Spatiotemporal Chaos: Complex Dynamics in a Model of Coupled Smooth Muscle Cells

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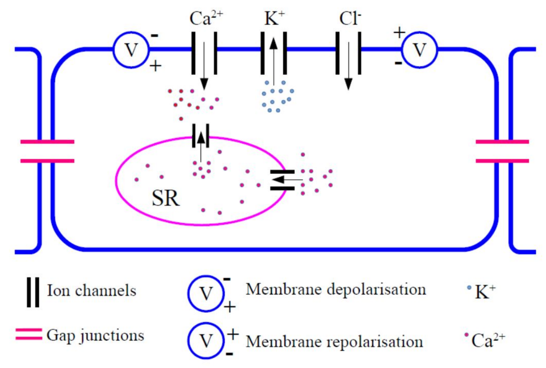
Overview

- We formulate a reaction-diffusion system to investigate spontaneous excitation-contraction coupling (ECC) in smooth muscle cells.
- It may arise as a result of interaction between ion fluxes through the voltage-gated ion channels.
- *In vivo* studies showed that spontaneous ECC depends on transmural pressure.
- We investigate mathematically the influence of model parameters on ionic mechanisms and ECC activity of smooth muscle cells in feline cerebral arteries.



Reaction-diffusion system

The equations of the spatial diffusion of the action potential in transmembrane of the SMCs without any external stimulation are described as

$$\frac{\partial V}{\partial \tau} = D\frac{\partial^2 V}{\partial X^2} - \bar{g}_L(V - \bar{v}_L) - \bar{g}_K N(V - \bar{v}_K) - \bar{g}_{ca} M_\infty(V)(V-1) \quad (1)$$

$$\frac{\partial N}{\partial \tau} = \lambda_N(V)\big(N_\infty(V) - N\big) \quad (2)$$

where V is the membrane potential, N is the fraction of open potassium channels, and

$$M_\infty(V) = 0.5\left(1 + \tanh\left(\frac{V - \bar{v}_1}{\bar{v}_2}\right)\right)$$

$$N_\infty(V) = 0.5\left(1 + \tanh\left(\frac{V - \bar{v}_3}{\bar{v}_4}\right)\right)$$

$$\lambda_N(V) = \psi \cosh\left(\frac{V - \bar{v}_3}{2\bar{v}_4}\right)$$

Numerical Simulations

In Eqs (1)–(2), $\bar{v}_1$ and $\bar{v}_3$ are pressure dependent parameters, thus they are considered as bifurcation parameters. For the range of values of $\bar{v}_1$ and $\bar{v}_3$ considered, the systems either converge to a steady state (absence of ECC) or oscillate (presence of ECC).

