Alexandria Engineering Journal (2018) 57, 1519–1524

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Alexandria Engineering Journal

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ORIGINAL ARTICLE

Chaos synchronization and parameter identification of a finance chaotic system with unknown parameters, a linear feedback controller

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Received 31 December 2016; revised 11 March 2017; accepted 29 March 2017
Available online 29 April 2017

KEYWORDS

Chaos synchronization;
Linear feedback controller;
Finance chaotic system;
Parameter identification

Abstract In this research study, an adaptive linear feedback controller is presented for controlling the behavior of a financial chaotic system and identical/non-identical synchronization with unknown system parameters. An adaptive linear feedback controller is introduced based on the Lyapunov stability theorem, which is added to the nonlinear chaotic systems to achieve synchronization. Furthermore, the disparity amount of system parameters is estimated simultaneously. Identical and non-identical synchronizations are followed by some numerical simulations to verify the validity of the proposed method. The results show the effectiveness of the theoretical discussions.

1. Introduction

Since the pioneer work by Pecora and Carroll [1], control and synchronization of chaotic systems has received great attention due to their potential real world applications in engineering, physics, chemistry, control theory and secure communications, up to now, many chaotic systems have been developed and studied by the researchers. Lü chaotic system [2], Chen chaotic system [3], Lorenz chaotic system [4], Genesio chaotic system [5], Liu chaotic system [6], supply chain chaotic system [7], economic and finance chaotic systems [8,9] are some of them. Among these chaotic systems, the economics and finance chaotic systems, which were discovered in 1985, have great impact on prominent economics at present. Since the economic crisis, the role of chaos has great attention, which shows the existence of butterfly effect and chaos in the finance system.

During the past two decades, many control and synchronization methods have been introduced to synchronize (hyper)chaotic systems. Active method [10,11], adaptive method [12–14], projective method [15,16], phase synchronization [17], lag synchronization [18], backstepping method [19], sliding mode method [20–22], state feedback control [23], nonlinear feedback control [24,25] and linear feedback controller [26,27] are some of the investigated methods. The majority of the aforementioned synchronization methods provide a nonlinear feedback controller to control the behavior of chaotic systems. Undoubtedly, the linear feedback controller is easier to be implemented than other nonlinear methods. Linear feedback control has been successfully utilized for the control by Chua in [28], Lorenz [29], Liu [30] and so many others. Therefore, the concentration of this paper is to design a linear

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2017.03.041
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feedback controller to synchronize the behavior of a finance chaotic system with identical or non-identical synchronization.

The reminder of this paper is organized as follows: Section 2 presents the chaos synchronization between a class of chaotic systems via linear feedback controller. Then, identical and non-identical synchronization of a finance chaotic system is addressed in Section 3, followed by some numerical simulations. Finally, some concluding remarks are given in Section 4.

2. Synchronization

In this section, adaptive synchronization of a finance chaotic system is performed via designing a linear feedback controller. During this section, the parameters of the leader chaotic system are considered unknown and are identified with a parameter estimation law, which is designed based on the proposed linear feedback controller.

Consider a class of (hyper) chaotic systems as follows:

$$\dot{\mathbf{x}} = \mathbf{A}f(\mathbf{x}) + F(\mathbf{x}) \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ is the state variables vector of the system, $f(\mathbf{x}) \in \mathbf{R}^{n\times 1}$ and $F(\mathbf{x}) \in \mathbf{R}^{n\times n}$ are the linear and nonlinear functions of system state variables $\mathbf{x}$, respectively. $\mathbf{A} = diag\{\alpha_1, \alpha_2, \ldots, \alpha_n\} \in \mathbf{R}^{n\times n}$, which $\alpha_1, \ldots, \alpha_n$ are parameters of the system.

