## Homework Set #7

## Problem 1

Consider a sequence of random variables  $X_1, X_2, ..., X_i, ...,$  for example denoting the monthly profits of a supermarket chain. Suppose that  $X_i \sim (m, \sigma^2)$  for all i and that the correlation coefficient between  $X_i$  and  $X_j$ ,  $\rho_{ij}$ , depends only on the time lag |i-j| as

$$\rho_{ij} = 0.8^{|i-j|}$$

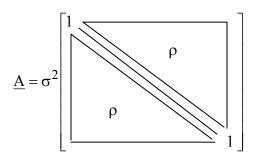
Using conditional SM analysis, calculate and plot, as a function of  $k \ge 1$ , the variances of  $(X_{i+k}|X_i)$  and  $(X_{i+k}|X_{i,1})$ . Comment on the results.

## Problem 2

X is an unknown quantity, say the compressive strength of a concrete column, with mean value m and variance  $\sigma^2$ . Several indirect measurements of X, in the form  $Z_i = X + \varepsilon_i$  for i = 1, ..., n, are made through a nondestructive technique.

Under the assumption that the  $\varepsilon_i$  are iid measurement errors with zero mean and common variance  $\sigma_{\varepsilon}^2$ , use conditional SM analysis to find the variance of  $(X|Z_1,...,Z_n)$ . Plot this conditional variance against n for  $\sigma^2=1$  and  $\sigma_{\varepsilon}^2$  either 1 or 0.2.

Useful result on the inverse of covariance matrices with a special "equicorrelated" structure. The inverse of an  $n \times n$  matrix  $\underline{A}$  of the type:



is:

$$\underline{A}^{-1} = \frac{1}{\sigma^2 (1-\rho)[1+(n-1)\rho]} \begin{bmatrix} [1+(n-2)\rho] & & & \\ & -\rho & & \\ & & -\rho & & \\ & & & [1+(n-2)\rho] \end{bmatrix}$$