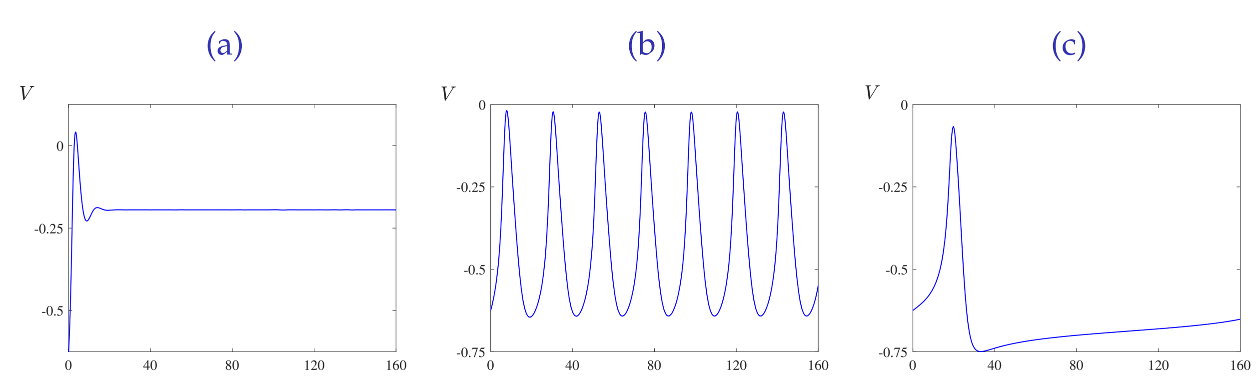


Figure 1: Time evolution of the membrane potential V for (a) $\bar{v}_1 = -0.4375$; (b) $\bar{v}_1 = -0.3125$; (c) $\bar{v}_1 = -0.25$.

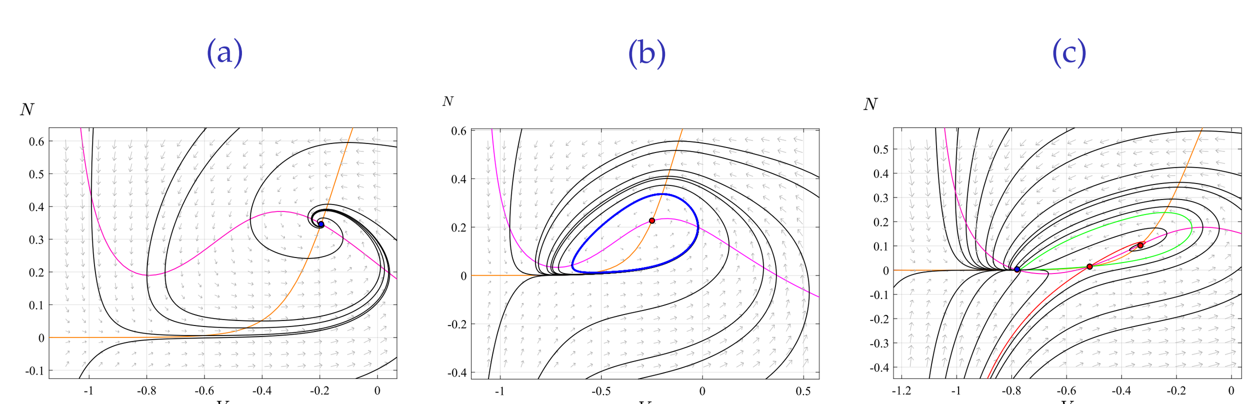


Figure 2: The phase plane for (1)–(2) for corresponding to the values of with (a) $\bar{v}_1$ in Fig.1a. The magenta and orange curves are the nullclines for V and N. The black curves are the solution trajectories. The blue curve is a stable periodic orbit. The blue and red circles are stable and unstable equilibria. The red and green curves in (c) are the stable and unstable manifolds of the saddle point.

Bifurcation Analysis

1. Type I excitability: Saddle-node on an invariant circle (SNIC) bifurcation
2. Type II excitability: Hopf bifurcation

One-parameter Continuation: Tuning of model parameters induced Type I and type II excitability.

