COMBINATION SYNCHRONIZATION OF MULTIPLE CHAOTIC SYSTEMS WITH UNCERTAIN PARAMETERS USING ADAPTIVE HYBRID MODIFIED PROJECTIVE CONTROL METHOD

Hamed Tirandaz, Ali Karami-Mollaee

Electrical and Computer Engineering Faculty, Hakim Sabzevari University
Sabzevar, Iran

(Received October 16, 2017; accepted November 28, 2017)

This research addresses the chaotic synchronization of an uncertain chaotic system as the master system with multiple chaotic systems as slave systems, simultaneously. Adaptive control and parameter estimation laws based on the hybrid modified projective synchronization (HMPS) method are derived to synchronize the states of a master chaotic system with a proportional rate of their corresponding states from multi-slave chaotic systems. The feasibility and stability of the proposed scheme are analytically proved by means of the Lyapunov stability theorem. Furthermore, the validity of the proposed HMPS method and the theoretical discussions are verified by numerical simulations. The simulation results confirm the effectiveness of the proposed method.

DOI:10.5506/APhysPolB.49.59

1. Introduction

Chaos phenomenon usually appears in a nonlinear dynamical system under some special circumstances. Sensitivity to the initial conditions, various unstable periodic orbits and rich frequencies are three important properties of chaotic systems, which persuade studying control and synchronization of these dynamical systems. Chaos exists in a lot of real world dynamical systems. Furthermore, many practical application in secure communication and data encryption arise which benefit some sort of projective synchronization method, such as secure communication in [1, 2] and image encryption in [3].

Since the introductory work by Pecora *et al.* [4] for synchronization of chaotic systems, the problem of synchronization of a chaotic system with another chaotic system by means of designing an appropriate controller has become a challenging research topic. As a result, various synchronization schemes have been presented by researchers including active synchronization schemes [5, 6], adaptive schemes [7–9], sliding mode schemes [10–12], impulsive schemes [13, 14] and projective schemes [15–18]. Among these primary methods, projective synchronization methods have attracted considerable attention due to the dependency of the system errors to a scaling factor λ. Easy implementation and efficiently in practical applications are other benefits of the projective methods. Moreover, many types of synchronization approaches are a typical generalization of projective schemes. When scaling factor λ is considered as 1, the method can also be called as complete synchronization and if $\lambda = -1$, it gives the anti-synchronization scheme. Furthermore, modified projective synchronization [11, 19], function projective synchronization [20, 21], generalized projective synchronization [17, 22] and lag projective synchronization [23] methods are some of the projective related methods.

A master–slave synchronization task, in general, consists of a slave chaotic system, which has to force the behavior of its state variables to follow the motion trajectories of a master system state variables. This goal can be realized by means of designing an appropriate feedback controller. Due to its importance in science and industry, many researchers devoted their investigations to develop efficient controllers in order to synchronize two chaotic systems. Readers can refer to references [4–18] for more details. However, the problem of chaos synchronization between a chaotic system as master system and multiple slave chaotic systems is rarely studied. In addition, controlling the behavior of a chaotic system with multiple chaotic systems provides a more accurate synchronization scheme than other master–slave synchronization methods. Therefore, the main focus of this paper is to address this subject.

There are few works devoted to the synchronization between multiple chaotic systems. In [24], a task-oriented approach was proposed to control a master manipulator arm and multi-slave arms, each of which has six or more degrees of freedom. They have also addressed the controlling problem of a single-master multi-slave manipulators via virtual internal model (VIM) in [25]. Controlling the behavior of a gyroscope mechanical system as a master with multiple slave nonlinear gyroscope systems is another example, which is introduced in [26] using the mechanics fundamentals. Multi-synchronization of two chaotic systems is investigated in [27], where a linear combination of the master system state variables is synchronized with a linear combination of the slave system state variables. In [28], an active backstepping controller is designed for combination synchronization of three classic chaotic systems. Backstepping synchronization of Three Josephson Junctions is studied in [29] using function projective combination scheme.

Function combination synchronization is used in [30] of three chaotic complex systems. Combination–combination synchronization for three and four nonlinear complex chaotic systems is investigated in [31] and [32], respectively. In their work, the authors utilize projective method to achieve synchronization. Compound synchronization for four chaotic systems in [33], compound combination synchronization of five chaotic systems in [34] and dual combination synchronization of six chaotic systems in [35] are the recently published articles in the subject of compound synchronization. However, to the best knowledge of the present authors, the synchronization problem of a chaotic system as a master and multiple chaotic systems as multi-slave systems is not addressed yet. Actually, the main drawback of the previous research is that they are not flexible enough for different number of chaotic systems with different number of state variables. Therefore, the aim of this paper is to provide a combination synchronization approach, which can be utilize to synchronize between a chaotic system as a master system with multiple another chaotic systems as slave systems. A control law and parameter estimation strategy are introduced based on the HMPS method, adaptive control theory and the Lyapunov stability theory, to achieve such a synchronization scheme.

