

## Methods and tools for big data

Manuel – Summer 2023

### Notes

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## 0. Course information

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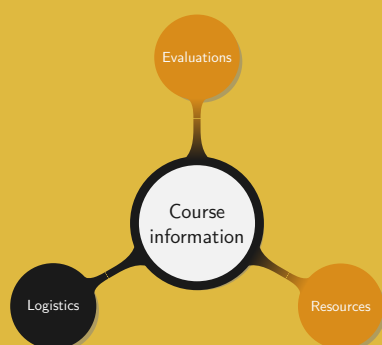
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### Chapter organisation



### Notes

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Basic information

Teaching team:

- Instructor: Manuel (charlem@sjtu.edu.cn)
- Teaching assistant: Yuxuan (zyxdenny@sjtu.edu.cn)

Important rules:

- When contacting a TA for an important matter, cc the instructor
- Prepend [ECE472] to the subject, e.g. Subject: [ECE472] Grades
- Use SJTU jBox service to share large files (> 2 MB)

Never send large files by email

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Course schedule

Course arrangements:

- Lectures:
  - Monday 12:10 – 13:40
  - Wednesday 12:10 – 13:40
- Labs: Friday 18:20 – 20:40

Office hours:

- Anytime on Piazza
- On appointment

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Course objectives

Primary goals:

- Understand how big data sets are analysed in practice
  - Be able to use Hadoop
  - Learn how to work in the Hadoop ecosystem
- Be able to performed advanced data analysis on large data sets
  - Get good foundations on big data analysis
  - Be able to design, implement, and use advanced algorithm in Spark

*Be able to analyse any given dataset, regardless of there size*

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Course workflow

Learning strategy:

- Course side:
  - Understand the new issues appearing as datasets grow
  - Be able to setup a Hadoop cluster and use it
  - Understand why traditional algorithms fail on big data
  - Be able to implement advanced algorithms for big data
- Personal side:
  - Derive algorithms for big data
  - Use and work "inside" Hadoop, Drill, and Spark
  - Relate known strategies to new problems
  - Perform extra research

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Course outcomes

Detailed goals:

- Understand the basic logic behind Hadoop
- Have a general knowledge of the Hadoop ecosystem
- Be familiar with the basic Hadoop components: HDFS, YARN, and MapReduce
- Understand the structure of Drill and Spark
- Be able to work in Hadoop and “extend” its functionalities
- Know what tool to use for common specific purposes related to the study of big data
- Be familiar with common dimension reduction techniques
- Understand the limitations when facing “real” big data
- Be able to run basic data analysis on big data

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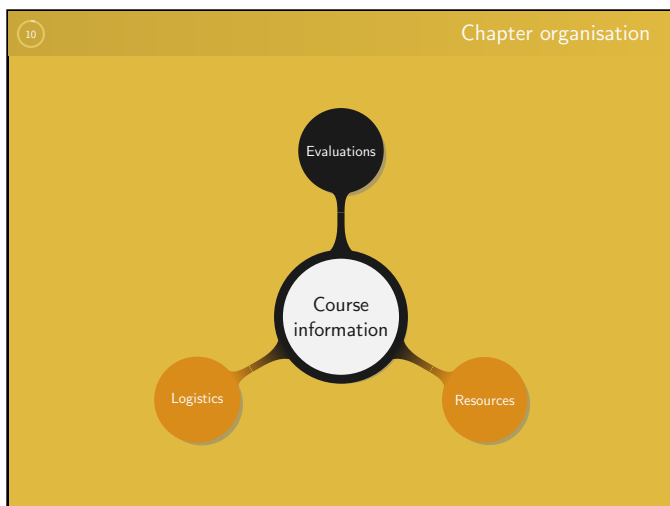
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Assignments

Homework:

- Total: 6
- Content: basic Hadoop, algorithms, Spark

Labs:

- Total: 12
- Content: guided sessions to setup and work with Hadoop, and Spark

Projects:

- Total: 1
- Content: analysis of some big dataset

Challenge:

- Total: 1
- Content: compare theory and practice in Hadoop and Spark implementations

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Grading policy

Grade weighting:

- Midterm exam: 25%
- Final exam: 25%
- Projects: 30%
- Homework: 10%
- Labs: 10%

Assignment submissions: –10% per day, not accepted after 3 days

*Grades will be curved with the median in the range  $\llbracket B, B+ \rrbracket$*

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Honor Code

General rules:

- Not allowed:
  - Reuse the code or work from other students or groups
  - Reuse the code or work from the internet
  - Share too many details on how to complete a task
- Allowed:
  - Reuse part the course or textbooks and quoting the source
  - Share ideas and understandings on the course
  - Provide hints on where or how to find information

