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Adaptive Fuzzy Fault Tolerant Control Using Dynamic Sliding Mode

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Abstract: In this paper, an actuator fault tolerant control (FTC) has been designed using dynamic sliding mode control (DSMC) for single input nonlinear systems which are affected from uncertainty. In DSMC the switching of input control signal is removed due to the integrator which is placed before the input control signal of the plant hence, its result is reducing of chattering. However, in DSMC the augmented system (the system plus the integrator) is one dimension bigger than the actual system and then, the plant model should be completely known. To overcome on this problem, an adaptive fuzzy observer (AFO) has been proposed. Finally, we show the combination result of DSMC and AFO is the compensation of the fault by input control signal of the plant, while the performance and robustness of the closed loop system are reserved. The main property of the proposed approach is its simplicity and we only use system output for FTC and moreover, the fault diagnosis (detection, isolation and reconstruction) is done at the same time. The upper bound of the uncertainty is not used in DSMC and AFO, which is important in practical implementation. Simulation results show the advantages of the proposed approach.

Keywords: Adaptive fuzzy observer, fault tolerant control, nonlinear system, sliding mode control.

1. INTRODUCTION

1.1. Fault detection and isolation (FDI):

A fault is defined as an un-permitted deviation of at least one characteristic property or parameter of the system from the acceptable, usual or standard condition [1]. When, the system endures an abnormal condition (as an example a malfunction in the actuators or sensors) a fault is supposed to occur [1–4]. When faults occur in a system and do not detected online, the effect of system operation may be devastating and can cause calamitous accident and system performance may be unacceptable. Therefore, FDI techniques are important in practical systems [3–6]. For this reasons, FDI is a vital component for all systems to improve the reliability, safety and efficiency. Advanced methods of FDI become increasingly important for many technical processes [3]. The main task of FDI is to infer the something is wrong and to determine which system or component has a fault [4].Quick detection helps to minimize the maintenance and repair costs of the system and contributes towards increased system validity [7]. FDI approaches are classified into three major categories: Signal-based approach, model-based approach and parameter estimation approach [8]. The progress of modelling techniques allows the possibility of using model-based FDI approaches, which have been considered a very effective method both in theory and practice [1–5, 7, 8]. But more recent works are focused on approaches which reconstruct faults. Not only these approaches detect and isolate the source of the faults, they also provide additional insight about the fault e.g., shape and magnitude [1–5, 7–9].

1.2. Fault tolerant control (FTC):

A step ahead after FDI is FTC, which is a control system (controller) that can automatically compensate the effect of faults in system and also can maintain the overall system stability and acceptable performance in the event of component failures [10, 11]. In general, FTC can be achieved either passively by use of a feedback control law that is robust to possible system faults [11] or actively by means of the combination of FDI and FTC [12, 13]. Most of works in this literature are passively but some recent results on the integration of FDI and FTC can be found in [13–17]. As a result, a passively FTC consists of three parts: a sufficiently robust controller, a sensitive FDI system and a reconfiguration mechanism which can recover the system performance after the event of fault as much as possible [10].

1.3. Dynamic sliding mode control (DSMC):

Many works are focused on fault reconstruction using sliding mode (see [18–20] and references therein). It has been shown that the SMC, because of its invariance property, is a powerful tool in facing structured or unstructured uncertainties, disturbances, and noises that always pro-

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duce difficulties in the realization of designed controller for real systems [21, 22]. Invariance means insensitivity to parameter variations and is stronger than robustness [21, 22]. The greatest shortcoming of SMC is chattering, the high (but finite) frequency oscillations with small amplitude that produce heat losses in electrical power circuits and wear mechanical parts [23]. Chattering is often due to the excitation of high frequency un-modelled (ignored) dynamics (sensors, actuators and plant) [22–24]. One phenomenon which can excite these dynamics is high frequency switching of input control signal [22–24]. In DSMC an integrator is placed in front of the system to be controlled and time derivative of control input, is acted as the control variable for the augmented system [23] and then chattering is reduced. With appropriate definition of the sliding surface and input control signal for augmented system, the faulty system is controlled properly during the event of fault and we show that the fault is compensated by input control signal. Accordingly, for reaching to the main purpose (FDI+FTC), design of DSMC is challenging and sliding surface should be estimated because we only have the output of system.

1.4. Adaptive fuzzy observer (AFO):

Many works are done with the combination of SMC and fuzzy observer. But, in most of them the Sign function is available directly in input control signal and then, the chattering remain as a problem [25–27]. In some of the other works which are using fuzzy system for FTC the chattering is also a problem [16].

Then, the challenge of fault tolerant control can be solved by fuzzy systems [28, 29]. To overcome the problem of sliding surface estimation in DSMC, this paper proposes using a new AFO. Hence, an online training law has been proposed to update the fuzzy parameters. This observer estimates the unknown parts of the sliding surface only using the output of the nonlinear system and hence, sliding surface is available. Due to the simplicity of the proposed approach and do not using of fault and disturbance bound, this approach is applicable to the real system from the application point of view.

Moreover, based our present knowledge the last work in using of the DSMC for fault reconstruction is in [30] which a linear system is considered and assumed the system states is available.