The reminder of this research is organized as follows: The hybrid modified projective synchronization problem between a typical class of chaotic systems consisting a master system and multiple slave systems are discussed in Section 2. In this section, the appropriate feedback controller and parameter estimation laws are introduced. Furthermore, the feasibility of the proposed synchronization scheme is verified by the Lyapunov stability theorem. Then, some numerical simulations are presented in Section 3 to confirm the feasibility and the stability of the theoretical discussions presented in the previous section. Finally, a brief discussion and some concluding remarks are presented in Section 4.

2. Synchronization

Consider a typical class of one master and m slaves (haper)chaotic systems as follows:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = f(\boldsymbol{x},t)\boldsymbol{\Phi}_c + F(\boldsymbol{x},t)\,, \\ \dot{\boldsymbol{y}}_{\boldsymbol{i}}(t) = g_i(\boldsymbol{y_i},t)\boldsymbol{\Phi_i}(t) + G_i(\boldsymbol{y_i},t) + \boldsymbol{u_i}(t)\,, \qquad i = 1,2,\cdots,m\,, \end{cases} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^T \in \boldsymbol{R}^{n\times 1}$ stands for the state variables vector of the master system and $\boldsymbol{y_i} = (y_{i1}, y_{i2}, \cdots, y_{in})^T \in \boldsymbol{R}^{n\times 1}$ denotes the state variables vector of the i^{th} slave system. $\boldsymbol{\Phi_i}(t) = (\phi_{i1}(t), \phi_{i2}(t), \cdots, \phi_{in}(t)) \in \boldsymbol{R}^{n\times 1}$ for $i = 1, 2, \cdots, n$ denotes the uncertainty function of the system parameters vectors $\boldsymbol{\Phi}_c = (\phi_{1c}, \phi_{2c}, \cdots, \phi_{nc}) \in \boldsymbol{R}^{n\times 1}$. $f(\boldsymbol{x}, t), g_i(\boldsymbol{y_i}, t) \in$

$\boldsymbol{R}^{n\times n}$ and $F(\boldsymbol{x},t), G_i(\boldsymbol{y}_i,t) \in \boldsymbol{R}^{n\times 1}$ are some sort of linear and nonlinear matrix of functions, respectively, and $\boldsymbol{u}(t)$ indicates the forcing controller of the slave system to be designed. In the presented HMPS method, an appropriate control law $\boldsymbol{u} = (\boldsymbol{u_1}, \boldsymbol{u_2}, \cdots, \boldsymbol{u_m})$ would be designed in which the states of the leader attractor $\boldsymbol{x_i}$ are synchronized with a proportional combination of the states of follower attractors $\boldsymbol{y_i}$ of system (1). Thus, the HMPS errors can be defined as follows:

$$\boldsymbol{e_j} = x_j - \sum_{k=1}^{n} \lambda_{kj} y_{kj}\,, \qquad \forall j = 1, 2, \cdots, n\,, \tag{2}$$

where $\boldsymbol{\lambda_j} = (\lambda_{j1}, \cdots, \lambda_{jm})$ denotes the modified scaling factor, which is defined in such a way that $\lambda_{j1} + \cdots + \lambda_{jm} = 1$: for all $j = 1, \cdots, n$. Then, the dynamical representation of system errors (2) can be given as follows:

$$\dot{\boldsymbol{e}}_{\boldsymbol{j}} = \dot{x}_j - \sum_{k=1}^{n} \lambda_{kj} \dot{y}_{kj}\,, \qquad \forall j = 1, 2, \cdots, n\,. \tag{3}$$

The main object of this study is to derive an appropriate control law and a parameter estimation law to control the behavior of multiple chaotic systems to asymptotically track the state variables trajectories of a leader chaotic system. In other words, the system errors (2) asymptotically approach zero as time tends to infinity, *i.e.* $\lim_{t\to\infty} |e_j(t)| = 0$. To this end, in the following theorem, some controllers are defined to provide such criteria.