Assume the dynamical system (1), as the leader system. Then the identical follower (hyper)chaotic system can be given as follows:

$$\dot{\mathbf{y}} = (\mathbf{A} + \mathbf{\Delta A})f(\mathbf{y}) + F(\mathbf{y}) + \mathbf{u} \tag{2}$$

where y_1, y_2 and y_3 are the state variables of the follower chaotic system (2). $\mathbf{\Delta A} = diag\{\Delta\alpha_1, \Delta\alpha_2, \ldots, \Delta\alpha_n\}$ represents the disparity amount of the system parameter vector $\mathbf{A}$. $\mathbf{u} = (u_1, u_2, \ldots, u_n)^T$ indicates the linear feedback controller vector, which has to be designed to force the motion trajectories of the follower chaotic system state variables (2) to track the trajectories of the state variables of the leader chaotic system (2). Let the error vector be as follows:

$$\mathbf{e} = \mathbf{y} - \mathbf{x} \tag{3}$$

where $\mathbf{e} = (e_1, e_2, e_3)^T$ denotes the system errors vector. Then, the dynamical representation of system errors can be obtained by $\dot{\mathbf{e}} = \dot{\mathbf{y}} - \dot{\mathbf{x}}$, which can be described by considering the leader finance chaotic system (1) and the follower chaotic system (2) as follows:

$$\dot{\mathbf{e}} = (\mathbf{A} + \mathbf{\Delta A})f(\mathbf{y}) + F(\mathbf{y}) + \mathbf{u} - \mathbf{A}f(\mathbf{x}) - F(\mathbf{x}) \tag{4}$$

Then we have the following definition, which represents the ultimate goal of synchronization via designing an appropriate linear feedback controller.

Definition 1. For the leader chaotic system (1) and the follower chaotic system (2), if there exists a linear feedback controller $\mathbf{u}$, which makes the following limit true. Then, it is been said that the chaos synchronization between the leader chaotic system (1) and the follower chaotic system (2) can be achieved.

Assumption 1. The nonlinear function vector $F(\mathbf{x})$ would satisfies to the Lipschitz condition as:

$$\|F(\mathbf{y}) - F(\mathbf{x})\| < L\|\mathbf{y} - \mathbf{x}\| \tag{5}$$

where L is a Lipschitz constant and $\|.\|$ denotes the 2-norm. In the following theorem, an appropriate feedback controller is introduced to satisfy the definition. 1 requirements.

Remark 1. Due to the boundedness of a chaotic system, the nonlinear part $F(.)$ of a chaotic system is bounded too. Then, assumption. 1 is reasonable.

Theorem 1. *The state variables trajectories of the leader chaotic system (1) will be tracked by the follower chaotic system (2), for any initial state variables* $\mathbf{x}(0)$ *and* $\mathbf{y}(0)$, *if the adaptive control law is designed based on the following linear feedback controller as follows*:

$$\mathbf{u} = -(\mathbf{A} + \mathbf{\Delta A})(f(\mathbf{y}) - f(\mathbf{x})) - Ke \tag{6}$$

where K *is a positive constant. In addition, the parameter identification can be described as:*

$$\dot{\mathbf{\Delta A}} = e^T f(\mathbf{x}) - \mathbf{\Psi}^T(\mathbf{\Delta A}) \tag{7}$$

which $\mathbf{\Psi} = (\psi_1, \psi_2, \ldots, \psi_n)$ *is a constant vector with positive constant elements.*

Proof. Consider the Lyapunov stability function as follows:

$$V = \frac{1}{2}\left(\mathbf{e}^T\mathbf{e} + (\mathbf{\Delta A})^T(\mathbf{\Delta A}) + \mathrm{L}^T\mathrm{L}\right)^2 \tag{8}$$

Which V is positive definite. Then, the dynamical representation of V can be obtained as follows:

$$\begin{aligned}\dot{V} &= \mathbf{e}^T\dot{\mathbf{e}} + (\mathbf{\Delta A})^T\dot{\mathbf{\Delta A}} + \mathrm{L}^T\,\dot{\mathrm{L}} \\ &= \mathbf{e}^T\{(\mathbf{A} + \mathbf{\Delta A})g(\mathbf{y}) + G(\mathbf{y}) + \mathbf{u} - \mathbf{A}f(\mathbf{x}) - F(\mathbf{x})\} \\ &\quad + (\mathbf{\Delta A})^T\dot{\mathbf{\Delta A}} \\ &\leqslant -K\mathbf{e}^T\mathbf{e} - \mathbf{\Psi}(\mathbf{\Delta A})^T(\mathbf{\Delta A}) + L\|\boldsymbol{y} - \boldsymbol{x}\| + \mathrm{L}^T\,\dot{\mathrm{L}}\end{aligned} \tag{9}$$