1.5. Paper organization:

In Section 2, preliminaries background and problem formulation are defined. In Section 3, an AFO is proposed to solve the problem of DSMC and in Section 4, design of control variable, using the sliding surface, is described. Finally, in Section 5 we discuss simulation results to verify theoretical concepts presented in previous sections. The conclusion is given in Section 6.

2. PROBLEM FORMULATION

Consider the following time varying canonical form affine single input nonlinear system:

$$\begin{aligned}
&\dot{x}_i = x_{i+1} : i = 1,2,\ldots,n-1,\\
&\dot{x}_n = h(x,t) + u + d^T(x,u,t)x + f(x,u,t),\\
&y = x_1,\\
&x = [x_1, x_2, x_3, \ldots, x_n]^T,\\
&\dot{x} = [x_2, x_3, x_4, \ldots, x_{n+1}]^T.
\end{aligned} \tag{1}$$

Such that, $y \in R^1$ is the measurable system output, $x \in R^{n\times 1}$ is the un-accessible vector of system state, $u \in R^1$ is the scalar control input, $d \in R^{n\times 1}$ is bounded uncertainty with unknown bound $\bar{d}$ i.e., $\|d\| \le \bar{d} < \infty$ and $f(x,u,t) \in R^1$ is bounded fault signal and also assume that the system dynamic model $h(x,t)$ is unknown unbounded function where $h(x = 0.t) = 0$ i.e. $x_i = 0 :\ i = 1,2,\cdots,n$ is the equilibrium point of time varying system when fault signal is zero. The other form of this equation is as follows:

$$\begin{aligned}
&\dot{x} = Ax + B\,(u + g(x,u,t)),\\
&y = C^T x,
\end{aligned} \tag{2}$$

where

$$g(x,u,t) = h(x,t) + d^T(x,u,t)x + f(x,u,t) + \sum_{i=1}^{n} a_i x_i, \tag{3}$$

and

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 0 & 1 \\ -a_1 & -a_2 & \ldots & -a_{n-1} & -a_n \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \tag{4}$$

Assume that a_i are selected such that the pair (A,B) is minimum phase i.e., controllable and observable. Now, consider the last component of the system equation. From 1 one can write:

$$x_{n+1} = h(x,t) + u + d^T(x,u,t)x + f(x,u,t). \tag{5}$$

Our purpose is to use DSMC for finding a suitable smooth u such that, $x = [x_1, x_2, x_3, \cdots, x_n]^T$ and x_{n+1} converge to zero. In fact, DSMC is used to stabilize the zero equilibrium point of the system. Based on (5), in this case

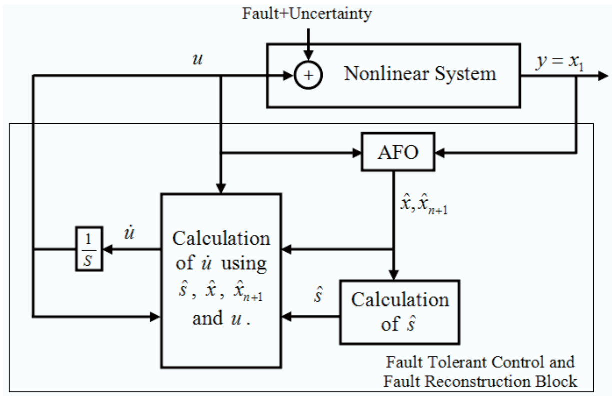

Fig. 1. The proposed fault tolerant control approach via dynamic sliding mode based adaptive fuzzy observe.

$u+f=0$ and then $f(x,u,t)=-u$ or the fault reconstruction is done as well as the FTC or in the other word the fault is compensated by input control signal without loss of the robustness and performance. Moreover, in DSMC the switching is filtered due to the integrator which is placed before the input control of the system and we will have a smooth u and then, chattering is alleviated. For this purpose, an appropriate sliding surface is defined as follows:

$$s=\lambda^T x+\lambda_{n+1}x_{n+1}, \lambda=[\lambda_1,\lambda_2,\lambda_3,\ldots,\lambda_n]^T. \tag{6}$$

Remark 1: $x=[x_1,x_2,x_3,\cdots,x_n]^T$ and x_{n+1} converge to zero if sliding surface becomes zero and the coefficients $\lambda_1,\lambda_2,\cdots,\lambda_{n+1}$ are properly chosen such that the zero dynamics of equation $s=0$ are zero or the polynomial $\lambda_{n+1}S^n+\lambda_n S^{n-1}+\cdots+\lambda_2 S+\lambda_1=0$ is Hurwitz.

But, there is a problem in calculation of this surface. Only the output of system is available and also variable x_{n+1} depends to the unknown function h, uncertainty d, and fault signal f (refer to (5)). For solve this problem we propose a new scheme of AFO which has been described in the next sections. Moreover, the overall diagram of the proposed approach has been depicted in Fig. 1 which, consist of two designing steps: AFO and DSMC and we will describe each step of this figure in the next sections.

Remark 2: In the other chattering reduction methods, such as boundary layer and adaptive boundary layers, the sliding surface is defined as $s=\lambda^T x$. In this case $x=[x_1,x_2,x_3,\cdots,x_n]^T$ converges to zero if s becomes zero and the coefficients $\lambda_1,\lambda_2,\cdots,\lambda_n$ are chosen such that the polynomial $\lambda_n S^{n-1}+\ldots+\lambda_2 S+\lambda_1=0$ is Hurwitz. But x_{n+1} will not converge to zero and then, equality $u+f=0$ is not reliable.