Theorem 2.1. *The chaotic synchronization of the leader–follower attractors (1) with the leader state variables vector $\boldsymbol{x}$, the follower state variables vectors $\boldsymbol{y_i}$, the modified projective scaling factors λ_{ij}, and the system parameters vector $\boldsymbol{\Phi}_c$ will be achieved for every initial state variables $\boldsymbol{x}(0)$ and $\boldsymbol{y}_i(0)$, if the adaptive feedback controller and the parameter estimation law are defined as follows:*

$$\boldsymbol{u_i} = -G_i(\boldsymbol{y_i},t) - g_i(\boldsymbol{y_i},t)\boldsymbol{\Phi}(t) + \sum_{k=1}^{n} \Lambda_i f_{ki}(\boldsymbol{x},t)\phi_{ic}(t) + \frac{1}{n}\Lambda_i F(\boldsymbol{x},t) + K\boldsymbol{e_i}\,, \tag{4}$$

and

$$\dot{\phi}_i(t) = \boldsymbol{e_i} \sum_{k=1}^{n} f_{ki}(\boldsymbol{x},t)\,, \tag{5}$$

where $\Lambda_i = diag\{\frac{1}{\lambda_{i1}}, \frac{1}{\lambda_{i2}}, \cdots, \frac{1}{\lambda_{in}}\}$ and $K = (k_{i1}, k_{i2}, \cdots, k_{in})$ are positive constant coefficients.

Proof. Assume the Lyapunov candidate function here as

$$V = \frac{1}{2}\sum_{i=1}^{n} \boldsymbol{e}_i^2 \,. \tag{6}$$

By differentiating equation (6) with respect to the time domain, one can get

$$\dot{V} = \sum_{i=1}^{n} \boldsymbol{e}_i \dot{\boldsymbol{e}}_i \,. \tag{7}$$

Substituting the presented control law (4) and the parameter estimation equation (5) into equation (7) yields

$$\dot{V} = -\sum_{i=1}^{n} k_i \boldsymbol{e}_i^2 \,, \tag{8}$$

which is negative definite when the constants k_i are positively valued and the proof is complete. Therefore, according to Theorem 2.1., hybrid modified projective synchronization of a typical uncertain master chaotic system with an arbitrary combination of chaotic systems can be achieved by designing an appropriate control law based on equation (4) and parameter estimation law according to equation (5), which guarantees that the synchronization errors in equation (2) asymptotically approach zero. Furthermore, the estimation of master system parameters converges to their unknown true values. □

3. Numerical results

In this section, the results of numerical simulations of chaotic systems are presented to verify the feasibility and the effectiveness of the proposed method.

3.1. Chaos synchronization of gyroscope chaotic system

In this subsection, the problem of the single-master, multi-slave system synchronization between the gyroscope chaotic system [36] and multiple slave systems is presented via designing an appropriate hybrid modified projective feedback controller. Multi-slave HMPS is performed by Lü [37], Chen [38] and Liu [39] as multi-slave chaotic systems.

In [36], a typical structure of a gyroscope chaotic system is introduced. The dynamical representation of gyroscope chaotic system presented in [36] can be rewritten in the following dynamical form:

$$\begin{cases} \dot{x}_1 = x_2\,, \\ \dot{x}_2 = -\alpha\frac{(1-\cos x_1)^2}{\sin^3 x_1} + \beta\sin x_1 - c_1 x_2 - c_2 x_2^3 + f\sin(\omega t)\sin x_1\,, \end{cases} \tag{9}$$

where the state variables x_1 and x_2 represent the value of rotation angle and its dynamical values of gyroscope system, respectively. α, β, c_1, c_2 and f are the unknown parameters of the system. The chaotic representation of gyroscope system (9) is shown in Fig. 1, with system parameters $\alpha = 100, \beta = 1, c_1 = 0.5, c_2 = 0.05, f = 35.5$ and $\omega = 2$, and the initial-state variables as: $x_1 = 6$ and $x_2 = 5.7$.

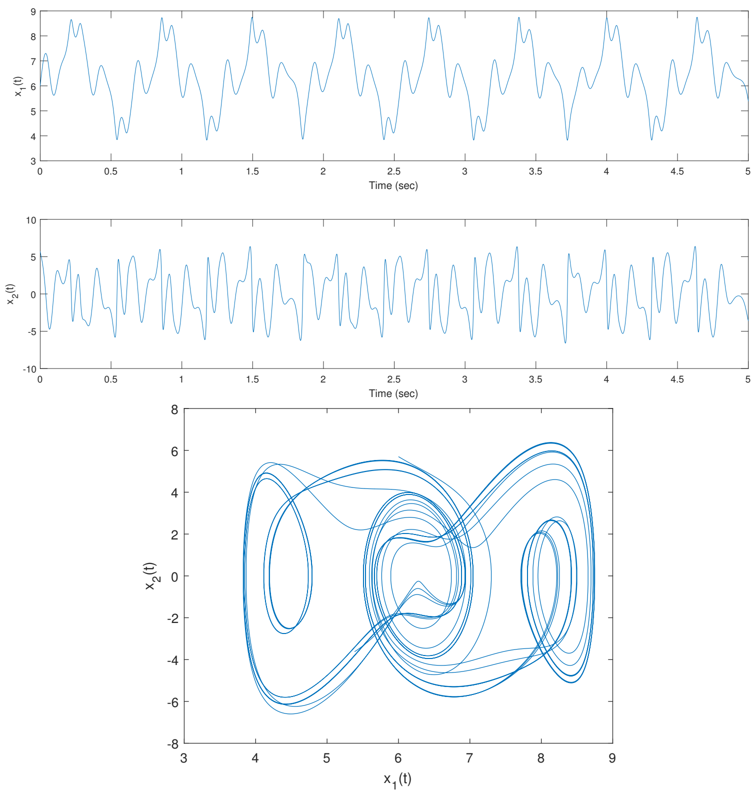

Fig. 1. Phase portraits of the gyroscope chaotic system.