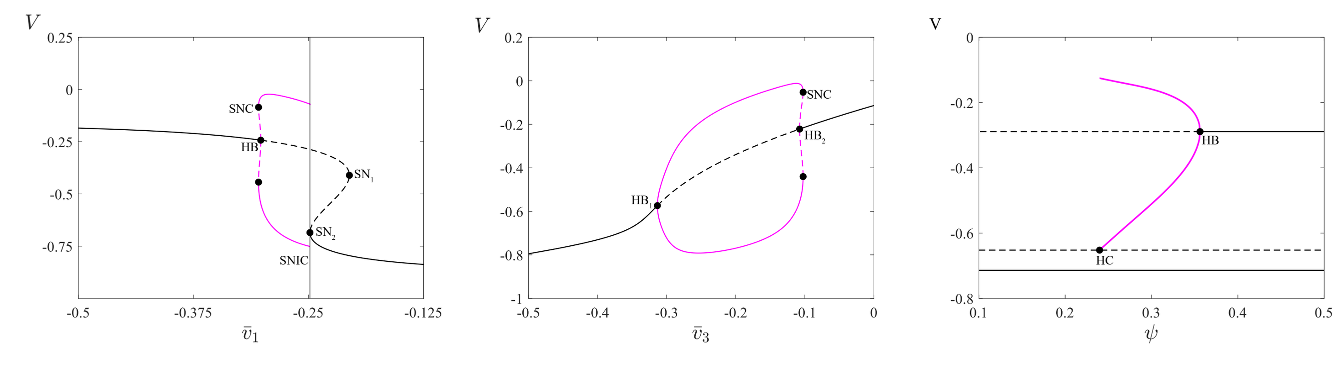


Figure 3: One-parameter bifurcation diagrams of the membrane potential V with $\bar{v}_1$, $\bar{v}_3$ and ψ as the bifurcation parameters.

Two-parameter Continuation: Simultaneous variation of two parameters shows the switches between the types of excitability.

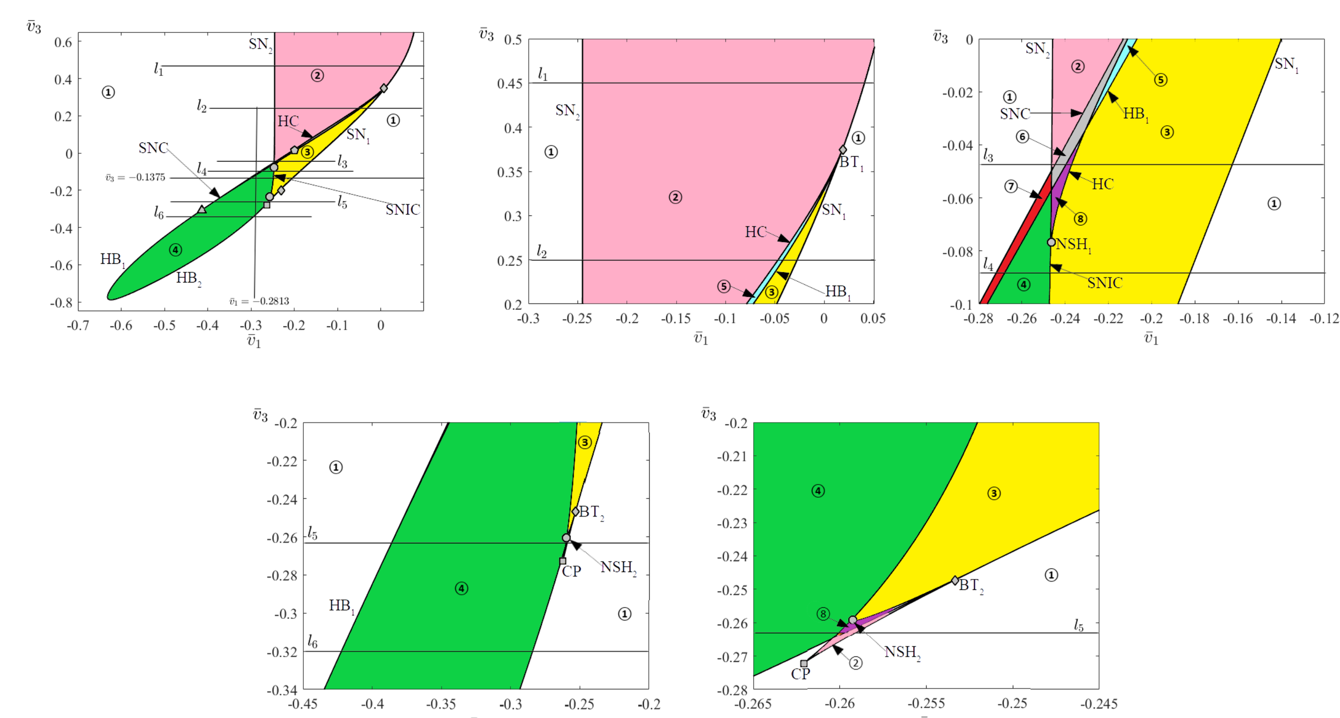


Figure 4: Codimension-2 bifurcation diagram in $(\bar{v}_1, \bar{v}_3)$-plane. The codimension-2 bifurcations are: Bogdanov-Takens-BT, Generalised Hopf-GH, Cusp point-CP

