Actuator Fault Diagnosis and Accommodation in Interconnected Nonlinear Dynamical Systems

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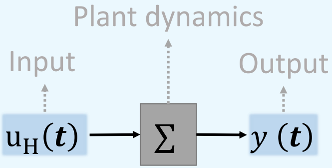
Outline

Dynamical Systems

System Structure



Dynamical Systems

System Structure

A class of nonlinear dynamical systems:

(state-space representation)

$$\Sigma : \begin{cases} \dot{x}(t) = \overbrace{Ax(t) + B\big(u_H(t) + u_F(t)\big)}^{\text{Linear term}} + \overbrace{g\big(x(t)\big)}^{\text{Nonlinearity}} + \overbrace{\eta(x(t), t)}^{\text{Modelling uncertainty and disturbance}} + f_p(x(t), t) \\ y(t) = h\big(x(t)\big) + \underbrace{\varepsilon_y(t)}_{\text{Measurement noise}} + f_y(x(t), u(t), t) \end{cases}$$

Dynamical Systems System Structure

A class of nonlinear dynamical systems:

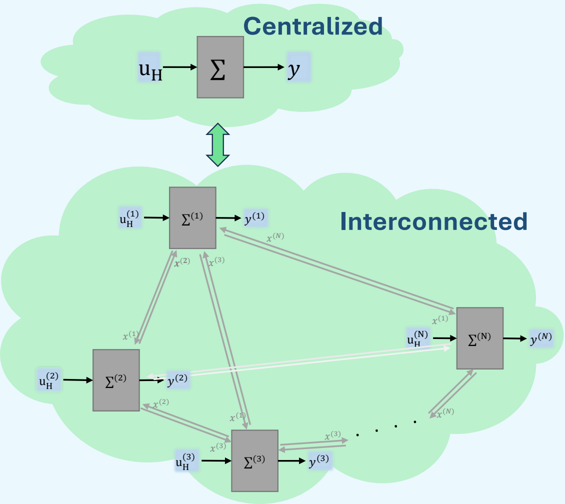
(state-space representation)

Fault types

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + B\big(u_H(t) + \overbrace{u_F(t)}^{\text{Actuator fault(s)}}\big) + g\big(x(t)\big) + \eta\big(x(t), t\big) + \overbrace{f_p\big(x(t), t\big)}^{\text{Process fault(s)}} \\ y(t) = h\big(x(t)\big) + \varepsilon_y(t) + \underbrace{f_y\big(x(t), u(t), t\big)}_{\text{Sensor fault(s)}} \end{cases}$$

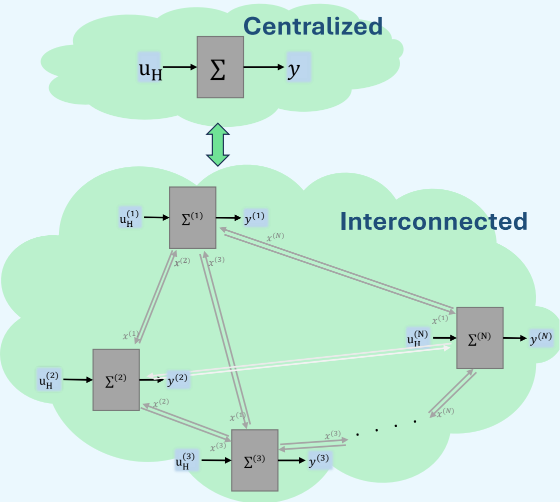
Dynamical Systems

Centralized vs. Interconnected



Dynamical Systems

Centralized vs. Interconnected



The interconnected representation:
- Provides Simplicity in Monitoring & Control
- Improves fault isolability by addressing fault propagation

- **Objectives**
  - **Actuator Fault Detection and Isolation**
  - **Actuator and Sensor Fault Isolation**
  - **Actuator Fault Accommodation**

Actuator Fault Detection and Isolation in a Class of Nonlinear Interconnected Systems

Objectives:

- Actuator Fault Detection
- Actuator Fault Isolation

Literature Review: Actuator Fault Detection

Existing Research:

[1]. Keliris et al., 2013,
A distributed fault detection **filtering** approach for a class of interconnected continuous-time nonlinear systems, *IEEE Trans. Autom. Control*.

[2]. Reppa et al., 2014,
Adaptive approximation for multiple sensor fault detection and isolation of nonlinear uncertain systems, *IEEE Trans. Neural Netw. Learn. Syst*.

[3]. Keliris et al., 2015,
A robust nonlinear observer-based approach for distributed fault detection of input–output interconnected systems, *Automatica.*

[4]. Keliris et al., 2017,
An **integrated learning and filtering approach** for fault diagnosis of a class of nonlinear dynamical systems, *IEEE Trans. Neural Netw. Learn. Syst.*

New Contribution:

We propose a **new filtering** scheme to improve fault detection in nonlinear interconnected systems.

Literature Review: Actuator Fault Isolation

[5]. Reppa V, et al., 2014,
Decentralized isolation of multiple sensor faults in large-scale interconnected nonlinear systems, *IEEE Trans. Autom. Control*.

- **Reasoning-based decision scheme**
- **Fault signature matrix**
- **Detection decision**
- **Consistency test**

Dynamical system

$$\Sigma^{(I)}: \quad \dot{x}^{(I)}(t) = A^{(I)}x^{(I)}(t) + \gamma^{(I)}\left(x^{(I)}(t), u^{(I)}(t)\right) + h^{(I)}\left(x^{(I)}(t), u^{(I)}(t), z^{(I)}(t)\right) + \eta^{(I)}\left(x^{(I)}(t), u^{(I)}(t), z^{(I)}(t), t\right)$$

$$\mathcal{S}^{(I)}: \quad y^{(I)}(t) = C^{(I)}x^{(I)}(t) + d^{(I)}(t) + f^{(I)}(t)$$

Literature Review: Actuator Fault Isolation

[6]. Kościelny JM, Bartyś M., 2023,
A new method of diagnostic row reasoning based on trivalent residuals, *Expert Syst. Appl.*

▶ Reasoning-based decision scheme

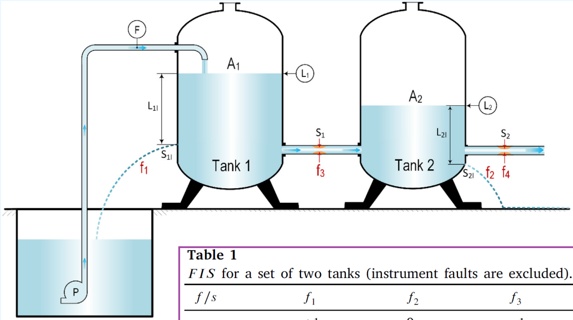


Table 1
FIS for a set of two tanks (instrument faults are excluded).

f/s	f_1	f_2	f_3	f_4
s_1	+1	0	−1	0
s_2	0	+1	+1	−1
s_3	+1	+1	0	−1

Table 2
Conflict sets $C(s_j = v_p)$ in the case of bi-valued diagnostic signals.

S	V	$C(s_j = v_p)$
s_1	1	$\{f_1, f_3\}$
s_2	1	$\{f_2, f_3, f_4\}$
s_3	1	$\{f_1, f_2, f_4\}$

Table 3
Conflict sets $C(s_j = v_p)$ in the case of three-valued diagnostic signals.

S	V	$C(s_j = v_p)$
s_1	−1	$\{f_3\}$
	+1	$\{f_1\}$
s_2	−1	$\{f_4\}$
	+1	$\{f_2, f_3\}$

Problem Formulation

System Description

The I-th subsystem considered in our study

$$\Sigma^{(I)}: \begin{cases} \dot{x}^{(I)} = A^{(I)}x^{(I)} + g^{(I)}\left(x^{(I)}, u^{(I)}\right) + h^{(I)}\left(x^{(I)}, z^{(I)}, u^{(I)}\right) + \eta^{(I)}\left(x^{(I)}, z^{(I)}, t\right) \\ y^{(I)} = x^{(I)} + \xi_y^{(I)}, \qquad\qquad I = 1, \cdots, N \end{cases}$$

Variables:

$$x^{(I)}, y^{(I)} \in \mathbb{R}^{n_I}, \quad z^{(I)} \in \mathbb{R}^{\bar{n}_I}, \quad u^{(I)} \in \mathbb{R}^{m_I}, \quad y^{(I)} \in \mathbb{R}^{n_I}$$

(Multiple Input Multiple Output per Subsystem)

Functions:

$A^{(I)}x^{(I)}$ (Known linear term)

$g^{(I)}: \mathbb{R}^{n_I} \times \mathbb{R}^{m_I} \to \mathbb{R}^{n_I}$ (Known nonlinear term)

$h^{(I)}: \mathbb{R}^{n_I} \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^{m_I} \to \mathbb{R}^{n_I}$ (Known interconnection term)

$\eta^{(I)}: \mathbb{R}^{n_I} \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^{+} \to \mathbb{R}^{n_I}$ (Unknown uncertainty term)

$\xi_y^{(I)}(t) \in \mathbb{R}^{n_I}$ (Sensor noise)

Problem Formulation

System Description

The I-th subsystem considered in our study

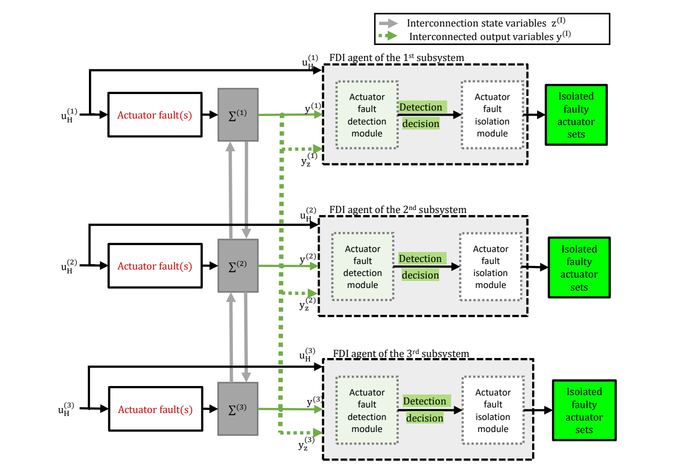
$$\Sigma^{(I)}: \begin{cases} \dot{x}^{(I)} = A^{(I)}x^{(I)} + g^{(I)}\left(x^{(I)}, u^{(I)}\right) + h^{(I)}\left(x^{(I)}, z^{(I)}, u^{(I)}\right) + \eta^{(I)}\left(x^{(I)}, z^{(I)}, t\right) \\ y^{(I)} = x^{(I)} + \xi_y^{(I)}, \qquad\qquad\qquad\qquad I = 1, \cdots N \end{cases}$$

Assumptions:

- The state $x^{(I)}$ remains bounded before and after the occurrence of any actuator fault,
- $g^{(I)}$ Lipschitz in $x^{(I)}$,
- $h^{(I)}$ Lipschitz in both $x^{(I)}$ and $z^{(I)}$,
- $\eta^{(I)}$ Bounded, i.e., $|\eta^{(I)}| \leq \bar{\eta}^{(I)}$ where $\bar{\eta}^{(I)}$ is a Lipschitz in $x^{(I)}$,
- $\xi_y^{(I)}$ Bounded, i.e., $|\xi_y^{(I)}| \leq \bar{\xi}_y^{(I)}$.

