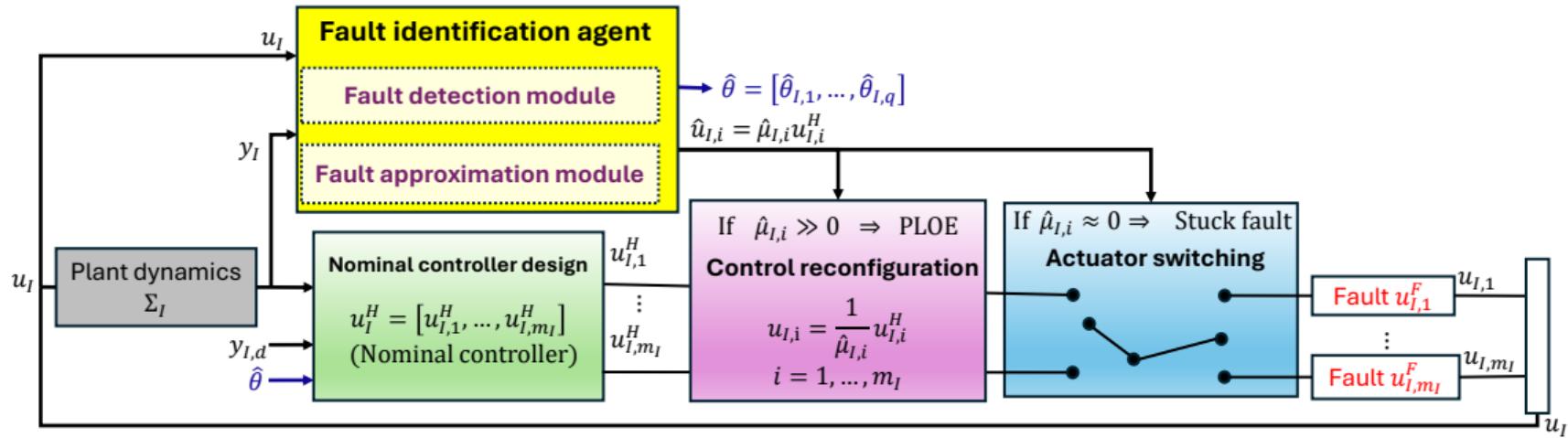
System Timeline Schedule



Control objective:

- ▶ The output signal $y_I(t)$ tracks desired trajectory $y_{I,d}(t)$.
- ▶ All signals are bounded in healthy/faulty conditions.

Actuator Fault Accommodation Architecture



System description

Consider the I -th subsystem $\Sigma_I \quad I = 1, \dots, N$:

$$\Sigma_I : \begin{cases} \dot{x}_I = f_I(x_I) + \sum_{i=1}^{m_I} g_{I,i}(x_I) u_{I,i} + \underbrace{\sum_{j=1}^{n_I} d_{I,i}(x_I)}_{\text{Unknown}} \underbrace{\eta_I(x_I)}_{\text{Unknown}} + \sum_{k=1}^{n_I} q_{I,k}(x_I) \underbrace{\psi_I(X_I)}_{\text{Unknown}} \\ y_I = h_I(x_I) \end{cases}$$

Assumptions:

- ▶ **The triple $(f_I, g_{I,i}, h_I)$ has relative degree $r_{I,i}$ for all $i = 1, \dots, m_I$,**
- ▶ **The triple $(f_I, d_{I,j}, h_I)$ has relative degree j for all $j = 1, \dots, n_I$,**
- ▶ **The triple $(p_I, q_{I,k}, h_I)$ has relative degree k for all $k = 1, \dots, n_I$,**

System description

$$\Sigma_I: \begin{cases} \dot{x}_I = f_I(x_I) + \sum_{i=1}^{m_I} g_{I,i}(x_I) \textcolor{blue}{u}_{I,i} + \sum_{j=1}^{n_I} d_{I,i}(x_I) \eta_{I,j}(x_I) + \sum_{k=1}^{n_I} q_{I,k}(x_I) \psi_{I,k}(X_I) \\ y_I = h_I(x_I), \end{cases}$$

$$u_I = u_I^H + u_I^F,$$

How to distinguish PLOE from TLOE:

We have

$$u_{I,i}(t) = \mu_{I,i}(t) u_{I,i}^H(t) + \bar{u}_{I,i} \quad i = 1, \dots, m_I,$$

- **TLOE of actuator $u_{I,i}$, occurs when $\mu_{I,i} \approx 0$.**
- **PLOE of actuator $u_{I,i}$, occurs when $0 \ll \mu_{I,i} < 1$.**

System Normalization

System description

$$\Sigma_I: \begin{cases} \dot{x}_I = f_I(x_I) + \sum_{i=1}^{m_I} g_{I,i}(x_I)u_{I,i} + \sum_{j=1}^{n_I} d_{I,i}(x_I)\eta_{I,j}(x_I) + \sum_{k=1}^{n_I} q_{I,k}(x_I)\psi_{I,k}(X_I) \\ y_I = h_I(x_I), \end{cases}$$

System Normalization

$$\xi_{Iij} = L_{f_I}^{(j-1)} h_I(x_I) \Big|_{x=\Phi_I^{-1}(\xi_I, \zeta_I)} \quad j = 1, \dots r_{I,i}, i = 1, \dots m_I$$

↑↓

$$\xi_{I,i} \triangleq [h_I(x_I), L_{f_I} h_I(x_I), \dots, L_{f_I}^{(r_i-1)} h_I(x_I)]_{x=\Phi_I^{-1}(\xi_I, \zeta_I)}^T \in \mathbb{R}^{r_i}$$

