

Team Notebook

HCMUTE.3SAT

November 7, 2024

Contents

1 DP	2	2.12 RURQ	6	5 Maths	12
1.1 Divide and Conquer	2	2.13 Treap	6	5.1 Matrix	12
1.2 SOS	2	2.14 Trie	7	5.2 Mod Int	12
2 Data Structures	2	2.15 Wavelet	8	5.3 Precal Modulo Inverse	12
2.1 CentroidDecomp	2	3 Geometry	8	5.4 Rabin-Miller	12
2.2 CoordinateCompression	2	3.1 Basic	8	6 Miscellaneous	13
2.3 DSU	2	3.2 ConvexHull	8	6.1 Clion	13
2.4 FakeFenwick	3	3.3 LineContainer	9	6.2 Debug	13
2.5 Fenwick	3	4 Graphs	9	6.3 mt19937	13
2.6 HLD	3	4.1 Articular	9	6.4 time	13
2.7 LinkCutTree	4	4.2 Bellman-Ford	9	7 Strings	13
2.8 OrderedSet	5	4.3 Bridge	9	7.1 Hash	13
2.9 PURQ	5	4.4 Kosaraju	9	7.2 KMP	13
2.10 PersistentSegmentTree	5	4.5 Matching	10	7.3 Z Function	14
2.11 RMQ	6	4.6 Maximum Flow	10		
		4.7 TwoSatSolver	11		

1 DP

1.1 Divide and Conquer

```
int m, n;
vector<long long> dp_before(n), dp_cur(n);

long long C(int i, int j);

// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr) {
    if (l > r)
        return;

    int mid = (l + r) >> 1;
    pair<long long, int> best = {LLONG_MAX, -1};

    for (int k = optl; k <= min(mid, optr); k++) {
        best = min(best, {(k ? dp_before[k - 1] : 0) + C(k, mid), k});
    }

    dp_cur[mid] = best.first;
    int opt = best.second;

    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}

int solve() {
    for (int i = 0; i < n; i++)
        dp_before[i] = C(0, i);

    for (int i = 1; i < m; i++) {
        compute(0, n - 1, 0, n - 1);
        dp_before = dp_cur;
    }

    return dp_before[n - 1];
}
```

1.2 SOS

```
// sum over subsets O(N * 2^N)
for (int i = 0; i < (1LL << n); i++) {
    sos[i] = cnt[i];
}
for (int i = 0; i < n; i++) {
```

```
    for (int mask = 0; mask < (1LL << n); mask++) {
        if (mask & (1 << i)) {
            sos[mask] += sos[mask ^ (1LL << i)];
        }
    }
}

// list all over subsets O(3^N)
for (int i = 0; i < (1 << n); i++) {
    for (int j = (i - 1) & i; j >= 0; j = (j - 1) & i) { sos[i] += a[j]; }
}
}
```

2 Data Structures

2.1 CentroidDecomp

```
vector<vector<int>> adj;
vector<bool> is_removed;
vector<int> subtree_size;

/** DFS to calculate the size of the subtree rooted at 'node' */
int get_subtree_size(int node, int parent = -1) {
    subtree_size[node] = 1;
    for (int child : adj[node]) {
        if (child == parent || is_removed[child]) { continue; }
        subtree_size[node] += get_subtree_size(child, node);
    }
    return subtree_size[node];
}

/**
 * Returns a centroid (a tree may have two centroids) of the
 * subtree
 * containing node 'node' after node removals
 * @param node current node
 * @param tree_size size of current subtree after node
 * removals
 * @param parent parent of u
 * @return first centroid found
 */
int get_centroid(int node, int tree_size, int parent = -1) {
    for (int child : adj[node]) {
        if (child == parent || is_removed[child]) { continue; }
        if (subtree_size[child] * 2 > tree_size) {
            return get_centroid(child, tree_size, node);
        }
    }
}
```

```
}
return node;
}

/** Build up the centroid decomposition recursively */
void build_centroid_decomp(int node = 0) {
    int centroid = get_centroid(node, get_subtree_size(node));

    // do something

    is_removed[centroid] = true;

    for (int child : adj[centroid]) {
        if (is_removed[child]) { continue; }
        build_centroid_decomp(child);
    }
}
```

2.2 CoordinateCompression

```
template<typename T>
struct CC {
    vector<T> val;
    bool ckcc = 1;
    void insert(T num) { val.push_back(num); ckcc = 0; }
    void cc() { sort(all(val)), val.erase(unique(all(val)), val.end()), ckcc = 1; }
    int idx(T num) { assert(ckcc); return lower_bound(all(val), num) - val.begin() + 1; }
};
```

2.3 DSU

```
struct DSU {
    int n;
    vector<int> par, sz;

    DSU(int n): n(n) {
        par = sz = vector<int> (n + 1);
        fill(all(sz), 1LL);
        iota(all(par), 0LL);
    }

    int find(int v) {
        while (v != par[v]) v = par[v] = par[par[v]];
    }
}
```

```

        return v;
    }

    bool join(int u, int v) {
        u = find(u);
        v = find(v);
        if (u == v) return false;
        if (sz[u] < sz[v]) swap(u, v);
        sz[u] += sz[v];
        par[v] = u;
        return true;
    }

    int same(int u, int v) {
        return find(u) == find(v);
    }
};

```

2.4 FakeFenwick

```

struct FakeFenwick {
    vector<vector<int>> fw, val;
    int n;
    FakeFenwick() {}
    FakeFenwick(int n): n(n), val(n + 1, vector<int>()),
        fw(n + 1) {}
    bool iscc = 0;
    void fakeU(int x, int y) {
        iscc = 0;
        for (; x <= n; x += x & -x) val[x].push_back(y);
    }
    void cc() {
        if (iscc) return;
        for (int x = 1; x <= n; x++) {
            sort(all(val[x]));
            val[x].erase(unique(all(val[x])), val[x].end());
            fw[x].resize(val[x].size() + 1);
        }
        iscc = 1;
    }
    void update(int x, int y, int v) {
        assert(iscc);
        for (; x <= n; x += x & -x) {
            int yy = upper_bound(all(val[x]), y) -
                val[x].begin();
            for (; yy <= val[x].size(); yy += yy &
                -yy) {