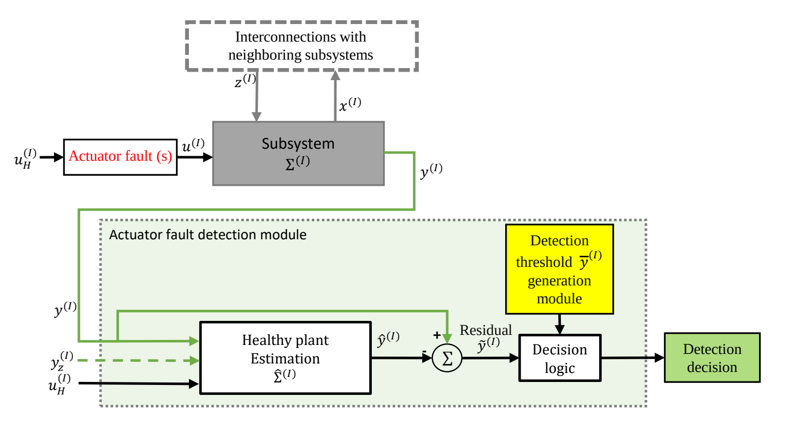
Problem formulation

The architecture of the proposed actuator FDI scheme:



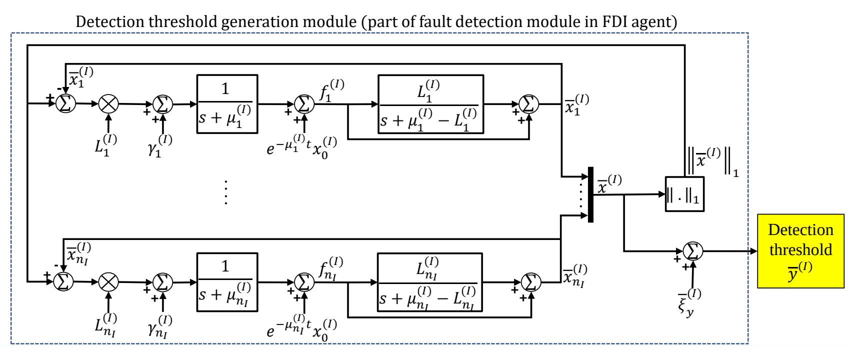
Actuator Fault Detection

Actuator fault detection agent of the I-th subsystem:



Adaptive Threshold

– **Generated through a new filtering technique.**



Actuator Fault Propagation

It is guaranteed that the occurrence of an actuator fault can only be detected by the local detection agent of the faulty subsystem;

$$\Sigma^{(I)} : \begin{cases} \dot{x}^{(I)} = f^{(I)}(x^{(I)}, z^{(I)}, \overbrace{u_f^{(I)}}^{\text{Faulty}}) \\ y^{(I)} = x^{(I)} + \xi_{y_i}^{(I)} \end{cases}$$

$$\Sigma^{(J)} : \begin{cases} \dot{x}^{(J)} = f^{(J)}(x^{(J)}, z^{(J)}, \overbrace{u_H^{(J)}}^{\text{Healthy}}) \\ y^{(J)} = x^{(I)} + \xi_{y_i}^{(I)} \end{cases} \Rightarrow \begin{array}{c} |\tilde{y}_j^{(J)}| < \bar{y}_j^{(J)} \\ \forall j = 1, \cdots, n_j \end{array}$$

Actuator Fault Isolation

- **Signature Matrix:** $F = [F_{ij}]_{n_I \times |\mathcal{F}|}$,

$$F_{ij} = \begin{cases} 0 & \textbf{Fault } \mathcal{F}_{c_j} \textbf{ has not any affect on residual } \tilde{y}_i^{(I)} \\ -1 & \textbf{Fault } \mathcal{F}_{c_j} \textbf{ has negative affect on residual } \tilde{y}_i^{(I)} \\ +1 & \textbf{Fault } \mathcal{F}_{c_j} \textbf{ has positive affect on residual } \tilde{y}_i^{(I)} \\ * & \textbf{Fault } \mathcal{F}_{c_j} \textbf{ has unknown affect on residual } \tilde{y}_i^{(I)}, \end{cases}$$

- **Detection Decision:** $\mathcal{E}^{(I)}(t) = [\epsilon_1^{(I)}, \cdots, \epsilon_{n_I}^{(I)}]$

$$\epsilon_i^{(I)}(t) = \begin{cases} +1 & \tilde{y}_i^{(I)}(t) > \bar{y}_i^{(I)}(t) \\ -1 & \tilde{y}_i^{(I)}(t) < -\bar{y}_i^{(I)}(t) \\ 0 & |\tilde{y}_i^{(I)}(t)| < \bar{y}_i^{(I)}(t) \end{cases} \qquad i = 1, \cdots, n_I.$$

Actuator Fault Isolation

Consistent set: (Can result in false alarms in AFI)

$$\mathcal{D}_s(t) = \left\{ \mathcal{F}_{c_k}^{(I)} \middle| \begin{cases} \varepsilon_i(t) = F_{ik} \text{ or } F_{ik} = * & \forall i \in \mathcal{J} \\ F_{ik} = 0 & \forall i \notin \mathcal{J}. \end{cases} \right\}, \quad \begin{array}{l} \mathcal{J} = \left\{ j \middle| |\tilde{y}_j^{(I)}| > \bar{y}_j^{(I)} \right\} \\ k \in \{1, \cdots, 2^{m_I} - 1\} \end{array}$$

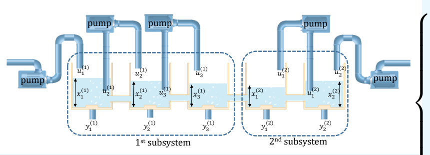
Weakly consistent set: (Contain the accurate faulty actuator set)

$$\mathcal{D}_w^{(I)}(t) = \left\{ \mathcal{F}_{c_k}^{(I)} \middle| \varepsilon_j^{(I)}(t) = F_{jk}^{(I)} \text{ or } F_{jk}^{(I)} = *, \quad \forall j \in \mathcal{J}^{(I)}, \ k \in \{1, \cdots, 2^{m_I} - 1\} \right\}$$

Diagnosed fault set(s): (Accurate isolated faulty actuator set)

$$\mathcal{F}_w^{(I)} \triangleq \bigcap_{i=1}^{2^{m_I}-1} \left\{ \mathcal{F}_{c_i}^{(I)} \middle| \mathcal{F}_{c_i}^{(I)} \in \mathcal{D}_w^{(I)}(t) \right\}.$$

Five-Tank System



$$\begin{cases} \dot{x}_1 = \frac{1}{S_t}\left(u_1 - u_2 - r_1 - \psi(x_2, x_1)\right) \\ \dot{x}_2 = \frac{1}{S_t}\left(u_2 - u_3 - r_2 + \psi(x_2, x_1) - \psi(x_3, x_2)\right) \\ \dot{x}_3 = \frac{1}{S_t}\left(u_3 - r_3 + \psi(x_3, x_2) - \psi(x_4, x_3)\right) \\ \dot{x}_4 = \frac{1}{S_t}\left(u_4 - r_4 + \psi(x_5, x_4) + \psi(x_4, x_3)\right) \\ \dot{x}_5 = \frac{1}{S_t}\left(u_5 - u_4 - r_5 + \psi(x_5, x_4)\right) \\ y_i = x_i + \xi_{y,i} \quad i = 1, \cdots, 5 \end{cases}$$

where $\psi(v, w) = cs_p\, sign\,(w - v)\sqrt{2g\,|w - v|}$

Faulty actuator: $\{u_1\}$

Time interval	$[0, T_0^{(1)}) = [0, 5)$ sec	$[T_0^{(1)}, T_{d,1}^{(1)}) = [5, 5.3)$ sec	$[T_{d,1}^{(1)}, 35.1) = [5.3, 35.1)$ sec	$[35.1, 60]$ sec
Actual faulty actuators	–	$\mathcal{F}_{\mathbf{c_1}}^{(\mathbf{1})} = \{\mathbf{u_1^{(1)}}\}$	$\mathcal{F}_{\mathbf{c_1}}^{(\mathbf{1})} = \{\mathbf{u_1^{(1)}}\}$	–
Observed pattern $\mathcal{E}^{(1)}(t)$	$[0,\ 0,\ 0]^T$		$[-1,\ 0,\ 0]^T$	$[0,\ 0,\ 0]^T$
Diagnosis sets $\mathcal{D}_s^{(1)}(t)$ (consistency)	–		$\{\mathcal{F}_{c_1}^{(1)}\}$	–
Diagnosis sets $\mathcal{D}_w^{(1)}(t)$ (weak consistency)	–		$\{\mathcal{F}_{c_1}^{(1)}, \mathcal{F}_{c_4}^{(1)}, \mathcal{F}_{c_5}^{(1)}, \mathcal{F}_{c_7}^{(1)}\}$	–
Faulty identified set $\mathcal{F}_{\mathbf{w}}^{(\mathbf{1})}(\mathbf{t})$	–		$\mathcal{F}_{\mathbf{c_1}}^{(\mathbf{1})} = \{\mathbf{u_1^{(1)}}\}$	–

[1] H. Tirandaz, C. Keliris, and M. M. Polycarpou, “Actuator Fault Detection and Isolation in a Class of Nonlinear Interconnected Systems", *International Journal of Control*, 2024.

Actuator and Sensor Fault Isolation in a Class of Nonlinear Dynamical Systems

Objectives:

- **Identification of Fault Type: Sensor(s) or Actuator(s)**
- **Isolation of Faulty Component(s)**

Literature Review

[1]. Trunov A.B., Polycarpou M. M., 2000,
Automated fault diagnosis in nonlinear multivariable systems using a learning methodology, *IEEE Trans. Neural Netw.*

Dynamical system:

$$\dot{z} = Az + \alpha(y,u) + \eta_z(z,u,t) + \underline{\mathcal{B}_z(t-T_z)f_z(y,u)}$$
$$y = Cz + \eta_y(z,u,t) + \underline{\mathcal{B}_y(t-T_y)f_y(u)} \qquad (1)$$

A. Nonlinear Estimation Model

$$\dot{\hat{z}} = A\hat{z} + \alpha(y,u) + K(y-\hat{y}) + \underline{\hat{f}_z(y,u;\hat{\theta}_z)} + \Omega_z\dot{\hat{\theta}}_z + \Omega_y\dot{\hat{\theta}}_y, \qquad \hat{z}(0) = z_0 \qquad (3)$$
$$\dot{\Omega}_z = A_0\Omega_z + Z_z(y,u;\hat{\theta}_z), \qquad \Omega_z(0) = 0 \qquad (4)$$
$$\dot{\Omega}_y = A_0\Omega_y - KZ_y(u;\hat{\theta}_y), \qquad \Omega_y(0) = 0 \qquad (5)$$
$$\hat{y} = C\hat{z} + \underline{\hat{f}_y(u;\hat{\theta}_y)} \qquad (6)$$

[2]. Talebi HA, et al.,2008,
A recurrent neural-network-based sensor and actuator fault detection and isolation for nonlinear systems with application to the satellite's attitude control subsystem, *IEEE Trans. Neural Netw.*

– **Neural-Network-Based Scheme**

Dynamical system:

$$\dot{x} = Ax + g(x,u) + \eta_x(x,u,t) + T_A(x,u)$$
$$y = Cx + \eta_y(x,u,t) + T_S(x,u) \qquad (1)$$

Estimation model:

$$\dot{\hat{x}} = A\hat{x} + g(\hat{x},u) + \hat{T}_A(\hat{x},u,W_1)$$
$$\hat{y} = C\hat{x} + \hat{T}_S(\hat{x},u,W_2) \qquad (6)$$

Literature Review

[3]. Zhang X, et al.,2008,
Design and analysis of a fault isolation scheme for a class of uncertain nonlinear systems, *Annual Reviews in Control*.

