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Emergence of spatiotemporal patterns in pacemaker coupled excitable cells

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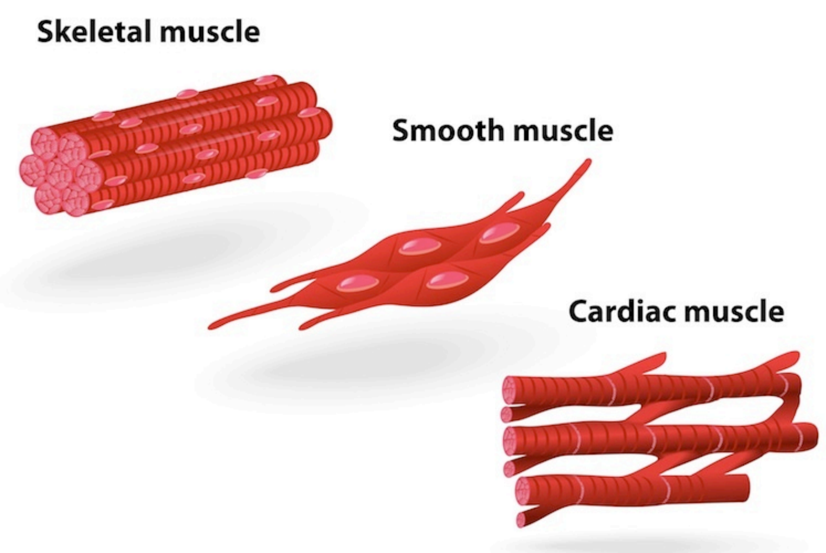
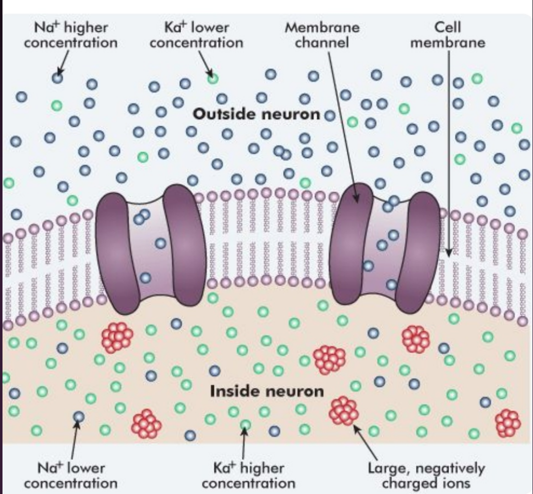
2018 NZMS Colloquium

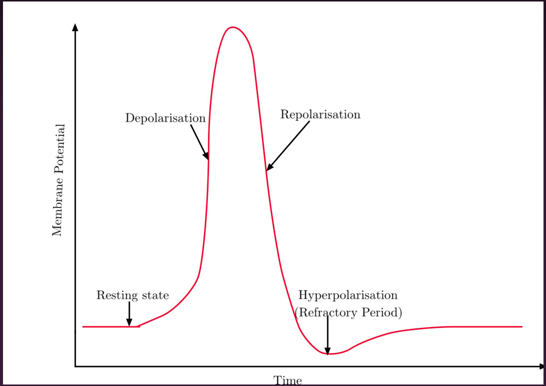
4th December, 2018

Excitable cells



Cellular excitability





Canonical models

- Hodgkin-Huxley model
- Fitzhugh-Nagumo model
- Morris-Lecar model

General model

$$C\frac{dV}{dt} = -\sum_j I_j(V, P) + I_{ext}$$

$$\frac{dP_j}{dt} = \omega(V)\left(\tilde{P}_j(V) - P_j\right)$$

Model formulation

$$C\frac{dv}{dt} = -g_L(v - v_L) - g_K n(v - v_K) - g_{Ca} m_\infty (v - v_{Ca})$$
$$\frac{dCa_i}{dt} = \left(-\alpha g_{Ca} m_\infty (v - v_{Ca}) - k_{Ca} Ca_i\right) \rho$$
$$\frac{dn}{dt} = \lambda_n \left(n_\infty(v, Ca_i) - n\right)$$

where,

$$m_\infty(v) = 0.5\left(1 + \tanh\left(\frac{v - v_1}{v_2}\right)\right)$$
$$n_\infty(v, Ca_i) = 0.5\left(1 + \tanh\left(\frac{v - v_3}{v_4}\right)\right)$$
$$\lambda_n = \phi_n \cosh\left(\frac{v - v_3}{2v_4}\right)$$