```

```

                fw[x][yy] += v;
            }
        }
    }
    int get(int x, int y) {
        assert(iscc);
        int res = 0;
        for (; x; x -= x & -x) {
            int yy = upper_bound(all(val[x]), y) -
                val[x].begin();
            for (; yy; yy -= yy & -yy) {
                res += fw[x][yy];
            }
        }
        return res;
    }
};

```

2.5 Fenwick

```

template<typename T>
struct fenwick {
    vector<T> f;
    int n;
    fenwick(int n): n(n) {
        f.resize(n + 1);
    }
    void add(int i, T v) {
        dbg(i);
        for (; i <= n; i += i & -i) {
            f[i] += v;
            dbg(i, v);
        }
    }
    T get(int i) {
        T ans = 0;
        for (; i >= 1; i -= i & -i) ans += f[i];
        return ans;
    }
    T find(T k) {
        T sum = 0;
        int pos = 0;
        for (int i = __lg(n); i >= 0; i--)
        {
            if (pos + (1 << i) < n && sum + f[pos + (1 << i)] < k)
            {
                sum += f[pos + (1 << i)];
                pos += (1 << i);
            }
        }
    }
};

```

```

    }
}
return pos + 1;
}
};

```

2.6 HLD

```

struct HLD {
    int n;
    int root;
    vector<int> sz, par, dep, top, in, out, seq;
    vector<vector<int>> adj;
    int curdfs = 0;
    HLD(int n): n(n) {
        par = sz = dep = top = in = out = seq = vector<int>(n + 1);
        adj.resize(n + 1);
    }
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void start(int root = 1) {
        this->root = root;
        par[root] = root;
        top[root] = root;
        dfssz(root);
        dfshld(root);
    }
    void dfssz(int u) {
        if (u != root) adj[u].erase(find(all(adj[u]),
            par[u]));
        sz[u] = 1;
        for (int &v : adj[u]) {
            dep[v] = dep[u] + 1;
            par[v] = u;
            dfssz(v);
            sz[u] += sz[v];
            if (sz[v] > sz[adj[u][0]]) swap(adj[u][0], v);
        }
    }
    void dfshld(int u) {
        in[u] = ++curdfs;
        seq[in[u]] = u;
        for (int v : adj[u]) {
            top[v] = (v == adj[u][0] ? top[u] : v);
            dfshld(v);
        }
    }
};

```

```

    }
    out[u] = curdfs;
}
int lca(int u, int v) {
    while (top[u] != top[v]) {
        if (dep[top[u]] > dep[top[v]]) swap(u, v);
    }
    v = par[top[v]];
    return (dep[u] < dep[v] ? u : v);
}
bool isancestor(int u, int v) {
    return in[u] <= in[v] && out[v] <= out[u];
}
};

```

2.7 LinkCutTree

```
typedef long long ll;
```

```

typedef struct snode *sn;
struct snode {
    ////////////////////////////////////////////////// VARIABLES
    sn p, c[2];    // parent, children
    bool flip = 0; // subtree flipped or not
    int sz;        // # nodes in current splay tree
    ll sub, vsub = 0; // vsub stores sum of virtual children
    ll val;        // value in node
    snode(int _val) : val(_val) {
        p = c[0] = c[1] = NULL;
        calc();
    }
    friend int getSz(sn x) { return x ? x->sz : 0; }
    friend ll getSub(sn x) { return x ? x->sub : 0; }
    void prop() { // lazy prop
        if (!flip) return;
        swap(c[0], c[1]);
        flip = 0;
        for (int i = 0; i < 2; i++)
            if (c[i]) c[i]->flip ^= 1;
    }
    void calc() { // recalc vals
        for (int i = 0; i < 2; i++)
            if (c[i]) c[i]->prop();
        sz = 1 + getSz(c[0]) + getSZ(c[1]);
        sub = val + getSub(c[0]) + getSub(c[1]) + vsub;
    }
    ////////////////////////////////////////////////// SPLAY TREE OPERATIONS
    int dir() {
        if (!p) return -2;

```

```

        for (int i = 0; i < 2; i++)
            if (p->c[i] == this) return i;
        return -1; // p is path-parent pointer
    } // -> not in current splay tree
    // test if root of current splay tree
    bool isRoot() { return dir() < 0; }
    friend void setLink(sn x, sn y, int d) {
        if (y) y->p = x;
        if (d >= 0) x->c[d] = y;
    }
    void rot() { // assume p and p->p propagated
        assert(!isRoot());
        int x = dir();
        sn pa = p;
        setLink(pa->p, this, pa->dir());
        setLink(pa, c[x ^ 1], x);
        setLink(this, pa, x ^ 1);
        pa->calc();
    }
    void splay() {
        while (!isRoot() && !p->isRoot()) {
            p->p->prop(), p->prop(), prop();
            dir() == p->dir() ? p->rot() : rot();
            rot();
        }
        if (!isRoot()) p->prop(), prop(), rot();
        prop();
        calc();
    }
    sn fbo(int b) { // find by order
        prop();
        int z = getSZ(c[0]); // of splay tree
        if (b == z) {
            splay();
            return this;
        }
        return b < z ? c[0]->fbo(b) : c[1]->fbo(b - z - 1);
    }
    ////////////////////////////////////////////////// BASE OPERATIONS
    void access() { // bring this to top of tree, propagate
        for (sn v = this, pre = NULL; v; v = v->p) {
            v->splay(); // now switch virtual children
            if (pre) v->vsub -= pre->sub;
            if (v->c[1]) v->vsub += v->c[1]->sub;
            v->c[1] = pre;
            v->calc();
            pre = v;
        }
        splay();
        assert(!c[1]); // right subtree is empty