- **Generalized observer scheme (FIEs are sensitive to all but one fault)**

Dynamical system:

$$\dot{x} = Ax + \gamma(y,u) + \eta(x,u,t) + \underline{\beta_x(t - T_x)\phi(y,u)}$$
$$y = Cx + d(x,u,t) + \underline{\beta_y(t - T_y)F\theta(t)} \qquad (1)$$

Fault set: (2)

$$\mathcal{F}_P \triangleq \{\phi^1(y,u), \ldots, \phi^N(y,u)\}.$$

Fault Isolation Decision Scheme: *If, for each* $r \in \{1, \ldots, N+l\} \backslash \{s\}$, *there exist some finite time* $t^r > T_d$ *and some* $j \in \{1, \ldots, l\}$, *such that* $|\epsilon^r_{y_j}(t^r)| > \mu^r_j(t^r)$, *then the occurrence of fault s is concluded.*

$p = 1, \ldots, N,$ the pth process fault,

$q = N+1, \ldots, N+l,$ the qth sensor fault,

Literature Review

[4]. Keliris C, et al., 2016,
An integrated learning and filtering approach for fault diagnosis of a class of nonlinear dynamical systems, *IEEE Trans. Neural Netw. Learn. Syst.*

– **Dedicated observer scheme (FIEs are sensitive to single fault)**

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + f(x(t), u(t)) + \eta(x(t), u(t)) \\ \qquad + \beta^x(t - T_0^x)\phi(x(t), u(t)) & (1) \\ y(t) = x(t) + \xi(t) + \beta^y(t - T_0^y)\sigma(t) & (2) \end{cases}$$

V. Fault Type Identification

A. Process Fault

$$\dot{\hat{x}}_p(t) = g(y(t), u(t)) + \hat{\eta}(y(t), u(t), \hat{\theta}(t)) \quad (28)$$
$$\epsilon_p(t) \triangleq H_1(s)[y(t) - \hat{x}_p(t)] \quad (29)$$
$$r_p(t) \triangleq H_2(s)[\epsilon_p(t)] \quad (30)$$

B. Sensor Faults

$$\dot{\hat{x}}_s(t) = A\hat{x}_s(t) + f(\hat{x}_s(t), u(t)) + \hat{\eta}_L(y(t), u(t)) + \Lambda\epsilon_s^y(t) \quad (33)$$
$$\hat{y}_s(t) = \hat{x}_s(t) + \hat{\sigma}(t) \quad (34)$$
$$\dot{\hat{\sigma}}(t) = \mathcal{P}_\sigma(\Gamma^y\epsilon_s^y(t)) \quad (35)$$

Fault Identification Logic:

$$|r_p^{(k)}(t)| \le \bar{r}_p^{(k)}(t) \text{ for all } k = 1, \dots, n \text{ for all } t > T_d + T_{L,2}.$$

Fault Identification Logic:

$$|\epsilon_s^y(t)| \le \bar{\epsilon}_s^y(t) \text{ and } |r_s^{(k)}(t)| \le \bar{r}_s^{(k)}(t) \quad \text{for all } k = 1, \dots, n \text{ for all } t > T_d.$$

Literature Review

[5]. Reppa V, et al., 2014,
Decentralized isolation of multiple sensor faults in large-scale interconnected nonlinear systems, *IEEE Trans. Autom. Control*.

– **Reasoning-based decision scheme**

Dynamical system:

$$\Sigma^{(I)}: \quad \dot{x}^{(I)}(t) = A^{(I)}x^{(I)}(t) + \gamma^{(I)}\left(x^{(I)}(t), u^{(I)}(t)\right) + h^{(I)}\left(x^{(I)}(t), u^{(I)}(t), z^{(I)}(t)\right) + \eta^{(I)}\left(x^{(I)}(t), u^{(I)}(t), z^{(I)}(t), t\right) \quad (1)$$

$$\mathcal{S}^{(I)}: \quad y^{(I)}(t) = C^{(I)}x^{(I)}(t) + d^{(I)}(t) + f^{(I)}(t) \quad (4)$$

Assumption 1:
the corresponding reference signals of the interconnection variables $z^{(I)}(t)$, denoted by $\boxed{z_r^{(I)}(t)\text{, are known}}$.

$$|z^{(I)}(t) - z_r^{(I)}(t)| \leq \boxed{\bar{z}^{(I)}}, \forall t \geq 0.$$

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[1] Keliris C, et al.,2015,
Distributed fault diagnosis for process and sensor faults in a class of interconnected input–output nonlinear discrete-time systems, *Int. J. Control*.

Literature Review

[6]. Reppa V, et al., 2017,
Optimization of observer design for sensor fault detection of nonlinear systems, *Proc. IEEE Conf. Decis. Control*.

- **Bias (and abrupt) faults**
- **Sensor fault detection**
- **Single sensor fault**
- **Detectability**

Dynamical system:

$$\dot{x}(t) = Ax(t) + \gamma(x(t), u(t)) + \eta(t) \qquad (1)$$

$$y(t) = x(t) + d(t) + f(t), \qquad (2)$$

Abrupt & Bias Faults:

$$f(t) = f_0 + \tilde{f}(t), \qquad (34)$$

$f_0 \in \mathbb{R}^n$ is the (unknown) non-zero offset

Corollary 3.4: Every single sensor fault $f_i = f_{0i} + \tilde{f}_i$ with $|f_{0i}| \geq f_{0i}^{\min}$ and $|\tilde{f}_i| \leq \bar{f}_i^*$ is strongly detectable, if $|V_j^{-1}A_i| > |V_j^{-1}|\Gamma$.

$$f_{0i}^{\min} = \min\{f_i : (|\lambda_j^{-1}V_j^{-1}A_i| - |\lambda_j^{-1}V_j^{-1}|\Gamma)|f_i| > c\} \qquad (53)$$

$$c = |\lambda_j^{-1}V_j^{-1}| \left(2\bar{\eta} + \Gamma\left(2\left\|\bar{d}\right\| + \bar{f}_i^*\right)\right) + \left(|V_j^{-1} - \lambda_j^{-1}V_j^{-1}A| + |V_j^{-1}|\right)\left(2\bar{d} + [0, \ldots, \bar{f}_i^*, \ldots, 0]^\top\right) \qquad (54)$$

Literature Review

[7]. Chen W, Saif M., 2007,
Adaptive actuator fault detection, isolation and accommodation in uncertain systems, *Int. J. Control*.

- **Actuator fault detection and isolation**
- **Known and distinguishable fault functions**

$$\dot{x} = Ax - ay + \underline{b_1 u_1} + \cdots + \underline{b_m u_m}$$
$$y = x_1,$$

$$u_1(t) = u(t),\ u_2(t) = f_2(t) + u(t), \ldots, u_m(t) = f_m(t) + u(t),$$

$$u_j(t) = \bar{u}_j, \quad t \geq t_j, \quad j \in 1, 2, \ldots, m \qquad (29)$$

$$f_j(t) = df_j \sin(\omega_j t), \qquad 2 \leq j \leq m$$

$C_m^1 + \cdots + C_m^{m-1}$ adaptive estimates need to isolate the faults.

Key Contribution

- **Address Sensor Fault Propagation**

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- **Identification of Fault Type (Sensor/Actuator)**

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- Address Sensor Fault Propagation
- **Identification of Fault Type (Sensor/Actuator)**
  - **No Need for Persistency of Excitation**

Definition (Persistence of excitation):
A signal $y(t)$ is persistently exciting if there are positive scalars $\eta_1, \eta_2, \mathcal{T} \in \mathbb{R}^+$ such that

$$\eta_1 I \leq \int_t^{t+\mathcal{T}} y(\tau) y^T(\tau) d\tau \leq \eta_2 I \qquad \forall t \in \mathbb{R}^+.$$

Persistently of exciting $\Longleftrightarrow$ sufficient variability over time

Key Contribution

- Address Sensor Fault Propagation
- Identification of Fault Type (Sensor/Actuator)
- **Accurate Isolation of Multiple Faulty Sensors: a Decentralized scheme**

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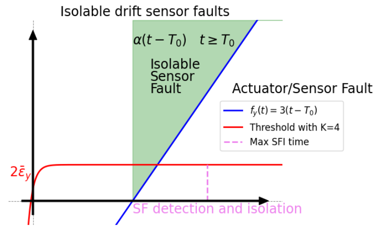
- Address Sensor Fault Propagation
- Identification of Fault Type (Sensor/Actuator)
- **Accurate Isolation of Multiple Faulty Sensors: a Decentralized scheme**

Sensor fault isolability $(\mathbf{T_0} = \mathbf{T_d} = \mathbf{T_I}) \iff |\mathbf{f_{y,i}}| > 4\bar{\xi}_{\mathbf{y,i}}$

Key Contribution

- Address Sensor Fault Propagation
- Identification of Fault Type (Sensor/Actuator)
- **Accurate Isolation of Multiple Faulty Sensors: a Decentralized scheme**

KIOS Testbed simulation results



WaterSafe state vector
$[\mathcal{L}_1, \mathcal{Q}_1, \mathcal{L}_2, \mathcal{Q}_2, \mathcal{L}_3, \mathcal{Q}_3, \mathcal{L}_4, \mathcal{Q}_4]$

It can easily show that drift faults greater than $3(t - T_0)$ for all $T_0 = 5$ can be detectable for states (tank levels):
$[\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4]$

$$\tilde{y}_{si}^{F_y}(T_d) - \tilde{y}_{si}^{f_y}(T_d) = F_{y_i}(T_d) - f_{y_i}(T_d) - \int_{T_0}^{T_d} e^{A_{k_i}(T_d - \tau)} \left[g(\hat{x}_{s_i}^{F_y}, \underline{y}_i) - g(\hat{x}_{s_i}^{f_y}, \underline{y}_i) + K_i(F_{y_i}(\tau) - f_{y_i}(\tau)) \right] d\tau$$

Key Contribution

- Address Sensor Fault Propagation
- Identification of Fault Type (Sensor/Actuator)
- Accurate Isolation of Multiple Faulty Sensors: a Decentralized scheme
- **Isolation of Multiple Faulty Actuators: an Adaptive Approximation Scheme**

Problem Formulation

System Description

$$\boldsymbol{\Sigma}: \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + Bu_F(t) + g(x(t)) + \eta(x,t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0)f_y(x,t), \end{cases}$$

Variables:

$$x, y \in \mathbb{R}^n, u \in \mathbb{R}^m.$$

(Actuators and sensors can be faulty)

Functions:

Ax (known linear term)

$g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ (known nonlinear term)

$\eta : \mathbb{R}^n \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^+ \to \mathbb{R}^n$ (unknown uncertainty term)

$\xi_y(t) \in \mathbb{R}^n$ (sensor noise)

$f_y(x,t) \in \mathbb{R}^n$ (sensor fault)

Problem Formulation

System Description

$$\boldsymbol{\Sigma}: \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + Bu_F(t) + g(x(t)) + \eta(x,t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0) f_y(x,t), \end{cases}$$

Assumptions:

- g Differentiable and hence Lipschitz in x
- Modeling uncertainty and measurement noise boundedness:
  $|\eta_i(x,t)| \leq \bar{\eta}_i \quad i = 1, \cdots, n, \quad \bar{\eta}_i$ known constant!
  $|\xi_{y,i}(t)| \leq \bar{\xi}_{y,i} \quad i = 1, \cdots, n, \quad \bar{\xi}_{y,i}$ known constant!
- Sensor and actuator fault boundedness:
  $|f_{y,i}| < \bar{f}_{y,i} \quad i = 1, \cdots, n \quad \bar{f}_{y,i}$ known constant!
  $|u_{F,i}(t)| < \bar{u}_{F,i} \quad i = 1, \cdots, n \quad \bar{u}_{F,i}$ known constant!
- Sensor fault isolability: Sensor fault $f_{y,i}$ will be isolated if we have
  $|f_{y,i}| > 4\bar{\xi}_{y,i} \quad i \in \{1, \cdots, n\}.$

Problem Formulation

System Description

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + Bu_F(t) + g(x(t)) + \eta(x,t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0) f_y(x,t), \end{cases}$$

- **Actuator fault structure:**

$$u_F(t) \triangleq -\boldsymbol{\Upsilon}(t - T_0)\Lambda u_H(t),$$

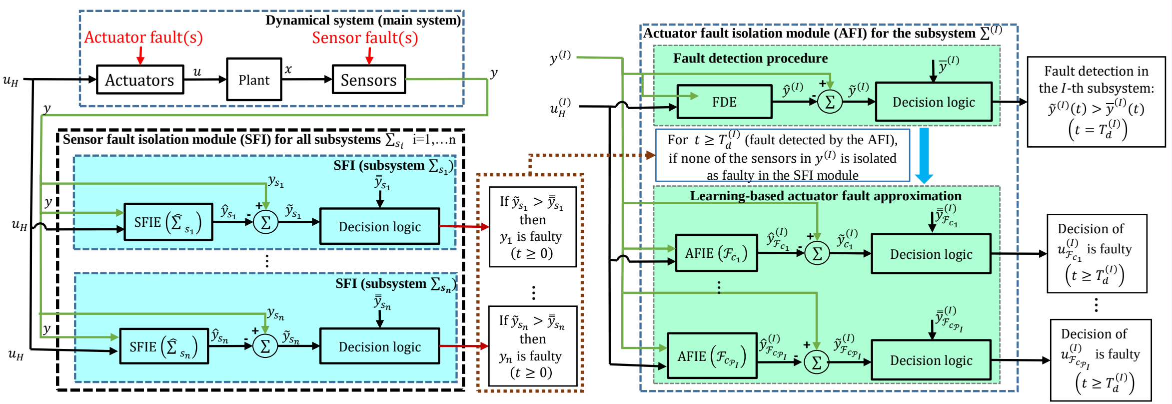
$$\Upsilon_i(t - T_{0,i}) = \begin{cases} 0 & \text{for all } t < T_{0,i} \\ 1 - exp(-\delta_i(t - T_{0,i})) & \text{for all } t \geq T_{0,i}, \end{cases}$$