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Honor Code

Documents allowed during the exams:

- Midterm: none
- Final: a single A4 paper sheet with original handwritten notes

Group works:

- Every student in a group is responsible for his group's submission
- If a student breaks the Honor Code, the whole group is guilty

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Special circumstances

Contact us as early as possible when:

- Facing special circumstances, e.g. full time work, illness
- Feeling late in the course
- Feeling to work hard without any result

Any late request will be rejected

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Chapter organisation

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graph TD
    CI((Course information)) --- E((Evaluations))
    CI --- L((Logistics))
    CI --- R((Resources))
  
```

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Canvas

Information and documents available on the Canvas platform:

- Course materials:
  - Syllabus
  - Lecture slides
  - Homework
  - Labs
  - Projects
- Course information:
  - Announcements
  - Notifications
  - Grades
  - Polls

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References

Useful places where to find information:

- Hadoop the definitive guide*
- Spark the definitive guide*
- Machine learning, an algorithmic perspective*
- Introduction to Data Mining*, by Tan et al..
- Mining of Massive Datasets*, by Leskovec et al.. by White
- Search information online, i.e.  $\{websites \setminus \{non-English\ websites}\}$

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Key points – Key points

- Work regularly, do not wait the last minute/day
- Respect the Honor Code
- Go beyond what is taught
- Do not learn, understand
- Keep in touch with us
- Advice and suggestions are always much appreciated

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1. Introduction to Hadoop

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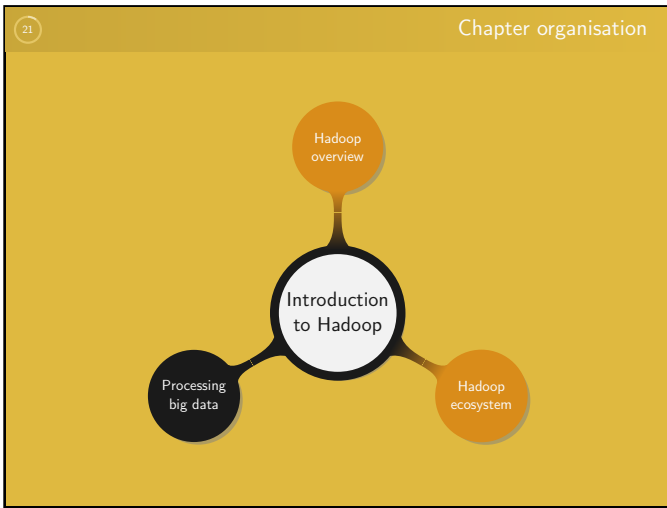
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22 Data analysis

Generated data is often:

- Stored, e.g. in databases
- Preprocessed, e.g. cleaned
- Analysed, e.g. machine learning

Most common advanced analytics:

- Supervised learning: predict a label based on some features
- Recommendation: suggest product based on users' behaviour
- Unsupervised learning: discover structure in the data
- Graph analytics: searching for patterns

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23 Von Neumann bottleneck

Problem for a regular computer:	Mitigating the problem:
<ul style="list-style-type: none"><li>• Fast CPU</li><li>• Large memory</li><li>• Limited throughput</li></ul>	<ul style="list-style-type: none"><li>• Use caching</li><li>• Apply branch prediction</li><li>• Parallel read using RAID</li></ul>

Example. The speed of a disc read decreased relatively over time:

- 1990: 1.5 GB HDD at 4.4 MB/s
- Today: 1 TB HDD at 100 MB/s

*Scanning a whole disc in 1990 took 5 min, today it takes over 2.5 h!*

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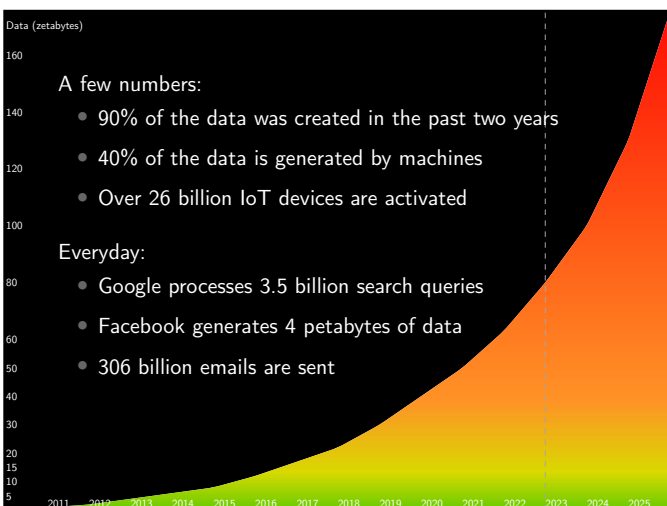
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# How to store and process data as it grows very big?

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## Storing data

### Relational Database Management Systems:

- Data size: gigabytes
- Access: interactive and batch
- Update: read/write small proportions of the data
- Structure: schema defined at writing time
- Efficiency: low-latency retrieval for small amount of data