```

```

    }
    void makeRoot() {
        access();
        flip ^= 1;
        access();
        assert(!c[0] && !c[1]);
    }
    ////////////////////////////////////////////////// QUERIES
    friend sn lca(sn x, sn y) {
        if (x == y) return x;
        x->access(), y->access();
        if (!x->p) return NULL;
        x->splay();
        return x->p ? x : y; // y was below x in latter case
    } // access at y did not affect x -> not connected
    friend bool connected(sn x, sn y) { return lca(x, y); }
    // # nodes above
    int distRoot() {
        access();
        return getSz(c[0]);
    }
    sn getRoot() { // get root of LCT component
        access();
        sn a = this;
        while (a->c[0]) a = a->c[0], a->prop();
        a->access();
        return a;
    }
    sn getPar(int b) { // get b-th parent on path to root
        access();
        b = getSZ(c[0]) - b;
        assert(b >= 0);
        return fbo(b);
    } // can also get min, max on path to root, etc
    ////////////////////////////////////////////////// MODIFICATIONS
    void set(int v) {
        access();
        val = v;
        calc();
    }
    friend void link(sn x, sn y, bool force = 0) {
        assert(!connected(x, y));
        if (force) y->makeRoot(); // make x par of y
        else {
            y->access();
            assert(!y->c[0]);
        }
        x->access();
        setLink(y, x, 0);
        y->calc();

```

```

}
friend void cut(sn y) { // cut y from its parent
    y->access();
    assert(y->c[0]);
    y->c[0]->p = NULL;
    y->c[0] = NULL;
    y->calc();
}
friend void cut(sn x, sn y) { // if x, y adj in tree
    x->makeRoot();
    y->access();
    assert(y->c[0] == x && !x->c[0] && !x->c[1]);
    cut(y);
}
};

```

2.8 OrderedSet

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace std;
using namespace __gnu_pbds;
template <class type1>
using ordered_set = tree<type1, null_type, less_equal<type1>,
    >, rb_tree_tag, tree_order_statistics_node_update>;
.insert(x); // insert x
.order_of_key(val + 1) - .order_of_key(val) > 0; // check
    val exists
.erase(.find_by_order(.order_of_key(c))) // erase c , should
    check exists before
.find_by_order(pos) // s[pos] : 0-indexed
.order_of_key(x) // n.o element less than x

```

2.9 PURQ

```

struct Info {
    int val;
    bool empty;
    Info(): val(0), empty(1) { }
    Info(int val): val(val), empty(0) { }
    friend Info operator + (Info lef, Info rig) {
        if (lef.empty || rig.empty) {
            return lef.empty ? rig : lef;
        }
        Info res;
        res.empty = 0;

```

```

        res.val = lef.val + rig.val;
        return res;
    }
};

struct Segtree {
    vector<Info> st;
    int n;
#define lc id << 1
#define rc id << 1 | 1
#define mi ((l + r) >> 1)
    Segtree() {}
    Segtree(int n): n(n), st(n << 2, Info(0)) {}
    void update(int i, int x, int id, int l, int r) {
        if (l == r) {
            st[id].val += x;
            return;
        }
        if (i <= mi) update(i, x, lc, l, mi);
        else update(i, x, rc, mi+1, r);
        st[id] = st[lc] + st[rc];
    }
    Info get(int u, int v, int id, int l, int r) {
        if (u > r || l > v) {
            return Info();
        }
        if (u <= l && r <= v) {
            return st[id];
        }
        return get(u, v, lc, l, mi) + get(u, v, rc, mi+1, r);
    }
#undef lc
#undef rc
#undef mi
};

```

2.10 PersistentSegmentTree

```

struct PSTNode {
    int val;
    PSTNode *lef, *rig;
    PSTNode (int val) {
        this->val = val;
        lef = rig = nullptr;
    }
    PSTNode (PSTNode *lef, PSTNode *rig) {
        this->lef = lef;
        this->rig = rig;

```

```

        val = 0;
        if (lef) val += lef->val;
        if (rig) val += rig->val;
    }
    PSTNode (PSTNode *p) {
        this->lef = p->lef;
        this->rig = p->rig;
        this->val = p->val;
    }
};

struct PST {
    int n, nv;
    vector<PSTNode*> rt;
    PST() {}
    PST(int n, int nv, vector<int> vec) {
        this->n = n;
        this->nv = nv;
        rt.resize(nv + 1);
        rt[0] = build(1, n, vec);
    }

    PSTNode *build(int l, int r, vector<int> &vec) {
        if (l == r) {
            return new PSTNode(vec[l]);
        }
        int mi = (l + r) >> 1;
        return new PSTNode(build(l, mi, vec), build(mi+1, r, vec));
    }

    int get(int i, PSTNode *p, int l, int r) {
        if (l == r) {
            return p->val;
        }
        int mi = (l + r) >> 1;
        if (i <= mi) return get(i, p->lef, l, mi);
        else return get(i, p->rig, mi+1, r);
    }

    int get(int i, int v) {
        return get(i, rt[v], 1, n);
    }

    PSTNode *update(int i, int val, PSTNode *p, int l,
        int r) {
        if (l == r) {
            return new PSTNode(val);
        }
        int mi = (l + r) >> 1;
        if (i <= mi) return new PSTNode(update(i, val,
            p->lef, l, mi), p->rig);

```