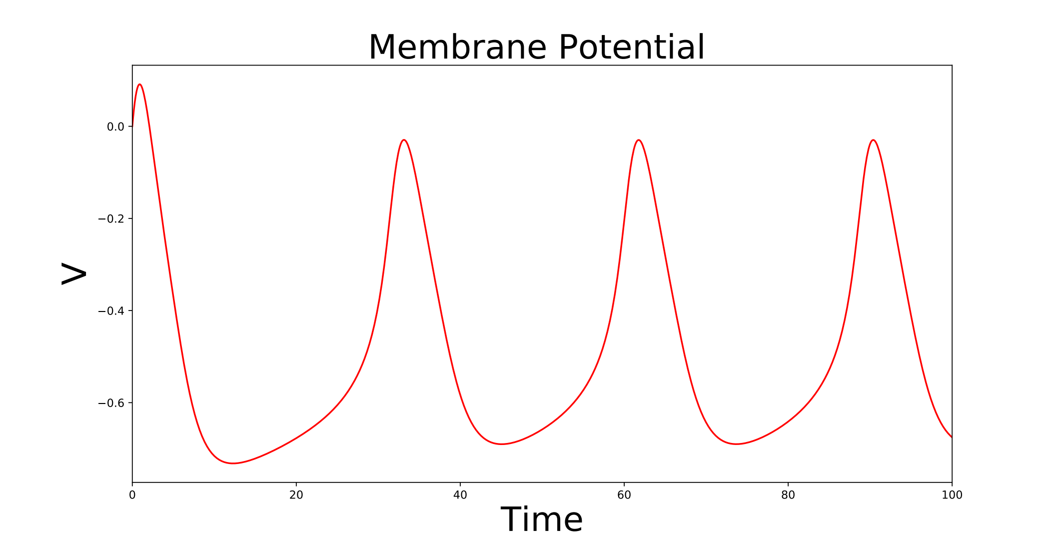
Reduced and non-dimensionalised model

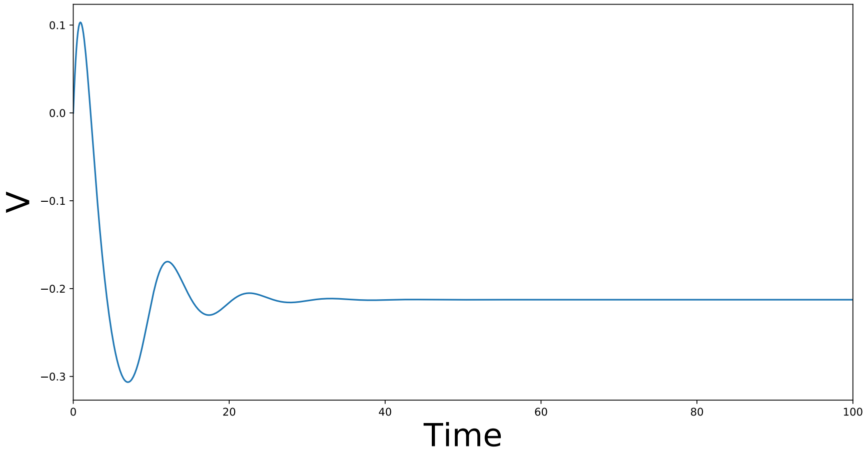
$$
\begin{aligned}
\frac{dV}{dT} &= -\tilde{g}_L(V-\tilde{v}_L) - \tilde{g}_k N(V-\tilde{v}_k) - \tilde{g}_{ca}\tilde{m}_\infty(V-1) \\
\frac{dN}{dT} &= \psi\lambda(V)(\tilde{n}_\infty - N)
\end{aligned}
$$