- **Sensor fault structure (abrupt & bias faults)**

$$\beta_i(t - \mathcal{T}_{0,i}) = \begin{cases} 0 & \text{if } t < \mathcal{T}_{0,i} \\ 1 & \text{if } t \geq \mathcal{T}_{0,i} \end{cases}$$

Actuator/Sensor Fault Isolation Modules

The architecture of actuator and sensor fault isolation modules



Actuator/Sensor Fault Isolation Modules

Sensor Fault Isolation Module:

- **SFI Decomposition Procedure**
- **A Modified Luenberger estimator design**
- **Reasoning-based sensor fault isolation**

Actuator Fault Isolation Module:

- **AFI Decomposition Procedure**
- **Fault Detection Procedure** (designed in our first work)
- **Adaptive Approximation-Based Actuator Fault Estimation and Isolation**

SFI Decomposition Procedure

Decompose the main system into n subsystems:

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu_H(t) + Bu_F(t) + g(x(t)) + \eta(x,t) \\ y(t) = x(t) + \xi_y(t) + \beta(t - \mathcal{T}_0) f_y(x,t), \end{cases}$$

$$\Downarrow$$

$$\Sigma_{s_i} : \begin{cases} \dot{x}_i = a_{ii}x_i + A_i\underline{x}_i + B_i u_H + B_i u_F + g_i(x) + \eta_i(x,t) \\ y_i = x_i + \xi_{y,i} + \beta_i(t - \mathcal{T}_{0,i}) f_{y,i}(x,t), \end{cases} \qquad i = 1, \cdots, n$$

where variables:

$x_i \in I\!R$ (i-th state signal)

$y_i \in I\!R$ (i-th output signal)

$\underline{x}_i \triangleq [x_1, \cdots, x_{i-1}, 0, x_{i+1}, \cdots, x_n]^T$

$A_i = [a_{i1}, \cdots, a_{in}]$ (i-th row of A)

$B_i = [b_{i1}, \cdots, b_{im}]$ (i-th row of B)

A Modified Luenberger Estimator Design

By utilizing the Hurwitz stability in [2], we have

$$A - \bar{\mathcal{K}} = \Pi \triangleq [\pi_{ij}]_{n\times n} = \begin{bmatrix} \boxed{\pi_{11}} & \pi_{12} & \cdots & \pi_{1i} & \cdots & \pi_{1n} \\ \pi_{21} & \boxed{\pi_{22}} & \cdots & \pi_{2i} & \cdots & \pi_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \pi_{i1} & \pi_{i2} & \cdots & \boxed{\pi_{ii}} & \cdots & \pi_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{ni} & \cdots & \boxed{\pi_{nn}} \end{bmatrix}$$

[2] W. Zhang, et al. Sufficient conditions for Hurwitz stability of matrices. *Lat. Am. Appl. Res.*, 2008.

A Modified Luenberger estimator design

$$\Sigma_{s_i}: \begin{cases} \dot{x}_i = a_{ii}x_i + A_i\underline{x}_i + B_iu_H + B_iu_F + g_i(x) + \eta_i(x,t) \\ y_i = x_i + \xi_{y,i} + \beta_i(t - \mathcal{T}_{0,i})f_{y,i}(x,t), \end{cases}$$

$$\Downarrow$$

$$\hat{\Sigma}_{s_i}: \begin{cases} \dot{\hat{x}}_{s_i} = a_{ii}\hat{x}_{s_i} + A_i\underline{y}_i + B_iu_H + g_i(\hat{x}_{s_i}, \underline{y}_i) + \mathcal{K}_{ii}(y_i - \hat{y}_{s_i}) \\ \hat{y}_{s_i} = \hat{x}_{s_i}, \end{cases}$$

where the estimator gain $\mathcal{K}_{ii}$ has to be selected such that:

$$\mathcal{K}_{ii} > a_{ii} + L_{g_i},$$

$$|a_{ii} - \mathcal{K}_{ii}| > \sum_{j=1, j\neq i}^{n} |a_{ij}|.$$

Sensor Fault Isolation

Reasoning-based sensor fault isolation

Theorem 3.1 (SFI Threshold for the i-th Sensor)

If the i-th sensor is healthy, then we have

$$|\tilde{y}_{s_i}(t)| \leq \bar{y}_{s_i}(t) \qquad \forall t \geq 0,$$

where

$$\bar{y}_{s_i}(t) \triangleq \bar{x}_{s_i}(t) + \bar{\xi}_{y,i}(t),$$

$$\bar{x}_{s_i}(t) \triangleq \mathcal{C}_i(t) + \int_0^t e^{(L_{g_i}+\mathcal{A}_{ii})(t-\tau)}\mathcal{C}_i(\tau)d\tau,$$

$$\mathcal{C}_i(t) \triangleq e^{\mathcal{A}_{ii}t}\bar{x}_{i,0} + \int_0^t e^{\mathcal{A}_{ii}(t-\tau)}\left[\gamma_i - \mathcal{A}_{ii}\bar{\xi}_{y,i}\right]d\tau,$$

$$\gamma_i \triangleq |B_i|\bar{u}_F + \sum_{j=1, j\neq i}^{n} |a_{ij}|(\bar{\xi}_{y,j} + \bar{f}_{y,j}) + L_{g_i}\sum_{j\in\mathscr{Z}_i}(\bar{\xi}_{y,j} + \bar{f}_{y,j}) + \bar{\eta}_i + a_{ii}\bar{\xi}_{y,i},$$

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If the i-th sensor is healthy, then
$\forall t_{0,i} > 0, \ \& \ \forall \epsilon_i > 0 \ \ \exists \mathcal{M}_{ii} > 0,$ such that:

$$\forall \mathcal{K}_{ii} > \mathcal{M}_{ii}, \Rightarrow |\tilde{y}_{s_i}(t)| < 2\bar{\xi}_{y,i} + \epsilon_i \quad \forall t > t_{0,i}$$

Assumption
Sensor fault isolability:

$$|f_{y,i}| > 4\bar{\xi}_{y,i}$$

Actuator Fault Isolation

AFI decomposition procedure

Actuator faults can only be detected in their corresponding subsystems.

$$\Downarrow$$

Each actuator has to appear only in one subsystem:

$u_i \in \mathcal{U}^{(I)}$ **iff** $u_i \notin \mathcal{U}^{(J)}$, **for all** $I, J \in \{1, \cdots, N\}, I \neq J.$

Fault detection procedure

The fault detection scheme in our previous work is utilized for actuator fault detection.

Adaptive Approximation-Based Actuator Fault Estimation and Isolation

The $r_{_I}$-th adaptive estimator $\hat{\Sigma}^{(I)}_{\mathcal{F}_{r_I}}$ $r_{_I} = 1, \cdots, \mathcal{P}_{_I}$ are given by:

$$\hat{\Sigma}^{(I)}_{\mathcal{F}_{r_I}} : \begin{cases} \dot{\hat{x}}^{(I)}_{\mathcal{F}_{r_I}} = A^{(I)}\hat{x}^{(I)}_{\mathcal{F}_{r_I}} + B^{(I)}(\hat{u}^{(I)}_{\mathcal{F}_{r_I}} + \hat{\hat{u}}^{(I)}_{\mathcal{F}_{r_I}}) + p(\hat{x}^{(I)}_{\mathcal{F}_{r_I}}) \\ \qquad +q^{(I)}(\hat{x}^{(I)}_{\mathcal{F}_{r_I}}, y^{(I)}_z) + K^{(I)}(y^{(I)} - \hat{y}^{(I)}_{\mathcal{F}_{r_I}}) + \Omega^{(I)}_{\mathcal{F}_{r_I}} \dot{\hat{u}}^{(I)}_{\mathcal{F}_{r_I}} \\ \hat{y}^{(I)}_{\mathcal{F}_{r_I}}(t) = \hat{x}^{(I)}_{\mathcal{F}_{r_I}}(t) \\ \dot{\Omega}^{(I)}_{\mathcal{F}_{r_I}} = A_k^{(I)}\Omega^{(I)}_{\mathcal{F}_{r_I}} + B^{(I)}_{\mathcal{F}_{r_I}} \\ \dot{\hat{u}}^{(I)}_{\mathcal{F}_{r_I}} = \mathcal{P}\{\Gamma^{(I)}_{\mathcal{F}_{r_I}}(\Omega^{(I)}_{\mathcal{F}_{r_I}})^T D^{(I)}[\tilde{y}^{(I)}_{\mathcal{F}_{r_I}}]\}, \end{cases}$$

where

$$\dot{\hat{u}}^{(I)}_{\mathcal{F}_{r_I}} = \Gamma^{(I)}_{\mathcal{F}_{r_I}}(\Omega^{(I)}_{\mathcal{F}_{r_I}})^T D[\tilde{y}^{(I)}_{\mathcal{F}_{r_I}}] - \mathcal{X}^{(I)}_{\mathcal{F}_{r_I}}\Gamma^{(I)}_{\mathcal{F}_{r_I}} \frac{\hat{u}^{(I)}_{\mathcal{F}_{r_I}} \hat{u}^{(I)^T}_{\mathcal{F}_{r_I}}}{\hat{u}^{(I)}_{\mathcal{F}_{r_I}} \Gamma^{(I)}_{\mathcal{F}_{r_I}} \hat{u}^{(I)}_{\mathcal{F}_{r_I}}} \mathscr{P}^{(I)},$$

Adaptive Approximation-Based Actuator Fault Estimation and Isolation

AFI Threshold Design: Considering the accurate faulty actuator set is $\mathcal{F}_{k_I}^{(I)}$, and the fault is detected at time $T_d^{(I)}$, then, we have

$$|\tilde{y}^{(I)}_{\mathcal{F}_{k_I},i}(t)| \leq \bar{\bar{y}}^{(I)}_{\mathcal{F}_{k_I},i}(t) \qquad \forall i = 1, \cdots, n_I \quad \forall t \geq T_d^{(I)} + t_D^{(I)},$$

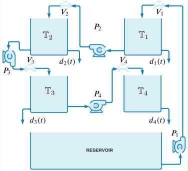
where

$$\bar{\bar{y}}^{(I)}_{\mathcal{F}_{k_I}}(t) \triangleq \bar{\bar{x}}^{(I)}_{\mathcal{F}_{k_I}}(t) + \bar{\xi}_y^{(I)}(t) \quad \forall t \geq T_d^{(I)} + t_D^{(I)},$$

$$\bar{\bar{x}}^{(I)}_{\mathcal{F}_{k_I},i}(t) \triangleq \mathbb{E}^{(I)}_{\mathcal{F}_{k_I},i}(t) + L_i^{(I)} \int_{T_d^{(I)}}^{t} e^{(a_{k,i}^{(I)} + L_i^{(I)})(t-\tau)} \mathbb{E}^{(I)}_{\mathcal{F}_{k_I},i}(\tau) d\tau,$$

$$\mathbb{E}^{(I)}_{\mathcal{F}_{k_I},i}(t) \triangleq \left|\Omega^{(I)}_{\mathcal{F}_{k_I},i}\right| \kappa^{(I)}_{\mathcal{F}_{k_I}} + \int_{T_d^{(I)}}^{t} e^{a_{k,i}^{(I)}(t-\tau)} \mu^{(I)}_{\mathcal{F}_{k_I},i} d\tau + e^{a_{k,i}^{(I)}(t-T_d^{(I)})} \bar{x}^{(I)}_{d,i},$$