3. AFO DESIGN FOR ESTIMATION OF SLIDING SURFACE

Since x and x_{n+1} cannot be evaluated correctly, an AFO is suggested to estimate them. According to the fuzzy theorems, Gaussian fuzzy basis functions (GFBF) can approximate any real continuous function with any arbitrary accuracy. This means that the GFBF has universal approximation property [31]. In order to estimate the nonlinear function g, a singleton fuzzifier with product inference engine and a defuzzifier as weight sum of each output rule is used.

$$\begin{aligned}&\dot{\hat{x}}=A\hat{x}+B\left(u+\hat{g}\right),\\&\hat{g}=\ \hat{w}^T\xi(\hat{x},u),\end{aligned} \tag{7}$$

where $\hat{w}\in R^{m\times 1}$ is the estimation weight vector and $\xi(.):R^{n+1}\to R^{m\times 1}$ is the activation function vector. Due to the approximation capability of the fuzzy networks, there exists an ideal weight vector w with arbitrary large enough dimension m such that the system (2) can be written as follow.

$$\begin{aligned}&\dot{x}=Ax+B\left(u+g\right),\\&g=w^T\xi(x,u)+\varepsilon,\end{aligned} \tag{8}$$

where ε is an arbitrary small reconstruction error with bound B_ε, i.e. $|\varepsilon|\leq B_\varepsilon$. Now, the following estimator is proposed.

$$\begin{aligned}&\dot{\hat{x}}=A\hat{x}+B\left(u+\hat{w}^T\xi(\hat{x},u)+r\right)+T(y-\hat{y}),\\&\hat{y}=C^T\hat{x},\end{aligned} \tag{9}$$

where r is robustifying term and matrix A and observer gain vector $T\in R^{n\times 1}$ are chosen such that $A_s=A-TC^T$ is stable i.e., for any symmetric positive definite matrix Q, there exist a symmetric positive definite matrix P satisfying the following Lyapunov equation

$$A_s^T P+PA_s=-Q. \tag{10}$$

Note that by using Kalman-Yakubovich lemma [32], there exist positive definite matrices P and Q such that $PB=C$. By subtracting (9) from (8), we obtain

$$\begin{aligned}\dot{\tilde{x}}=&Ax+B\left(u+w^T\xi(x,u)+\varepsilon\right)-A\hat{x}\\&-B\left(u+\hat{w}^T\xi(\hat{x},u)+r\right)-T(y-\hat{y})\\=&A_s\tilde{x}+B\left(w^T\xi(x,u)+\varepsilon-\hat{w}^T\xi(\hat{x},u)-r\right),\end{aligned} \tag{11}$$

where $\tilde{x}=x-\hat{x}$ and $\tilde{w}=w-\hat{w}$ are the state and parameter estimation errors.

$$\begin{aligned}\dot{\tilde{x}}=&A_s\tilde{x}+B\left(w^T\xi(x,u)+\varepsilon-\hat{w}^T\xi(\hat{x},u)\right.\\&\left.-r-w^T\xi(\hat{x},u)+w^T\xi(\hat{x},u)\right)\\=&A_s\tilde{x}+B\left(\tilde{w}^T\hat{\xi}+w^T\tilde{\xi}+\varepsilon-r\right).\end{aligned} \tag{12}$$

Such that $\tilde{\xi}=\xi-\hat{\xi}$, $\xi=\xi(x,u)$ and $\hat{\xi}=\xi(\hat{x},u)$.

Theorem 1: Using the following adaptive weight law:

$$\dot{\hat{w}} = k_w \hat{\xi} \tilde{y} - 4k_e k_w |\tilde{y}| \hat{w}, \tag{13}$$

where $r = k_x \tilde{y}$ and $\tilde{y} = y - \hat{y}$. Moreover, k_w and k_e are arbitrary positive scalar constants and k_x is positive scalar. Then, estimation error $\tilde{x}(t)$ converges to zero if $k_x \to \infty$.

Proof: Consider the following Lyapunov function

$$V(t) = \frac{1}{2}\tilde{x}^T P \tilde{x} + \frac{1}{2k_w}\tilde{w}^T \tilde{w}. \tag{14}$$

Taking the derivative of $V(t)$ yields

$$\dot{V}(t) = \frac{1}{2}\dot{\tilde{x}}^T P \tilde{x} + \frac{1}{2}\tilde{x}^T P \dot{\tilde{x}} + \frac{1}{k_w}\tilde{w}^T \dot{\tilde{w}}. \tag{15}$$

Substituting (10) and (12) in the above equation follow that

$$\begin{aligned}\dot{V}(t) = &-\frac{1}{2}\tilde{x}^T Q \tilde{x} + \tilde{x}^T PB(w^T \tilde{\xi} + \varepsilon) - k_x \tilde{x}^T CC^T \tilde{x} \\ &+ \tilde{w}^T \left(\frac{1}{k_w}\dot{\tilde{w}} + \hat{\xi} B^T P \tilde{x} \right)\end{aligned} \tag{16}$$

using $\dot{\tilde{w}} = -\dot{\hat{w}}$ and the tuning law (13) in the above equation gives

$$\begin{aligned}\dot{V}(t) = &-\tfrac{1}{2}\tilde{x}^T Q \tilde{x} + \tilde{x}^T PB(w^T \tilde{\xi} + \varepsilon) \\ &-k_x \tilde{x}^T CC^T \tilde{x} + 4k_e |\tilde{y}| \tilde{w}^T \hat{w}\end{aligned} \tag{17}$$