where,

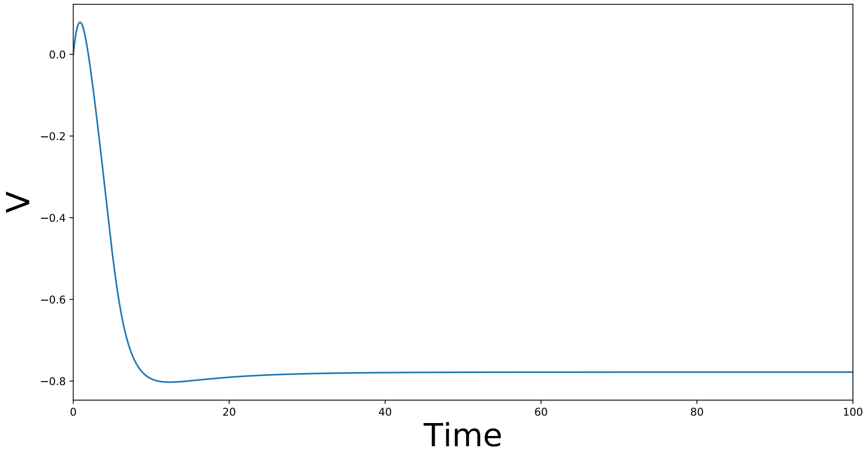
$$
\begin{aligned}
\tilde{m}_\infty(V) &= 0.5(1+\tanh(Vv_a - v_b)) \\
\tilde{n}_\infty(V) &= 0.5\left(1+\tanh\left(Vv_c - v_d\right)\right) \\
\lambda(V) &= \cosh\left(\frac{Vv_c - v_d}{2}\right)
\end{aligned}
$$

Numerical Analysis





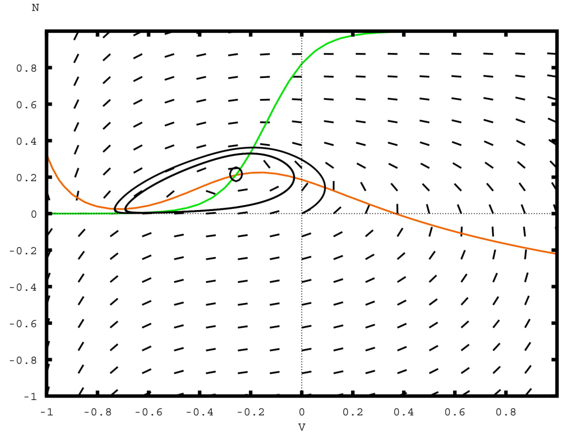
V
0.1
0.0
−0.1
−0.2
−0.3
0
20
40
60
80
100
Time



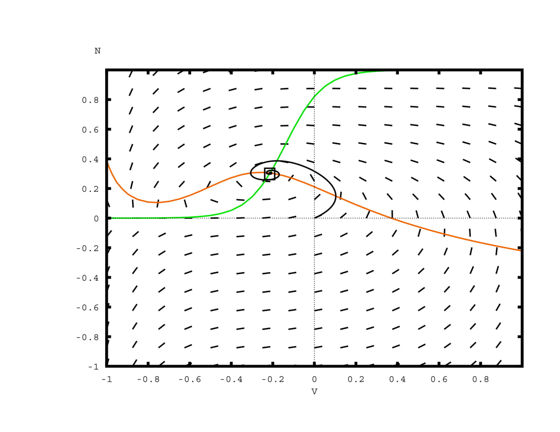
0.0
−0.2
−0.4
−0.6
−0.8
V
0
20
40
60
80
100
Time