The KIOS Water Systems Testbed



$$\Sigma_I : \begin{cases} \dot{\mathcal{L}}_I(t) = \frac{1}{S_I}\left[\mathcal{Q}_I(t) - \mathcal{Q}_{I+1}(t) - d_I(t)\right] \\ \dot{\mathcal{Q}}_I(t) = \Psi_A\left(\mathcal{Q}_I(t), u_I(t)\right) \end{cases} \quad I = 1, 2, 3, 4,$$

$\mathcal{Q}_{I+1}(t)$ denotes the outflow rate of tank $\mathbb{T}_I$ at time t

Local Feedback Controller (LFC)

$$\Psi_A^{(I)}(x_2^{(I)}, u_I) \triangleq \begin{cases} \lambda_I u_I - \alpha_I x_2^{(I)} & 0.1 < u_I \leq 1 \\ -\vartheta_I x_2^{(I)} & 0 \leq u_I \leq 0.1, \end{cases}$$

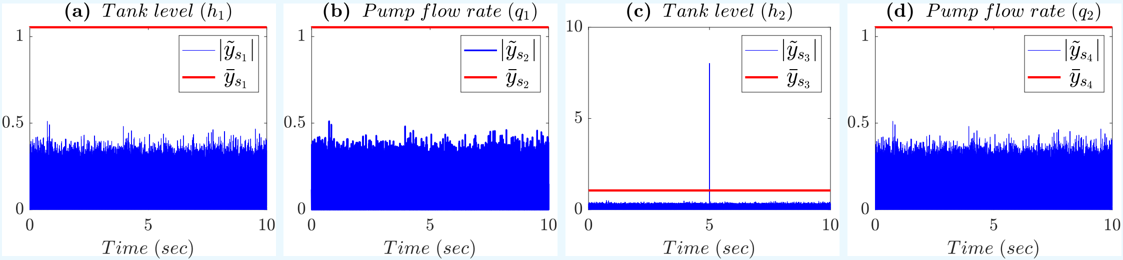
[1] Vrachimis, Stelios, et al. "WaterSafe: A water network benchmark for fault diagnosis research." IFAC-PapersOnLine 55.6 (2022): 655-660.

Isolation of Faulty Sensors

The faulty sensors are $\mathbf{y_3}$, $\mathbf{y_7}$ and $\mathbf{y_8}$.

The faults $f_{y,3} = 8$, $f_{y,7} = 3$ and the intermittent fault as $f_{y,8} = 5 \quad t \in [2j, 2j + 0.5]$ for all $j = 1, 2, 3,$

SFI results of the SFI module for subsystems $\Sigma_{s_i} \quad i = 1, \cdots, 8$



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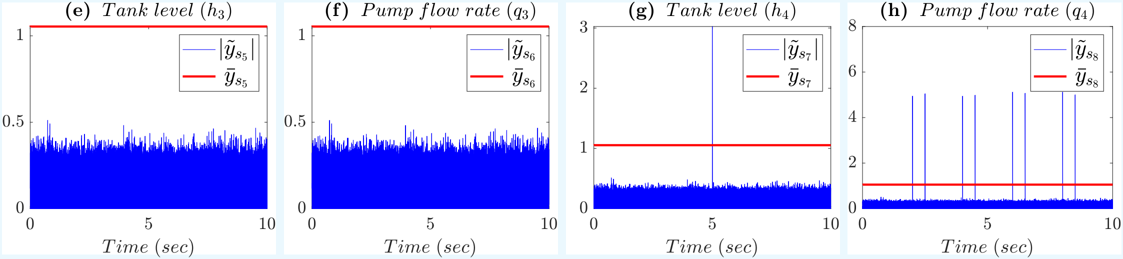

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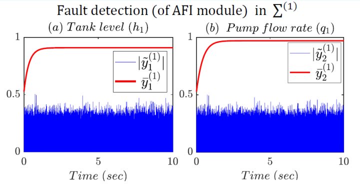
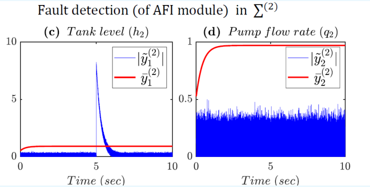
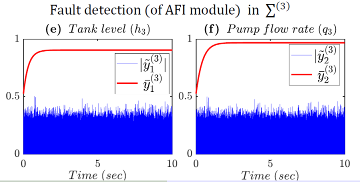
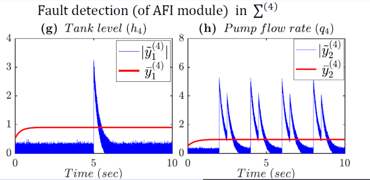
AFI results (fault detection) of the AFI module for $\Sigma^{(i)} \quad i = 1, \cdots, 4$.





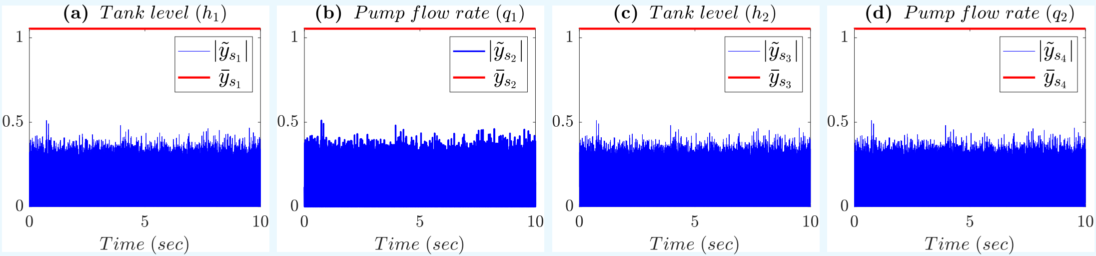




Isolation of Faulty Actuator

The faulty actuators are $\mathbf{u}_1$ and $\mathbf{u}_2$ (i.e., $\mathbf{u}_1^{(1)}$ and $\mathbf{u}_1^{(2)}$).

- SFI results of the SFI Module for subsystems $\Sigma_{s_i} \quad i = 1, 2, 3, 4$

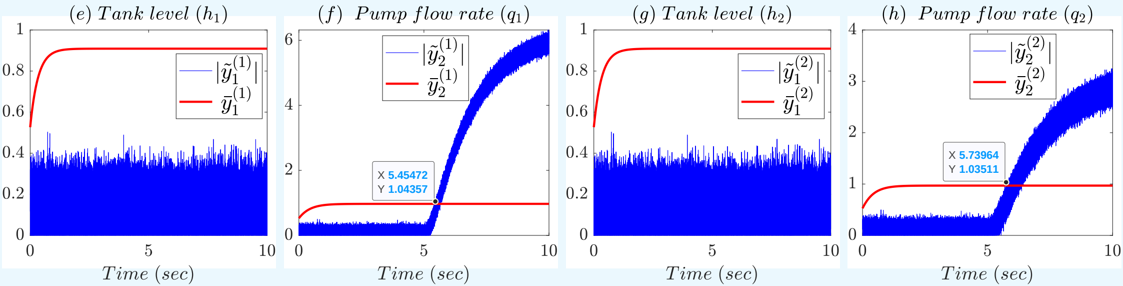


[2] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*, 2024.

Isolation of Faulty Actuator

The faulty actuators are $\mathbf{u}_1$ and $\mathbf{u}_2$ (i.e., $\mathbf{u}_1^{(1)}$ and $\mathbf{u}_1^{(2)}$).

- Fault detection result of the AFI Module for subsystems $\Sigma^{(1)}$ and $\Sigma^{(2)}$



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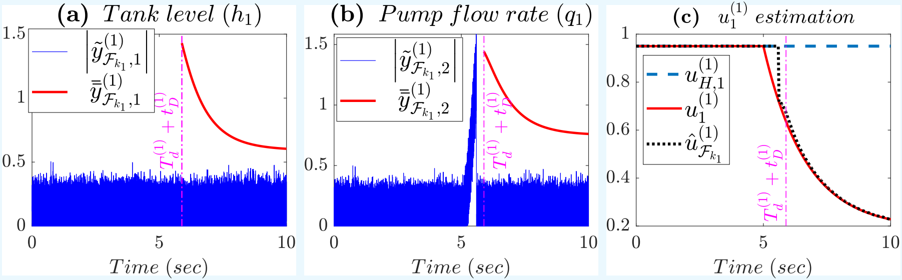
Isolation of Faulty Actuator

The faulty actuators are $\mathbf{u_1}$ and $\mathbf{u_2}$ (i.e., $\mathbf{u_1^{(1)}}$ and $\mathbf{u_1^{(2)}}$).

- AFI results of the AFI Module for subsystems $\Sigma^{(1)}$



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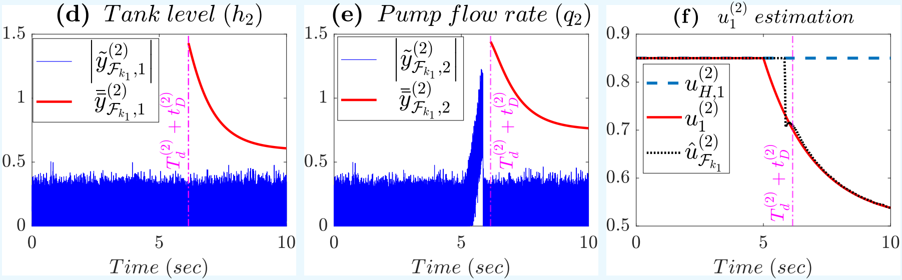
[2] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*, 2024.

Isolation of Faulty Actuator

The faulty actuators are $\mathbf{u}_1$ and $\mathbf{u}_2$ (i.e., $\mathbf{u}_1^{(1)}$ and $\mathbf{u}_1^{(2)}$).

- AFI results of the AFI Module for subsystems $\Sigma^{(2)}$



[2] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*, 2024.

Actuator Fault Accommodation for a Class of Nonlinear Uncertain Interconnected Systems: An Adaptive Command Filtering-Based Scheme

Objectives:

- Nominal Controller Design
- Actuator Fault Accommodation

Literature Review

[1]. Lai G, et al., 2018,
Adaptive compensation for infinite number of actuator failures based on tuning function approach, *Automatica*.

- **Actuator fault compensation scheme**
- **Actuators with the same relative degrees**
- **No need for FDI**

Dynamical system:

$$\dot{x}_i = x_{i+1} + \theta^T \varphi_i(\bar{x}_i), \quad i = 1, \ldots, n-1$$

$$\dot{x}_n = \varphi_0(\bar{x}_n, \xi) + \theta^T \varphi_n(\bar{x}_n, \xi) + \sum_{j=1}^{m} b_j \beta_j(\bar{x}_n, \xi) u_j$$

$$\dot{\xi} = \psi(\bar{x}_n, \xi) + \phi(\bar{x}_n, \xi)\theta$$

$$y = x_1,$$

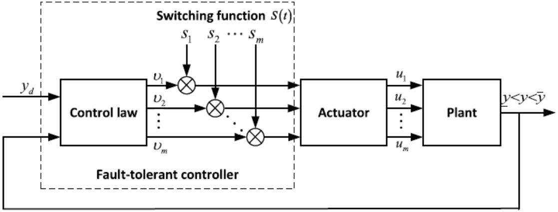
Literature Review

[2]. Ruan Z, et al., 2020,
Performance-guaranteed fault-tolerant control for uncertain nonlinear systems via learning-based switching scheme, *IEEE Trans. Neural Netw. Learn. Syst.*, [1]

- **Actuators with the same relative degrees**
- **Switching-based scheme**
- **No need for FDI**

Dynamical system:

$$\dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t)$$
$$\dot{x}_n = f_n(\bar{x}_n) + g_n^T(\bar{x}_n)u + d_n(t)$$
$$y = x_1$$



[1] Another switching scheme is proposed in:
Yang, Qinmin, Shuzhi Sam Ge, and Youxian Sun. "Adaptive actuator fault tolerant control for uncertain nonlinear systems with multiple actuators." Automatica 60 (2015): 92-99.