### Limitations of databases:

- Hard drive seek time increases slower than data transfer rate
- Data is often unstructured
- Slow to process as designed for read/write many times

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## Processing data

### High-performance computing (HPC):

- Distributes computation across a cluster of machines
- Uses message passing interface
- Fits compute-bound jobs
- Data-flow controlled by programmer

### Limitations of HPC:

- Handling of node or process failure
- Require very high network bandwidth
- Expensive infrastructures, complex to extend
- Low level APIs

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## Sequential vs. parallel algorithms

### Random Access Machine (RAM) model:

- A processor with a memory attached to it
- Each operation has a constant cost
- Runtime is proportional to the number of operations

### Parallel Random Access Machine (PRAM) model:

- Several processors with one or more memory modules attached
- Need to specify how to deal with concurrent writes
- Each operation has a constant cost
- Runtime is defined when the slowest processor completes

*When dealing with "real" big data we need a distributed system*

## Notes

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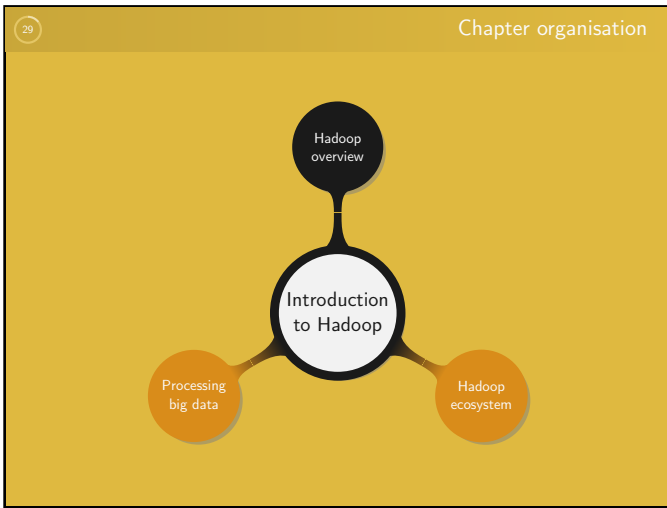
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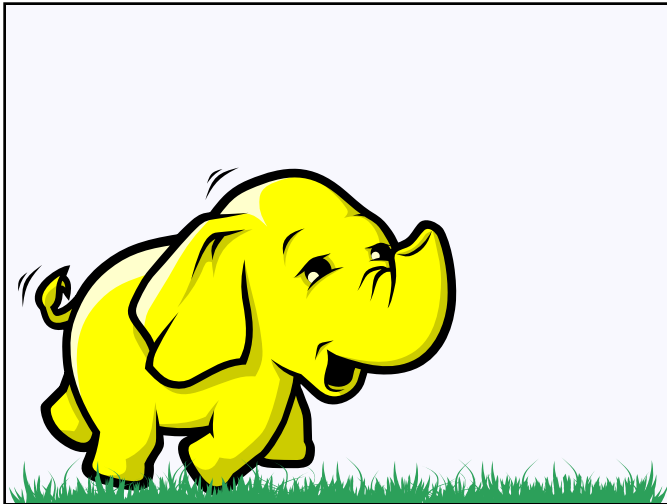
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31 A short history

The birth of Hadoop:

- 2002: Nutch, an open source web search engine
- 2003: paper describing Google File System (GFS)
- 2004:
  - NDFS: open source implementation of GFS for Nutch
  - Paper describing data processing on large clusters (MapReduce)
- 2005: open source implementation of MapReduce for Nutch
- 2006:
  - NDFS and MapReduce moved out of Nutch
  - Hadoop 0.1.0 released
  - Hadoop is run in production at Yahoo!

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32 A short history

The adolescence of Hadoop:

- 2007 – 2008:
  - Number of companies using Hadoop jumps from 3 to over 20
  - Creation of Cloudera, first Hadoop distributor
- 2009:
  - MapR, new Hadoop distributor
  - HDFS and MapReduce become separate projects
- 2010 – 2011:
  - Many new “components” added to the Hadoop ecosystem
  - Receive two prizes at the Media Guardian Innovation Awards

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A short history

The maturity of Hadoop:

- 2012:
  - Hadoop 1.0 released
  - YARN ready to replace MapReduce (Hadoop 2.0)
- 2013 – 2014:
  - More than half of the Fortune 50 use Hadoop
  - Spark and Drill added to the Hadoop ecosystem
- 2017: Hadoop 3.0 released

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Hadoop's goals

Context where to adopt Hadoop:

- Massive amount of data to analysed
- Data stored over hundreds or thousands of computers
- Computation must be completed even if some nodes fail
- Cluster composed of commodity or high-end hardware

Hadoop's records:

- 2006: sort 1.8 TB of data in less than 48 h
- 2008: sort 1 TB of data in 209 s
- 2009: sort 1 TB of data in 62 s
- 2014: sort 100 TB of data in less than 23 min 30s

*The end goal is to efficiently analyse massive amount of data*

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Hadoop framework

Hadoop is composed of core modules:

- Hadoop common: base libraries and utilities used by other modules
- Hadoop Distributed File System (HDFS): distributed file system
- Hadoop MapReduce: implementation of the MapReduce model
- Apache Yet Another Resource Negotiator (YARN): manages the cluster resources and schedules the user's tasks