Transitions between Types of Excitability

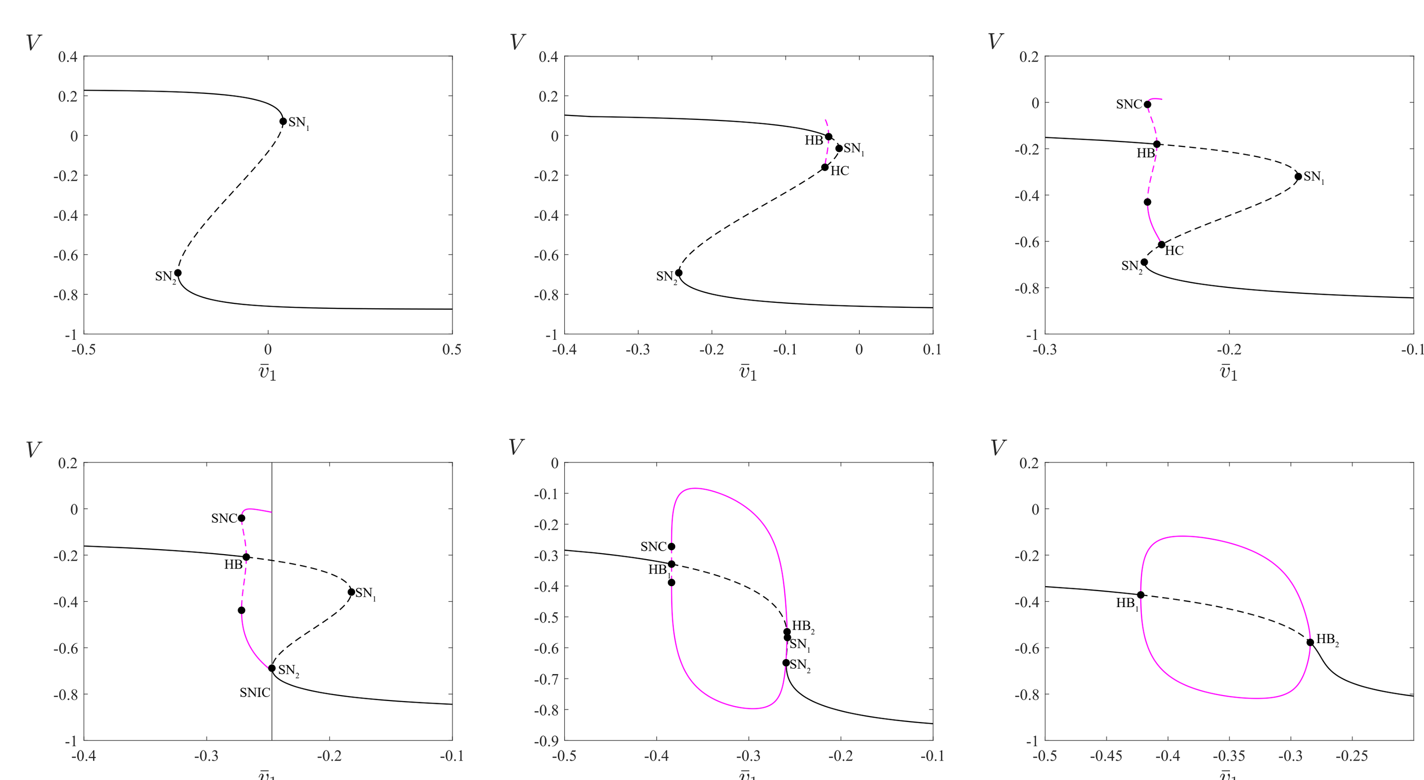


Figure 5: One-parameter bifurcation diagrams along l_1–l_6 in Figure 4 showing transition from type I and type II excitability.