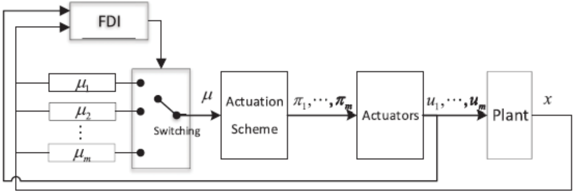
Literature Review

[3]. Zhang K, et al., 2021.
Fault-tolerant control for systems with unmatched actuator faults and disturbances, *IEEE Trans. Autom. Control*.

- **Actuators with various relative degrees**
- **Centralized scheme**
- **Sliding surface FTC scheme**
- **It utilizes FDI information**
- **No modeling uncertainty**

Consider a class of nonlinear over-actuated systems

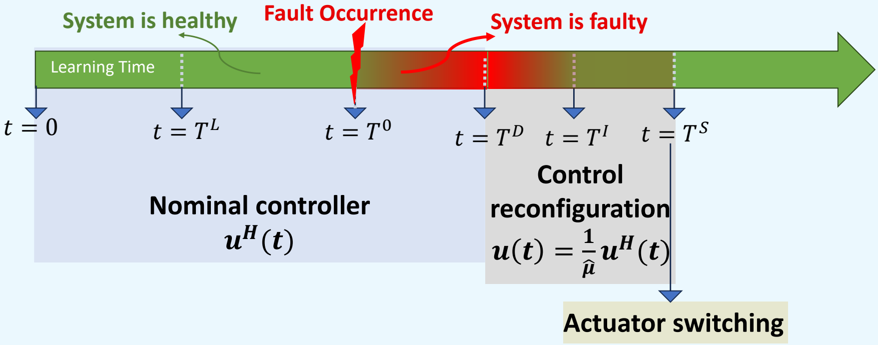
$$\dot{\zeta} = f(\zeta) + \sum_{i=1}^{m} g_i(\zeta)u_i(t) + \sum_{i=1}^{n} D_i(\zeta)d_i(t)$$
$$y = h(\zeta) \qquad (1)$$



Key contributions

1. **A distributed backstepping control scheme** is designed for systems with multiple fault-prone actuators, ensuring bounded signals in both healthy and faulty conditions.
2. **The "explosion of complexity"** issue associated with backstepping design is mitigated by applying a finite-time command filter technique.
3. **An adaptive approximation method** is proposed, overcoming limitations of existing uncertainty estimation techniques and accurately estimating both local and interconnected uncertainties.
4. **A unified actuator fault accommodation scheme** is introduced in this work that addresses both partial and total loss of actuator effectiveness, stabilizing the system and handling multiple fault-prone actuators, modeling uncertainty, and unknown interconnections.

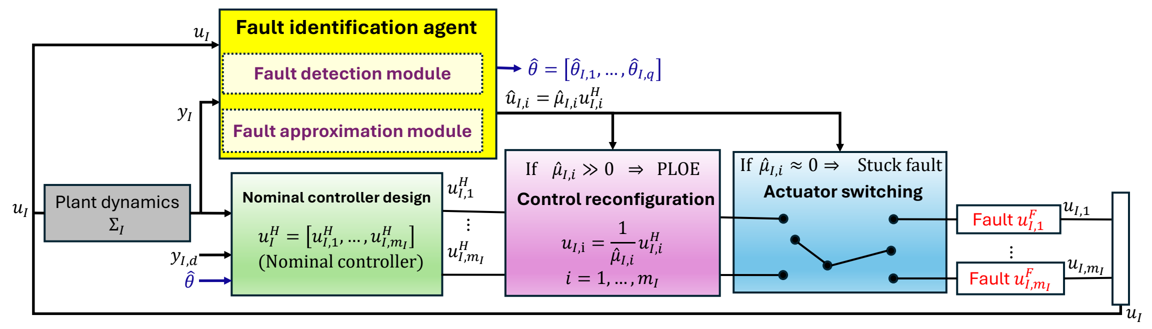
System Timeline Schedule

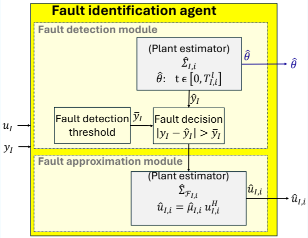


Control objective:

- **The output signal $y_I(t)$ tracks desired trajectory $y_{I,d}(t)$.**
- **All signals are bounded in healthy/faulty conditions.**

Actuator Fault Accommodation Architecture





System description

Consider the I-th subsystem $\Sigma_I \quad I = 1, \cdots, N$:

$$\Sigma_I : \begin{cases} \dot{x}_I = f_I(x_I) + \sum_{i=1}^{m_I} g_{I,i}(x_I)u_{I,i} + \overbrace{\sum_{j=1}^{n_I} d_{I,i}(x_I)\, \underbrace{\eta_I(x_I)}_{\text{Unknown}}}^{\text{Modeling Uncertainty}} + \overbrace{\sum_{k=1}^{n_I} q_{I,k}(x_I)\, \underbrace{\psi_I(X_I)}_{\text{Unknown}}}^{\text{Interconnection}} \\ y_I = h_I(x_I) \end{cases}$$

Assumptions:

- **The triple ($f_I, g_{I,i}, h_I$) has relative degree $r_{I,i}$ for all $i = 1, \cdots, m_I$,**
- **The triple ($f_I, d_{I,j}, h_I$) has relative degree j for all $j = 1, \cdots, n_I$,**
- **The triple ($p_I, q_{I,k}, h_I$) has relative degree k for all $k = 1, \cdots, n_I$,**

Problem formulation

System description

$$\Sigma_I : \begin{cases} \dot{x}_I = f_I(x_I) + \sum_{i=1}^{m_I} g_{I,i}(x_I) u_{I,i} + \sum_{j=1}^{n_I} d_{I,i}(x_I)\eta_{I,j}(x_I) + \sum_{k=1}^{n_I} q_{I,k}(x_I)\psi_{I,k}(X_I) \\ y_I = h_I(x_I), \end{cases}$$

$$u_I = u_I^H + u_I^F,$$

How to distinguish PLOE from TLOE:
We have

$$u_{I,i}(t) = \mu_{I,i}(t) u_{I,i}^H(t) + \bar{u}_{I,i} \quad i = 1, \cdots, m_I,$$

- **TLOE of actuator** $u_{I,i}$**, occurs when** $\mu_{I,i} \approx 0.$
- **PLOE of actuator** $u_{I,i}$**, occurs when** $0 << \mu_{I,i} < 1.$

System Normalization

System description

$$\Sigma_I : \begin{cases} \dot{x}_I = f_I(x_I) + \sum_{i=1}^{m_I} g_{I,i}(x_I)u_{I,i} + \sum_{j=1}^{n_I} d_{I,i}(x_I)\eta_{I,j}(x_I) + \sum_{k=1}^{n_I} q_{I,k}(x_I)\psi_{I,k}(X_I) \\ y_I = h_I(x_I), \end{cases}$$

System Normalization

$$\xi_{Iij} = L_{f_I}^{(j-1)} h_I(x_I)\Big|_{x=\Phi_I^{-1}(\xi_I,\zeta_I)} \qquad j = 1, \cdots r_{I,i}, i = 1, \cdots m_I$$

$$\Updownarrow$$

$$\xi_{I,i} \triangleq [h_I(x_I), L_{f_I} h_I(x_I), \cdots, L_{f_I}^{(r_i-1)} h_I(x_I)]^T_{x=\Phi_I^{-1}(\xi_I,\zeta_I)} \in \mathbb{R}^{r_i}$$

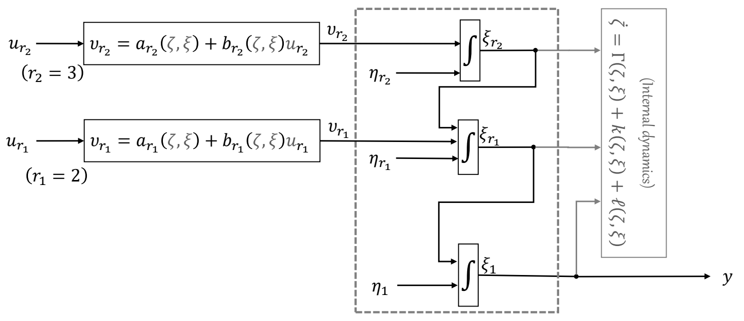
Then, by utilizing the above transformation, the actuator $u_{I,i} \quad i \in \{1, \cdots, m_I\}$ can be designed to control subsystem Σ_I.

System Normalization

System description

$$\Sigma_I : \begin{cases} \dot{x}_I = f_I(x_I) + \sum_{i=1}^{m_I} g_{I,i}(x_I) u_{I,i} + \sum_{j=1}^{n_I} d_{I,i}(x_I) \eta_{I,j}(x_I) + \sum_{k=1}^{n_I} q_{I,k}(x_I) \psi_{I,k}(X_I) \\ y_I = h_I(x_I), \end{cases}$$

System Normalization



Adaptive approximation

Objective:

We aim to estimate the unknown local and interconnected terms.

Why estimation?!

The majority of fault diagnosis investigations consider modeling uncertainty to be bounded by a known bound, i.e.,

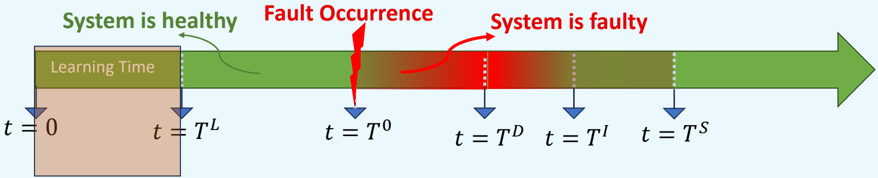
$$|\eta(\mathbf{x})| \leq \bar{\eta} \qquad \forall \mathbf{x} \in \mathcal{X}$$

where $\bar{\eta}$ is a known constant/function.

Key challenges:

- **Unknown bounds for unknown uncertainties,**
- **Large-magnitude bounds,**
- **Focuses on stability & not guarantees optimality.**

System Timeline Schedule



T^L Learning time

To address modeling uncertainty and unknown interconnection terms:

- **Approximation** $\left(\mathbf{0 \leq t \leq T^L}\right)$

$$\hat{\Xi}(x) \approx \hat{\theta}(t)\Omega(x) \quad \text{where} \quad \hat{\theta}(t) = \mathcal{P}\left(\Gamma\Omega(t)e\big(y(t)\big)\right)$$

- **Estimation** $\left(\mathbf{t \geq T^L}\right)$

$$\hat{\Xi}(x) \approx \hat{\theta}^*\Omega(x)$$

Objective:

We aim to estimate the unknown local and interconnected terms.

Literature review:

[1]. Caccavale, F., et al., 2009,,
Actuators fault diagnosis for robot manipulators with uncertain model, , *Control Eng. Pract.*.

- **An SVM-based offline scheme considering five fault-free trajectories is applied for uncertainty estimation.**

[2]. Reppa, V., et al., 2014.,
Adaptive approximation for multiple sensor fault detection and isolation of nonlinear uncertain systems. , *IEEE Trans. Neural Netw. Learn. Syst.*

- **The variables are considered to be known limited intervals, i.e.,** $x_4 \in [-0.2, 0.2]$**.**

[3] Keliris, C., et al., 2017,,
An integrated learning and filtering approach for fault diagnosis of a class of nonlinear dynamical systems, *IEEE Trans. Neural Netw. Learn. Syst.*.

- **The variables are considered known limited intervals, i.e.,** $x_4 \in [-0.25, 0.25]$**.**

Objective:

We aim to estimate the unknown local and interconnected terms.