In addition, let us consider the Lü [37], Chen [38] and Liu [39] chaotic systems as three slave chaotic systems as follows:

— Lü chaotic system [37] is considered as the first slave system

$$\boldsymbol{y_1} : \begin{cases} \dot{y_1} = \alpha_1(y_2 - y_1)\,, \\ \dot{y_2} = \alpha_2 y_2 - y_1 y_3\,, \\ \dot{y_3} = y_1 y_2 - \alpha_3 y_3\,, \end{cases} \tag{10}$$

where α_1, α_2 and α_3 are the parameters of the Lü chaotic system as the first slave system. The phase portrait of the Lü chaotic system based on equation (10) is depicted in Fig. 2 with the system parameters: $\alpha_1 = 36, \alpha_2 = 30$, and $\alpha_3 = 20$.

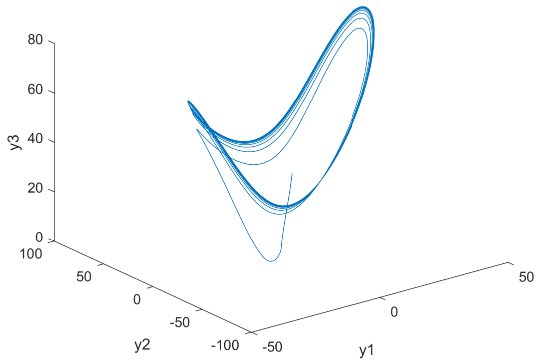

Fig. 2. Time portrait of the Lü chaotic system.

— Chen chaotic system [38] is used as the second slave system

$$\boldsymbol{y_2} : \begin{cases} \dot{y_1} = \beta_1(y_2 - y_1)\,, \\ \dot{y_2} = (\beta_3 - \beta_1)y_1 - y_1y_3 + \beta_3y_2\,, \\ \dot{y_3} = y_1y_2 - \beta_2y_3\,, \end{cases} \tag{11}$$

where β_1, β_2 and β_3 are the parameters of the Chen chaotic system as the second slave system. The phase portrait of the Chen chaotic system according to equation (11) is shown in Fig. 3 with the system parameters: $\beta_1 = 35, \beta_2 = 3$ and $\beta_3 = 28$.

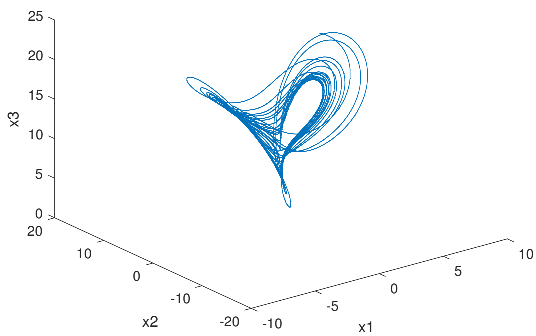

Fig. 3. Time portrait of the Chen chaotic system.

— Liu chaotic system [39] is chosen as the third slave system

$$\boldsymbol{y_3}: \begin{cases} \dot{y_1} = y_3 - \gamma_1 y_1 + y_1 y_2\,, \\ \dot{y_2} = 1 - \gamma_2 y_2 - y_1^2\,, \\ \dot{y_3} = -y_1 - \gamma_3 y_3\,, \end{cases} \tag{12}$$

where γ_1, γ_2 and γ_3 are the parameters of the Chen chaotic system as the third slave system. The phase portrait of the Liu chaotic system (12) is presented in Fig. 4 with the system parameters: $\gamma_1 = 0.2, \gamma_2 = 0.5$ and $\gamma_3 = 0.1$.

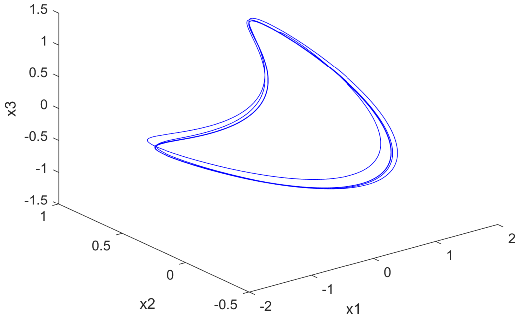

Fig. 4. Time portrait of the Liu chaotic system.

