Spatiotemporal Pattern Formation in a Model of Electrically Coupled Smooth Muscle Cells

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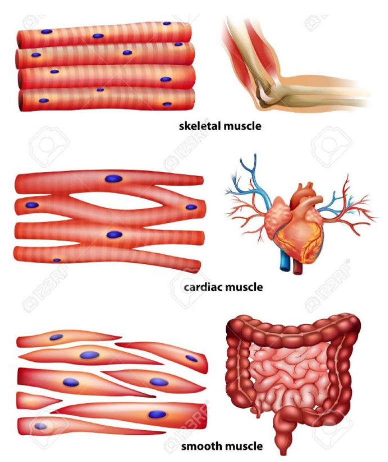
Introduction

- Electro-mechanical coupling (EMC) is the contraction of muscle cell as a result of the excitability of the cell membrane in response to an external stimulation.
- In some muscle cells, for example smooth muscle cell (SMC), EMC activity is spontaneous due to ion fluxes in the cell membrane through the voltage-gated ion channels.
- This type of behaviour of the muscle cell is known as *pacemaker dynamics*.

Better understanding of this cellular behaviour may provide new approach to treatment of diseases associated with the smooth muscle and other muscle cells.

Aims



- *In vivo* studies showed that pacemaker EMC activity observed in a arterial muscle cells depend on transmural pressure .
- Upon elevation of transmural pressure, spontaneous electrical firing is observed and the blood vessel constricts.

Our aims:

- To investigate mathematically how parameters involved in the equations governing transmural pressure influence the ionic mechanisms and EMC activity of smooth muscle cells in feline cerebral arteries
- To study the collective behaviour of the SMCs by using a reaction-diffusion system and incorporating gap junction coupling between cells.

Single Cell Dynamics

The dynamics related to the transitions between firing and resting states in excitable cells are characterized by distinct bifurcation types and spiking frequency.

- Modulation of model parameters induces type I and type II excitability.
- Simultaneous variation of two parameters shows the switches between the types of excitability.
- The type of excitability observed can affect the spatiotemporal behaviour of coupled cells.

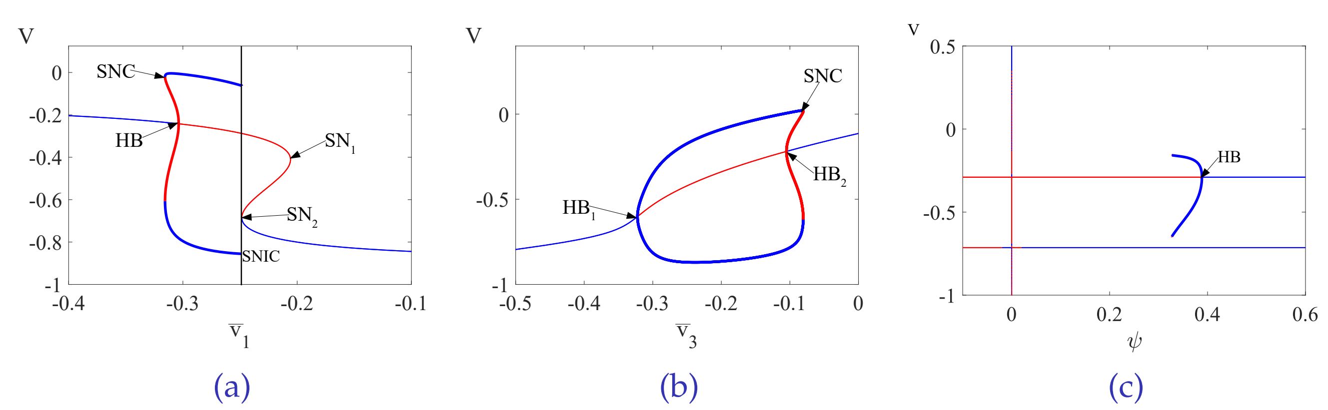


Figure 1: Bifurcation diagram of the membrane potential with $\overline{v}_1$, $\overline{v}_3$, and ψ as bifurcation parameters respectively.