```

        else return new PSTNode(p->lef, update(i, val,
            p->rig, mi+1, r));
    }

    void update(int i, int val, int v) {
        rt[v] = update(i, val, rt[v], 1, n);
    }

    void copy(int t1, int t2) {
        rt[t2] = rt[t1];
    }
};

```

2.11 RMQ

```

struct RMQ {
    int n, lg;
    vector<int> v;
    vector<vector<int>> st;
    RMQ(vector<int> &v): v(v), n(v.size() - 1) {
        lg = __lg(n) + 1;
        st.resize(n + 1, vector<int> (lg + 1));
        for (int i = 1; i <= n; i++) {
            st[i][0] = v[i];
        }
        for (int i = 1; i <= lg; i++) {
            for (int j = 1; j + (1LL << i) - 1 <= n; j++) {
                st[j][i] = min(st[j][i - 1], st[j + (1LL << (i - 1))][i - 1]);
            }
        }
    }
    int get(int l, int r) {
        assert(l <= r);
        int len = __lg(r - l + 1);
        return min(st[l][len], st[r - (1 << len) + 1][len]);
    }
};

```

2.12 RURQ

```

struct Tag {
    int add;
    Tag(): add(0) {}
};

```

```

Tag(int add): add(add) {}
};

struct Info {
    int val;
    bool empty = 1;
    Info(): val(0) {
        empty = 1;
    }
    Info(int val): val(val) {
        empty = 0;
    }
    friend Info operator + (Info lef, Info rig) {
        if (lef.empty || rig.empty) {
            return lef.empty ? rig : lef;
        }
        Info res;
        res.empty = 0;
        res.val = lef.val + rig.val;
        return res;
    }
};

struct Segtree {
    vector<Info> st;
    vector<Tag> lz;
    int n;
    #define lc id << 1
    #define rc id << 1 | 1
    #define mi ((l + r) >> 1)
    Segtree() {}
    Segtree(int n): n(n), st(n << 2, Info(0)), lz(n << 2, Tag()) {}
    void push(int id, int l, int r) {
        if (lz[id].add == 0) return;
        int x = lz[id].add;
        lz[id].add = 0;
        update(l, mi, x, lc, l, mi);
        update(mi+1, r, x, rc, mi+1, r);
    }
    void update(int u, int v, int x, int id, int l, int r) {
        if (u > r || l > v) {
            return;
        }
        if (u <= l && r <= v) {
            st[id].val += x * (r - l + 1);
            lz[id].add += x;
            return;
        }
    }
};

```

```

push(id, l, r);
update(u, v, x, lc, l, mi);
update(u, v, x, rc, mi+1, r);
st[id] = st[lc] + st[rc];
}
Info get(int u, int v, int id, int l, int r) {
    if (u > r || l > v) {
        return Info();
    }
    if (u <= l && r <= v) {
        return st[id];
    }
    push(id, l, r);
    return get(u, v, lc, l, mi) + get(u, v, rc, mi+1, r);
}
#undef lc
#undef rc
#undef mi
};

```

2.13 Treap

```

struct Treap {
    Treap *lef, *rig;
    Treap *par;
    int num;
    int prio;
    int sz;
    Treap(int num):
        num(num),
        prio(rnd()),
        sz(1),
        par(nullptr),
        lef(nullptr),
        rig(nullptr) {}
};

int getsz(Treap *p) {
    return (p ? p->sz : 0);
}

void pull(Treap *p) {
    if (!p) return;
    p->sz = getsz(p->lef) + 1 + getsz(p->rig);
    p->par = nullptr;
    if (p->lef) p->lef->par = p;
    if (p->rig) p->rig->par = p;
}

```

```

void merge(Treap *&p, Treap *lef, Treap *rig) {
    if (!lef || !rig) {
        p = (lef ? lef : rig);
        return;
    }
    if (lef->prio > rig->prio) {
        merge(lef->rig, lef->rig, rig);
        p = lef;
    }
    else {
        merge(rig->lef, lef, rig->lef);
        p = rig;
    }
    pull(p);
}

void split(Treap *p, Treap *&lef, Treap *&rig, int k) {
    if (!p) {
        lef = rig = nullptr;
        return;
    }
    if (getsz(p->lef) < k) {
        split(p->rig, p->rig, rig, k - 1 - getsz(p->lef));
        lef = p;
    }
    else {
        split(p->lef, lef, p->lef, k);
        rig = p;
    }
    pull(p);
}

int index(Treap *p, Treap *pre = nullptr) {
    if (!p) return 0LL;
    if (pre && p->lef == pre) {
        return index(p->par, p);
    }
    else {
        return index(p->par, p) + getsz(p->lef) + 1;
    }
}

```

2.14 Trie

```

struct trie {
    trie* ch[2];
    int sum, cnt, end;
    trie() {
        ch[0] = ch[1] = nullptr;
    }

```

```

        sum = cnt = end = 0;
    }
};

int MXVAL = 0; // 2^(MX + 1) - 1

trie *rt = new trie();
void add(int num) {
    auto p = rt;
    for (int i = 30; i >= 0; i--) {
        int b = (num >> i) & 1LL;
        if (p->ch[b] == nullptr) p->ch[b] = new trie();
        p = p->ch[b];
        p->cnt++;
        p->sum += num;
    }
    p->end++;
}

int find(int num) {
    auto p = rt;
    for (int i = 30; i >= 0; i--) {
        int b = (num >> i) & 1LL;
        if (p->ch[b] == nullptr) return false;
        p = p->ch[b];
    }
    return p->end;
}

bool del(trie *p, int num, int i, int d) {
    if (i >= 0) {
        int b = (num >> i) & 1LL;
        if (del(p->ch[b], num, i-1, d)) p->ch[b] = nullptr;
    }
    else {
        p->end -= d;
    }
    if (p != rt) {
        p->cnt -= d;
        p->sum -= num * d;
        if (p->cnt == 0) {
            delete(p);
            return true;
        }
    }
    return false;
}

void del(int num, int t) {
    int d = find(num);
    if (d == 0) return;
    if (t == 0) del(rt, num, 30, 1);
    else del(rt, num, 30, d);
}

```