Phase plane analysis

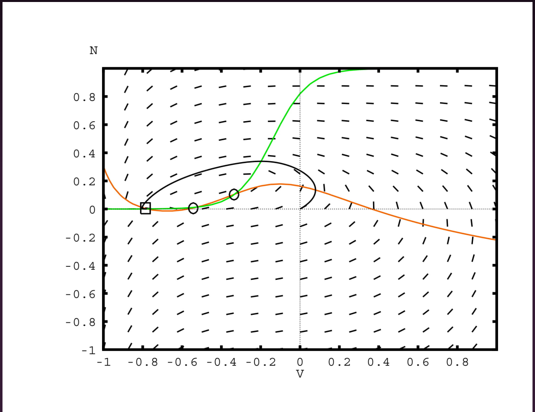
$v_b = -0.8999$



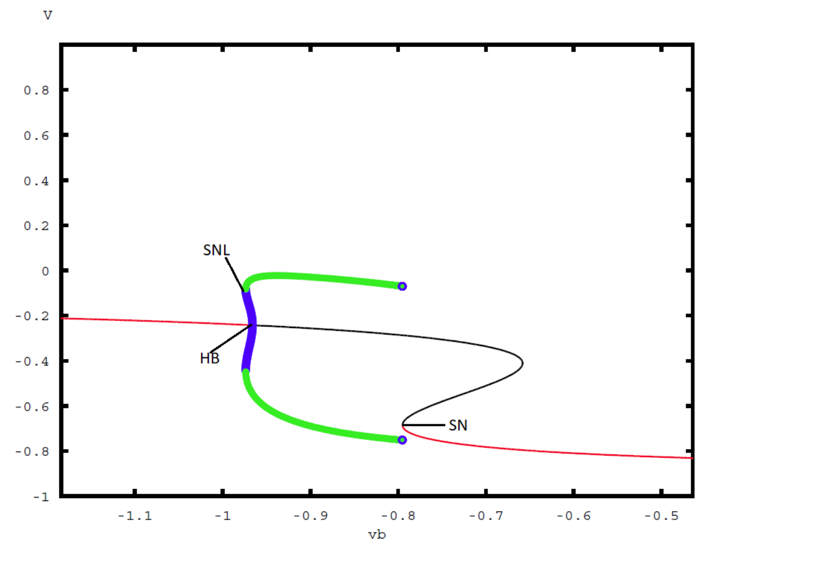
$v_b = -1.184$



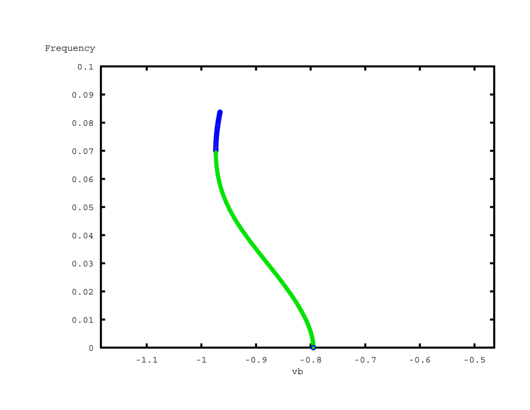
$v_b = -0.7088$



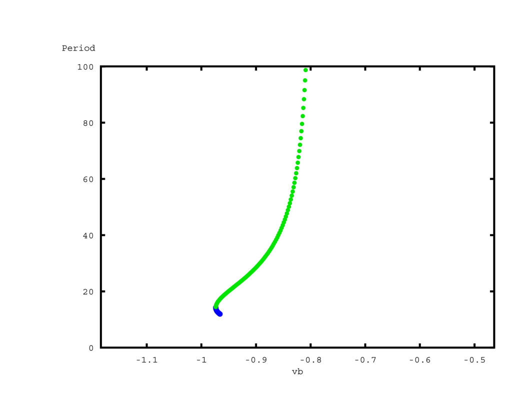
Bifurcation analysis



Frequency



Period



Reaction-diffusion model

$$\frac{\partial V}{\partial T} = D\frac{\partial^2 V}{\partial X^2} - \tilde{g}_L(V - \tilde{v}_L) - \tilde{g}_k N(V - \tilde{v}_k) - \tilde{g}_{ca}\tilde{m}_\infty(V - 1)$$
$$\frac{\partial N}{\partial T} = \psi\lambda(V)(\tilde{n}_\infty - N)$$