Consider the aforementioned gyroscope chaotic system with uncertainty in its parameters as the master system, and the Lü, Chen and Liu chaotic systems as the multi-slave chaotic systems. Then, the control and parameter estimation laws are derived as equations (13) and (14), respectively

$$\begin{aligned} &\boldsymbol{u_1}: \begin{cases} u_{11} = -\alpha_1(y_2 - y_1) + \frac{1}{3\lambda_{11}} x_2 + k_{11} e_1\,, \\ u_{12} = -\alpha_2 y_2 + y_1 y_3 - \frac{1}{\lambda_{12}} \left(\alpha(t) \frac{(1-\cos x_1)^2}{\sin^3 x_1} - \beta(t) \sin x_1 \right) + k_{21} e_2\,, \end{cases} \\ &\boldsymbol{u_2}: \begin{cases} u_{21} = -\beta_1(y_2 - y_1) + \frac{1}{3\lambda_{21}} x_2 + k_{21} e_1\,, \\ u_{22} = (\beta_1 - \beta_3) y_1 + y_1 y_3 - \phi_{23} y_2 - \frac{1}{\lambda_{22}} \left(c_1(t) x_2 + \hat{c}_2(t) x_2^3 \right) + k_{22} e_2\,, \end{cases} \\ &\boldsymbol{u_3}: \begin{cases} u_{31} = -y_3 + \gamma_1 y_1 - y_1 y_2 + \frac{1}{3\lambda_{31}} x_2 + k_{31} e_1\,, \\ u_{32} = -1 + \gamma_2 y_2 + y_1^2 + \frac{1}{\lambda_{32}} \left(f(t) \sin(\omega t) \sin x_1 \right) + k_{32} e_2\,, \end{cases} \end{aligned} \tag{13}$$

and

$$\begin{cases} \dot{\alpha}(t) = -e_2 \frac{(1-\cos x_1)^2}{\sin^3 x_1}\,, \\ \dot{\beta}(t) = +e_2 \sin x_1\,, \\ \dot{c}_1(t) = -e_2 x_2\,, \\ \dot{c}_2(t) = -e_2 x_2^3\,, \\ \dot{f}(t) = +e_2 \sin \omega t \sin x_1\,, \end{cases} \tag{14}$$

where $\alpha(t), \beta(t), c_1(t), c_2(t)$ and $f(t)$ are the estimation of the master system parameters α, β, c_1, c_2 and f, respectively, and k_{ij} are positive constants for all $i, j = 1, 2, 3$. The numerical simulations of the proposed method are carried out with considering the $\lambda_{ij} = 0.5$ and $k_{ij} = 2$ for all $i, j = 1, 2, 3$. The constant parameters of the master system (9) are taken as: $\alpha = 80, \beta = 2, c_1 = 0.07, c_2 = 0.6$ and $f = 28$. In addition, the constant values of the slave chaotic systems, equations (10), (11) and (12) are chosen as: $\alpha_1 = 36, \alpha_2 = 30, \alpha_3 = 20$, $\beta_1 = 35, \beta_2 = 3, \beta_3 = 28$ and $\gamma_1 = 0.2, \gamma_2 = 0.5, \gamma_3 = 0.1$, respectively. Finally, the initial values of the estimations of system parameters are: $\alpha(t) = 70, \beta(t) = 1.5, c_1(t) = 0.04, c_2(t) = 0.2$ and $f(t) = 20$.

The simulation is performed by assuming the initial system state values as: $x_1(0) = 2$, $x_2(0) = 3$ and $x_3(0) = 25$, for master chaotic system in equation (9) and $y_{11}(0) = 8, y_{12}(0) = 14, y_{13}(0) = 3$ and $y_{21}(0) = 12, y_{22}(0) = 6$, $y_{23}(0) = 4$ and $y_{31}(0) = 10, y_{32}(0) = 16, y_{33}(0) = 2$, for slaves chaotic systems in equations (10), (11) and (12), respectively. The numerical simulation result of the synchronization scheme is shown in Fig. 5.