```

}

int kth(int k) {
    auto p = rt;
    int res = 0;
    for (int i = 30; i >= 0; i--) {
        for (int b = 0; b < 2; b++) {
            if (p->ch[b]) {
                if (p->ch[b]->cnt >= k) {
                    p = p->ch[b];
                    res += (b << i);
                    break;
                }
                else {
                    k -= p->ch[b]->cnt;
                }
            }
        }
    }
    return res;
}

int sum(int u, int v, trie *p, int l, int r) {
    if (u > r || l > v || p == nullptr) {
        return 0;
    }
    if (u <= l && r <= v) {
        return p->cnt;
    }
    int mi = (l + r) / 2;
    return sum(u, v, p->ch[0], l, mi) + sum(u, v, p->ch[1], mi + 1, r);
}

int sum(int l, int r) {
    return sum(l, r, rt, 0, MXVAL);
}

int xr(int num) {
    auto p = rt;
    int res = 0;
    for (int i = 30; i >= 0; i--) {
        int b = (num >> i) & 1LL;
        if (p->ch[1 ^ b] != nullptr) {
            p = p->ch[1 ^ b];
            res |= (1 << i);
        }
        else {
            p = p->ch[b];
        }
    }
    return res;
}

```

2.15 Wavelet

```

struct wavelet {
    wavelet *lc, *rc;
    vector<int> pf;
    int lo, hi;

    wavelet(vector<int> a): wavelet(1 + all(a), *min_element(1
        + all(a)), *max_element(1 + all(a))){}
    wavelet(vector<int>::iterator l, vector<int>::iterator r,
        int lo, int hi): lo(lo), hi(hi) {
        if (lo == hi || l == r) return;
        int mi = lo + (hi - lo)/2;
        pf.reserve(r - l + 1);
        pf.push_back(OLL);
        for (auto it = l; it != r; it++) {
            pf.push_back(pf.back() + (*it <= mi));
        }
        auto m = stable_partition(l, r, [&](int num) { return num
            <= mi; });
        lc = new wavelet(l, m, lo, mi);
        rc = new wavelet(m, r, mi+1, hi);
    }
    // tim so nho thu k trong doan (l, r);
    int kth(int l, int r, int k) {
        if (l > r) return 0;
        if (lo == hi) {
            return lo;
        }
        int lef = pf[r] - pf[l-1];
        if (lef >= k) return lc->kth(pf[l-1] + 1, pf[r], k);
        else return rc->kth(l - pf[l-1], r - pf[r], k - lef);
    }
    // dem so > k trong doan (l, r)
    int count(int l, int r, int num) {
        if (l > r || hi <= num) return OLL;
        if (lo > num) {
            return (r - l + 1);
        }
        return lc->count(pf[l-1] + 1, pf[r], num) + rc->count(l -
            pf[l-1], r - pf[r], num);
    }
};

```

3 Geometry

3.1 Basic

```

struct Point {
    double x, y;
    Point() { x = y = 0.0; }
    Point(double x, double y) : x(x), y(y) {}

    Point operator + (const Point &a) const { return Point(x
        + a.x, y + a.y); }
    Point operator - (const Point &a) const { return Point(x
        - a.x, y - a.y); }
    Point operator * (double k) const { return Point(x * k, y
        * k); }
    Point operator / (double k) const { return Point(x / k, y
        / k); }
};

struct Line { // Ax + By = C
    double a, b, c;
    Line(double a = 0, double b = 0, double c = 0) : a(a), b(b), c(c) {}
    Line(Point A, Point B) {
        a = B.y - A.y;
        b = A.x - B.x;
        c = a * A.x + b * A.y;
    }
};

Line Perpendicular_Bisector(Point A, Point B) {
    Point M = (A + B) / 2;
    Line d = Line(A, B);
    // the equation of a perpendicular line has the form: -Bx
    + Ay = D
    double D = -d.b * M.x + d.a * M.y;
    return Line(-d.b, d.a, D);
}

```

3.2 ConvexHull

```

// Kiu im
struct Point {
    int x, y;
};

// Tch c hng ca AB v AC
long long cross(const Point &A, const Point &B, const Point
    &C) {

```

```

    return 1LL * (B.x - A.x) * (C.y - A.y) - 1LL * (C.x - A.x
        ) * (B.y - A.y);
}

// A -> B -> C i theo th t theo chiu kim ng h
(-1), thng hng (0), ngc chiu kim ng h
(1)
int ccw(const Point &A, const Point &B, const Point &C) {
    long long S = cross(A, B, C);
    if (S < 0) return -1;
    if (S == 0) return 0;
    return 1;
}

// Tr v bao li vi th t cc im c
lit k ngc chiu kim ng h
vector<Point> convexHull(vector<Point> p, int n) {
    // a im c tung nh nht (v tri nht )
    ln u tp
    for (int i = 1; i < n; ++i) {
        if (p[0].y > p[i].y || (p[0].y == p[i].y && p[0].x >
            p[i].x)) {
            swap(p[0], p[i]);
        }
    }

    // Sp xp cc im I theo gc to bi trc
    honh theo chiu dng v OI
    sort(p.begin() + 1, p.end(), [&p](const Point &A, const
        Point &B) {
        int c = ccw(p[0], A, B);
        if (c > 0) return true;
        if (c < 0) return false;
        return A.x < B.x || (A.x == B.x && A.y < B.y);
    });

    // Tp bao li
    vector<Point> hull;
    hull.push_back(p[0]);

    // Dng bao li
    for (int i = 1; i < n; ++i) {
        while (hull.size() >= 2 && ccw(hull[hull.size() - 2],
            hull.back(), p[i]) < 0) {
            hull.pop_back();
        }
        hull.push_back(p[i]);
    }
    return hull;
}

```


3.3 LineContainer