Reaction-diffusion system

The equations of the spatial diffusion of the action potential in transmembrane of the SMCs without any external stimulation are described as

$$\frac{\partial V}{\partial \tau} = D\frac{\partial^2 V}{\partial X^2} - \overline{g}_L(V-\overline{v}_L) - \overline{g}_K N(V-\overline{v}_K) - \overline{g}_{ca} M_\infty(V)(V-1) \quad (1a)$$

$$\frac{\partial N}{\partial \tau} = \lambda_N(V)\big(N_\infty(V) - N\big) \quad (1b)$$

where V is the membrane potential, N is the fraction of open potassium channels, and

$$M_\infty(V) = 0.5\left(1+\tanh\left(\frac{V-\overline{v}_1}{\overline{v}_2}\right)\right)$$

$$N_\infty(V) = 0.5\left(1+\tanh\left(\frac{V-\overline{v}_3}{\overline{v}_4}\right)\right)$$

$$\lambda_N(V) = \psi\cosh\left(\frac{V-\overline{v}_3}{2\overline{v}_4}\right)$$

with no-flux boundary conditions and initial conditions:

$$V(0,X) = V_0 + Ae^{-\left(\frac{X-X_0}{\sigma}\right)^2} \text{ and } N(0,X) = N_0, \forall\, X\in\Omega$$

Computational Simulation Results

Simulations are performed in MATLAB using the method of lines. Variation of model parameters results in wide range of spatiotemporal patterns including stationary inhomogeneous patterns, travelling pulses and fronts with spatiotempoaral chaos. In Figures 2-3, the colorbar corresponds to the values of $V(X,\tau)$.