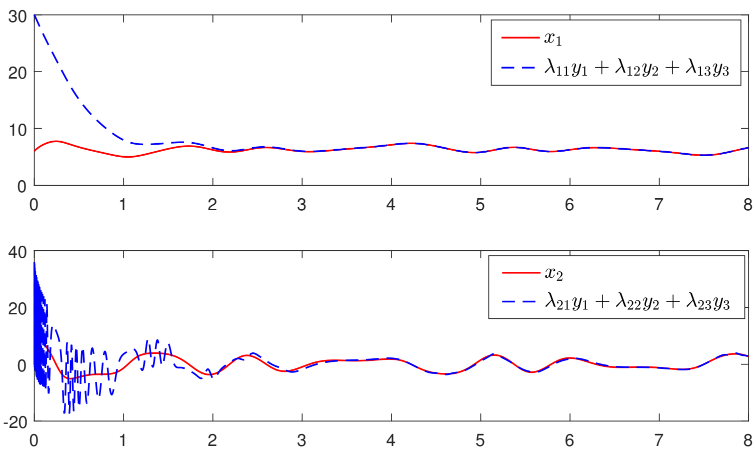

Fig. 5. Chaos synchronization of gyroscope chaotic system with multi-slave systems.

3.2. Chaotic example

In this part, adaptive hybrid modified projective synchronization between the Lorenz chaotic system [40] as master system, the Lü [37], Chen [38] and Liu [39] chaotic systems as three slave chaotic systems are presented. The Lorenz chaotic system is as follows:

$$\boldsymbol{x}: \begin{cases} \dot{x}_1 = -\phi_1 x_1 + \phi_1 x_2\,, \\ \dot{x}_2 = \phi_2 x_1 - x_2 - x_1 x_3\,, \\ \dot{x}_3 = x_1 x_2 - \phi_3 x_3\,, \end{cases} \tag{15}$$

where x_1, x_2 and x_3 are the state variables of the system, and ϕ_1, ϕ_2 and ϕ_3 are the three constant and unknown parameters of the system, which have to be estimated beside synchronization. The chaotic behavior of the system is shown in Fig. 6, with the system parameters: $\phi_1 = 11, \phi_2 = 27, \phi_3 = 2.7$.

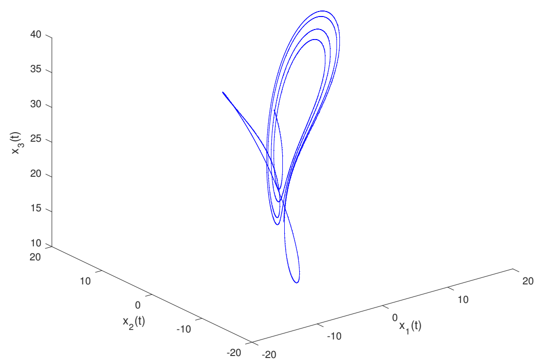

Fig. 6. Time portrait of the Lorenz chaotic system.

Furthermore, if the Lü, Chen and Liu chaotic systems are the multi-slave chaotic systems, then the control and the parameter estimation laws can be achieved based on the general proposed controller in equation (4) and system parameter estimation in equation (5) as follows:

$$\begin{aligned}
&\boldsymbol{u_1}: \begin{cases} u_{11} = -\alpha_1(y_2 - y_1) - \frac{1}{\lambda_{11}}\left((x_2 - x_1)\phi_1(t)\right) + k_{11}e_1\,, \\ u_{12} = -\alpha_2 y_2 + y_1 y_3 + \frac{1}{3\lambda_{12}}\left(-x_2 - x_1 x_3\right) + k_{21}e_2\,, \\ u_{13} = -y_1 y_2 + \alpha_3 y_3 + \frac{1}{3\lambda_{13}}\left(x_1 x_2\right) + k_{13}e_3\,, \end{cases} \\
&\boldsymbol{u_2}: \begin{cases} u_{21} = -\beta_1(y_2 - y_1) + k_{21}e_1\,, \\ u_{22} = (\beta_1 - \beta_3)y_1 + y_1 y_3 - \phi_{23} y_2 - \frac{1}{\lambda_{22}}\left(\phi_2(t)x_1 + \frac{1}{3}(x_2 + x_1 x_3)\right) + k_{22}e_2\,, \\ u_{23} = -y_1 y_2 + \beta_2 y_3 + \frac{1}{3\lambda_{23}}\left(x_1 x_2\right) + k_{23}e_3\,, \end{cases} \\
&\boldsymbol{u_3}: \begin{cases} u_{31} = -y_3 + \gamma_1 y_1 - y_1 y_2 + k_{31}e_1\,, \\ u_{32} = -1 + \gamma_2 y_2 + y_1^2 + \frac{1}{3\lambda_{32}}\left(-x_2 - x_1 x_3\right) + k_{32}e_2\,, \\ u_{33} = +y_1 + \gamma_3 y_3 - \frac{1}{\lambda_{33}}\left(-\phi_3(t)x_3 - \frac{1}{3}x_1 x_2\right) + k_{33}e_3\,, \end{cases}
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
\dot{\phi}_1(t) &= -(x_2 - x_1)e_1\,, \\
\dot{\phi}_2(t) &= -x_1 e_2\,, \\
\dot{\phi}_3(t) &= x_3 e_3\,,
\end{aligned} \tag{17}$$