Languages:

- Mainly Java
- Some C
- Shell scripts for command line utilities

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HDFS overview

Characteristics of HDFS:

- Large files: at least hundreds of megabytes to terabytes
- Streaming data access: write once, read many times
- Commodity hardware: inexpensive common hardware

Limitations of HDFS:

- High throughput at the expense of latency
- The "Master node" keeps the filesystem metadata in memory
- Write always in append mode, by a single writer

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MapReduce overview

Programming paradigm composed of three main steps:

- Map:
  - A master node distributes the work and ensures exactly one copy of the redundant data is processed
  - Each worker node considers its local data and transforms it into key-value pairs
- Shuffle: each worker node redistributes its pairs based on the keys
- Reduce: each worker node combines a set of pairs into a smaller one

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MapReduce overview

MapReduce requirements:

- Mapping operations must be independent of each others
- Parallelism is limited by the number of sources and nearby CPUs
- Either all the output sharing the same key must be processed by a single reducer or the reduction must be associative

MapReduce benefits:

- Highly scalable on commodity hardware
- Possible to recover for partial failure
- Great efficiency due to parallelism

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YARN overview

A *container* is an environment with restricted resources where application-specific processes are run

YARN provides two types of daemons:

- Resource manager:
  - One per cluster
  - Manages the resources for the whole cluster
- Node manager:
  - One per cluster node
  - Launches and monitors containers

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MapReduce vs. YARN

In Hadoop 1, MapReduce:

- Directly interacts with the filesystem
- Manages resources

In Hadoop 2, YARN:

- Manages the resources
- Interacts with the filesystem
- Hides low level details from the user
- Offers an intermediate layer supporting many other distributed programming paradigms

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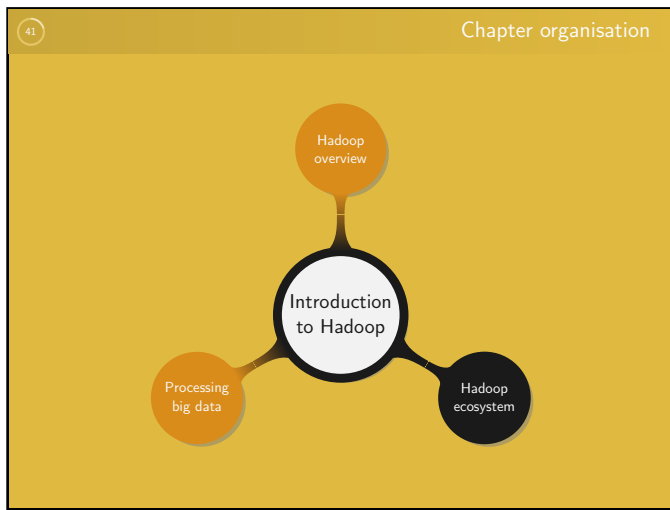
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42 Mesos

Goal: global scalable resource manager, not restricted to Hadoop

Mesos scheduling:

- Determine the available resources
- Offer “various options” to an application scheduler
- Allow any number of scheduling algorithm to be developed, plugged, and used simultaneously
- Each framework decides what scheduling algorithm to use
- Mesos allocates resources across the schedulers, resolves conflicts, and ensures a fair share of the resources

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43 Myriad

Goal: use Mesos to manage YARN resource requests

Simplified strategy:

- 1 A job requests resources to YARN
- 2 YARN uses the Myriad scheduler to allocate resources
- 3 Myriad scheduler matches requests to Mesos’ resources offers
- 4 YARN allocates the resources

Benefits:

- Get the best from both worlds
- Give more flexibility to YARN

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44 Spark

Goals:

- Be a full replacement for MapReduce
- Efficiently support multi-pass applications
- Write and read from the disk as little as possible
- As much as possible take advantage of the memory

Main ideas:

- Resilient Distributed Dataset (RDD): contains the data to be transformed or analysed