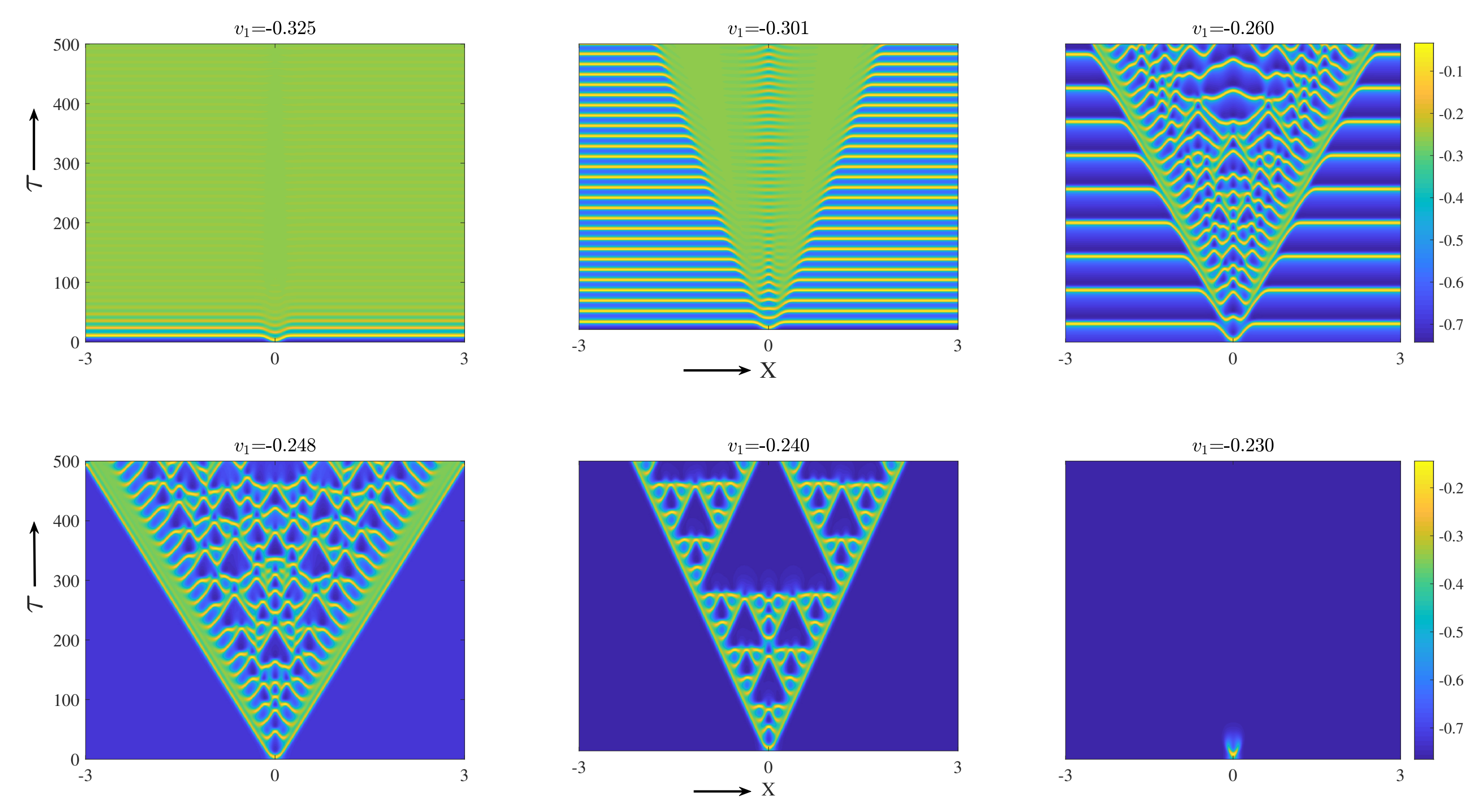


Figure 2: Space-time plot of the membrane potential V for selected values of parameter v_1

It can be observed from Figure 2 that the patterns transition from periodic oscillations of a spatially constant solutions to chaotic spatiotemporal structure behind travelling fronts in opposite direction, then to the rest state.