where $\phi_1(t), \phi_2(t)$ and $\phi_3(t)$ are the estimation of the master system parameters ϕ_{1c}, ϕ_{2c} and ϕ_{3c}, respectively, and k_{ij} are positive constants for all $i, j = 1, 2, 3$. For numerical simulations, it was assumed that the $\lambda_{ij} = 0.5$ and $k_{ij} = 2$ for all $i, j = 1, 2, 3$, $\phi_{1c} = 11, \phi_{2c} = 27$ and $\phi_{3c} = 2.7$. In addition, the constant values of the slave chaotic systems in equations (10), (11) and (12) are chosen as: $\alpha_1 = 36, \alpha_2 = 30, \alpha_3 = 20$, $\beta_1 = 35, \beta_2 = 3, \beta_3 = 28$ and $\gamma_1 = 0.2, \gamma_2 = 0.5, \gamma_3 = 0.1$, respectively. The initial values of the estimations of system parameters are: $\phi_1(0) = 0.4, \phi_2(0) = 0.1$ and $\phi_3(0) = 0.3$.

The simulation is performed by considering the initial values of the system state variables as: $x_1(0) = 2, x_2(0) = 3$ and $x_3(0) = 25$, for master chaotic system in equation (15) and $y_{11}(0) = 8, y_{12}(0) = 14, y_{13}(0) = 3$, and $y_{21}(0) = 12, y_{22}(0) = 6, y_{23}(0) = 4$ and $y_{31}(0) = 10, y_{32}(0) = 16, y_{33}(0) = 2$, for slave chaotic systems in equations (10), (11) and (12), respectively. The obtained synchronization scheme is shown in Fig. 7. In addition, the amount of disparity between unknown master system parameters and their corresponding estimated values are depicted in Fig. 8. The simulation results confirm the effectiveness of the proposed synchronization scheme.

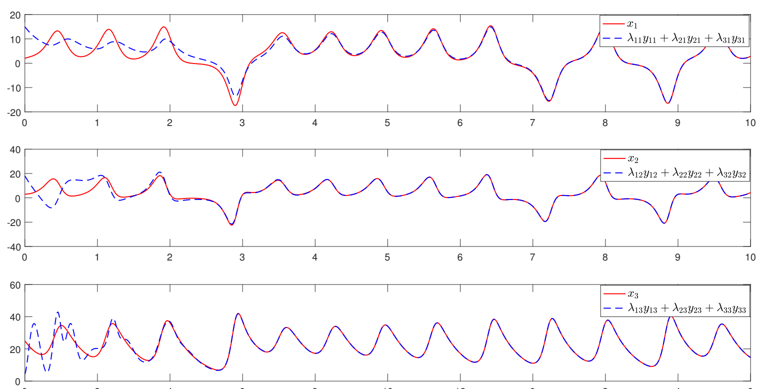

Fig. 7. Time portrait of the master–slaves systems state values along the time domain.

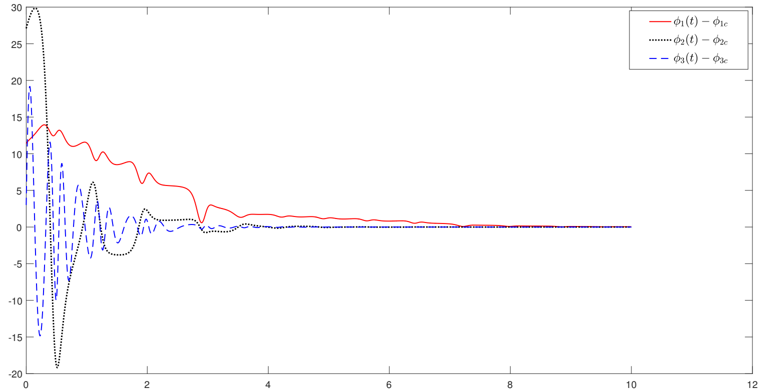

Fig. 8. Time portrait of the parameter estimation errors along the time domain.

4. Conclusion

This paper has focused on synchronization of a chaotic system as a master system with multiple chaotic systems as slave systems. To this end, a typical class of chaotic system has been considered for either master system or slave systems, then complete synchronization has been carried out with a hybrid modified projective synchronization scheme. Control and parameter estimation laws have been derived to achieve such a synchronization goal. The validity of the proposed controller has been verified by means of the Lyapunov stability theorem. Furthermore, the numerical simulation results have also confirmed the effectiveness of the theoretical discussions.

REFERENCES

[1] Z. Li, D. Xu, *Chaos, Solitons Fract.* **22**, 477 (2004).

[2] X. Wu, H. Wang, H. Lu, *Nonlinear Anal. Real World Appl.* **13**, 1441 (2012).