Key challenges:

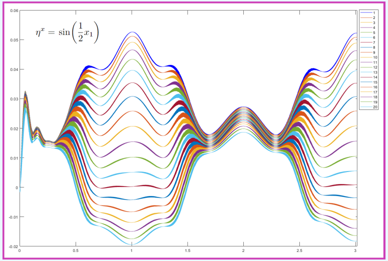
- **Persistency of excitation,**
- **The convergence of $\hat{\theta}(t)$ to the optimal value θ^* cannot be guaranteed at finite time, i.e., $\dot{\tilde{\theta}} \to 0$ but $\lim_{t \to T^l_{I,i}} \hat{\theta}(t) \neq \theta^*$,**

Example:

Modeling uncertainty is:

$$\eta(x_1) = \sin\left(\tfrac{1}{2}x_1\right)$$

$$\approx \theta_0 + \theta_1 x^1 + \cdots + \theta_{19} x^{19},$$



Adaptive approximation

We aim to estimate the unknown local and interconnected terms simultaneously as

$$\Xi_{Iik}(\xi_{I,i},\zeta_{I,i},X_I) \triangleq \overbrace{p^d_{I,k}(\xi_{I,i},\zeta_{I,i})\eta^x_{Iik}(\xi_{I,i},\zeta_{I,i})}^{\text{Local modeling uncertainty}} + \overbrace{p^q_{I,1}(\xi_{I,i},\zeta_{I,i})\psi_{I,1}(X_I)}^{\text{Unknown interconnected term}}$$

$$\Xi_{Iik}(\xi_{I,i},\zeta_{I,i},X_I) \triangleq \hat{\Xi}_{Iik}(\xi_{I,i},\zeta_{I,i},X_I;\theta^*_{Iik}) + e_{\Xi_I,k}(\xi_{I,i},\zeta_{I,i},X_I)$$

$$\hat{\Xi}_{I,k}(\xi_{I,i},\zeta_{I,i},X_I;\theta^*_{Iik}) = \Pi_{I,k}(\xi_{I,i},\zeta_{I,i},X_I)\theta^*_{Iik}$$

Assumption

In our work, the unknown function Ξ_{Iik} is considered to be

$$\Xi_{Iik}(\varkappa_1,\cdots,\varkappa_{\mathcal{N}}) = \Xi_{Iik}(\varkappa_1)+\cdots+\Xi_{Iik}(\varkappa_{\mathcal{N}}).$$

Hence, we have $\Pi_{I,k}(\xi_{I,i},\zeta_{I,i},X_I) = \sum_{j=1}^{\mathcal{N}_{I,k}} \Pi_{I,k}(\varkappa_j)$.

Adaptive approximation

Dynamical System Estimation

$$\hat{\Sigma}_{I,i}: \begin{cases} \dot{\hat{\xi}}_{Iij} = \xi_{Ii\{j+1\}} + \sum_{j=1}^{\mathcal{N}_{I,k}} \Pi_{I,j}(\varkappa_{I,k}^{(j)})\hat{\theta}_{Iij}^{(j)} + k_{Ijj}(\xi_{Iij} - \hat{\xi}_{Iij}) & j = 1, \cdots, r_{I,i} - 1 \\ \dot{\hat{\xi}}_{Iir_i} = \zeta_{Ii1} + p_{I,r_i}^{g}(\xi_{I,i}, \zeta_{I,i})u_{I,i} + \sum_{j=1}^{\mathcal{N}_{I,k}} \Pi_{I,r_i}(\varkappa_{I,i}^{(j)})\hat{\theta}_{Iir_i}^{(j)} + k_{Ir_ir_i}(\xi_{Iir_i} - \hat{\xi}_{Iir_i}) \\ \zeta_{Iij} = \Gamma_{Iij}(\xi_{I,i}, \zeta_{I,i}) & j = 1, \cdots, n_I - r_{I,i} \\ \hat{y}_I = \hat{\xi}_{Ii1}, \end{cases}$$

where

$$\dot{\hat{\theta}}_{Iik}^{(j)}(t) = \mathcal{P}_p\left(\Gamma_{I,k}\Pi_{I,k}^{T}(\xi_{I,i}, \zeta_{I,i}, X_I)\tilde{\xi}_{I,k}\right) \qquad t \leq T_{I,i}^{l} \qquad \begin{array}{l} \text{for all } k = 1, \cdots, r_{I,i}, \\ \text{for all } j = 1, \cdots, \mathcal{N}_{I,k}. \end{array}$$

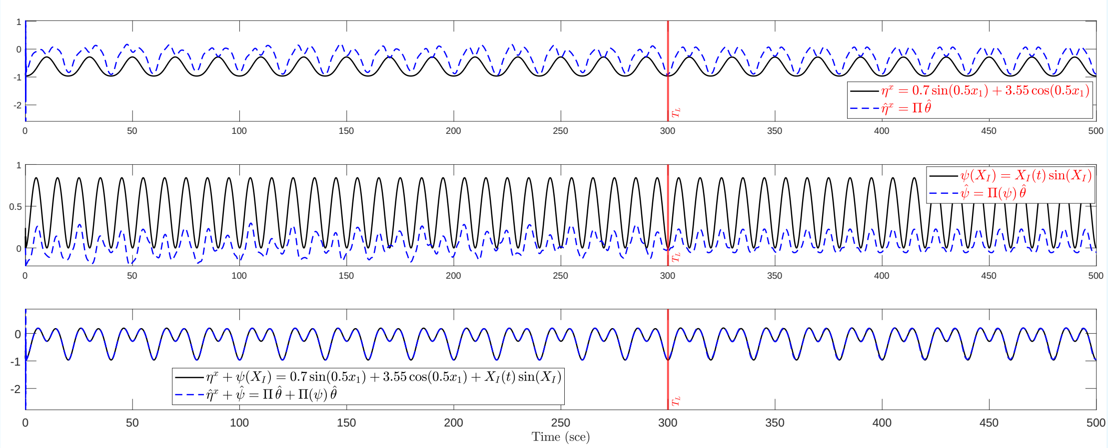
Adaptive approximation

Challenge: $\lim_{t \to T^l_{I,i}} \hat{\theta}(t) \neq \theta^*$

Solution: By assembling data points $\hat{\eta}_{I,k}(\varkappa^{(j)}_{I,k}(t_r)) = \sum_{s=1}^{q} \Pi'_{Iks}(\varkappa^{(j)}_{I,k}(t_r)) \cdot \hat{\theta}^{(j)}_{Iik,s}(t_r)$ where $\Pi_{I,k} = [\Pi'_{Ik1}, \cdots, \Pi'_{Ikq}]$, we have

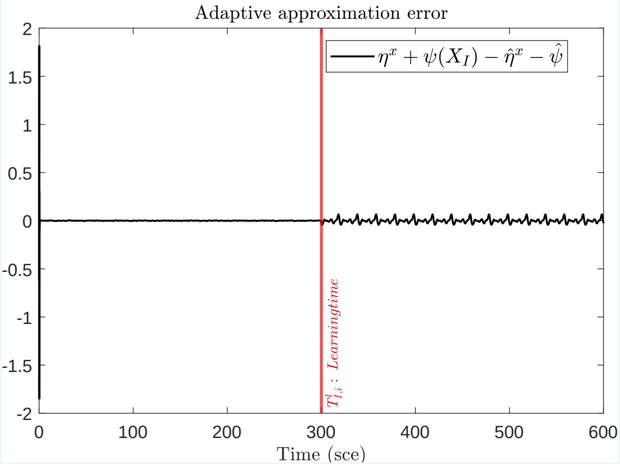
$$\underbrace{\begin{bmatrix} \hat{\Xi}^{(j)}_{I,k}(\varkappa^{(j)}_{I,k}(t_1)) \\ \hat{\Xi}^{(j)}_{I,k}(\varkappa^{(j)}_{I,k}(t_2)) \\ \vdots \\ \hat{\Xi}^{(j)}_{I,k}(\varkappa^{(j)}_{I,k}(t_m)) \end{bmatrix}}^{\hat{\Xi}^{(j)}_{I,k}} \approx \overbrace{\begin{bmatrix} \Pi'_{Ik1}(\varkappa^{(j)}_{I,k}(t_1)) & \cdots & \Pi'_{Ikq}(\varkappa^{(j)}_{I,k}(t_1)) \\ \Pi'_{Ik1}(\varkappa^{(j)}_{I,k}(t_2)) & \cdots & \Pi'_{Ikq}(\varkappa^{(j)}_{I,k}(t_2)) \\ \vdots & \ddots & \vdots \\ \Pi'_{Ik1}(\varkappa^{(j)}_{I,k}(t_m)) & \cdots & \Pi'_{Ikq}(\varkappa^{(j)}_{I,k}(t_m)) \end{bmatrix}}^{\boldsymbol{\Pi'_{I,k}(\varkappa^{(j)}_{I,k})}} \overbrace{\begin{bmatrix} \hat{\theta}^{(j)*}_{Iik,1} \\ \hat{\theta}^{(j)*}_{Iik,2} \\ \vdots \\ \hat{\theta}^{(j)*}_{Iik,q} \end{bmatrix}}^{\boldsymbol{\hat{\theta}^{(j)*}_{Iik}}},$$

Adaptive approximation



$$\overbrace{\eta^x = 2\sin(0.5x_1) + \cos(0.5x_1)}^{\textbf{Modeling uncertainty}}, \qquad \overbrace{\psi(X_1) = 3X_1\sin(X_1)}^{\textbf{Interconnection term}}$$

Adaptive approximation



$$\underbrace{\eta^x = 2\sin(0.5x_1) + \cos(0.5x_1)}^{\textbf{Modeling uncertainty}}, \qquad \overbrace{\psi(X_1) = 3X_1\sin(X_1)}^{\textbf{Interconnection term}}$$

Nominal Controller Design

Distributed command-filter-based backstepping controller $\mathbf{u_{I,i} = u^H_{I,i}}$:

$$u^H_{I,i} \triangleq \frac{-1}{p^g_{I,r_i}(\xi_{I,i},\zeta_{I,i})}\Big[L^{(r_i)}_{f_I} h_I(x_I) + \frac{1}{2}\nu_{Iir_i} + l_{Iir_i} z_{Iir_i} + \ell_{Iir_i}\sin\Big(\gamma_{Iir_i}\Big)$$
$$+\gamma_{Ii\{r_i-1\}} + \nu_{Ii\{r_i-1\}} + \sum_{j=1}^{\mathcal{N}_{I,r_i}} \Pi_{I,r_i}(\varkappa^{(j)}_{I,r_i}(t))\hat{\theta}^{(j)*}_{Iir_i} - \dot{\alpha}_{Ii\{r_i-1\}} - y^{(r_i)}_{I,d}\Big],$$

The virtual control law $\alpha_{\mathbf{Iik}}$:

$$\alpha_{Iik} \triangleq -\nu_{Ii\{k-1\}} - \frac{1}{2}\nu_{Iik} - \sum_{j=1}^{\mathcal{N}_{I,k}} \Pi_{I,k}(\varkappa^{(j)}_{I,k}(t))\hat{\theta}^{(j)*}_{Iik} - l_{Iik} z_{Iik} + \dot{\alpha}_{Ii\{k-1\}} - \ell_{Iik}\sin\Big(\gamma_{Ii1k}\Big),$$

Fault Type Identification

Control Reconfiguration (will be addressed after fault detection)

$$\hat{\Sigma}_{\mathcal{F}_{I,i}}:\begin{cases}\dot{\hat{\xi}}_{\mathcal{F}_{Iir_i}} = \zeta_{Ii1} + p^{g}_{I,r_i}(\xi_{I,i},\zeta_{I,i})\hat{u}_{I,i} + \Pi_{I,r_i}(\xi_{I,i},\zeta_{I,i},X_{I})\hat{\theta}_{Iir_i} + k_{Ir_ir_i}(\xi_{Iir_i} - \hat{\xi}_{\mathcal{F}_{Iir_i}}) + \Omega_{\mathcal{F}_{Iir_i}}\dot{\hat{\mu}}_{I,i}\\ \dot{\Omega}_{\mathcal{F}_{Iir_i}} = k_{Ir_ir_i}\Omega_{\mathcal{F}_{Iir_i}} + p^{g}_{I,r_i}(\xi_{I,i},\zeta_{I,i})u^{H}_{I,i}\\ \hat{u}_{I,i} = \hat{\mu}_{I,i}u^{H}_{I,i},\end{cases}$$

$$\dot{\hat{\mu}}_{I,i} = \mathscr{Q}_{I,i} - \mathcal{X}_{\mathcal{F}_{I,i}}\Gamma_{\mathcal{F}_{I,i}}\frac{\hat{\mu}_{I,i}\hat{\mu}^{T}_{I,i}}{\hat{\mu}^{T}_{I,i}\Gamma_{\mathcal{F}_{I,i}}\hat{\mu}_{I,i}}\mathscr{Q}_{I,i}, \quad \text{where} \quad \mathscr{Q}_{I,i} \triangleq \Gamma_{\mathcal{F}_{I,i}}(\Omega_{\mathcal{F}_{I,i}})^{T}\tilde{y}_{\mathcal{F}_{I,i}}$$