Spatiotemporal Patterns: Varying ψ

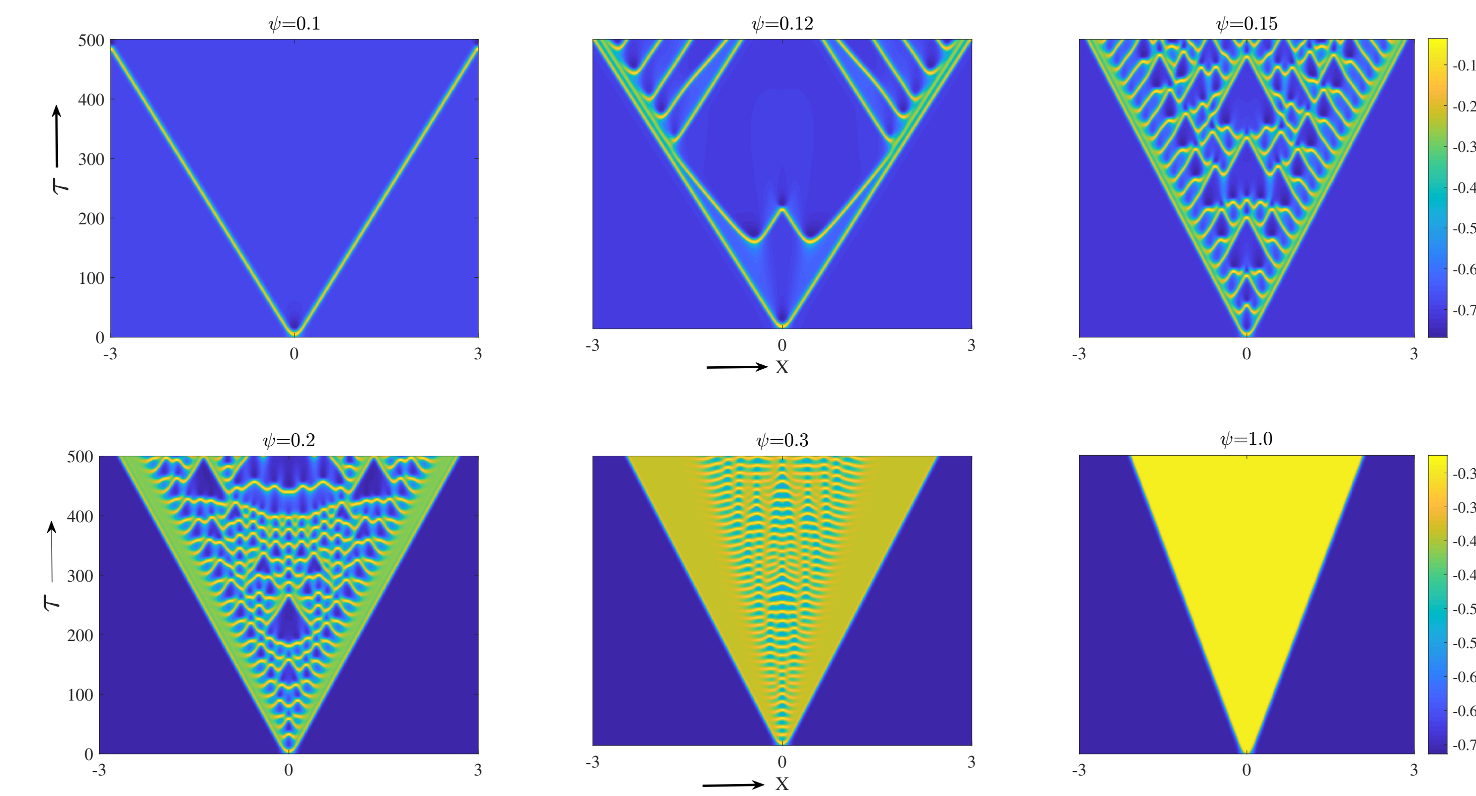


Figure 3: Space-time plot of the membrane potential V for selected values of parameter ψ

Also, Figure 3 reveals that the patterns transition from stable pulses to spatiotemporal chaos behind two propagating fronts, and to stable fronts travelling in opposite direction.

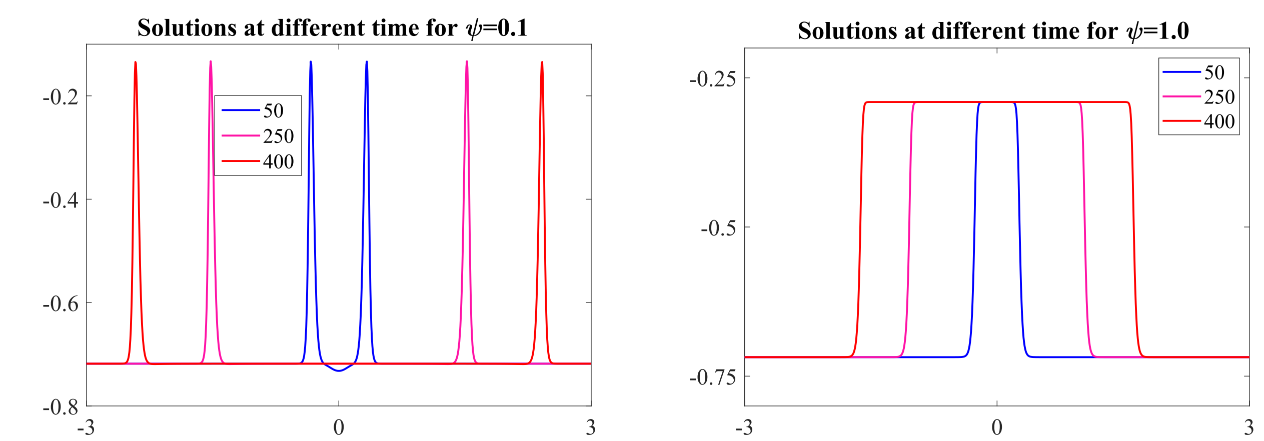


Figure 4: Solutions of the membrane potential for $\psi = 0.1$ and $\psi = 1.0$ in Figure 3

Summary and Conclusion

- We investigated the role of physiological parameters on EMC activity of SMCs in feline cerebral arteries.
- We found that the EMC is regulated by model parameters not external sources.
- Our results indicate that in some parameter regimes the coupled cells exhibit spatiotemporal chaos.
- These results could be useful in improving the understanding of physiological responses and disorders in smooth muscles.

Future Work

It remains to analyse the spectral stability of the travelling wave solutions observed in the model and modify the model by incorporating the Na^+ inward current.

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