Considering the feedback controller presented in (6), the dynamical system errors presented in (7), and Lipschitz constant L as $\dot{\mathrm{L}} = -\boldsymbol{e}^T\|\boldsymbol{y} - \boldsymbol{x}\|$, the derivative of V in Eq. (9), will be described as follows:

$$\dot{V} \leqslant -K\mathbf{e}^T\mathbf{e} - \mathbf{\Psi}(\mathbf{\Delta A})^T(\mathbf{\Delta A}) \tag{10}$$

Then, it is clear that $\dot{V}$ is negative definite. Then adaptive synchronization between the leader chaotic system (1) and the follower chaotic system (2) will be achieved by the proposed linear feedback controller (6) and the parameter estimation (7). So the theorem is proved. In the following section, the effectiveness of the proposed linear feedback controller method is verified numerically by identical synchronization of a finance chaotic system. □

3. Synchronization of finance chaotic system

Generally, a wide variety of chaotic systems can be represented in the form of (1). For example, consider the finance chaotic system presented in [31] as follows:

$$\begin{aligned}\dot{x}_1 &= x_3 + (x_2 - a)x_1 \\ \dot{x}_2 &= 1 - bx_2 - x_1^2 \\ \dot{x}_3 &= -x_1 - cx_3\end{aligned} \tag{11}$$

which $\dot{x}_1, \dot{x}_2$ and $\dot{x}_3$ are the dynamical representations of the state variables x_1 as the interest rate, x_2, as the investment demand, and x_3, as the price exponent, respectively. The system parameter a is the saving amount, b denotes the investment cost and c indicates the elasticity of the demands of commercials. The parameters of the system are considered positive constants. The phase portraits of the finance system (11) are shown in Fig. 1, with system parameters $a = 0.8, b = 0.2$ and $c = 1.9$, and initial values of state variables of system as $x_1(0) = -0.2, x_2(0) = 1.5, x_3(0) = 0.3$.

3.1. Identical synchronization

Consider the finance chaotic system in (11) as the leader system. Then, the follower chaotic system can be given as follows:

$$\begin{aligned}\dot{y}_1 &= y_3 + (y_2 - (a + \Delta a))y_1 + u_1 \\ \dot{y}_2 &= 1 - (b + \Delta b)y_2 - y_1^2 + u_2 \\ \dot{y}_3 &= -y_1 - (c + \Delta c)y_3 + u_3\end{aligned} \tag{12}$$

where y_1, y_2 and y_3 are the state variables of the follower chaotic system. $\Delta a, \Delta b$ and Δc are the disparity amounts of system parameters a, b and c, respectively. u_1, u_2 and u_3 are the feedback controllers related to the state variables y_1, y_2 and y_3, respectively, which can be obtained based on the designed control law in (6) as follows:

$$\begin{aligned}u_1 &= -y_3 + (a + \Delta a)y_1 + x_3 - (a + \Delta a)x_1 - k_1e_1 \\ u_2 &= -1 + (b + \Delta b)y_2 - (b + \Delta b)x_2 - k_2e_2 \\ u_3 &= +y_1 + (c + \Delta c)y_3 - x_1 - (c + \Delta c)x_3 - k_3e_3\end{aligned} \tag{13}$$

where k_1, k_2 and k_3 are the three positive constants which are defined based on the bond state variables x_1, x_2 and x_3, respectively. In addition, the system parameters can be estimated based on the following equation:

$$\begin{aligned}\dot{\Delta}a &= e_1x_1 - \psi_1(\Delta a) \\ \dot{\Delta}b &= e_2x_2 - \psi_2(\Delta b) \\ \dot{\Delta}c &= e_3x_3 - \psi_3(\Delta c)\end{aligned} \tag{14}$$

where ψ_1, ψ_2 and ψ_3 are the positive constants.