Now, consider the logical assumption that the actual weight w in (8) is norm bounded i.e., $\|w\| \le B_w$ and moreover the activation functions ξ or $\hat{\xi}$ are also bounded which means that $\|\xi\| \le B_\xi$ and $\|\hat{\xi}\| \le B_\xi$. Therefore, we can write $\left| w^T \tilde{\xi} + \varepsilon \right| \le 2B_w B_\xi + B_\varepsilon$. Here, consider the properties of positive definite matrices Q and P, and using $\hat{w} = w - \tilde{w}$, the above equation yields

$$\begin{aligned}\dot{V}(t) \le &-(0.5\underline{\sigma}(Q) + k_x)\|\tilde{x}\|^2 \\ &+ \overline{\sigma}(PB)(2B_w B_\xi + B_\varepsilon)\|\tilde{x}\| \\ &- 4k_e(\|\tilde{w}\|^2 - B_w\|\tilde{w}\|)|\tilde{y}|.\end{aligned} \tag{18}$$

Now, we define $B_{\tilde{x}}$ as follows:

$$\begin{aligned}B_{\tilde{x}} &= \frac{\overline{\sigma}(PB)(2B_w B_\xi + B_\varepsilon) + k_e B_w^2 \|C\|}{0.5\underline{\sigma}(Q) + k_x} \\ &= \frac{\overline{\sigma}(PB)(2B_w B_\xi + B_\varepsilon) + k_e B_w^2}{0.5\underline{\sigma}(Q) + k_x}.\end{aligned} \tag{19}$$

Such that $\overline{\sigma}$ and σ denote maximum and minimum singular value. Therefore:

$$\begin{aligned}\dot{V}(t) \le &-(0.5\underline{\sigma}(Q) + k_x)(\|\tilde{x}\| - B_{\tilde{x}})\|\tilde{x}\| \\ &- 4k_e\left(\|\tilde{w}\| - \frac{1}{2}B_w\right)^2 |\tilde{y}|,\end{aligned} \tag{20}$$

or

$$\dot{V}(t) \le -(0.5\underline{\sigma}(Q) + k_x)(\|\tilde{x}\| - B_{\tilde{x}})\|\tilde{x}\|. \tag{21}$$

Take $\omega(t) = (0.5\underline{\sigma}(Q) + k_x)(\|\tilde{x}\| - B_{\tilde{x}})\|\tilde{x}\|$ and suppose $\|\tilde{x}\| > B_{\tilde{x}}$ then, one can write $\dot{V} \le -\omega(t) \le 0$. Integration from zero to t yields:

$$0 \le \int_0^t \omega(\tau)d\tau \le \int_0^t \omega(\tau)d\tau + V(t) \le V(0) \tag{22}$$

when $t \to \infty$ the above integral exists and is less than or equal to $V(0)$. Since $V(0)$ is positive and finite, according to the Barbalat's lemma [32] we will have

$$\lim_{t\to\infty}\omega(t) = \lim_{t\to\infty}(0.5\underline{\sigma}(Q) + k_x)(\|\tilde{x}\| - B_{\tilde{x}})\|\tilde{x}\| = 0. \tag{23}$$

Since $(0.5\underline{\sigma}(Q) + k_x)$ is greater than zero, (23) implies decreasing $\|\tilde{x}\|$ until it becomes less than $B_{\tilde{x}}$ which its result is $\lim_{t\to\infty}\|\tilde{x}\| \le B_{\tilde{x}}$. This guarantees that $B_{\tilde{x}}$ is the upper bound of $\|\tilde{x}\|$ and from (19) it is clear that $\lim_{k_x\to\infty} B_{\tilde{x}} = 0$. Then, $\|\tilde{x}\|$ or $\tilde{x}$ will converge to zero if $k_x \to \infty$. □

Remark 3: The result of this theorem can be written as:

$$\lim_{k_x\to\infty}\tilde{x} = 0. \tag{24}$$

4. DESIGN OF INPUT CONTROL SIGNAL

Now, according to (9) we can write:

$$\begin{aligned}&\hat{x}_{n+1} = FA\hat{x} + u + \hat{w}^T\hat{\xi} + k_x(y - \hat{y}) + FT(y - \hat{y}), \\ &F = [0, 0 \ldots, 0, 1] \in R^n.\end{aligned} \tag{25}$$

Then, estimated sliding surface (6) can be written as follows.

$$\hat{s} = \lambda^T \hat{x} + \lambda_{n+1}\hat{x}_{n+1}. \tag{26}$$

Take derivative of this sliding surface.