- Transformation: modifies an RDD into a new one
- Action: analyses an RDD

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Drill

Goals:

- Integrate into Hadoop as a MapReduce replacement
- Be an interactive ad-hoc analysis system for read-only data
- Be easily expandable using storage plugins
- Enjoy data agility

Main ideas:

- Columnar execution: shredded, in-memory columnar data representation
- Runtime compilation and code generation: compile and re-compile queries at runtime
- Optimistic execution: stream data in memory to minimise disk usage

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Spark vs. Drill

When to use Spark or Drill:

- Drill is an ANSI SQL:2003
- Spark has SQL query capabilities
- Drill allows fine grained security at the file level
- Drill is best used as a distributed SQL query engine
- Spark is best used to perform complex math, statistics, or machine learning

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Flink

Basics on Flink:

- Allows the execution of dataflow programs following a data-parallel and pipelined approach
- Provides a high-throughput and low-latency streaming engine
- Nicely handles node failures
- Can connect to various storage types

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Tez

Basics on Tez:

- Targets batch and interactive data processing applications
- Intends to improve MapReduce paradigm
- Exposes more simple framework and API to write YARN applications
- Expresses computation as a dataflow graph

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HBase

Basics on HBase:

- NoSQL database system for distributed filesystems
- Low latency access to small amount of data in a large data set
- Fast scan across tables
- Random access to rows

Common use cases:

- Applications requiring sparse rows
- Not good for relational analytics and transactional needs

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Hive, Spark SQL, and Presto

Basics on Hive:

- Access SQL data in HDFS using an SQL-like query language HQL
- Convert queries to MapReduce, Tez, or Spark jobs
- Warning: does not fully comply to ANSI-standard SQL

Basics on Spark SQL (formerly Shark):

- Was initially a port of Hive to Spark
- Follows Spark in-memory computing model
- Is "mostly" compatible with HQL

Basics on Presto:

- Supports ANSI-SQL standard
- Uses a custom engine, not based on MapReduce
- Can access various data sources through storage plugins

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Serialization and storage

Avro:

- Input: a schema describing the data and the data
- Output: generates the code to read/write data

Parquet:

- Columnar storage format
- Complex to handle

Java Script Object Notation (JSON):

- Not part of Hadoop
- Often preferred to XML by Hadoop community
- Represent data using key-value pairs

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Management and monitoring

Ambari:

- Production-ready, easy to use web-based GUI for Hadoop
- Eases the installation and monitoring of a cluster

Zookeeper:

- Effective mechanism to store and share small amounts of states and configuration across the cluster
- Not a replacement for any key-value store
- Has built-in protections to prevent using it as large data-store
- Used as a coordination service

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Analytics helpers

Major analytics helpers:

- Pig: high-level language to speak to MapReduce
- Hadoop streaming: write mappers/reducers in any language
- Mahout: set of scalable machine-learning algorithms for Hadoop