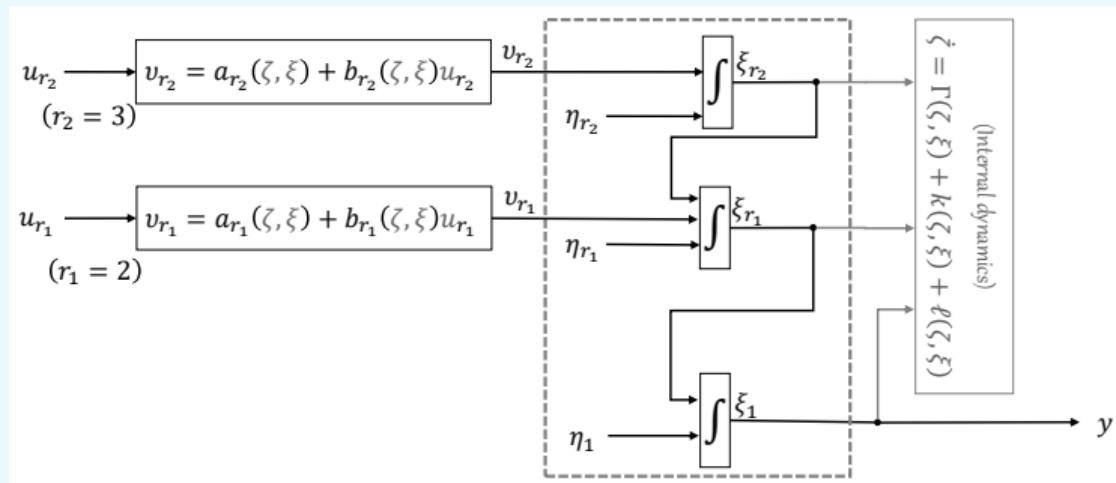
Then, by utilizing the above transformation, the actuator $u_{I,i} \quad i \in \{1, \dots, m_I\}$ can be designed to control subsystem Σ_I .

System Normalization

System description

$$\Sigma_I: \begin{cases} \dot{x}_I = f_I(x_I) + \sum_{i=1}^{m_I} g_{I,i}(x_I)u_{I,i} + \sum_{j=1}^{n_I} d_{I,i}(x_I)\eta_{I,j}(x_I) + \sum_{k=1}^{n_I} q_{I,k}(x_I)\psi_{I,k}(X_I) \\ y_I = h_I(x_I), \end{cases}$$

System Normalization



Objective:

We aim to estimate the unknown local and interconnected terms.

Why estimation?!

The majority of fault diagnosis investigations consider modeling uncertainty to be bounded by a known bound, i.e.,

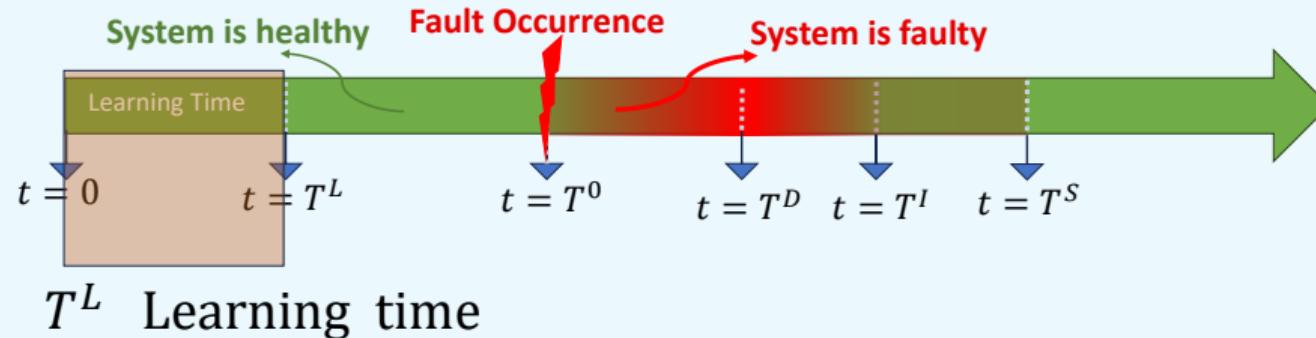
$$|\eta(\mathbf{x})| \leq \bar{\eta} \quad \forall \mathbf{x} \in \mathcal{X}$$

where $\bar{\eta}$ is a known constant/function.

Key challenges:

- Unknown bounds for unknown uncertainties,
- Large-magnitude bounds,
- Focuses on stability & not guarantees optimality.

System Timeline Schedule



To address modeling uncertainty and unknown interconnection terms:

➤ **Approximation ($0 \leq t \leq T^L$)**

$$\hat{\Xi}(x) \approx \hat{\theta}(t)\Omega(x) \quad \text{where} \quad \hat{\theta}(t) = \mathcal{P}(\Gamma\Omega(t)e(y(t)))$$

➤ **Estimation ($t \geq T^L$)**

$$\hat{\Xi}(x) \approx \hat{\theta}^*\Omega(x)$$

Objective:

We aim to estimate the unknown local and interconnected terms.

Literature review:

[1]. Caccavale, F., et al., 2009,,

Actuators fault diagnosis for robot manipulators with uncertain model, , *Control Eng. Pract.*..

- ▶ An SVM-based offline scheme considering five fault-free trajectories is applied for uncertainty estimation.

[2]. Reppa, V., et al., 2014.,

Adaptive approximation for multiple sensor fault detection and isolation of nonlinear uncertain systems. , *IEEE Trans. Neural Netw. Learn. Syst.*

- ▶ The variables are considered to be known limited intervals, i.e., $x_4 \in [-0.2, 0.2]$.

[3] Keliris, C., et al., 2017,,

An integrated learning and filtering approach for fault diagnosis of a class of nonlinear dynamical systems, *IEEE Trans. Neural Netw. Learn. Syst.*..

- ▶ The variables are considered known limited intervals, i.e., $x_4 \in [-0.25, 0.25]$.

Adaptive approximation

Objective:

We aim to estimate the unknown local and interconnected terms.