$$\begin{aligned}\dot{\hat{s}} = &\lambda^T \dot{\hat{x}} + \lambda_{n+1}\dot{\hat{x}}_{n+1} = \lambda^T \dot{\hat{x}} + \lambda_{n+1}\Big(FA\dot{\hat{x}} + \dot{u} + \dot{\hat{w}}^T\hat{\xi} \\ &+ \hat{w}^T\frac{\partial\hat{\xi}}{\partial\hat{x}}\dot{\hat{x}} + \hat{w}^T\frac{\partial\hat{\xi}}{\partial u}\dot{u} - k_x C^T\dot{\hat{x}} - FTC^T\dot{\hat{x}} + \phi\Big), \\ \phi = &k_x\dot{y} + FT\dot{y} = (k_x + FT)\dot{y},\end{aligned} \tag{27}$$

or

$$\begin{aligned}\dot{\hat{s}} = &\left(\lambda^T + \lambda_{n+1}FA + \lambda_{n+1}\hat{w}^T\frac{\partial\hat{\xi}}{\partial\hat{x}} - k_x C^T - FTC^T\right)\dot{\hat{x}} \\ &+ \lambda_{n+1}\left(1 + \hat{w}^T\frac{\partial\hat{\xi}}{\partial u}\right)\dot{u} + \lambda_{n+1}\dot{\hat{w}}^T\hat{\xi} + \lambda_{n+1}\phi.\end{aligned} \tag{28}$$

Such that $\dot{\hat{x}}$ can be calculated from (9) and the unknown terms is placed in ϕ. Variable ϕ is considered as uncertainty due to its dependency to unknown variable $\dot{y} = C^T\dot{x}$.

Remark 4: $\hat{x} = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \ldots, \hat{x}_n]^T$ and $\hat{x}_{n+1}$ converge to zero if $\hat{s}$ becomes zero and the coefficients $\lambda_1, \lambda_2, \ldots, \lambda_{n+1}$ are properly chosen such that the polynomial $\lambda_{n+1}S^n + \lambda_n S^{n-1} + \ldots + \lambda_2 S + \lambda_1 = 0$ is Hurwitz.

Theorem 2: The following dynamical equation causes the sliding surface $\hat{s}$ converges to zero.

$$\begin{aligned}\dot{u} = &-(\lambda_{n+1})^{-1}\left(1 + \hat{w}^T \frac{\partial \hat{\xi}}{\partial u}\right)^{-1}\left(\lambda_{n+1}\dot{\hat{w}}^T \hat{\xi}\right.\\ &+\delta_1\, sign(\hat{s}) + \delta_2\, \hat{s} + (\lambda^T + \lambda_{n+1} FA \\ &\left.+\lambda_{n+1}\hat{w}^T \frac{\partial \hat{\xi}}{\partial \hat{x}} - k_x C^T - FTC^T)\dot{\hat{x}}\right)\end{aligned} \tag{29}$$

and,

$$\delta_1 = \lambda_{n+1} B_\phi + \varepsilon,\ \varepsilon > 0 \text{ and } \delta_2 > 0, \tag{30}$$

where B_ϕ is bound of ϕ i.e., $|\phi| \le B_\phi$.

Proof: Consider the Lyapunov function $V = 0.5\hat{s}^2$. Then, $\dot{V} = \hat{s}\dot{\hat{s}}$ and moreover replacing $\dot{u}$ from (29) into (28) follows that

$$\dot{\hat{s}} = -\delta_1\, sign(\hat{s}) - \delta_2\, \hat{s} + \lambda_{n+1}\,\phi. \tag{31}$$

Hence

$$\begin{aligned}\dot{V} &= -\delta_1\,|\hat{s}| - \delta_2\,\hat{s}^2 + \lambda_{n+1}\,\phi\,\hat{s}\\ &\le -(\delta_1 - \lambda_{n+1}\,\phi))\,|\hat{s}|\\ &\le -\left(\delta_1 - \lambda_{n+1} B_\phi)\right)\,|\hat{s}|.\end{aligned} \tag{32}$$

Now, consider δ_1 in (30) then:

$$\dot{V} \le -\varepsilon\,|\hat{s}|. \tag{33}$$

Suppose t_f is the reaching time i.e., $\hat{s}(t_f) = 0$ then, it is easy to show that $t_f \le |\hat{s}(0)|/\varepsilon$. □

Lemma 1: Sliding surface s will converge to zero if $k_x \to \infty$.

Proof: Based on Theorem 1 and (24), we have $\lim_{k_x\to\infty} \tilde{x} = 0$ or $\lim_{k_x\to\infty} \dot{\tilde{x}} = 0$ and then:

$$\begin{aligned}|\tilde{s}| &= \lambda^T\,|x - \hat{x}| + \lambda_{n+1}\,|x_{n+1} - \hat{x}_{n+1}|\\ &= \lambda^T\,|\tilde{x}| + \lambda_{n+1}\left|F\,\dot{\tilde{x}}\right|.\end{aligned} \tag{34}$$

Therefore, $\lim_{k_x\to\infty} |\tilde{s}| = 0$ or $\lim_{k_x\to\infty} s = \hat{s}$ and equality $\hat{s}(t_f) = 0$ result in:

$$\lim_{k_x\to\infty} s = 0. \tag{35}$$

□

Remark 5: From (27) and (30) we can see that choosing a large value for k_x may cause increasing of B_ϕ. But, selection a small λ_{n+1} results a desired δ_1 which prevent high amplitude switching.