- MLlib: similar to Mahout, based on Spark (maintenance mode)
- Spark ML: similar to MLlib based on a higher level API
- Hadoop Image Processing Interface: package allowing to examine images and determine their differences and similarities

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Data transfer

Moving data to and from Hadoop:

- Sqoop: transfer data between HDFS and relational databases
- Flume: distributed system for collecting, aggregating, and moving large amount of data from various sources into HDFS
- Distributed Copy (DistCP):
  - Part of basic Hadoop tools
  - Used to move data between the clusters
  - Is the basis for more advanced Hadoop recovery tools

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From batch to realtime processing

Lambda data architecture:

- Setup three layers:
  - Batch layer: store all incoming data and batch process it
  - Speed layer: analyse incoming data in real time
  - Serving layer: serve curated data that can be analysed by other tools
- Drawback: maintain two code sets for batch and speed layers

Kappa data architecture:

- Not a replacement but an alternative to lambda architecture
- Layers: batch layer is removed compared to lambda architecture
- Suitable for systems with strict end-to-end latency requirements
- Drawback: replay the whole stream in case of error

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Realtime processing

Apache Storm:

- Distributed system for real-time processing of streaming data
- Able to process over a million records per second per cluster node
- Relies on Zookeeper for coordinating the nodes

Apache Kafka:

- Distributed platform used to create real-time streaming data pipelines
- Heavily relies on zerocopy (OS kernel level) to move data around
- Commonly used together with Spark, Flink, or Storm

Remote Dictionary Server (Redis):

- In-memory key-value data-store
- Extremely fast, simple, and versatile
- Benchmarked as the fastest DB in the world

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AI and ML

Ray:

- Spark's goal was to replace Hadoop and Ray's goal is to replace Spark
- Run fast machine learning or deep learning-based applications
- Hope to reach MPI power and granularity levels

TensorFlow:

- Python-friendly library for fast and easy machine learning computation
- Data is stored is a tensor
- Create dataflow graphs where the edges are tensors and vertices mathematical operations
- Can run on Hadoop (TonY), Spark, and Ray

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Hadoop ecosystem in VE472

Applications

MapReduceDrillSpark...

YARN

HDFS

Linux

Commodity hardware...

Linux

Commodity hardware...

Linux

Commodity hardware

Refer to Hadoop ecosystem table for more details

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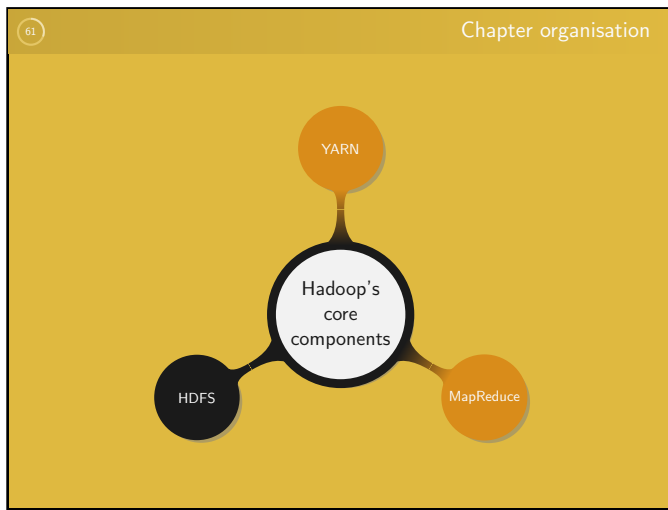
Key points

- What is big data?
- List limitations of common solutions when analysing large data sets?
- What is Hadoop?
- What are Hadoop's goals?
- Briefly describe the Hadoop ecosystem

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2. Hadoop's core components

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62 Distributed filesystems

Regular filesystems on a computer:

- Partition
- Hard drive
- LVM

Distributed filesystems:

- Spans several computers
- Has to deal with potential network issues

*Idea behind HDFS summarised on slide 1.36*

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63 File blocks

Blocks in HDFS:

- Default size of 128 MB
- Files smaller than a block size do not occupy the whole block
