QOSF task 1

Hamed Vakili

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First step we have to perform the following operation:

$$\sum |\Psi\rangle |0\rangle \rightarrow \sum |\phi\rangle |b\rangle \tag{1}$$

$|\phi\rangle$ and $|b\rangle$ are address and contents qubit, respectively. Where in case of (1 5 7 10) we can write the numbers in binary as ($|0001\rangle$ $|0101\rangle$ $|0111\rangle$ $|1010\rangle$). This process can be done by the following circuit: In order to do a Grover's

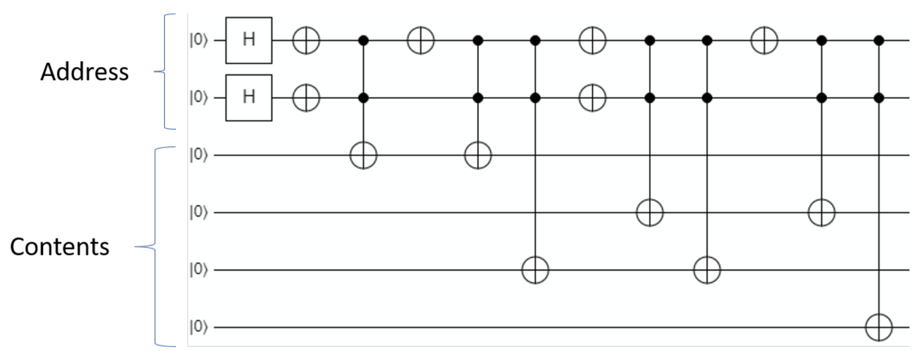

Figure 1: The out put of the circuit is $\sum |\phi\rangle |b\rangle = \frac{1}{2}(|00\rangle |0001\rangle + |01\rangle |0101\rangle + |10\rangle |0111\rangle + |11\rangle |1010\rangle)$

search for the desired answer an oracle qubit is also added. Since the number of answers M are more than half of the search space $M \geq N/2$, we have to first increase the search space to $2N$, this can be achieved by applying a Hadamard gate to the oracle qubit which then doubles the total number of states but keeping the number of answers intact. The circuit would be as shown:

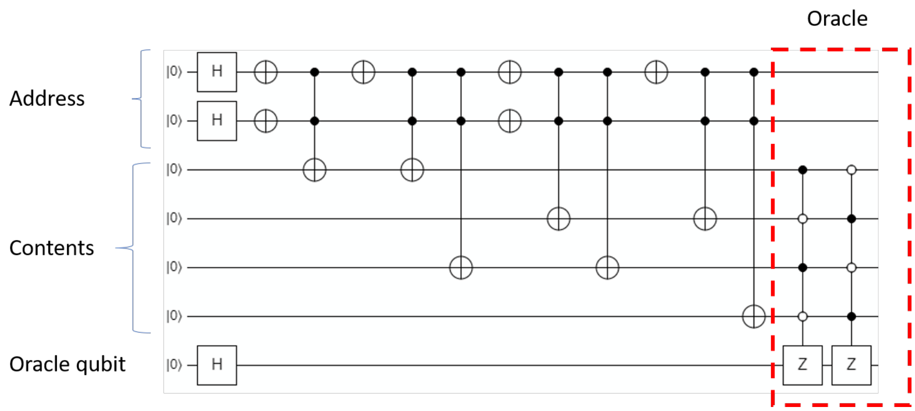

Figure 2: The oracle adds a negative phase for the desired states $|0101\rangle$ and $|1010\rangle$

The out put of above circuit is: $\frac{1}{\sqrt{8}}\big\{(|0\rangle+|1\rangle)\,|00\rangle\,|0001\rangle+(|0\rangle-|1\rangle)\,|01\rangle\,|0101\rangle+$

$(|0\rangle+|1\rangle)\,|10\rangle\,|0111\rangle+(|0\rangle-|1\rangle)\,|11\rangle\,|1010\rangle\big\}$.