5. SIMULATION RESULTS

To show the applicability and effectiveness of the proposed approach, we try to use a complicated example which have exponential sentences, multiplication, divided and time. Consider the following time varying single input nonlinear system.

$$\begin{aligned}&\dot{x}_1 = x_2,\\ &\dot{x}_2 = -{x_1}^2 \exp\left(t{x_2}^2\right) - \frac{0.25x_1x_2}{(1+x_1)}\\ &\qquad + u + d(x,u,t)x + f(x,u,t),\\ &y = x_1,\\ &x = [x_1, x_2]^T, \dot{x} = [x_2, x_3]^T.\end{aligned} \tag{36}$$

The parameters of sliding surface (26) are chosen as $\lambda_1 = 10, \lambda_2 = 10, \lambda_3 = 0.1$. Other design parameters are as $k_w = 5, k_x = 10, k_e = 3$ and B_ϕ has been considered as a constant such that $B_\phi = 20, \varepsilon = 0.1, \delta_2 = 20$. The initial conditions are chosen as $x_1(0) = 1, x_2(0) = -0.5$ and $\hat{x}_1(0) = 0, \hat{x}_2(0) = 0$ and $d = [1,1]^T$. To calculate derivative of input control signal $\dot{u}$, the initial value of u is needed which is set to zero i.e. $u(0) = 0$. The fault is as follow and will be applied to the system in time $t = 2$ i.e. in transient response of the system.

$$f = (u(t))^2 + x_1(t) + 0.1\sin(t) + 0.1\sin(2t). \tag{37}$$

Moreover

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix},\ T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{38}$$

We choose $m = 11$ and the initial value of weight vector w sets to zero and:

$$\begin{aligned}&\xi_i(\hat{x}, u) = \exp\left(-\left(\sqrt{\hat{x}_1^2 + \hat{x}_2^2 + u^2} + (i-6)\right)\Big/5\right),\\ &i = 1,2,\ldots,11,\end{aligned} \tag{39}$$

where $|\xi_i| \le 1$ which result that $\|\xi\| \le \sqrt{11}$ and $\left\|\hat{\xi}\right\| \le \sqrt{11}$ or $B_\xi = \sqrt{11}$. Simulation has been done by MATLAB with fixed step time of 0.01. The results of simulation are shown in Figs. 2-7. Figs. 2 and 3 show the system states which converge to zero even in the presence of fault and disturbance. In Fig. 4 sliding surface is depicted which also converge to zero in finite time. Fig. 5 shows the fault signal applied to the system after time 2 second. From Fig. 6 one can see that that the input control signal u is smooth and without switching. Fig. 7 shows the fault signal compared with input control signal and one can see that $u = -f$ and then, FDI and FTC are done correctly. Notice that the fault is happened before the steady state of the system which shows the capability of the proposed approach.

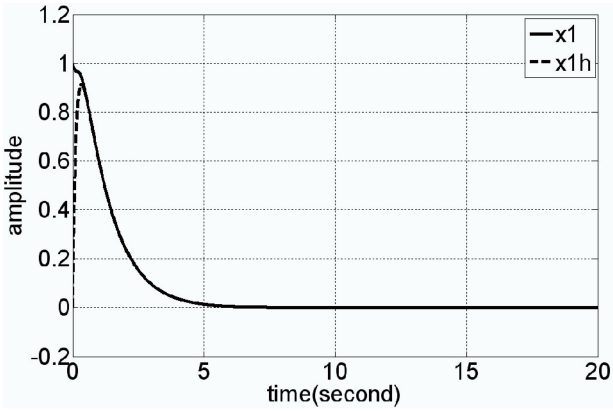

Fig. 2. First state and its estimation.

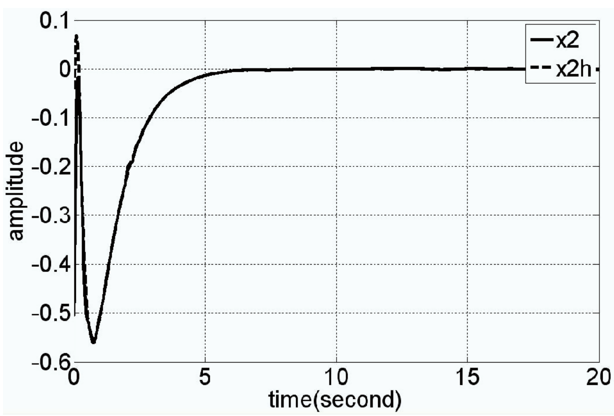

Fig. 3. Second state and its estimation.

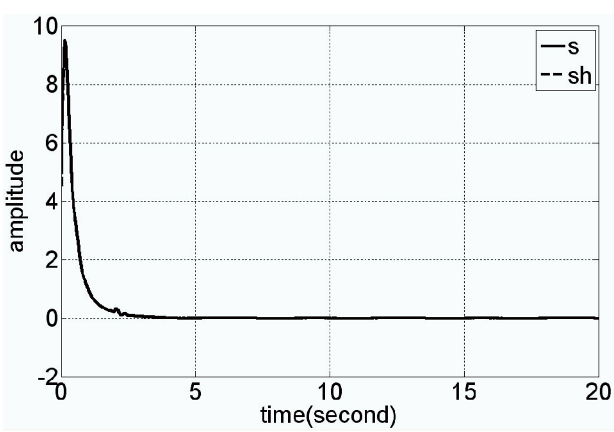

Fig. 4. Sliding surface and its estimation.