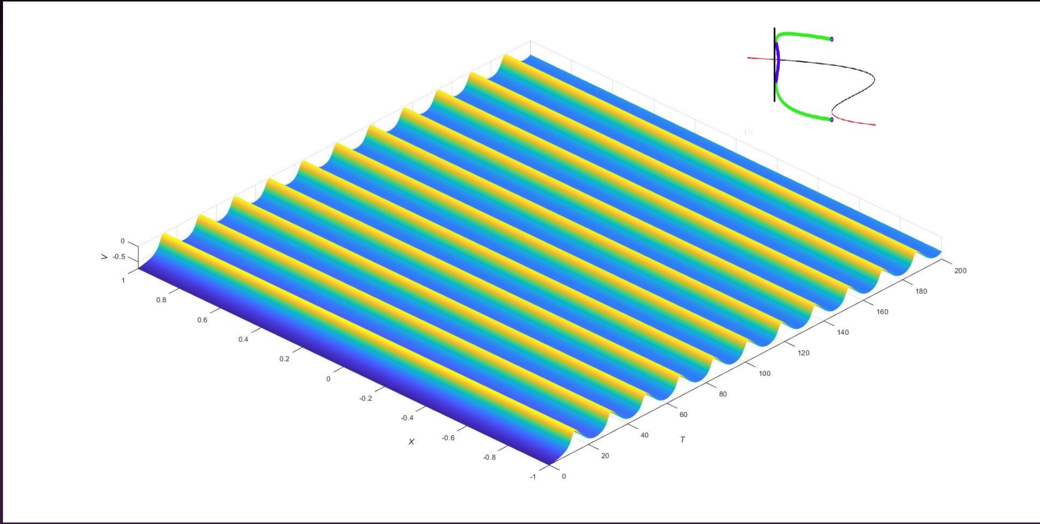
where D is the diffusion coefficient, with no-flux boundary conditions:

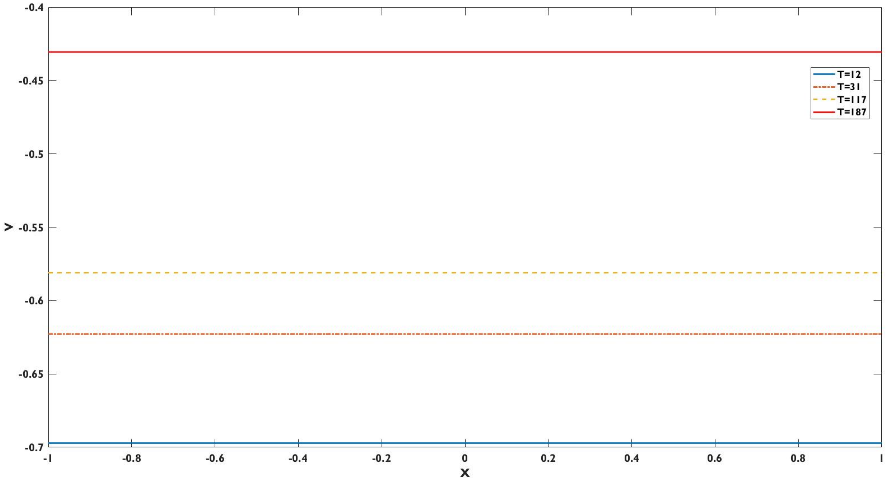
$$\frac{\partial V}{\partial \nu} = \frac{\partial N}{\partial \nu} = 0, \forall\, X \in \partial\Omega$$