Ongoing work

Spectral stability analysis of the travelling wave solutions observed in the model. In the future we will investigate the effects of ion conductances on pacemaker dynamics and modify the model by incorporating second inward Na^+

Reference

Fatoyinbo, H. O. et al. Numerical Bifurcation Analysis of Pacemaker Dynamics in a Model of Smooth Muscle Cells. *Bull Math Biol* 82, 95 (2020). doi.org/10.1007/s11538-020-00771-6

Spatiotemporal Patterns: Varying $\bar{v}_1$ and ψ

Simulations are performed in MATLAB using the method of lines. Variation of model parameters results in wide range of spatiotemporal patterns including stationary inhomogeneous patterns, travelling pulses and fronts with spatiotempoural chaos.

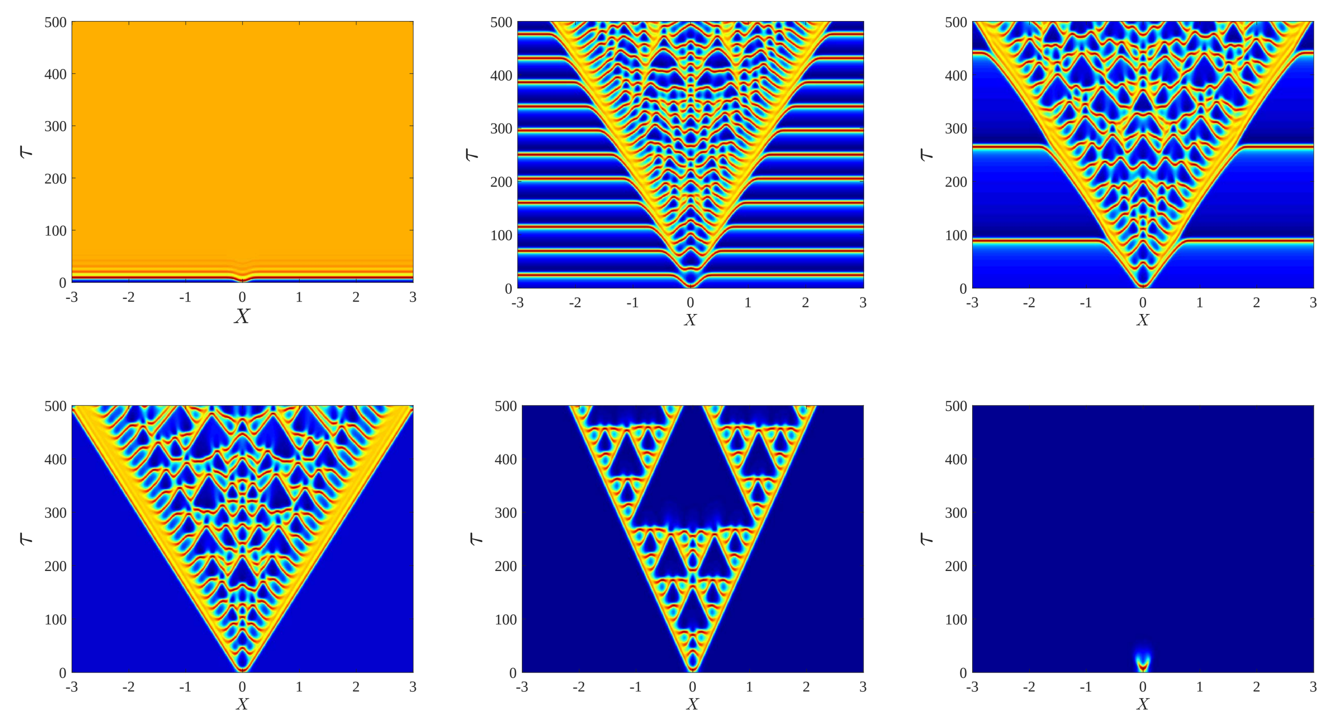


Figure 6: Space-time plot of the membrane potential V for selected values of parameter $\bar{v}_1$. It can be observed that the patterns transition from periodic oscillations of a spatially constant solutions to chaotic spatiotemporal structure behind travelling fronts in opposite direction, then to the rest state.

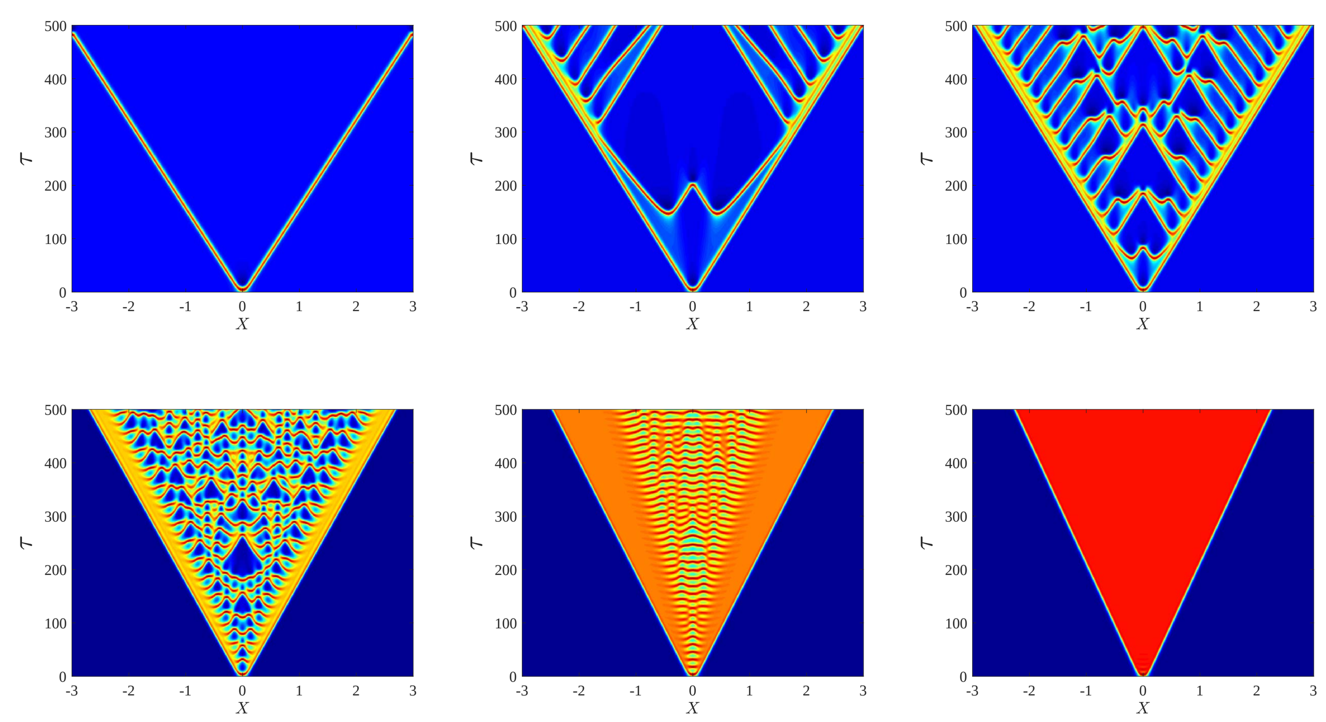


Figure 7: Space-time plot of the membrane potential V for selected values of parameter ψ. The patterns transition from stable pulses to spatiotemporal chaos behind two propagating fronts, and to stable fronts travelling in opposite direction.

