Probability and Distribution Unit

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1 Set Notation

Term	Symbol
Empty Set	$\emptyset$
Set of Natural Numbers	$\mathbb{N}$
Set of Integers	$\mathbb{Z}$
Set of Rational Numbers	$\mathbb{Q}$
Set of Real Numbers	$\mathbb{R}$
Set of Complex Numbers	$\mathbb{C}$
Is a member of	$\in$
Is not a member of	$\notin$
Owns	$\ni$
Is a proper subset of	$\subset$
Is a subset of	$\subseteq$
Is a proper superset of	$\supset$
Is a superset of	$\supseteq$
Set Union	$\cup$
Set Intersection	$\cap$

2 Probability and Distribution Notation

Term	Symbol
Event A	A
Sample Space	S
Event A not occurring	A'
Event A or B occurring	$A \cup B$
Event A and B occurring	$A \cap B$
Event A occurring give Event B occurred	$A\|B$
Odds	$h : k$
Number of Outcomes for Event A	$n(A)$
Probability of Event A occurring	$P(A)$
Probability of Success	p
Probability of Failure	q
Probability of Event X = x	$P(X = x)$
Expected Value of Event X	$E(X)$

3 Probability Rules

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A') = 1 - P(A)$$

$$h : k, P(A) = \frac{h}{h+k}$$

Mutually Exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Non-Mutually Exclusive

$$P(A \cup B) = P(A) + P(B)$$

Independent Events

$$P(A \cap B) = P(A) \cdot P(B)$$
$$P(A' \cap B) = P(A') \cdot P(B) = (1 - P(A)) \cdot P(B)$$
$$P(A \cap B') = P(B') \cdot P(A) = (1 - P(B)) \cdot P(A)$$

Dependent Events

$$P(A \cap B) = P(B) \cdot P(A|B) = P(A) \cdot P(B|A)$$
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B|A)}{P(B)}$$
$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B) \cdot P(A|B)}{P(A)}$$

Total Probability Formula

$$P(A) = P(B) \cdot P(A|B) + P(B') \cdot P(A|B')$$
$$P(A) = P(A) \cdot P(B|A) + P(A') \cdot P(B|A')$$
$$P(A) = \Sigma P(B_i) \cdot P(A|B_i)$$

4 Probability Distribution

Random Variable
A random variable (X) has a single value (x) for each outcome in an experiment.

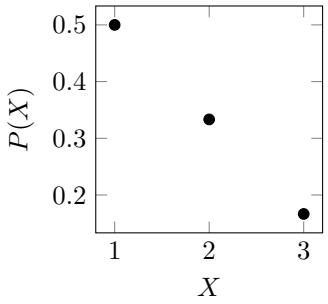
$$P(X), P(X = x_1), x_2, x_3, ..., x_n$$

Representations of Probability Distribution

Numeral

X	P(X)
1	1/2
2	1/3
3	1/6

Graphical



Algebraic

$$P(X = a) = equation, a = 1, 2, ..., n$$
$$P(X = a) = 5a^2, a = 1, 2, 3$$
$$P(X = 2) = 5 \cdot (2)^2 = 20$$

Uniform Probability Distribution
All outcomes are equally likely to occur for all values of X.

$$P(X) = \frac{1}{n(X)}$$

Unitary Condition
Holistic

$$P(X = x_1) + P(X = x_2) + ... + P(X = x_n) = \sum_{k=1}^{n} P(X = x_k) = 1$$

Expectation Expected outcome based on probability.

$$E(X) = x_1 P(X = x_1) + x_1 P(X = x_2) + ... + x_n P(X = x_n) = \sum_{k=1}^{n} x_k P(X = x_k)$$

5 Binomial Distribution

Conditions
Success or failure. All trials are independent and the probability of each trial is the same. The random variable is the number of successes in a given number of trials.

$$P(X) = \binom{n}{x} p^x q^{n-x}$$

$n = \#$ of trials
$x = \#$ of success

Expectation

$$E(X) = n \cdot p$$

6 Geometric Distribution

Success or failure. The probability of success is the same for each observation and observations are all independent. The goal is to find the number of trials until the first success.

$$P(X) = q^x p$$

$x = \#$ of trials before success

Expectation

$$E(X) = \sum_{k=0}^{\infty} k \cdot q^k p = \frac{q}{p} = \frac{1-p}{p}$$

7 Hypergeometric Distribution

Success or failure. Every trial is dependent, meaning the probability of success changes after each trial. The random variable is the number of successful trials in an experiment.

$$P(X) = \frac{\binom{a}{x}\binom{n-a}{r-x}}{\binom{n}{r}}$$

$x =$ given $\#$ of successes
$r = \#$ of dependent trials
$a = \#$ successful outcomes
$n = \#$ total outcomes

Expectation

$$E(X) = n \cdot \frac{a}{a+b} = \frac{r \cdot a}{n}$$