Practice Assignment: Combination with Identical Elements

Andy Yan

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1 Question

How many different sums of money can be formed from one \$2 bill, three \$5 bills, two \$10 bills, and one \$20 bill?

2 Approach

To find the number of distinct ways we can pick from the bills is simple. We can group the bills accordingly to their value.

$$\{(\$2), (\$5, \$5, \$5), (\$10, \$10), (\$20)\}$$

To find the total number of distinct possible picks we, multiply the number of ways to pick from each type of bill. For example, there are 2 ways to pick the \$2 bill, either don't pick it or pick it. Another example is there are 4 ways to pick the \$5 bill, no pick, 1 bill, 2 bills, or 3 bills. So we find the number of ways to pick the other bills and multiply them.

$$2 \cdot 4 \cdot 3 \cdot 2 = 48 \textbf{ distinct ways}$$

To summarize this formula, we can let n_i represent the total numbers in a subset and add 1 to include the option of no selection.

$$(n_1 + 1)(n_2 + 1)...(n_i + 1)...$$

3 Solution

However, 48 distinct ways to pick from the bills don't give us 48 **distinct sums**. For example, picking $\{\$10, \$10\}$ gives us the same sum as picking $\{\$20\}$. To solve this problem, we can combine the excess \$5 bills and \$10 bill into a \$20 bill $\$5 + \$5 + \$10 = \$20$. By doing this, we eliminate the possibility of the smaller bills creating duplicate values since the sum of 2,5,10 is smaller than 20. Our new set looks like this:

$$\{(\$2), (\$5), (\$10), (\$20, \$20)\}$$

Now we can apply the formula, however, we have to subtract 1 to avoid the null set case (\$0 sum):

$$(1 + 1)(1 + 1)(1 + 1)(2 + 1) - 1 = 2 \cdot 2 \cdot 2 \cdot 3 - 1 = 24 - 1 = 23 \textbf{ distinct sums}$$

Therefore, there are **23 different sums of money** formed from one \$2 bill, three \$5 bills, two \$10 bills, and one \$20 bill?