Transitions from Stable Pulses to Front and Back

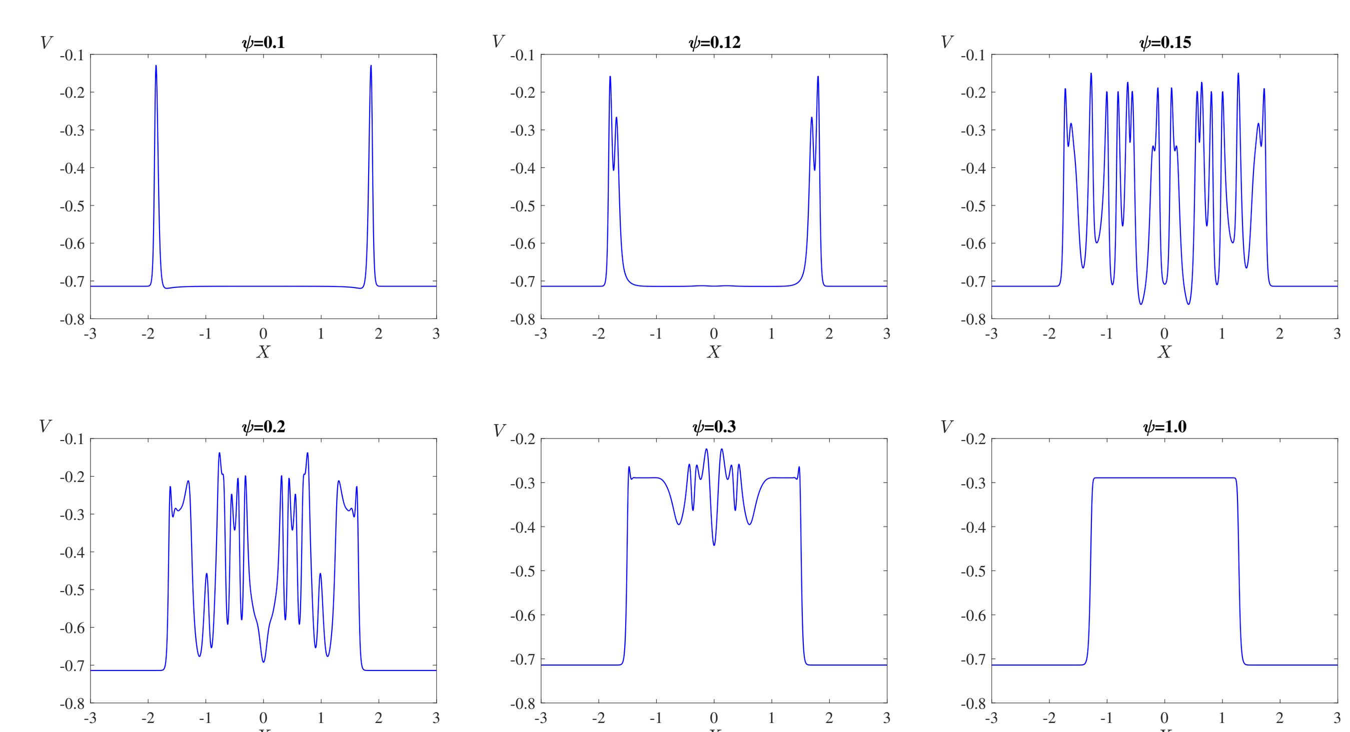


Figure 8: Solution profiles at time $\tau = 300$ showing the transitions from travelling pulses to spatiotemporal chaos and to fronts.

Conclusion

- It is found that the ECC is regulated by model parameters not external sources.
- The results indicate that in some parameter regimes the coupled cells exhibit spatiotemporal chaos.
- These results could be useful in improving the understanding of physiological responses and disorders in smooth muscle cells.

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