Which has two state with negative phase and total of 8 states. Finally, we apply the diffusion operator to pick the right answers. The final circuit would be as follows:

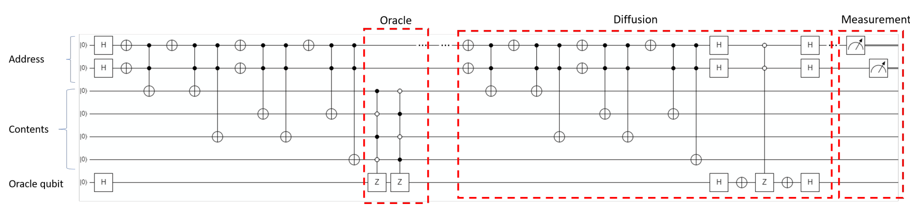

Figure 3: The out put before the measurements is: $\frac{1}{\sqrt{2}}(-|1000001\rangle-|1000011\rangle)$. By measuring the address qubit we get the index $\frac{1}{\sqrt{2}}(|01\rangle+|11\rangle)$

Bonus: To generalize the problem for values of size 2^n with m bits in length, the general circuit would have the following structure:

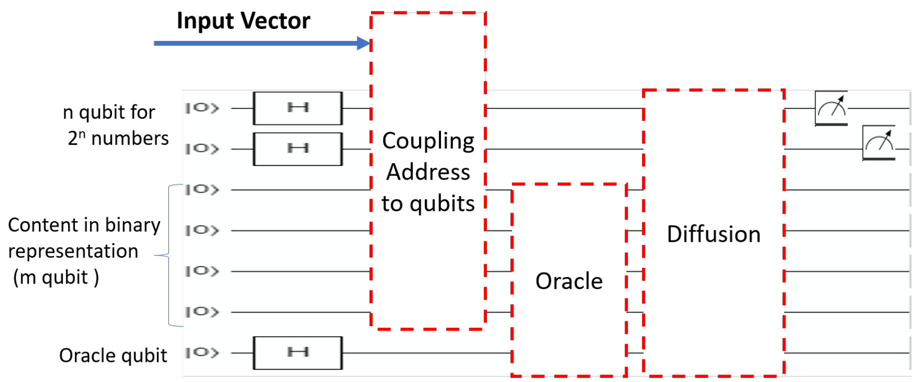

Figure 4: n address qubits and m content qubits.

The input vector would determine how the coupling between register and contents is going to be. Alternatively, it might be possible to explicitly store the information and do the calculation. In this way each number of vector is separately stored in a binary representation. The circuit would be as follows:

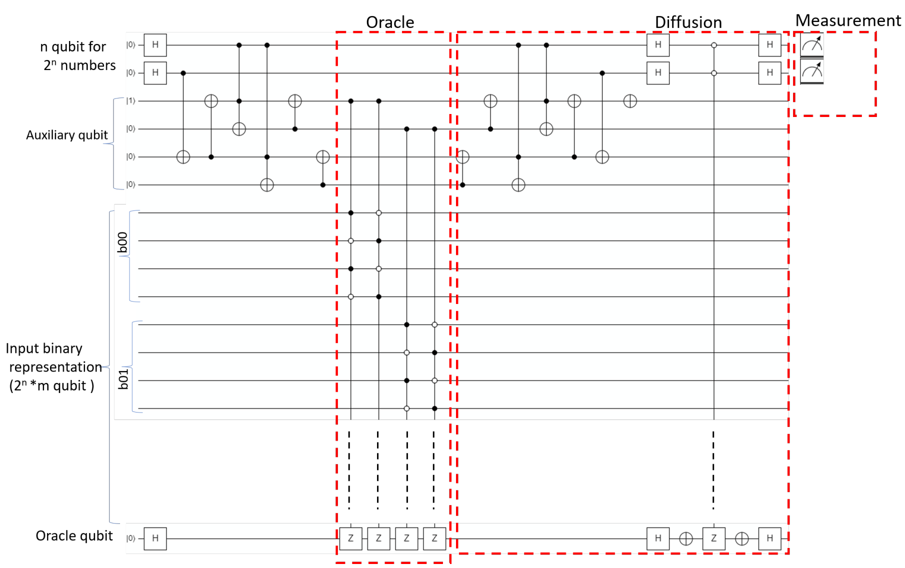

Figure 5: Explicitly storing vector of length 2^n with m bit length for each number.

However, in this method the number of bits required would be much more than previous version and is similar to classical computers. ow ever it might provide higher flexibility for a random input of size 2^n with m bits in length.