```
/**
 * Author: Simon Lindholm
 * Date: 2017-04-20
 * License: CC0
 * Source: own work
 * Description: Container where you can add lines of the
 *             form  $kx+m$ , and query maximum values at points  $x$ .
 * Useful for dynamic programming (''convex hull trick'').
 * Time:  $O(\log N)$ 
 * Status: stress-tested
 */
#pragma once

struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};
```

4 Graphs

4.1 Articular

```
(u == rt && ch[u].size() > 1) || (low[v] >= num[u] for some
v in ch[u])
```

4.2 Bellman-Ford

```
struct Edge {
    int a, b, cost;
};

int n, m;
vector<Edge> edges;
const int INF = 1000000000;

void solve()
{
    vector<int> d(n);
    vector<int> p(n, -1);
    int x;
    for (int i = 0; i < n; ++i) {
        x = -1;
        for (Edge e : edges) {
            if (d[e.a] + e.cost < d[e.b]) {
                d[e.b] = d[e.a] + e.cost;
                p[e.b] = e.a;
                x = e.b;
            }
        }
    }

    if (x == -1) {
        cout << "No negative cycle found.";
    } else {
        for (int i = 0; i < n; ++i)
            x = p[x];

        vector<int> cycle;
        for (int v = x;; v = p[v]) {
            cycle.push_back(v);
            if (v == x && cycle.size() > 1)
                break;
        }
        reverse(cycle.begin(), cycle.end());

        cout << "Negative cycle: ";
```

```
for (int v : cycle)
    cout << v << ' ';
cout << endl;
}
```

4.3 Bridge

```
low[u] = num[u]
```

4.4 Kosaraju

```
vector<vector<int>> adj, adj_rev;
vector<bool> used;
vector<int> order, component;

void dfs1(int v) {
    used[v] = true;

    for (auto u : adj[v])
        if (!used[u])
            dfs1(u);

    order.push_back(v);
}

void dfs2(int v) {
    used[v] = true;
    component.push_back(v);

    for (auto u : adj_rev[v])
        if (!used[u])
            dfs2(u);
}

int main() {
    int n;
    // ... read n ...

    for (;;) {
        int a, b;
        // ... read next directed edge (a,b) ...
        adj[a].push_back(b);
        adj_rev[b].push_back(a);
    }

    used.assign(n, false);
```

```

for (int i = 0; i < n; i++)
    if (!used[i])
        dfs1(i);

used.assign(n, false);
reverse(order.begin(), order.end());

for (auto v : order)
    if (!used[v]) {
        dfs2(v);

        // ... processing next component ...

        component.clear();
    }
}

```

4.5 Matching

```

class matching {
public:
    vector<vector<int>> g;
    vector<int> pa;
    vector<int> pb;
    vector<int> was;
    int n, m;
    int res;
    int iter;
    matching(int _n, int _m) : n(_n), m(_m) {
        assert(0 <= n && 0 <= m);
        pa = vector<int>(n, -1);
        pb = vector<int>(m, -1);
        was = vector<int>(n, 0);
        g.resize(n);
        res = 0;
        iter = 0;
    }
    void add(int from, int to) {
        assert(0 <= from && from < n && 0 <= to && to < m);
        g[from].push_back(to);
    }
    bool dfs(int v) {
        was[v] = iter;
        for (int u : g[v]) {
            if (pb[u] == -1) {
                pa[v] = u;
                pb[u] = v;
                return true;
            }
        }
    }
};

```

```

    }
    for (int u : g[v]) {
        if (was[pb[u]] != iter && dfs(pb[u])) {
            pa[v] = u;
            pb[u] = v;
            return true;
        }
    }
    return false;
}

int solve() {
    while (true) {
        iter++;
        int add = 0;
        for (int i = 0; i < n; i++) {
            if (pa[i] == -1 && dfs(i)) {
                add++;
            }
        }
        if (add == 0) {
            break;
        }
        res += add;
    }
    return res;
}

int run_one(int v) {
    if (pa[v] != -1) {
        return 0;
    }
    iter++;
    return (int)dfs(v);
}
};

```

4.6 Maximum Flow

```

// https://pastebin.com/exQM152L

template <typename T>
class flow_graph {
public:
    static constexpr T eps = (T) 1e-9;

    struct edge {
        int to;
        T c;
        T f;
    };
};

```

```

    int rev;
};

vector<vector<edge>> g;
vector<int> ptr;
vector<int> d;
vector<int> q;
vector<int> cnt_on_layer;
vector<int> prev_edge;
bool can_reach_sink;

int n;
int st, fin;
T flow;

flow_graph(int _n, int _st, int _fin) : n(_n), st(_st),
    fin(_fin) {
    assert(0 <= st && st < n && 0 <= fin && fin < n && st !=
        fin);
    g.resize(n);
    ptr.resize(n);
    d.resize(n);
    q.resize(n);
    cnt_on_layer.resize(n + 1);
    prev_edge.resize(n);
    flow = 0;
}

void clear_flow() {
    for (int i = 0; i < n; i++) {
        for (edge &e : g[i]) {
            e.f = 0;
        }
    }
    flow = 0;
}

void add(int from, int to, T forward_cap, T backward_cap)
{
    assert(0 <= from && from < n && 0 <= to && to < n);
    int from_size = g[from].size();
    int to_size = g[to].size();
    g[from].push_back({to, forward_cap, 0, to_size});
    g[to].push_back({from, backward_cap, 0, from_size});
}

bool expath() {
    fill(d.begin(), d.end(), n);
    q[0] = fin;
    d[fin] = 0;
};

```