- PLOE $0 \ll \hat{\mu}_{I,i} \leq 1 \;\Rightarrow$ Control reconfiguration: $u_{I,i} \triangleq \frac{1}{\hat{\mu}_{I,i}}u^{H}_{I,i}$
- TLOE $\hat{\mu}_{I,i} \approx 0 \;\Rightarrow$ Actuator switching $u_{I,i} \to u_{I,i+1}$

Simulation Results

Simulation Example: Consider a system consisting of two interconnected subsystems:

$$\Sigma_I : \begin{cases} \dot{x}_{I,1} = x_{I,3}^2 - 3x_{I,1} - x_{I,4} + u_{I,2} + \eta_{I,3}(x_I) \\ \dot{x}_{I,2} = -x_{I,1} - 6.3045u_{I,1} + 6.4654\,\eta_{I,2}(x_I) \\ \dot{x}_{I,3} = -x_{I,2} + 3x_{J,3}\sin\big(x_{J,3}\big) - 3.5544\,\eta_{I,1}(x_I) \\ \dot{x}_{I,4} = -5x_{I,4} - x_{I,1}sin(x_{I,1}) + \eta_{I,4}(x_I) \\ y_I = x_{I,3}, \end{cases} \qquad I = 1, 2,$$

Modeling uncertainty:

$$\eta_{I,1} = 0.7\sin(0.5x_1) + 3.55\cos(0.5x_1),$$

Unknown interconnected term:

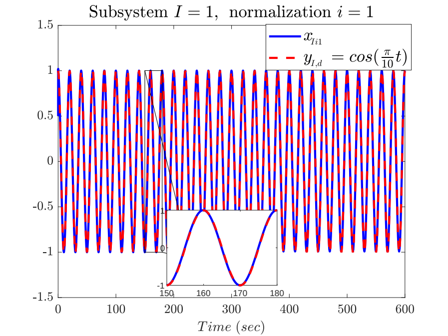
$$\psi_{I,1} = 3x_{J,3}sin(x_{J,3}),$$

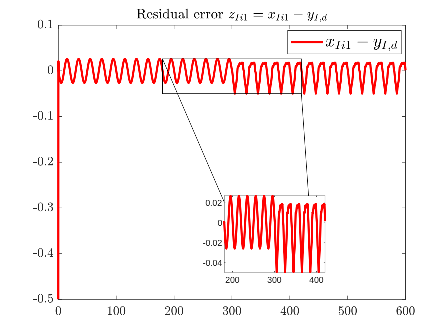
Simulation Results

Simulation 1: (Healthy condition)

No faults occur neither in subsystems Σ_1 or Σ_2.

Subsystem Σ_1 results:





Simulation Results

Simulation 1: (Healthy condition)

No faults occur neither in subsystems Σ_1 or Σ_2.

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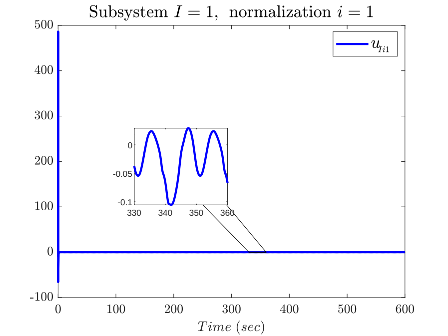
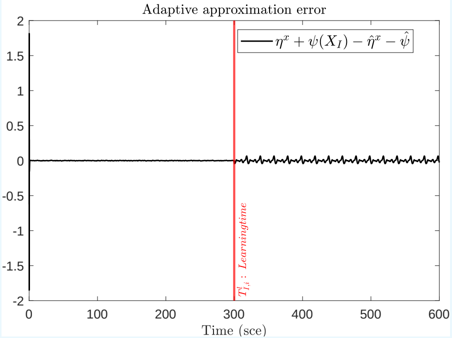


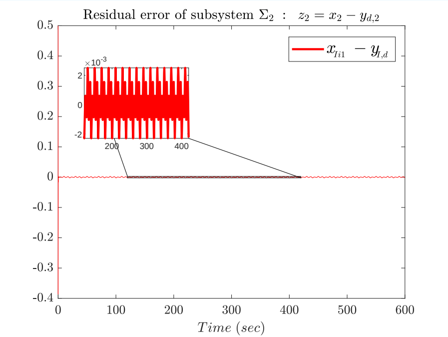


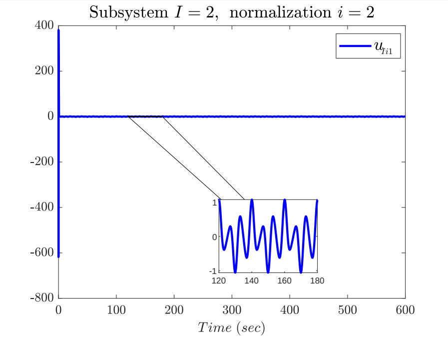
Simulation Results

Simulation 1: (Healthy condition)

No faults occur neither in subsystems Σ_1 or Σ_2.

Subsystem Σ_2 results:





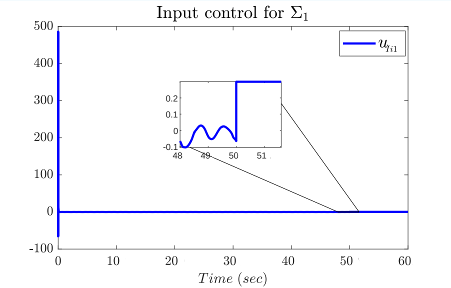
Simulation Results

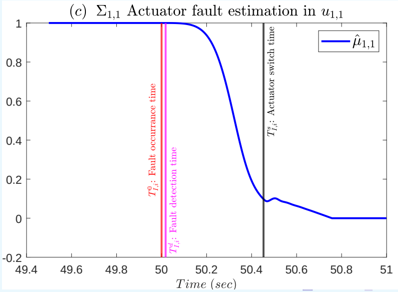
Simulation 2: (Stuck fault)

Actuator $u_{1,1}$ of Σ_1 become stuck at time $T^0_{I,1} = 50\,\text{sec}$

Fault occurrence time:	$T^0_{I,i} = 50.0\,sec$
Fault detection time:	$T^d_{I,i} = 50.017\,sec$
Stuck fault diagnosed time:	$T^s_{I,i} = 50.453\,sec$

Subsystem Σ_1 results:





Simulation Results

Simulation 2: (Stuck fault)

Actuator $u_{1,1}$ of Σ_1 become stuck at time $T^0_{I,1} = 50$ sec

Fault occurrence time:	$T^0_{I,i} = 50.0\, sec$
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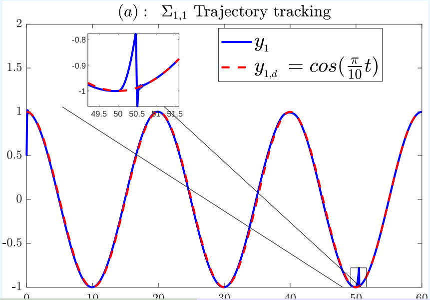
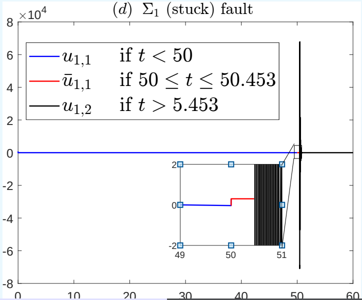
Subsystem Σ_1 results:





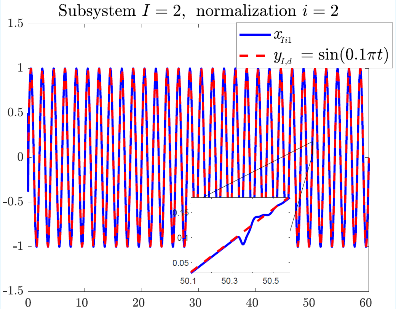
Simulation Results

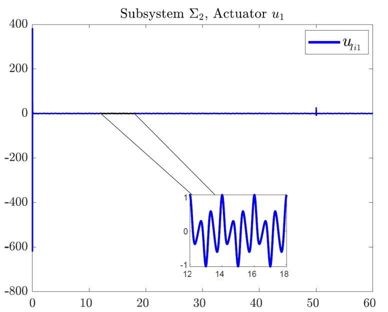
Simulation 2: (Stuck fault)

Actuator $u_{1,1}$ of Σ_1 become stuck at time $T^0_{I,1} = 50\,\text{sec}$

Fault occurrence time:	$T^0_{I,i} = 50.0\,sec$
Fault detection time:	$T^d_{I,i} = 50.017\,sec$
Stuck fault diagnosed time:	$T^s_{I,i} = 50.453\,sec$

Subsystem Σ_2 results: (fault propagation)





Simulation Results

Simulation 3: (Partial fault)

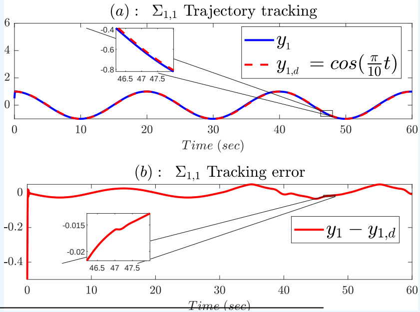
Actuator $u_{1,1}$ of Σ_1 become faulty (partial fault)

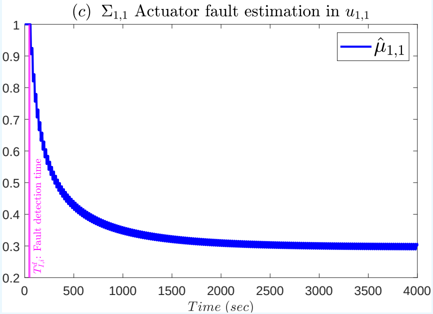
Fault occurrence time: $T^0_{I,i} = 47.0\, sec$

Fault detection time: $T^d_{I,i} = 47.251\, sec$

$u^F_{1,1} = -\frac{4}{5}u^H_{1,1}(t) \quad \Rightarrow \quad u_{1,1} = \frac{1}{5}u^H_{1,1}(t)$

Subsystem Σ_1 results:





[3] H. Tirandaz, C. Keliris, and M. M. Polycarpou, "Actuator fault accommodation for a class of nonlinear uncertain interconnected systems: An adaptive command filtering-based scheme ", *IEEE Transactions on Automation Science and Engineering, (Under Review)*, 2024.

Future Works:

Future work based on this thesis could focus on:

- **Actuator and Sensor Fault Accommodation**
- **Saturation-Based Actuator Fault Accommodation**
- **Decentralized Actuator Fault Diagnosis and Accommodation**

Publications:

1. Hamed Tirandaz, Christodoulos Keliris, and Marios Polycarpou. “ Actuator fault detection and isolation in a class of nonlinear interconnected systems." *International Journal of Control*,1-21,2024.
2. Hamed Tirandaz, Christodoulos Keliris, and Marios Polycarpou. “ Actuator and Sensor Fault Isolation in a Class of Large-Scale Nonlinear Dynamical Systems", *Journal of Automation and Intelligence*,3(2),57–72 2024.
3. Tirandaz, H., Keliris, C., & Polycarpou, M. M. Actuator fault accommodation for a class of nonlinear uncertain interconnected systems: An adaptive command filtering-based scheme . *IEEE Transactions on Automation Science and Engineering*, (Under Review).

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