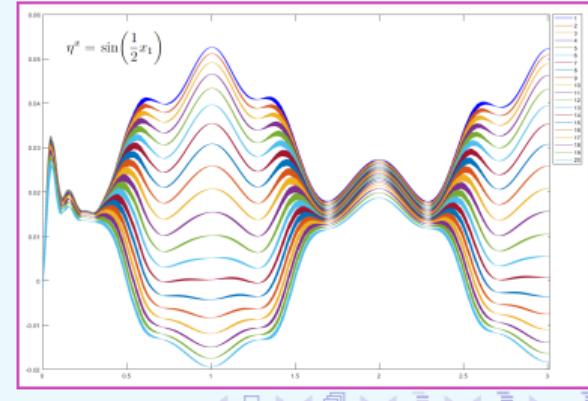
Key challenges:

- ▶ Persistency of excitation,
- ▶ The convergence of $\hat{\theta}(t)$ to the optimal value θ^* cannot be guaranteed at finite time, i.e., $\dot{\hat{\theta}} \rightarrow 0$ but $\lim_{t \rightarrow T_{I,i}^l} \hat{\theta}(t) \neq \theta^*$,

Example:

Modeling uncertainty is:

$$\begin{aligned}\eta(x_1) &= \sin\left(\frac{1}{2}x_1\right) \\ &\approx \theta_0 + \theta_1 x^1 + \cdots + \theta_{19} x^{19},\end{aligned}$$



Adaptive approximation

We aim to estimate the unknown local and interconnected terms simultaneously as

$$\begin{aligned}\Xi_{Iik}(\xi_{I,i}, \zeta_{I,i}, X_I) &\triangleq \overbrace{p_{I,k}^d(\xi_{I,i}, \zeta_{I,i})\eta_{Iik}^x(\xi_{I,i}, \zeta_{I,i})}^{\text{Local modeling uncertainty}} + \overbrace{p_{I,1}^q(\xi_{I,i}, \zeta_{I,i})\psi_{I,1}(X_I)}^{\text{Unknown interconnected term}} \\ \Xi_{Iik}(\xi_{I,i}, \zeta_{I,i}, X_I) &\triangleq \hat{\Xi}_{Iik}(\xi_{I,i}, \zeta_{I,i}, X_I; \theta_{Iik}^*) + e_{\Xi_{I,k}}(\xi_{I,i}, \zeta_{I,i}, X_I) \\ \hat{\Xi}_{I,k}(\xi_{I,i}, \zeta_{I,i}, X_I; \theta_{Iik}^*) &= \Pi_{I,k}(\xi_{I,i}, \zeta_{I,i}, X_I)\theta_{Iik}^*\end{aligned}$$

Assumption

In our work, the unknown function Ξ_{Iik} is considered to be

$$\Xi_{Iik}(\varkappa_1, \dots, \varkappa_N) = \Xi_{Iik}(\varkappa_1) + \dots + \Xi_{Iik}(\varkappa_N).$$

Hence, we have $\Pi_{I,k}(\xi_{I,i}, \zeta_{I,i}, X_I) = \sum_{j=1}^{N_{I,k}} \Pi_{I,k}(\varkappa_j)$.

Dynamical System Estimation

$$\hat{\xi}_{Iij} = \xi_{Ii\{j+1\}} + \sum_{j=1}^{\mathcal{N}_{I,k}} \Pi_{I,j}(\boldsymbol{\varkappa}_{I,k}^{(j)}) \hat{\theta}_{Iij}^{(j)} + k_{Ijj}(\xi_{Iij} - \hat{\xi}_{Iij}) \quad j = 1, \dots, r_{I,i} - 1$$
$$\hat{\xi}_{Iir_i} = \zeta_{Ii1} + p_{I,r_i}^g(\xi_{I,i}, \zeta_{I,i}) u_{I,i} + \sum_{j=1}^{\mathcal{N}_{I,k}} \Pi_{I,r_i}(\boldsymbol{\varkappa}_{I,i}^{(j)}) \hat{\theta}_{Iir_i}^{(j)} + k_{Irr_i}(\xi_{Iir_i} - \hat{\xi}_{Iir_i})$$
$$\zeta_{Iij} = \Gamma_{Iij}(\xi_{I,i}, \zeta_{I,i}) \quad j = 1, \dots, n_I - r_{I,i}$$
$$\hat{y}_I = \hat{\xi}_{Ii1},$$

where

$$\dot{\hat{\theta}}_{Ik}^{(j)}(t) = \mathcal{P}_p \left(\Gamma_{I,k} \Pi_{I,k}^T(\xi_{I,i}, \zeta_{I,i}, X_I) \tilde{\xi}_{I,k} \right) \quad t \leq T_{I,i}^l \quad \begin{array}{l} \text{for all } k = 1, \dots, r_{I,i}, \\ \text{for all } j = 1, \dots, \mathcal{N}_{I,k}. \end{array}$$