```

fill(cnt_on_layer.begin(), cnt_on_layer.end(), 0);
cnt_on_layer[n] = n - 1;
cnt_on_layer[0] = 1;
int beg = 0, end = 1;
while (beg < end) {
    int i = q[beg++];
    for (const edge &e : g[i]) {
        const edge &back = g[e.to][e.rev];
        if (back.c - back.f > eps && d[e.to] == n) {
            cnt_on_layer[d[e.to]]--;
            d[e.to] = d[i] + 1;
            cnt_on_layer[d[e.to]]++;
            q[end++] = e.to;
        }
    }
}
return (d[st] != n);
}

T augment(int &v) {
    T cur = numeric_limits<T>::max();
    int i = fin;
    while (i != st) {
        const edge &e = g[i][prev_edge[i]];
        const edge &back = g[e.to][e.rev];
        cur = min(cur, back.c - back.f);
        i = e.to;
    }
    i = fin;
    while (i != st) {
        edge &e = g[i][prev_edge[i]];
        edge &back = g[e.to][e.rev];
        back.f += cur;
        e.f -= cur;
        i = e.to;
        if (back.c - back.f <= eps) {
            v = i;
        }
    }
    return cur;
}

int retreat(int v) {
    int new_dist = n - 1;
    for (const edge &e : g[v]) {
        if (e.c - e.f > eps && d[e.to] < new_dist) {
            new_dist = d[e.to];
        }
    }
    cnt_on_layer[d[v]]--;

```

```

    if (cnt_on_layer[d[v]] == 0) {
        if (new_dist + 1 > d[v]) {
            can_reach_sink = false;
        }
    }
    d[v] = new_dist + 1;
    cnt_on_layer[d[v]]++;
    if (v != st) {
        v = g[v][prev_edge[v]].to;
    }
    return v;
}

T max_flow() {
    can_reach_sink = true;
    for (int i = 0; i < n; i++) {
        ptr[i] = (int) g[i].size() - 1;
    }
    if (expath()) {
        int v = st;
        while (d[st] < n) {
            while (ptr[v] >= 0) {
                const edge &e = g[v][ptr[v]];
                if (e.c - e.f > eps && d[e.to] == d[v] - 1) {
                    prev_edge[e.to] = e.rev;
                    v = e.to;
                    if (v == fin) {
                        flow += augment(v);
                    }
                    break;
                }
                ptr[v]--;
            }
            if (ptr[v] < 0) {
                ptr[v] = (int) g[v].size() - 1;
                v = retreat(v);
                if (!can_reach_sink) {
                    break;
                }
            }
        }
    }
    return flow;
}

vector<bool> min_cut() {
    max_flow();
    assert(!expath());
    vector<bool> ret(n);
    for (int i = 0; i < n; i++) {

```

```

        ret[i] = (d[i] != n);
    }
    return ret;
}
};

```

4.7 TwoSatSolver

```

struct TSS {
    int nvar, nvrt;
    vector<vector<int>> adj;
    vector<bool> res;
    TSS(int nvar): nvar(nvar), nvrt(nvar * 2), adj(nvrt + 1), res(nvar + 1) {}
    int conj(int u) {
        if (u > nvar) return u - nvar;
        return u + nvar;
    }
    int operator[](int i) { return res[i]; }
    void add(int u, bool nu, int v, bool nv) {
        if (nu) u = conj(u);
        if (nv) v = conj(v);
        adj[u].push_back(v);
    }
    bool solve() {
        vector<int> in(nvrt + 1), low(nvrt + 1), id(
            nvrt + 1);
        stack<int> st;
        int curdfs = 0, curidx = 2 * nvrt + 1;
        auto tarjan = [&](auto&& f, int u) -> void {
            in[u] = low[u] = ++curdfs;
            st.push(u);
            for (int v : adj[u]) {
                if (!in[v]) {
                    f(f, v);
                    low[u] = min(low[u], low[v]);
                } else if (!id[v]) {
                    low[u] = min(low[u], in[v]);
                }
            }
            if (low[u] == in[u]) {
                int v;
                do {
                    v = st.top();
                    st.pop();
                    id[v] = curidx;
                } while (v != u);
            }
        };
        for (int u = 0; u < nvar; u++) {
            tarjan(tarjan, u);
        }
    }
};

```

```

        --curidx;
    }
};
for (int i = 1; i <= nvrt; i++)
    if (!in[i]) tarjan(tarjan, i);
for (int i = 1; i <= nvar; i++) {
    if (id[i] == id[conj(i)]) return false;
    res[i] = id[i] > id[conj(i)];
}
return true;
}
};

```

5 Maths

5.1 Matrix

```

template<typename type>
struct Matrix {
    vector<vector<type>>> data;

    // S lng hng ca ma trn
    int row() const { return data.size(); }

    // S lng hng ca ma trn
    int col() const { return data[0].size(); }

    auto & operator [] (int i) { return data[i]; }

    const auto & operator [] (int i) const { return data[i]; }

    Matrix() = default;

    Matrix(int r, int c): data(r, vector<type>(c)) { }

    Matrix(const vector<vector<type>> &d): data(d) {

        // Kim tra cc hng c cng size khng v size c
        // ln hn 0 hay khng
        // Tuy nhin khng thc s cn thit , ta c
        // th b cc dng /**/ i
        /**/ assert(d.size());
        /**/ int size = d[0].size();
        /**/ assert(size);
        /**/ for (auto x : d) assert(x.size() == size);
    }

    // In ra ma trn .

```

```

friend ostream & operator << (ostream &out, const Matrix
&d) {
    for (auto x : d.data) {
        for (auto y : x) out << y << ' ';
        out << '\n';
    }
    return out;
}

// Ma trn n v
static Matrix identity(long long n) {
    Matrix a = Matrix(n, n);
    while (n--) a[n][n] = 1;
    return a;
}

// Nhn ma trn
Matrix operator * (const Matrix &b) {
    Matrix a = *this;

    // Kim tra iu kin nhn ma trn
    assert(a.col() == b.row());