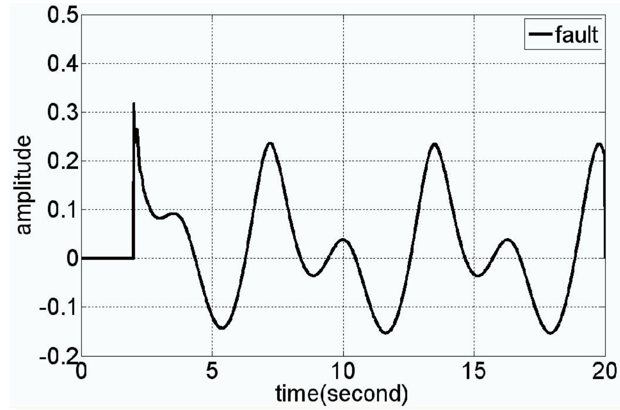

Fig. 5. Fault signal.

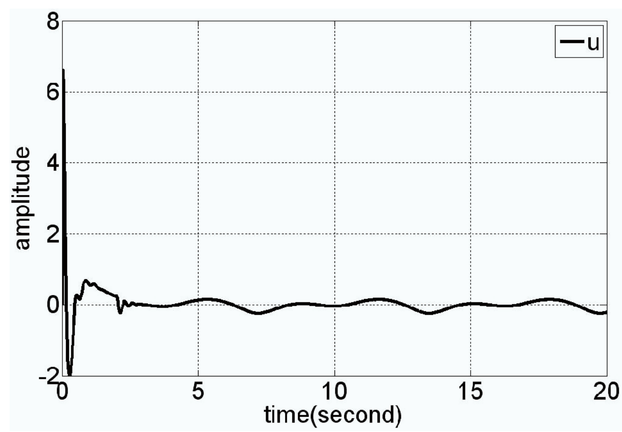

Fig. 6. Input control signal of the plant.

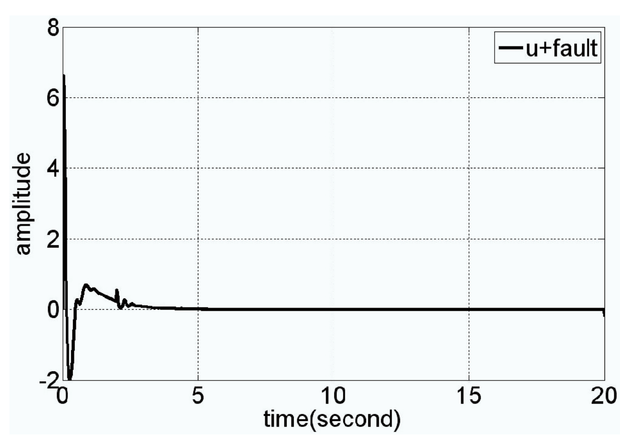

Fig. 7. Comparing input control signal and fault signal.

6. CONCLUSION

In this paper, a new approach is proposed for fault tolerant control (FTC) and fault reconstruction based on dynamic sliding mode controller (DSMC) and fuzzy systems. This approach is applicable to the single input single output (SISO) nonlinear systems. To solve the problem of DSMC we proposed an adaptive fuzzy observer (AFO). Because of using DSMC, chattering is reduced. Moreover, in the proposed approach, the upper bound of the uncertainty is not used for designing FTC and we only use the system output. Finally, the closed loop system is robust against fault because the fault signal is compensated by the input control signal. Then, proposed approach preserves all the main properties of SMC such as robustness and simplicity in design and implementation. Simulation results show the effectiveness of this approach. In the future works we can improve this approach to the multi input multi output (MIMO) systems. For this purpose an evolution to the proposed fuzzy system is needed.

REFERENCES

[1] H. Alwi, C. Edwards, and C. P. Tan, *Fault Detection and Fault-tolerant Control Using Sliding Modes*, Springer Science & Business Media, 2011.

[2] C. P. Tan and C. Edwards, "Sliding mode observers for detection and reconstruction of sensor faults," *Automatica*, vol. 38, no. 10, pp. 1815-1821, 2002. [click]

[3] X.-G. Yan and C. Edwards, "Nonlinear robust fault reconstruction and estimation using a sliding mode observer," *Automatica*, vol. 43, no. 9, pp. 1605-1614, 2007. [click]

[4] K. B. Goh, S. K. Spurgeon, and N. B. Jones, "Fault diagnostics using sliding mode techniques," *Control Engineering Practice*, vol. 10, no. 2, pp. 207-217, 2002. [click]

[5] J. Yang, F. Zhu, X. Wang, and X. Bu, "Robust sliding-mode observer-based sensor fault estimation, actuator fault detection and isolation for uncertain nonlinear systems," *International Journal of Control, Automation and Systems*, vol. 13, no. 5, pp. 1037-1046, 2015. [click]

[6] Y. Ren and D.-W. Ding, "Integrated fault detection and control for two-dimensional roesser systems," *International Journal of Control, Automation and Systems*, vol. 15, no. 2, pp. 722-731, 2017. [click]

[7] R. Isermann, "Model-based fault-detection and diagnosis-status and applications," *Annual Reviews in control*, vol. 29, no. 1, pp. 71-85, 2005. [click]

[8] H. Alwi, C. Edwards, and A. Marcos, "Fault reconstruction using a lpv sliding mode observer for a class of lpv systems," *Journal of the Franklin Institute*, vol. 349, no. 2, pp. 510-530, 2012. [click]

[9] B. Yan, Z. Tian, S. Shi, and Z. Weng, "Fault diagnosis for a class of nonlinear systems via eso," *ISA transactions*, vol. 47, no. 4, pp. 386-394, 2008.