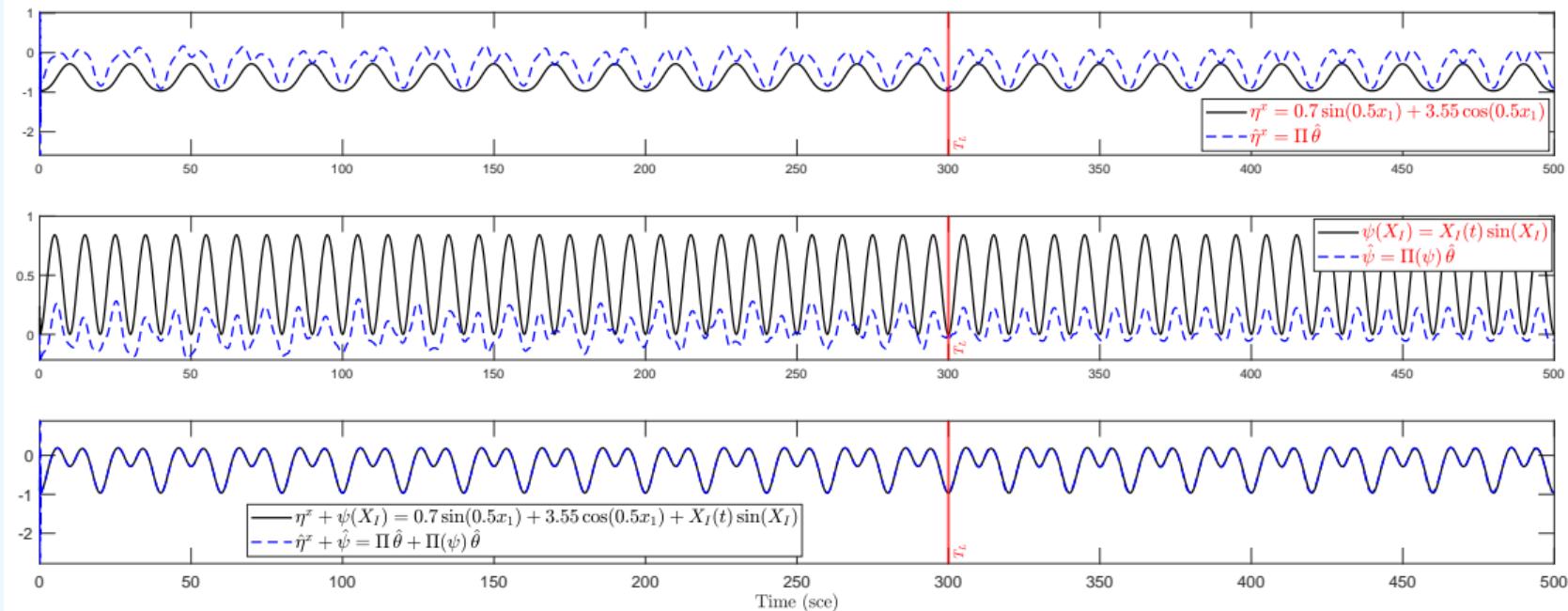
Adaptive approximation

Challenge: $\lim_{t \rightarrow T_{I,i}^l} \hat{\theta}(t) \neq \theta^*$

Solution: By assembling data points $\hat{\eta}_{I,k}(\varkappa_{I,k}^{(j)}(t_r)) = \sum_{s=1}^q \Pi'_{Ik,s}(\varkappa_{I,k}^{(j)}(t_r)) \cdot \hat{\theta}_{Ik,s}^{(j)*}(t_r)$ where $\Pi_{I,k} = [\Pi'_{Ik,1}, \dots, \Pi'_{Ik,q}]$, we have

$$\underbrace{\begin{bmatrix} \hat{\Xi}_{I,k}^{(j)} \\ \hat{\Xi}_{I,k}^{(j)}(\varkappa_{I,k}^{(j)}(t_1)) \\ \hat{\Xi}_{I,k}^{(j)}(\varkappa_{I,k}^{(j)}(t_2)) \\ \vdots \\ \hat{\Xi}_{I,k}^{(j)}(\varkappa_{I,k}^{(j)}(t_m)) \end{bmatrix}}_{\hat{\Xi}_{I,k}^{(j)}} \approx \underbrace{\begin{bmatrix} \Pi'_{Ik,1}(\varkappa_{I,k}^{(j)}(t_1)) & \cdots & \Pi'_{Ik,q}(\varkappa_{I,k}^{(j)}(t_1)) \\ \Pi'_{Ik,1}(\varkappa_{I,k}^{(j)}(t_2)) & \cdots & \Pi'_{Ik,q}(\varkappa_{I,k}^{(j)}(t_2)) \\ \vdots & \ddots & \vdots \\ \Pi'_{Ik,1}(\varkappa_{I,k}^{(j)}(t_m)) & \cdots & \Pi'_{Ik,q}(\varkappa_{I,k}^{(j)}(t_m)) \end{bmatrix}}_{\Pi'_{I,k}(\varkappa_{I,k}^{(j)})} \underbrace{\begin{bmatrix} \hat{\theta}_{Ik,1}^{(j)*} \\ \hat{\theta}_{Ik,2}^{(j)*} \\ \vdots \\ \hat{\theta}_{Ik,q}^{(j)*} \end{bmatrix}}_{\hat{\theta}_{Ik}^{(j)*}},$$

Adaptive approximation



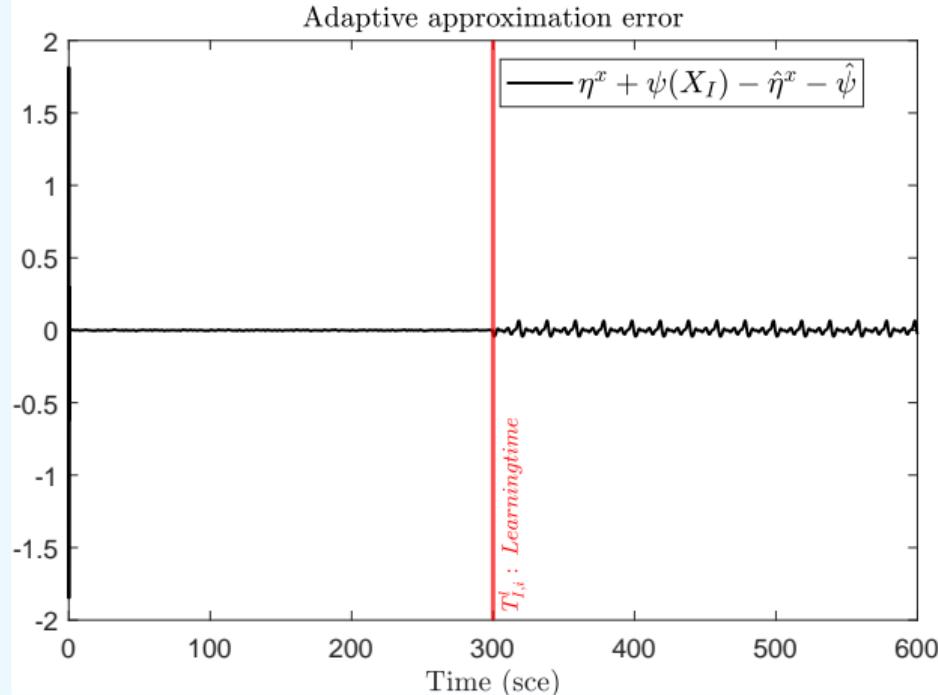
Modeling uncertainty

$$\overbrace{\eta^x = 2 \sin(0.5x_1) + \cos(0.5x_1),}^{}$$

Interconnection term

$$\overbrace{\psi(X_1) = 3X_1 \sin(X_1)}^{}$$

Adaptive approximation



Modeling uncertainty

$$\eta^x = 2 \sin(0.5x_1) + \cos(0.5x_1),$$

Interconnection term

$$\psi(X_1) = 3X_1 \sin(X_1)$$

Nominal Controller Design

Distributed command-filter-based backstepping controller $u_{I,i} = u_{I,i}^H$:

$$u_{I,i}^H \triangleq \frac{-1}{p_{I,r_i}^g(\xi_{I,i}, \zeta_{I,i})} \left[L_{f_I}^{(r_i)} h_I(x_I) + \frac{1}{2} \nu_{I,r_i} + l_{I,r_i} z_{I,r_i} + \ell_{I,r_i} \sin\left(\gamma_{I,r_i}\right) \right. \\ \left. + \gamma_{I,\{r_i-1\}} + \nu_{I,\{r_i-1\}} + \sum_{j=1}^{\mathcal{N}_{I,r_i}} \Pi_{I,r_i}(\varkappa_{I,r_i}^{(j)}(t)) \hat{\theta}_{I,r_i}^{(j)*} - \dot{\alpha}_{I,\{r_i-1\}} - y_{I,d}^{(r_i)} \right],$$

The virtual control law α_{Iik} :

$$\alpha_{Iik} \triangleq -\nu_{I,\{k-1\}} - \frac{1}{2} \nu_{Iik} - \sum_{j=1}^{\mathcal{N}_{I,k}} \Pi_{I,k}(\varkappa_{I,k}^{(j)}(t)) \hat{\theta}_{Iik}^{(j)*} - l_{Iik} z_{Iik} + \dot{\alpha}_{I,\{k-1\}} - \ell_{Iik} \sin\left(\gamma_{Iik}\right),$$

Fault Type Identification

Control Reconfiguration (will be addressed after fault detection)

$$\hat{\dot{\xi}}_{\mathcal{F}_{Iir_i}} = \zeta_{i1} + p_{I,r_i}^g(\xi_{I,i}, \zeta_{I,i})\hat{u}_{I,i} + \Pi_{I,r_i}(\xi_{I,i}, \zeta_{I,i}, X_I)\hat{\theta}_{Iir_i} + k_{Irr_i}(\xi_{Iir_i} - \hat{\xi}_{\mathcal{F}_{Iir_i}}) + \Omega_{\mathcal{F}_{Iir_i}}\dot{\hat{\mu}}_{I,i}$$
$$\hat{\Sigma}_{\mathcal{F}_{I,i}} : \begin{cases} \dot{\hat{\Omega}}_{\mathcal{F}_{Iir_i}} = k_{Irr_i}\Omega_{\mathcal{F}_{Iir_i}} + p_{I,r_i}^g(\xi_{I,i}, \zeta_{I,i})u_{I,i}^H \\ \hat{u}_{I,i} = \hat{\mu}_{I,i}u_{I,i}^H, \end{cases}$$

$$\dot{\hat{\mu}}_{I,i} = \mathcal{Q}_{I,i} - \mathcal{X}_{\mathcal{F}_{I,i}}\Gamma_{\mathcal{F}_{I,i}}\frac{\hat{\mu}_{I,i}\hat{\mu}_{I,i}^T}{\hat{\mu}_{I,i}^T\Gamma_{\mathcal{F}_{I,i}}\hat{\mu}_{I,i}}\mathcal{Q}_{I,i}, \text{ where } \mathcal{Q}_{I,i} \triangleq \Gamma_{\mathcal{F}_{I,i}}(\Omega_{\mathcal{F}_{I,i}})^T\tilde{y}_{\mathcal{F}_{I,i}}$$

- ▶ PLOE $0 \ll \hat{\mu}_{I,i} \leq 1 \Rightarrow$ Control reconfiguration: $u_{I,i} \triangleq \frac{1}{\hat{\mu}_{I,i}}u_{I,i}^H$
- ▶ TLOE $\hat{\mu}_{I,i} \approx 0 \Rightarrow$ Actuator switching $u_{I,i} \rightarrow u_{I,i+1}$

Simulation Results

Simulation Example: Consider a system consisting of two interconnected subsystems:

$$\Sigma_I : \begin{cases} \dot{x}_{I,1} = x_{I,3}^2 - 3x_{I,1} - x_{I,4} + u_{I,2} + \eta_{I,3}(x_I) \\ \dot{x}_{I,2} = -x_{I,1} - 6.3045u_{I,1} + 6.4654\eta_{I,2}(x_I) \\ \dot{x}_{I,3} = -x_{I,2} + 3x_{J,3} \sin(x_{J,3}) - 3.5544\eta_{I,1}(x_I) \quad I = 1, 2, \\ \dot{x}_{I,4} = -5x_{I,4} - x_{I,1} \sin(x_{I,1}) + \eta_{I,4}(x_I) \\ y_I = x_{I,3}, \end{cases}$$

Modeling uncertainty:

$$\eta_{I,1} = 0.7 \sin(0.5x_1) + 3.55 \cos(0.5x_1),$$

Unknown interconnected term:

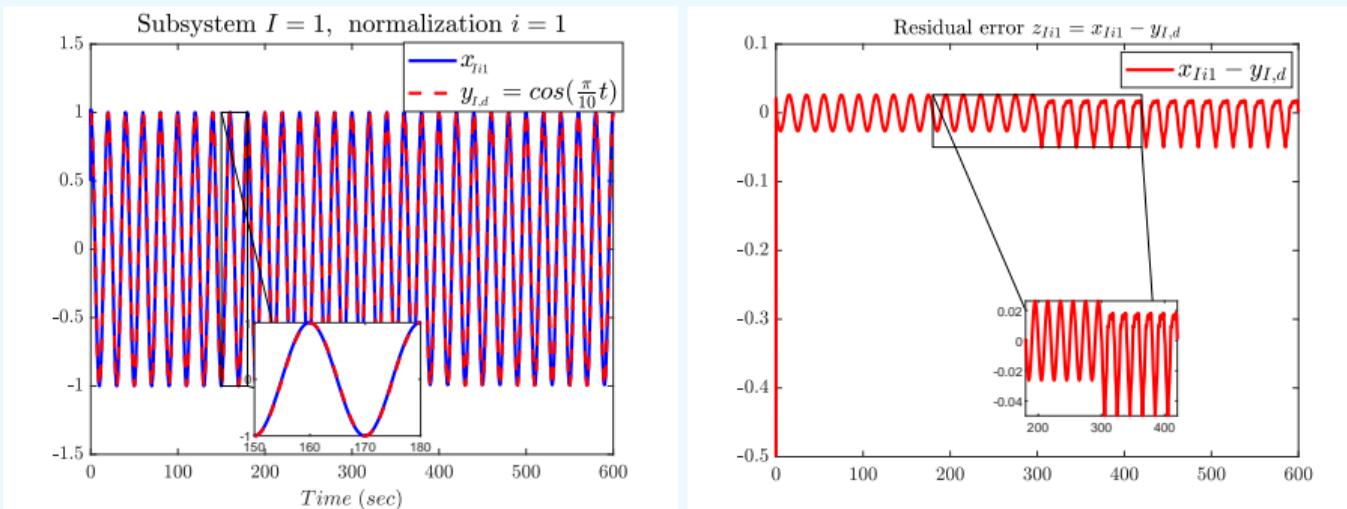
$$\psi_{I,1} = 3x_{J,3} \sin(x_{J,3}),$$

Simulation Results

Simulation 1: (Healthy condition)

No faults occur neither in subsystems Σ_1 or Σ_2 .

Subsystem Σ_1 results:

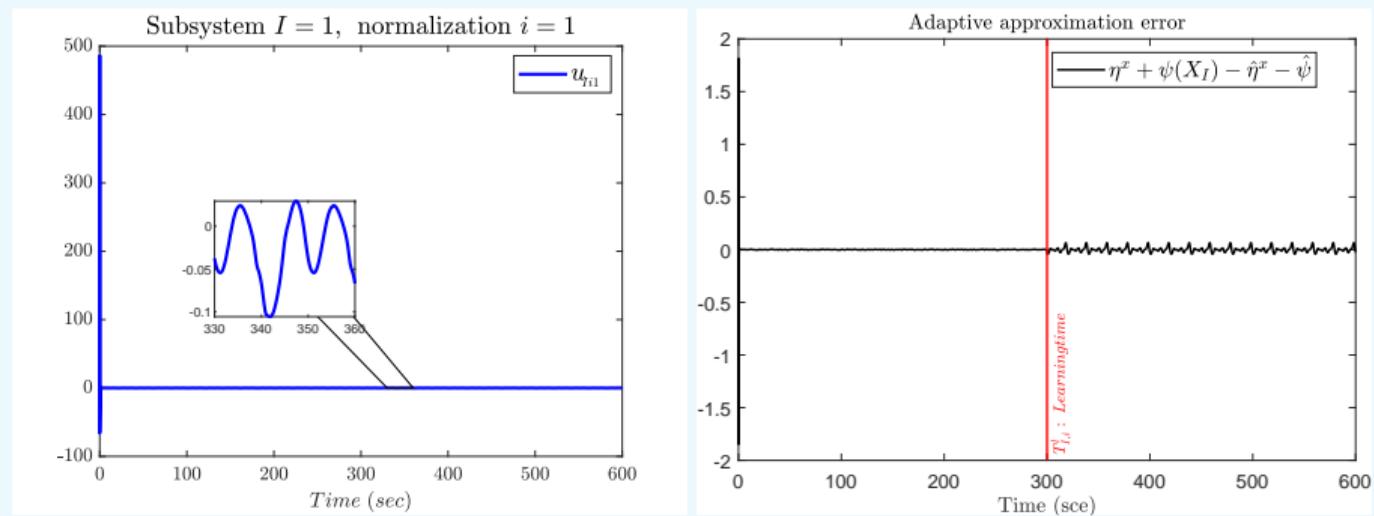


Simulation Results

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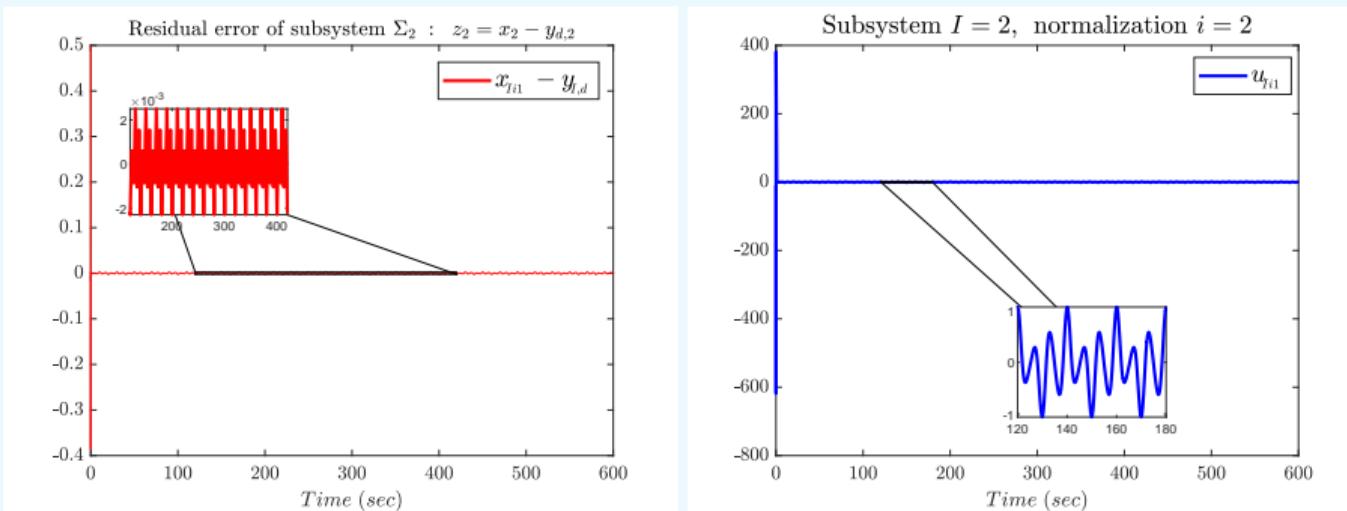