and initial conditions:

$$V(0, X) = V_0(X) \text{ and } N(0, X) = N_0(X), \forall\, X \in \Omega$$

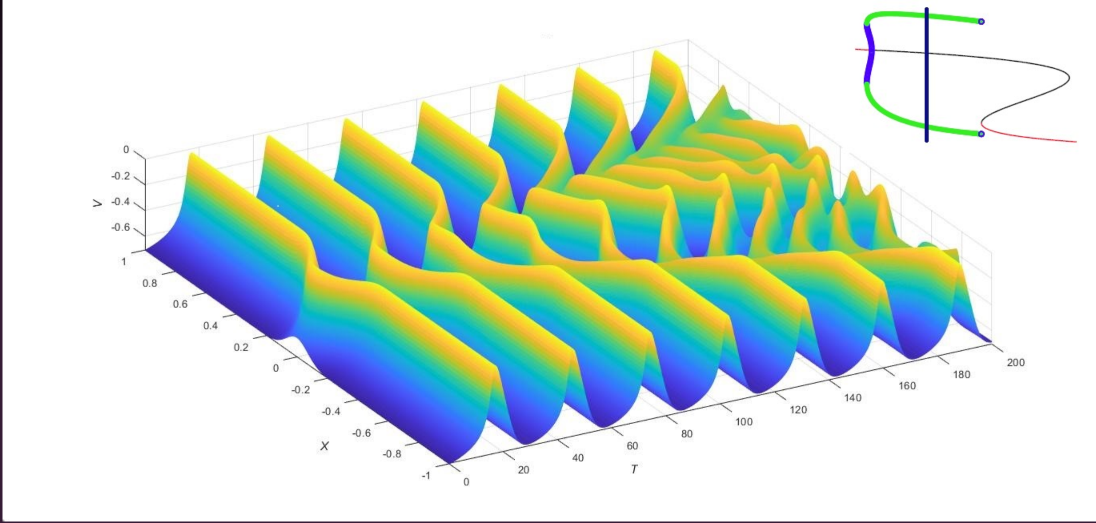
Stationary solutions

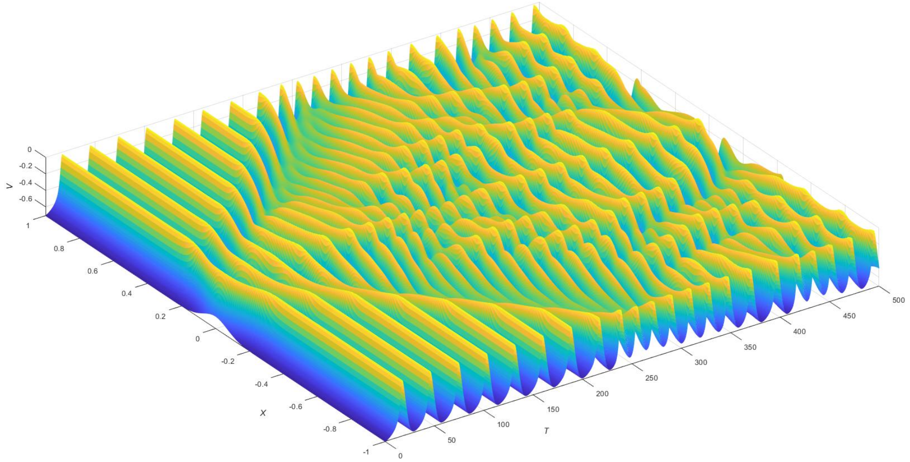




T=12
T=31
T=117
T=187
-0.4
-0.45
-0.5
-0.55
-0.6
-0.65
-0.7
y
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x

