Assignment: GOOOSE!

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1 Question

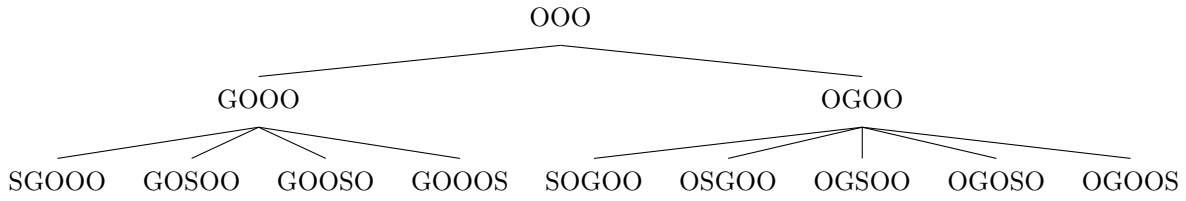
How many ways can you scramble all the letters in **GOOOSE**, such that **G** is always together with **O**, and **S** is never together with **E**?

2 Requirements

1. Sketch a tree of counting plan. [3K]
2. Check your tree diagram to see if you have counted some possibilities more than once.
3. Provide an algebraic solution based on your counting plan. That is, multiply all numbers on a single branch to get the branch number, and then add up all the branch numbers. [4A]
4. As verification, list all possible scrambles in an organized way based on your counting plan. [3K]

3 Counting Plan Tree

We start with our most repeated letter **O**, then we insert **G** so we don't break the rules, and then we insert S so when we are counting we know where **E** can go. All scrambles are reversible starting from the second layer.



4 Solution 1

From our counting plan, we are given all the scrambles (reversible) that don't contain **E**. We notice that all the scrambles that contain **G** in the first slot allow 3 possible slots for **E**. We also can see that if **G** and **S** are together there are only 3 possible slots for **E**. And for the rest of the cases where **G** and **S** are separated, there are only 4 possible slots for **E**. We also know that all of these scrambles are reversible.

$$\#ofScrambles = 2 \cdot (4 \cdot 3 + 2 \cdot 3 + 3 \cdot 4) = 2 \cdot 30 = 60 \textbf{ Scrambles}$$

5 Solution 2

Without using our counting plan, we can solve using combinatorics. First we can group **G** and **O** together into one letter **GO** leaving us with 5 letters to work with.
Our total scrambles is $2 \cdot 5!$ since we can swap **G** and **O**. However, we must take into count that we can't have **S** and **E** next to each other. The number of ways **S** and **E** can be next to each other is $2 \cdot \frac{5!}{2!}$. Now finally we have to divide everything by 2! because we have 3 of the same letter **O** and only two are identical since one is grouped with **O**. So our final formula is:

$$\frac{2 \cdot 5! - 2 \cdot \frac{5!}{2!}}{2!} = 60 \textbf{ Scrambles}$$

6 List of Solutions

egooos
egooso
egosoo
eogoos
eogoso
eogsoo
eoogos
eoogso
eooogs
eoosgo
eoosog
eosgoo
eosogo
eosoog
goeoos
goeoso
gooeos
goosoe
gosoeo
gosooe
oegoos
oegoso
oeogos
oeogso
oeoogs
oeosgo
oeosog
ogeoos
ogeoso
ogoeos
ogosoe
ogsoeo
ogsooe
ooegos
ooeogs
oogeos
oogsoe
oosgoe
oosoge
osgoeo
osgooe
osoego
osoeog
osogeo
osogoe
osooge
sgoeoo
sgooeo
sgoooe
soegoo
soeogo
soeoog
sogeoo
sogoeo
sogooe
sooego
sooeog
soogeo
soogoe
soooge