In the following part of this section, the chaos synchronization of the identical finance chaotic system is described by some numerical simulations.

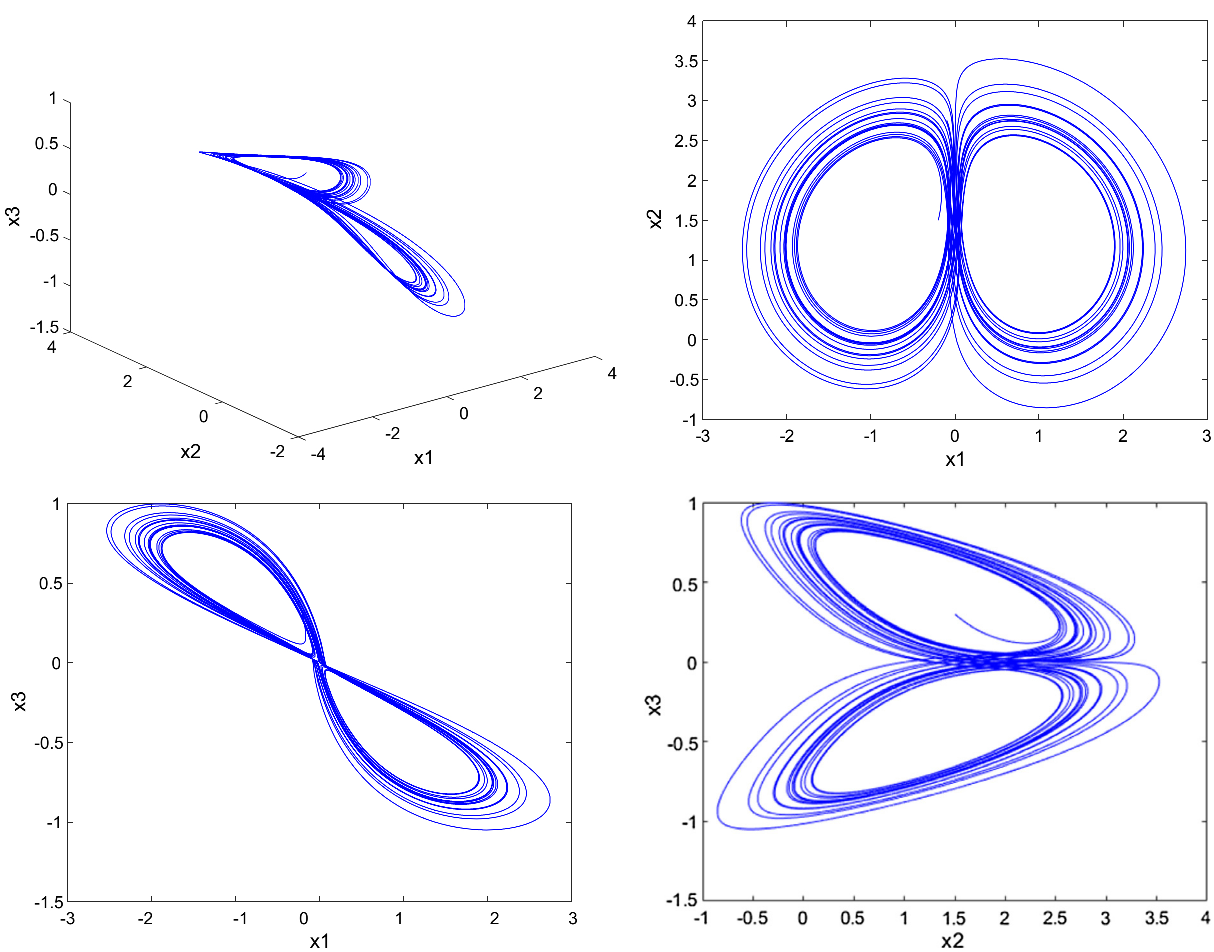

Figure 1 Some chaotic attractors of the finance chaotic system presented at Eq. (11).