[3] J. Chen, L. Jiao, J. Wu, X. Wang, *Nonlinear Anal. Real World Appl.* **11**, 3045 (2010).

[4] L.M. Pecora, T.L. Carroll, *Phys. Rev. Lett.* **64**, 821 (1990).

[5] J. Lei, X. Wang, Y. Lei, *Phys. Lett. A* **373**, 1249 (2009).

[6] M. Yassen, *Chaos, Solitons Fract.* **23**, 131 (2005).

[7] J. Lu, X. Wu, X. Han, J. Lü, *Phys. Lett. A* **329**, 327 (2004).

[8] J.H. Park, *Chaos, Solitons Fract.* **26**, 959 (2005).

[9] A.L. Fradkov, A.Y. Markov, *IEEE Trans. Circuits Syst.* **44**, 905 (1997).

[10] H.-T. Yau, *Chaos, Solitons Fract.* **22**, 341 (2004).

[11] N. Cai, Y. Jing, S. Zhang, *Commun. Nonlinear Sci. Numer. Simul.* **15**, 1613 (2010).

[12] D. Chen, R. Zhang, X. Ma, S. Liu, *Nonlinear Dynam.* **69**, 35 (2012).

[13] L. Run-Zi, *Phys. Lett. A* **372**, 648 (2008).

[14] B. Liu, X. Liu, G. Chen, H. Wang, *IEEE Trans. Circuits Syst.* **52**, 1431 (2005).

[15] Y. Tang, J.-a. Fang, *Phys. Lett. A* **372**, 1816 (2008).

[16] Y. Chen, X. Li, *Int. J. Mod. Phys. C* **18**, 883 (2007).

[17] J. Yan, C. Li, *Chaos, Solitons Fract.* **26**, 1119 (2005).

[18] H. Tirandaz, A. Hajipour, *Optik* **130**, 543 (2017).

[19] G.-H. Li, *Chaos, Solitons Fract.* **32**, 1786 (2007).

[20] Y. Yu, H.-X. Li, *Nonlinear Anal. Real World Appl.* **11**, 2456 (2010).

[21] H. Du, Q. Zeng, C. Wang, *Phys. Lett. A* **372**, 5402 (2008).

[22] P. Sarasu, V. Sundarapandian, *Int. J. Soft Comput.* **6**, 216 (2011).

[23] T.H. Lee, J.H. Park, *Chin. Phys. Lett.* **26**, 090507 (2009).

[24] K. Kosuge, J. Ishikawa, *Robot. Auton. Syst.* **12**, 95 (1994).

[25] K. Kosuge, J. Ishikawa, K. Furuta, M. Sakai, *Control of Single-master Multi-slave Manipulator System Using VIM*, in: *Robotics and Automation*, 1990, Proc. IEEE Int. Conf., pp. 1172–1177.

[26] H. Mylapilli, *J. Aerospace Eng.* **26**, 814 (2011).

[27] H. Salarieh, M. Shahrokhi, *J. Comput. Appl. Math.* **223**, 842 (2009).

[28] L. Runzi, W. Yinglan, D. Shucheng, *Chaos* **21**, 043114 (2011).

[29] K. Ojo, A. Njah, S. Ogunjo, O. Olusola, *Nonlinear Dyn. Syst. Theory* **14**, 119 (2014).

[30] J. Sun, J. Fang, Y. Wang, G. Cui, *Optik* **127**, 9504 (2016).

[31] X. Zhou, M. Jiang, Y. Huang, *Entropy* **15**, 3746 (2013).

[32] X. Zhou, L. Xiong, X. Cai, *Combination–Combination Synchronization of Four Nonlinear Complex Chaotic Systems*, in: *Abstract and Applied Analysis*, Vol. 2014, Hindawi Publishing Corporation, 2014.

[33] J. Sun, Q. Yin, Y. Shen, *Europhys. Lett.* **106**, 40005 (2014).

[34] J. Sun *et al.*, *Optik* **127**, 4136 (2016).

[35] J. Sun, S. Jiang, G. Cui, Y. Wang, *J. Comput. Nonlinear Dyn.* **11**, 034501 (2016).

[36] H.-K. Chen, T.-N. Lin, *J. Mech. Eng. Sci.* **217**, 331 (2003).

[37] J.H. Lü, G. C, *Int. J. Bifurcat. Chaos* **12**, 659 (2002).

[38] G. Chen, T. Ueta, *Int. J. Bifurcat. Chaos* **9**, 1465 (1999).

[39] C. Liu, T. Liu, L. Liu, K. Liu, *Chaos, Solitons Fract.* **22**, 1031 (2004).

[40] Y. Niu, X. Wang, *Int. J. Mod. Phys. B* **26**, 1250059 (2012).