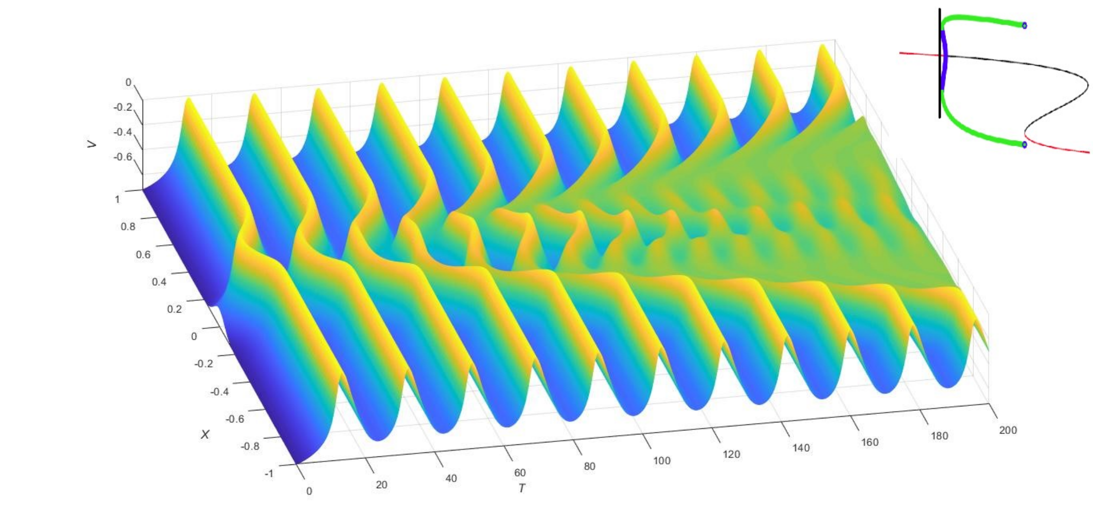
Oscillatory regime

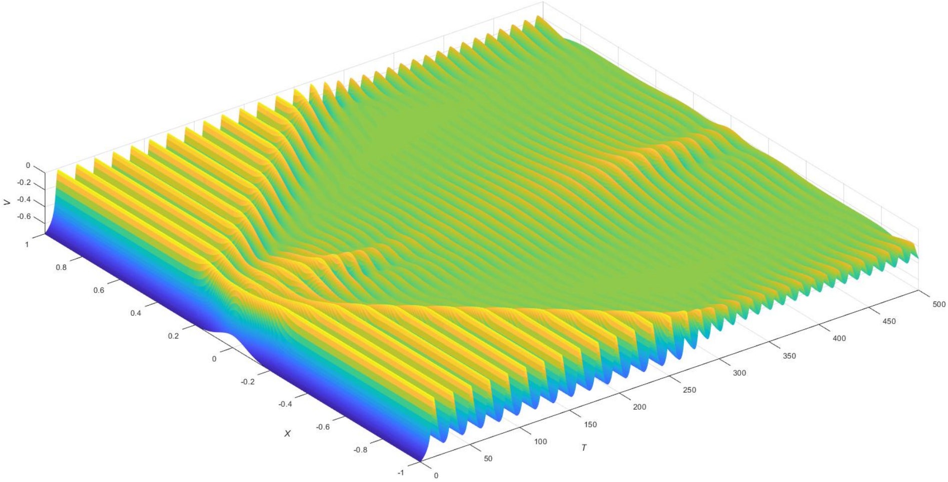




V
0
-0.2
-0.4
-0.6
X
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
T
0
50
100
150
200
250
300
350
400
450
500