For simulation purpose, the initial values of the leader and the follower chaotic systems are chosen as $\mathbf{x}(0) = (x_1(0), x_2(0), x_3(0)) = (2,6,4)$ and $\mathbf{y}(0) = (y_1(0), y_2(0), y_3(0)) = (7,1,0)$, respectively. The values of the unknown parameters are selected as $a = 0.8, b = 0.2$ and $c = 1.9$. In addition, the initial estimation of the system parameters is taken as $\Delta a = 0.5$, $\Delta b = 0.3$ and $\Delta c = 0.7$. The constants are chosen as $k_1 = 2$, $k_2 = 2, k_3 = 2, \phi_1 = 1.5, \phi_2 = 1.5$ and $\phi_3 = 1.5$. The simulation results obtained from the leader and the follower finance chaotic systems (11) and (12) are shown in Figs. 2 and 3. Fig. 2 depicts the motion trajectories of the leader and the follower chaotic system. Furthermore, the disparity amount of the parameter estimation is illustrated in Fig. 3. As it can be seen from these results, the synchronization of the leader chaotic system (11) and the follower chaotic system (12) is realized with the controller law (13) and the parameter estimation (14).

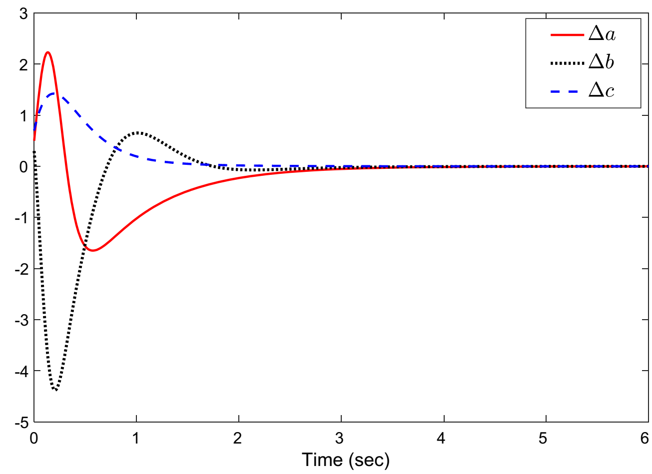

Figure 3 Error rate of parameter estimation from identical synchronization of finance chaotic system along the time.

3.2. Non-identical synchronization

Now, the non-identical synchronization of the finance chaotic system (11) is addressed in this subsection. Consider the finance chaotic system (11) as the leader system. Then, the non-identical follower chaotic system can be given as follows:

$$\begin{aligned} \dot{y}_1 &= (a + \Delta a)(y_2 - y_1) + u_1 \\ \dot{y}_2 &= (b + \Delta b)y_1 - y_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= y_1 y_2 - (c + \Delta c)y_3 + u_3 \end{aligned} \tag{15}$$

where y_1, y_2 and y_3 are the state variables of the follower chaotic system. $\Delta a, \Delta b$ and Δc are the disparity amount of system parameters a, b and c, respectively. u_1, u_2 and u_3 are the feedback controllers which can be presented as same as Eq. (13) based on the derived control law in (6). In addition, the parameter estimations can be given as same as Eq. (14). Then, the non-identical synchronization of finance chaotic system can be represented by some numerical simulations.