- A file can be larger than a whole disk
- Data and metadata handled separately
- Easy to implement fault tolerance and availability

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64 Nodes

Two types of node in a cluster:

- Namenode:
  - Maintains FS tree and metadata for all files and directories
  - Locally stores information in namespace image and edit log
  - Knows on which datanodes the blocks of a file are located
- Datanode:
  - Stores and retrieve blocks
  - Regularly reports the list of stored blocks to namenode
  - Can store certain blocks in cache

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Finding a file

A namenode has no persistent copy of where blocks are:

- Each datanode announces the blocks it has
- All the information is kept in memory by the namenode
- When a write occurs an entry is added to the edit log

*What to do if the namenode fails?*

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Memory limitations

A namenode stores all the blocks of all the files in its memory:

- Assume 1 GB of memory for 1 million blocks
- 200 nodes cluster, 24 TB each: ~12 GB of memory
- Cluster at Yahoo!: 25 PB → ~64 GB of memory
- Cluster at Facebook: 60 PB → ~156 GB of memory

*How about having more namenodes?*

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HDFS federation

Allowing more namenodes:

- Split the filesystem over several independent namenodes
- Each namenode has a namespace
- Each namespace has its own pool of blocks
- A namespace with a block pool is called namespace volume
- A datanode is not attached to a specific namespace volume

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High availability

Two namenodes in an active-passive mode:

- Passive node takes over in case of failure of the active one
- The two namenodes share the same edit log
- Only the active namenode can write to the edit log
- Passive namenode reads entries when written in edit log
- Datanodes send block reports to both namenodes
- Passive namenode also works as secondary namenode
- Clients must be configured to handle namenode failures

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69

Cluster balancing

The goal is to evenly spread blocks across the whole cluster:

Replicas' location:

- First: same node as the client
- Second: random, different rack from the first
- Third: same rack as the second but different node
- Others: random nodes in the cluster

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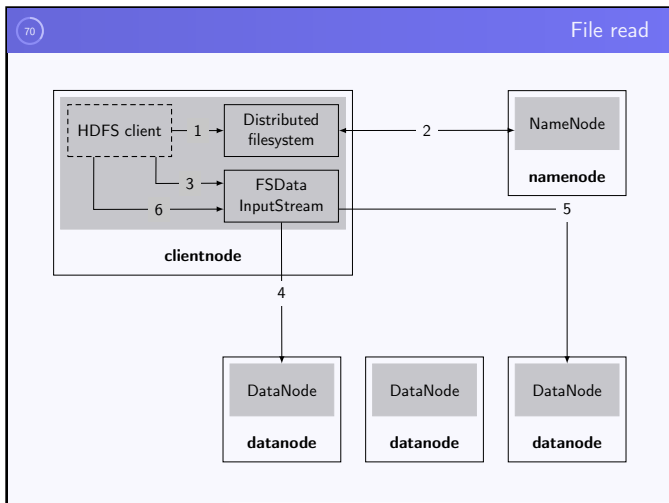
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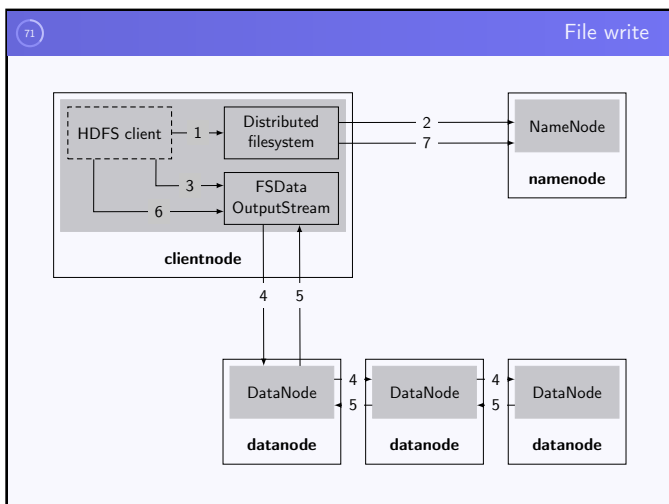
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72

Failing file write

When write on a data node fails:

- 1 Close the pipeline
- 2 Add packets in front of the acknowledgment queue
- 3 Inform the name node of the failing data node
- 4 Remove the faulty data node from the pipeline
- 5 Construct a new pipeline using only the healthy data nodes
- 6 Complete the writing of the block across the pipeline
- 7 Arrange for the replication of under-replicated blocks

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73
Other filesystems

Alternative filesystems built on top of Hadoop abstract filesystem:

- HAR: pack HDFS files into an archive
- View: used to create mount points for federated FS
- FTP: FS backed by an FTP server
- S3: FS based on Amazon S3
- Azure: FS developed by Microsoft Azure

Interfaces to access HDFS data:

- libhdfs: C library with API similar to the Java one
- NFS: use Hadoop NFS gateway
- FUSE: based on libhdfs

Notes

74
Chapter organisation