    Matrix c(a.row(), b.col());
    for (int i = 0; i < a.row(); ++i)
        for (int j = 0; j < b.col(); ++j)
            for (int k = 0; k < a.col(); ++k)
                c[i][j] += a[i][k] * b[k][j];
    return c;
}

// Ly tha ma trn
Matrix pow(long long exp) {

    // Kim tra iu kin ly tha ma trn (1 ma
    // trn vung)
    assert(row() == col());

    Matrix base = *this, ans = identity(row());
    for (; exp > 0; exp >>= 1, base = base * base)
        if (exp & 1) ans = ans * base;
    return ans;
}
};

```

5.2 Mod Int

```

/**
 * Description: Modular arithmetic. Assumes $MOD$ is prime.

```

```

* Source: KACTL
* Verification: https://open.kattis.com/problems/modulararithmetic
* Usage: mi a = MOD+5; inv(a); // 400000003
*/

template<int MOD> struct M {
    static const int mod = MOD;
    int v;
    explicit operator int() const { return v; }
    M():v(0) {}
    M(int _v):v((int)(_v%MOD)) { v += (v<0)*MOD; }
    M& operator+=(M o) {
        if ((v += o.v) >= MOD) v -= MOD;
        return *this; }
    M& operator-=(M o) {
        if ((v -= o.v) < 0) v += MOD;
        return *this; }
    M& operator*=(M o) {
        v = (int)((int)v*o.v%MOD); return *this; }
    friend M binpow(M a, int p) { assert(p >= 0);
        return p==0?1:binpow(a*a,p/2)*(p&1?a:1); }
    friend M inv(M a) { assert(a.v != 0); return binpow(a,MOD
        -2); }
    friend M operator+(M a, M b) { return a += b; }
    friend M operator-(M a, M b) { return a -= b; }
    friend M operator*(M a, M b) { return a *= b; }
    friend M operator/(M a, M b) { return a *= inv(b); }
    friend ostream & operator << (ostream &out, M a) { return
        (out << a.v); }
};

int const md = 1e9 + 7;
using mint = M<md>;

```

5.3 Precal Modulo Inverse

```

int n = 10, p = 1000000007;
int inv[n + 1];
inv[1] = 1;
for (int i = 2; i <= n; i++) inv[i] = 1LL * (p - p / i) *
    inv[p % i] % p;

```

5.4 Rabin-Miller

```

using u64 = uint64_t;
using u128 = __uint128_t;

```

```

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};

bool MillerRabin(u64 n, int iter=5) { // returns true if n
    is probably prime, else returns false.
    if (n < 4)
        return n == 2 || n == 3;

    int s = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }

    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}

```

6 Miscellaneous

6.1 Clion

```

set(GCC_COVERAGE_COMPILE_FLAGS "-O2 -Dbinhball")
set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} ${
    GCC_COVERAGE_COMPILE_FLAGS}" )

```

6.2 Debug

```

#ifdef hvmeqy
#define dbg(...) logger(__VA_ARGS__, __VA_ARGS__)
template<typename ...Args>
void logger(string vars, Args&&... values) {
    cerr << "[" << vars << " : ";
    string delim = "";
    (... , (cerr << delim << values, delim = ", "));
    cerr << "]" << '\n';
}
#else
#define dbg(...)
#endif

```

6.3 mt19937

```

mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch
    ().count());

```

6.4 time

```

cerr << '\n' << clock() * 1000.0 / CLOCKS_PER_SEC << "ms" <<
    '\n';

```

7 Strings

7.1 Hash

```

const int mod1 = 1035972859;
const int mod2 = 1704760909;
const int base = ;
struct hashing{
    int mod, n;

```

```

vector<int> h, power;
int binpow(int a, int n){
    if (n == 0)
        return 1LL;
    int res = binpow(a, n/2);
    if (n % 2)
        return res * res % mod * a % mod;
    return res * res % mod;
}

hashing() {}
hashing(string s, int mod): mod(mod){
    this->n = s.length();
    power = h = vector<int> (n);
    power[0] = 1;
    h[0] = s[0];
    for (int i = 1; i < n; i++){
        power[i] = power[i - 1] * base % mod;
        h[i] = (h[i - 1] * base + s[i]) % mod;
    }
}

int getH(int l, int r){
    return (!l ? h[r] : (h[r] - h[l - 1] * power[r
        - l + 1] + mod * mod) % mod);
}

};

struct bighash{
    bighash() {}
    hashing a, b;
    bighash(string s){
        a = hashing(s, mod1);
        b = hashing(s, mod2);
    }
    int get(int l, int r){
        return a.getH(l, r) * b.getH(l, r);
    }
}
};

```

7.2 KMP

```

int next_x[N];
int j = next_x[0] = 0;
n = s.length();
m = t.length();
for (int i = 1; i < n; i++){
    while (j > 0 && s[j] != s[i]){
        j = next_x[j - 1];
    }

```

```
    if (s[i] == s[j])
        j++;
    next_x[i] = j;
}
j = 0;
for (int i = 0; i < m; i++){
    while (j > 0 && s[j] != t[i])
        j = next_x[j - 1];
    if (s[j] == t[i])
        j++;
    if (j == n){
        cout << i - n + 2 << " ";
    }
}
```

```
}
```

7.3 Z Function

```
z[0] = n;
int l = 0, r = 0;
for (int i = 1; i < n; i++){
    if (i > r){
        l = r = i;
        while (r < n && s[r] == s[r-1]) ++r;
        z[i] = r - l;
        r-=1;
    }
}
```

```

}
else{
    int k= i - 1;
    if (z[k] < r - i + 1) z[i] = z[k];
    else{
        l = i;
        while (r < n && s[r] == s[r - 1]) r++;
        z[i]= r - l;
        r--;
    }
}
}
```