For simulation purpose, the initial values of the leader and the follower chaotic systems are chosen as $\mathbf{x}(0) = (x_1(0), x_2(0), x_3(0)) = (11, 7, 9)$ and $\mathbf{y}(0) = (y_1(0), y_2(0), y_3(0)) = (1, -2, 0)$, respectively. The values of the unknown parameters are selected as $a = 0.6, b = 0.3$ and $c = 1.7$. In addition, the initial estimation of the system parameters is taken as $\Delta a = 0.4$, $\Delta b = 0.5$ and $\Delta c = 0.6$. The constants are chosen as $k_1 = 2$, $k_2 = 2, k_3 = 2, \phi_1 = 1.5, \phi_2 = 1.5$ and $\phi_3 = 1.5$. The simulation results obtained from the non-identical leader and the follower finance chaotic systems are shown in Figs. 4 and 5. Fig. 4 depicts the motion trajectories of the leader and the follower chaotic system. Furthermore, the disparity amount of the parameter estimation is illustrated in Fig. 5. As it can be seen from these results, the synchronization of the leader chaotic system (11) and the follower chaotic system (15) is realized with the controller law (13) and the parameter estimation (14).

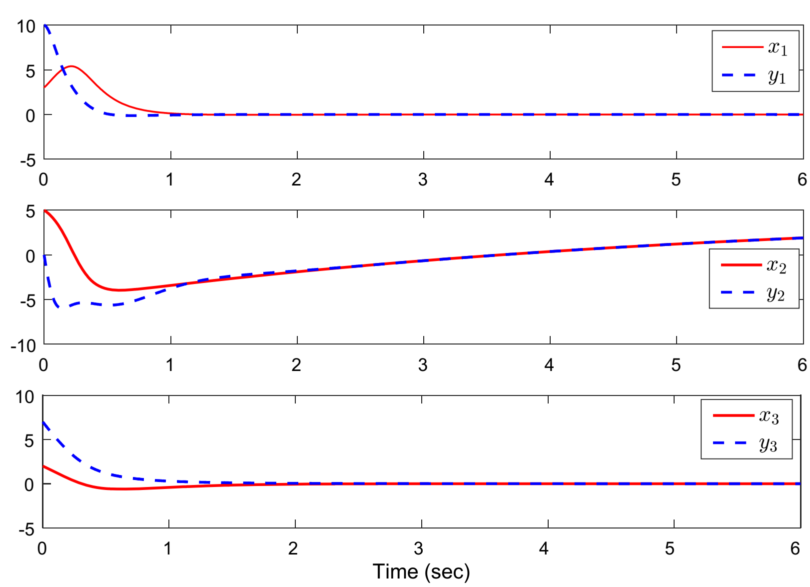

Figure 2 Motion trajectories of the state variables of the finance chaotic system obtained by identical synchronization along the time.

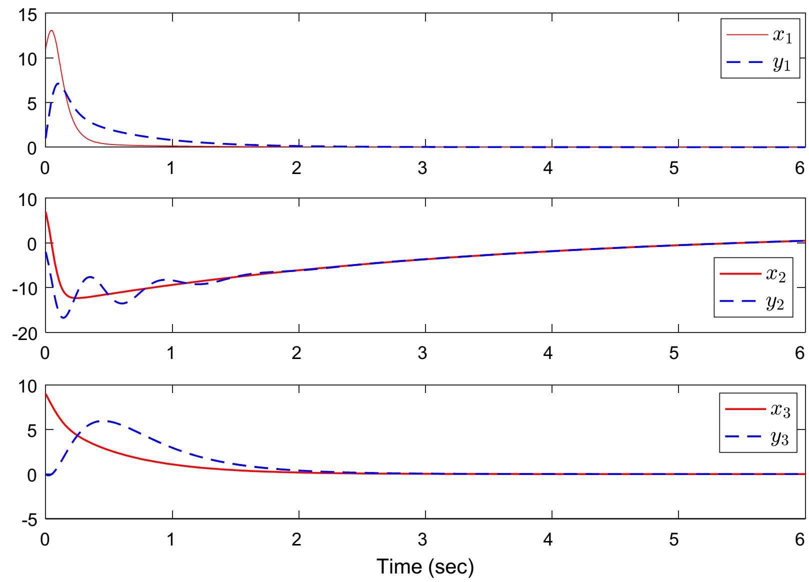

Figure 4 Motion trajectories of the state variables of the finance chaotic system obtained by non-identical synchronization along the time.