Simulation Results

Simulation 1: (Healthy condition)

No faults occur neither in subsystems Σ_1 or Σ_2 .

Subsystem Σ_2 results:



Simulation Results

Simulation 2: (Stuck fault)

Actuator $u_{1,1}$ of Σ_1 become stuck at time $T_{I,1}^0 = 50 \text{ sec}$

Fault occurrence time:

$$T_{I,i}^0 = 50.0 \text{ sec}$$

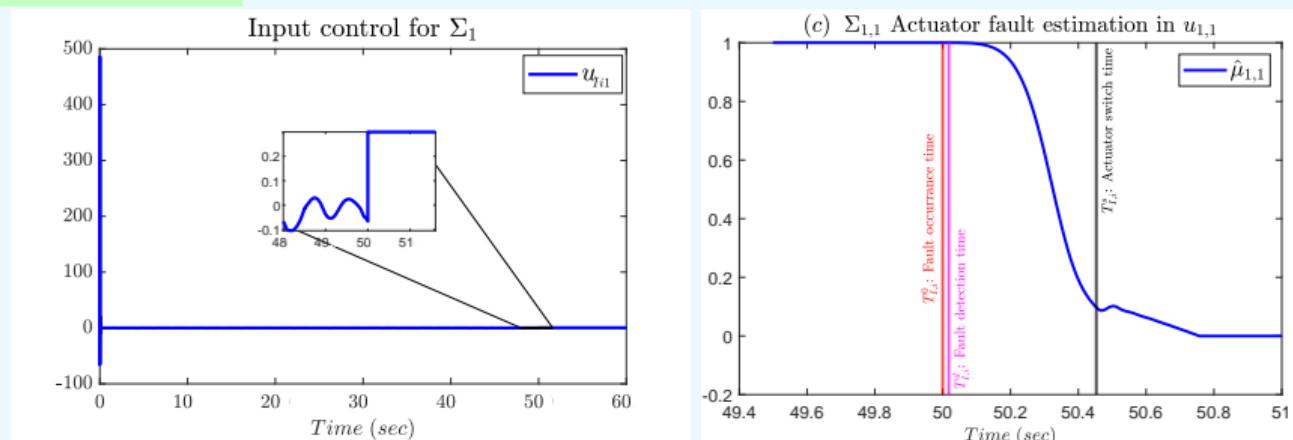
Fault detection time:

$$T_{I,i}^d = 50.017 \text{ sec}$$

Stuck fault diagnosed time:

$$T_{I,i}^s = 50.453 \text{ sec}$$

Subsystem Σ_1 results:



Simulation Results

Simulation 2: (Stuck fault)

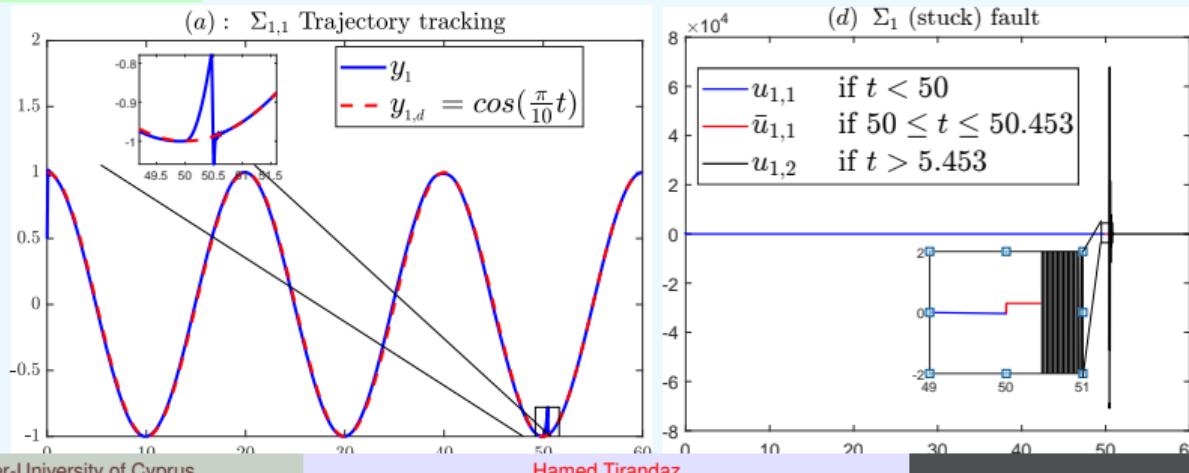
Actuator $u_{1,1}$ of Σ_1 become stuck at time $T_{I,i}^0 = 50 \text{ sec}$

Fault occurrence time: $T_{I,i}^0 = 50.0 \text{ sec}$

Fault detection time: $T_{I,i}^d = 50.017 \text{ sec}$

Stuck fault diagnosed time: $T_{I,i}^s = 50.453 \text{ sec}$

Subsystem Σ_1 results:



Simulation Results

Simulation 2: (Stuck fault)