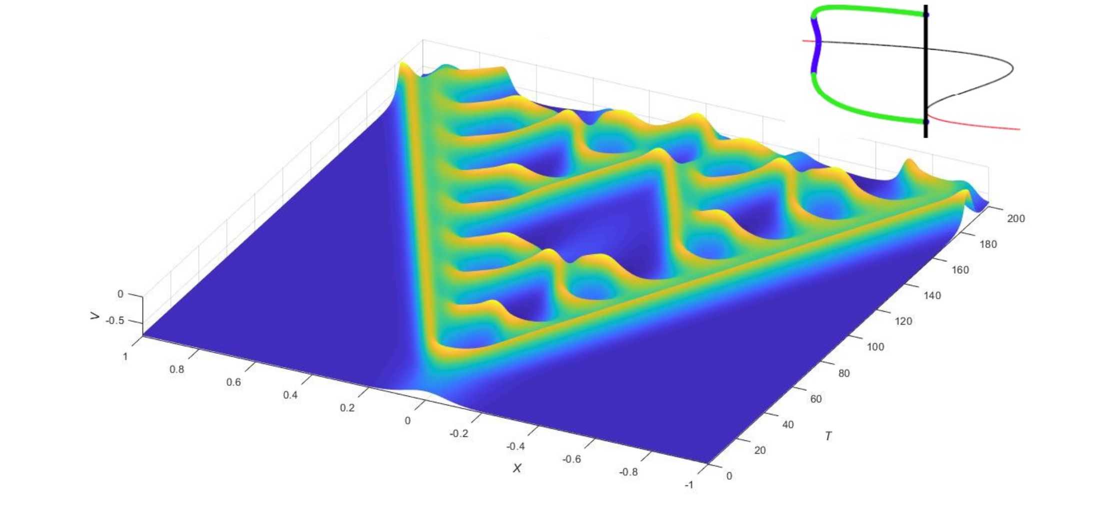
Bistability regime

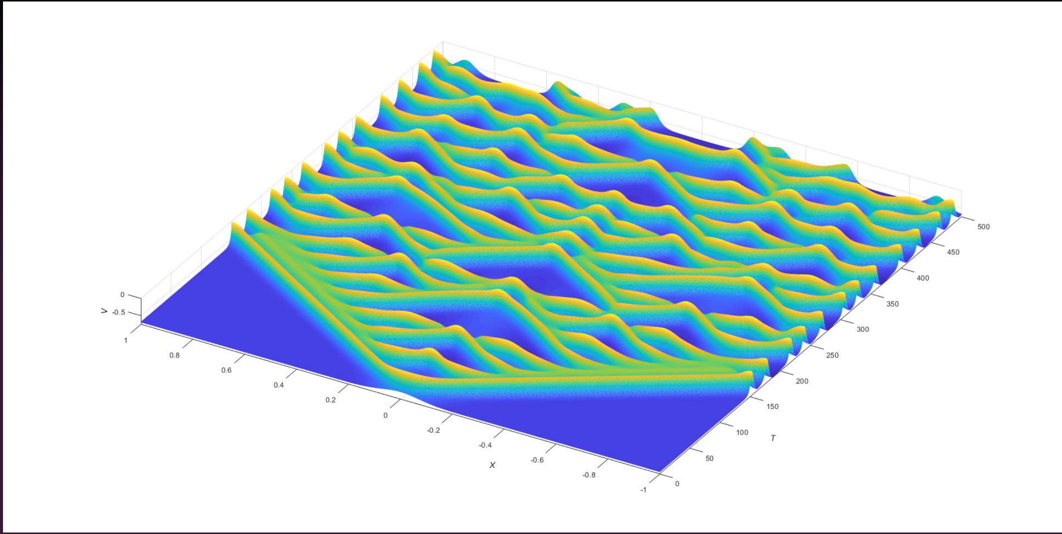




V
0
-0.2
-0.4
-0.6
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
X
0
50
100
150
200
250
300
350
400
450
500
T

Near the SNIC bifurcation





0
V
-0.5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
X
-0.6
-0.8
-1
0
50
100
150
200
250
300
350
400
450
500
T

Conclusion

Summary

- What is the essential origin of the complicated behaviour?
- Are the observed patterns mathematical artifact or physiological reality?

Future work

- Investigate travelling wave solutions
- Explore more on model analysis to understand the rationale behind the observed patterns
- Which type of pattern can we produce by varying other parameters of the model

Thank you for your attention!