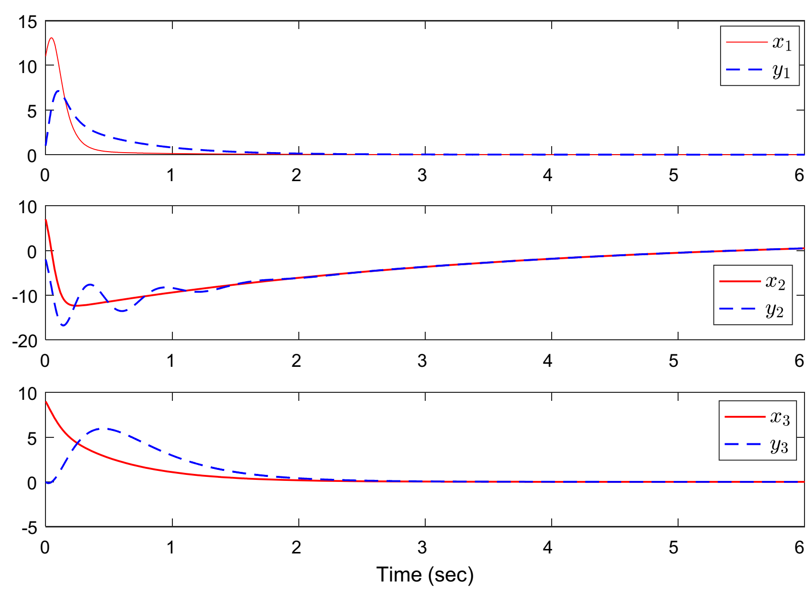

Figure 5 Error rate of parameter estimation from non-identical synchronization of finance chaotic system along the time.

4. Conclusion

In the present study, a new method for adaptive synchronization of a class of chaotic systems is presented. On the basis of the adaptive control theory and Lyapunov stability function, a new feedback linear control law and a new parameter estimation law are designed. Then, the proposed linear feedback controller scheme is evaluated by identical synchronization of the finance chaotic system with unknown system parameters. Moreover, some simulations are carried out to verify the validity of the proposed scheme. The numerical simulations show the effectiveness of the proposed linear feedback controller scheme from both speed and accuracy points of view.

References

[1] L.M. Pecora, T.L. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. 64 (8) (1990) 821.
[2] J. Lü, G. Chen, S. Zhang, The compound structure of a new chaotic attractor, Chaos, Solitons Fractals 14 (5) (2002) 669–672.
[3] G. Chen, T. Ueta, Yet another chaotic attractor, Int. J. Bifurcat. Chaos 9 (07) (1999) 1465–1466.
[4] Y. Zeng, S.N. Singh, Adaptive control of chaos in Lorenz system, Dyn. Control 7 (2) (1997) 143–154.
[5] R. Genesio, A. Tesi, Harmonic balance methods for the analysis of chaotic dynamics in nonlinear systems, Automatica 28 (3) (1992) 531–548.
[6] C. Liu, T. Liu, L. Liu, K. Liu, A new chaotic attractor, Chaos, Solitons Fractals 22 (5) (2004) 1031–1038.