[10] Y. Zhang and J. Jiang, "Active fault-tolerant control system against partial actuator failures," *IEE proceedings-Control Theory and Applications*, vol. 149, no. 1, pp. 95-104, 2002. [click]

[11] X.-J. Li and G.-H. Yang, "Robust adaptive fault-tolerant control for uncertain linear systems with actuator failures," *IET Control Theory & Applications*, vol. 6, no. 10, pp. 1544-1551, 2012. [click]

[12] X. Zhang, T. Parisini, and M. M. Polycarpou, "Adaptive fault-tolerant control of nonlinear uncertain systems: an information-based diagnostic approach," *IEEE Transactions on Automatic Control*, vol. 49, no. 8, pp. 1259-1274, 2004. [click]

[13] Y. Zhang and J. Jiang, "Bibliographical review on reconfigurable fault-tolerant control systems," *Annual Reviews in Control*, vol. 32, no. 2, pp. 229-252, 2008. [click]

[14] R. Kabore and H. Wang, "Design of fault diagnosis filters and fault-tolerant control for a class of nonlinear systems," *IEEE transactions on Automatic Control*, vol. 46, no. 11, pp. 1805-1810, 2001. [click]

[15] N. Eva Wu, Y. Zhang, and K. Zhou, "Detection, estimation, and accommodation of loss of control effectiveness," *International Journal of Adaptive Control and Signal Processing*, vol. 14, no. 7, pp. 775-795, 2000. [click]

[16] H. Li, P. Shi, and D. Yao, "Adaptive sliding-mode control of markov jump nonlinear systems with actuator faults," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 1933-1939, 2017. [click]

[17] A. Fekih, "Fault-tolerant flight control design for effective and reliable aircraft systems," *Journal of Control and Decision*, vol. 1, no. 4, pp. 299-316, 2014. [click]

[18] C. P. Tan and C. Edwards, "Sliding mode observers for robust detection and reconstruction of actuator and sensor faults," *International Journal of Robust and Nonlinear Control*, vol. 13, no. 5, pp. 443-463, 2003. [click]

[19] S. Barghandan, M. A. Badamchizadeh, and M. R. Jahed-Motlagh, "Improved adaptive fuzzy sliding mode controller for robust fault tolerant of a quadrotor," *International Journal of Control, Automation and Systems*, vol. 15, no. 1, pp. 427-441, 2017. [click]

[20] J. Yang, F. Zhu, X. Wang, and X. Bu, "Robust sliding-mode observer-based sensor fault estimation, actuator fault detection and isolation for uncertain nonlinear systems," *International Journal of Control, Automation and Systems*, vol. 13, no. 5, pp. 1037-1046, 2015. [click]

[21] A. Karami-Mollaee, N. Pariz, and H. Shanechi, "Position control of servomotors using neural dynamic sliding mode," *Journal of Dynamic Systems, Measurement, and Control*, vol. 133, no. 6, p. 061014, 2011.

[22] H. Lee and V. I. Utkin, "Chattering suppression methods in sliding mode control systems," *Annual Reviews in Control*, vol. 31, no. 2, pp. 179-188, 2007. [click]

[23] W. Perruquetti and J.-P. Barbot, *Sliding Mode Control in Engineering*, CRC Press, 2002.

[24] A. Levant, "Chattering analysis," *IEEE Transactions on Automatic Control*, vol. 55, no. 6, pp. 1380-1389, 2010. [click]

[25] H. Li, P. Shi, D. Yao, and L. Wu, "Observer-based adaptive sliding mode control for nonlinear markovian jump systems," *Automatica*, vol. 64, pp. 133-142, 2016. [click]

[26] H. Li, J. Wang, and P. Shi, "Output-feedback based sliding mode control for fuzzy systems with actuator saturation," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 6, pp. 1282-1293, 2016. [click]

[27] H. Li, J. Wang, H.-K. Lam, Q. Zhou, and H. Du, "Adaptive sliding mode control for interval type-2 fuzzy systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 12, pp. 1654-1663, 2016. [click]

[28] J. Hang, J. Zhang, and M. Cheng, "Application of multi-class fuzzy support vector machine classifier for fault diagnosis of wind turbine," *Fuzzy Sets and Systems*, vol. 297, pp. 128-140, 2016. [click]

[29] D. Zhai, L. An, J. Li, and Q. Zhang, *Adaptive fuzzy fault-tolerant control with guaranteed tracking performance for nonlinear strict-feedback systems*, *Fuzzy Sets and Systems*, vol. 302, pp. 82-100, 2016. [click]

[30] P. Ansari-Bonab, A. Karami-Mollaee, and M. A. Sadrnia, "Adaptive fuzzy dynamic sliding mode control based ltr observer for fault reconstruction," *Proc. of 13th Iranian Conference on Fuzzy Systems (IFSC)*, IEEE, pp. 1-5, 2013.

[31] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley & Sons, 2004.

[32] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*, vol. 199, Prentice-Hall, Englewood Cliffs, NJ, 1991.

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