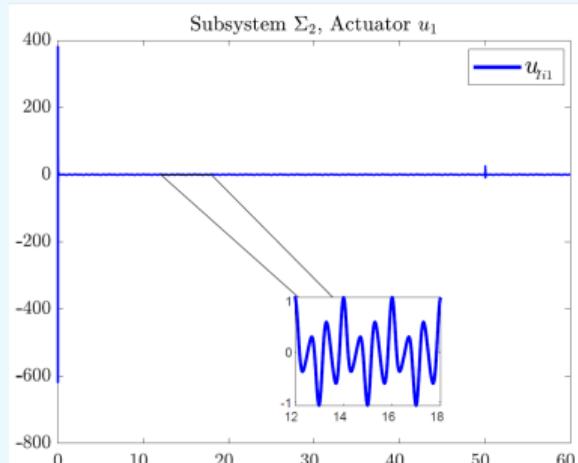
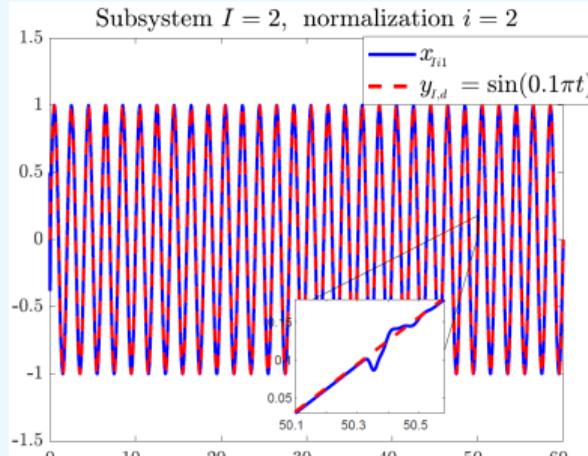
Actuator $u_{1,1}$ of Σ_1 become stuck at time $T_{I,1}^0 = 50 \text{ sec}$

Fault occurrence time: $T_{I,i}^0 = 50.0 \text{ sec}$

Fault detection time: $T_{I,i}^d = 50.017 \text{ sec}$

Stuck fault diagnosed time: $T_{I,i}^s = 50.453 \text{ sec}$

Subsystem Σ_2 results: (fault propagation)



Simulation Results

Simulation 3: (Partial fault)

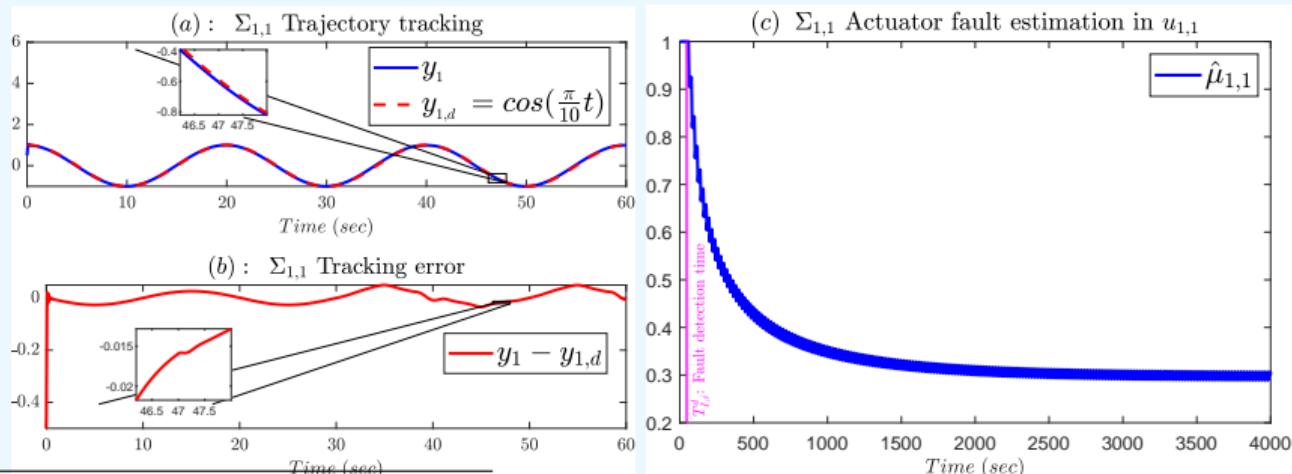
Actuator $u_{1,1}$ of Σ_1 become faulty (partial fault)

Fault occurrence time: $T_{I,i}^0 = 47.0 \text{ sec}$

Fault detection time: $T_{I,i}^d = 47.251 \text{ sec}$

$$u_{1,1}^F = -\frac{4}{5}u_{1,1}^H(t) \Rightarrow u_{1,1} = \frac{1}{5}u_{1,1}^H(t)$$

Subsystem Σ_1 results:



[3] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator fault accommodation for a class of nonlinear uncertain interconnected systems: An adaptive command filtering-based scheme", *IEEE Transactions on Automation Science and Engineering*, (Under Review), 2024.

Future Works:

Future work based on this thesis could focus on:

- ### ► Actuator and Sensor Fault Accommodation

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- ▶ Actuator and Sensor Fault Accommodation
 - ▶ Saturation-Based Actuator Fault Accommodation

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Future work based on this thesis could focus on:

- ▶ Actuator and Sensor Fault Accommodation
 - ▶ Saturation-Based Actuator Fault Accommodation
 - ▶ Decentralized Actuator Fault Diagnosis and Accommodation

Publications:

1. Hamed Tirandaz, Christodoulos Keliris, and Marios Polycarpou. "Actuator fault detection and isolation in a class of nonlinear interconnected systems." *International Journal of Control*, 1-21, 2024.
2. Hamed Tirandaz, Christodoulos Keliris, and Marios Polycarpou. "Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*, 3(2), 57–72 2024.
3. Tirandaz, H., Keliris, C., & Polycarpou, M. M. Actuator fault accommodation for a class of nonlinear uncertain interconnected systems: An adaptive command filtering-based scheme . *IEEE Transactions on Automation Science and Engineering*, (Under Review).

Thank you!