[7] S.E. Fawcett, M.A. Waller, Making sense out of chaos: why theory is relevant to supply chain research, J. Bus. Logist. 32 (1) (2011) 1–5.
[8] I. Evstigneev, M. Taksar, Dynamic interaction models of economic equilibrium, J. Econ. Dynam. Control 33 (1) (2009) 166–182.
[9] Q. Gao, J. Ma, Chaos and hopf bifurcation of a finance system, Nonlinear Dyn. 58 (1–2) (2009) 209–216.
[10] H. Agiza, M. Yassen, Synchronization of rossler and chen chaotic dynamical systems using active control, Phys. Lett. A 278 (4) (2001) 191–197.
[11] R.-A. Tang, Y.-L. Liu, J.-K. Xue, An extended active control for chaos synchronization, Phys. Lett. A 373 (16) (2009) 1449–1454.
[12] J. Sun, Y. Shen, Adaptive anti-synchronization of chaotic complex systems and chaotic real systems with unknown parameters, J. Vib. Control (2014), 1077546314557852.
[13] X.-F. Li, A.C.-S. Leung, X.-J. Liu, X.-P. Han, Y.-D. Chu, Adaptive synchronization of identical chaotic and hyper-chaotic systems with uncertain parameters, Nonlinear Anal. Real World Appl. 11 (4) (2010) 2215–2223.
[14] H. Tirandaz, A. Hajipour, Adaptive synchronization and anti-synchronization of TSUCS and lü unified chaotic systems with unknown parameters, Optik-Int. J. Light Electron Opt. (2017).
[15] J. Sun, Y. Shen, G. Zhang, Transmission projective synchronization of multi-systems with non-delayed and delayed coupling via impulsive control, Chaos Interdiscipl. J. Nonlinear Sci. 22 (4) (2012) 043107.
[16] M. Hu, Z. Xu, Adaptive feedback controller for projective synchronization, Nonlinear Anal. Real World Appl. 9 (3) (2008) 1253–1260.
[17] F. Dörfler, F. Bullo, Synchronization in complex networks of phase oscillators: a survey, Automatica 50 (6) (2014) 1539–1564.
[18] E. Shahverdiev, S. Sivaprakasam, K. Shore, Lag synchronization in time-delayed systems, Phys. Lett. A 292 (6) (2002) 320–324.
[19] Y. Yu, S. Zhang, Adaptive backstepping synchronization of uncertain chaotic system, Chaos, Solitons Fractals 21 (3) (2004) 643–649.
[20] H. Wang, X.-L. Zhang, X.-H. Wang, X.-J. Zhu, Finite time chaos control for a class of chaotic systems with input nonlinearities via TSM scheme, Nonlinear Dyn. 69 (4) (2012) 1941–1947.
[21] D.I.R. Almeida, J. Alvarez, Robust synchronization of nonlinear siso systems using sliding mode control, Nonlinear Dyn. 46 (3) (2006) 293–306.
[22] D. Efimov, A. Polyakov, L. Fridman, W. Perruquetti, J.-P. Richard, Delayed sliding mode control, Automatica 64 (2016) 37–43.
[23] A. Chen, J. Lu, J. Lü, S. Yu, Generating hyperchaotic lü attractor via state feedback control, Phys. A Statist. Mech. Appl. 364 (2006) 103–110.
[24] H.-H. Chen, G.-J. Sheu, Y.-L. Lin, C.-S. Chen, Chaos synchronization between two different chaotic systems via nonlinear feedback control, Nonlinear Anal. Theory, Methods Appl. 70 (12) (2009) 4393–4401.
[25] L. Ling, G. Zhi-An, Z. Chao, Synchronization between two different chaotic systems with nonlinear feedback control, Chin. Phys. 16 (6) (2007) 1603.
[26] G.-P. Jiang, G. Chen, W.K.-S. Tang, A new criterion for chaos synchronization using linear state feedback control, Int. J. Bifurcat. Chaos 13 (08) (2003) 2343–2351.
[27] K. Ding, Q.-L. Han, Master–slave synchronization criteria for chaotic hindmarsh–rose neurons using linear feedback control, Complexity (2015).
[28] C.-C. Hwang, H. Jin-Yuan, L. Rong-Syh, A linear continuous feedback control of chua's circuit, Chaos, Solitons Fractals 8 (9) (1997) 1507–1515.
[29] Y. Jianzu, T. Vincent, Investigation on control of chaos, Chin. J. Aeronaut. 10 (3) (1997) 74–79.
[30] M. Wang, X. Wang, Controlling liu system with different methods, Mod. Phys. Lett. B 23 (14) (2009) 1805–1818.
[31] J. Jian, X. Deng, J. Wang, Globally exponentially attractive set and synchronization of a class of chaotic finance system, in: International Symposium on Neural Networks, Springer, 2009, pp. 253–261.