```

graph TD
    YARN((YARN)) --- Core((Hadoop's core components))
    HDFS((HDFS)) --- Core
    MapReduce((MapReduce)) --- Core
    
```

Notes

75
Running an application

```

graph TD
    subgraph Client_node [Client node]
        AC[Application client]
    end
    subgraph RM_node [Resource manager node]
        RM[Resource manager]
    end
    subgraph NM1_node [Node manager node]
        NM1[Node manager]
        subgraph Container1 [Container]
            AP[Application process]
        end
    end
    subgraph NM2_node [Node manager node]
        NM2[Node manager]
        subgraph Container2 [Container]
            AM[Application master]
        end
    end
    AC -- 1 --> RM
    RM -- 2 --> NM2
    NM2 -- 3 --> RM
    NM2 -- 4 --> NM1
    NM1 -- 4 --> AP
    
```

Notes

76
Requesting resources

A request has two components:

- Amount of resources for each container
- Preferred location of the containers

Common strategy for a request:

- Announce all the resources requests at the beginning
- Dynamically request resources based on the needs

Notes

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Categories of application

YARN can be used in three ways by applications:

- One application per user job: simplest model
- One application per user session:
  - Containers can be reused between jobs
  - Possibility to cache data between jobs
- Long-running application shared among users:
  - Application master is always "on"
  - Application master acts as a coordinator for other applications

Notes

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YARN schedulers

Three schedulers available in YARN:

- FIFO: request served one by one in a queue
- Capacity:
  - Define queues based on the "size" of the jobs to complete
  - All the jobs start early
  - Resources are wasted when unused
- Fair:
  - Resources are dynamically balanced over all the jobs
  - All the resources are fully used
  - Delay due the resource reallocation

Notes

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Capacity scheduler

Capacity scheduler setup:

- Each queue is handled as a FIFO
- Queue elasticity
  - A single job cannot exceed the capacity of the queue
  - The capacity can be exceeded when several jobs wait and resources are available
- Containers are not preempted
- Can control:
  - The number of resources per user/application
  - The number of applications that can be run at a time
  - Access Control Lists (ACL) on the queues

*The challenge is to find a reasonable trade-off for the capacity*

Notes

80

Fair scheduler

Fair scheduler queues:

- One or more queues allowed:
  - Single queue: resources are fairly shared among all applications
  - Several queues, each having:
    - Its own scheduling policy
    - A max/min resources and number of applications
- Queues can be precisely configured using an allocation file
- By default a queue is dynamically created for each user

Notes

81

Fair scheduler

Preemption in the fair scheduler:

- Efficiency is reduced as killed containers must be restarted
- Two timeout settings  $t_1$  and  $t_2$  to trigger preemption:
  - Minimum share: if a queue waits longer than  $t_1$
  - Fair share: if a queue remains below half of its fair share for longer than  $t_2$
- Timeouts can be set per queue or globally

Notes

82

Delay scheduling

Locality problem when scheduling:

- An application requests a specific node
- The node is busy
- Should the application wait for the node or loosen its request?

YARN schedulers' approach:

- Every second each node manager sends a heartbeat reporting the running containers and available resources
- Capacity scheduler: wait for a predefined number of heartbeats before loosening the requirement
- Fair scheduler: wait for a predefined portion of nodes in the cluster to offer opportunities before loosening the requirement

Notes

83

Dominant resource fairness

*How to fairly share resources between applications when they do not use the same type of resources?*

Basic idea for two applications:

- Consider the proportion of resources requested for a container by each application
- Call the largest proportion the dominant resource and use it as measure of cluster usage
- Proportionally offer less containers to the more demanding application

Notes

84

Chapter organisation

```
graph TD; YARN --- Hadoop((Hadoop's core components)); HDFS --- Hadoop; MapReduce --- Hadoop;
```

Notes

86
MapReduce job

Parties involved in a MapReduce job:

- A client which initiates the job
- YARN resource manager
- YARN node manager
- MapReduce application master
- HDFS

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86
Job initialisation

Starting a MapReduce job:

- 1 Request a new application ID to the resource manager
- 2 Check the job parameters
- 3 Split the job into subtasks
- 4 Copy the splits and other necessary information to run the job onto the shared FS
- 5 Effectively submit the job on the resource manager

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87
Job startup

Running a MapReduce job:

- 1 YARN scheduler allocates a container
- 2 Application master launched by the resource manager
- 3 Setup the tasks
- 4 Retrieve the splits from the shared FS
- 5 Create a Map task for each split and specify the number of tasks for the Reduce part
- 6 Resources for
  - a Small tasks: run on the same node
  - b Large tasks: contact the resource manager for more containers
    - i Request resources for the maps (high priority)
    - ii Request resources for the reducers when enough maps have completed
- 7 Locate the data on the distributed FS and start the task

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88
Job failure

Potential points of failure:

- Task failure
- Application master failure
- Node manager failure: YARN level (no heartbeat received)
- Resource manager failure: YARN level (high availability mode)

Protection and progress monitoring:

- Tasks are run in a separate JVM: avoid crashing namenode
- Each task has a status and some counters
- Each task reports its progress to the application master
- Client application polls the application master every second to retrieve the latest status

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89

Task failure

Task failure occurrences and solutions:

- The map or reduce task fails:
  - When receiving a failure notice the application master marks the task as failed
  - The container is freed and resources released
- The JVM crashes: the node manager notices the application master of the failure
- A task hangs:
  - Tasks marked as failed if no report is received
  - The JVM is killed by the application master

*Failed tasks are rescheduled on a different nodemanager*

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90

Application master failure

Failure detection and recovery:

- The application master sends periodic heartbeats to the resource manager
- On failure a new instance is run on a new container
- The tasks progress is known so it is possible to resume without re-running the completed tasks
- Each task caches the application master's address
- Use a timeout after which the resource manager is contacted for the new application's master address

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91

Map and Reduce

The diagram illustrates the Map and Reduce tasks. The **Map task** takes **Input** (1) and processes it through a **Map function** (2). The output is written to a **Buffer in** memory (3). The buffer is then flushed to **Other reduces** (4). The **Reduce task** receives data from **Other maps** (4) and processes it through a **Reduce function** (6). The output is written to **Output** (7). The diagram also shows a **Mix of in-memory and on-disk** data (5) and a **Buffer in** memory (3).

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92

Speeding up a MapReduce job

Optimized configuration setup:

- Provide shuffle with as much memory as possible
- Keep enough memory for map and reduce functions
- Optimize the code with respect to memory consumption
- Minimize the number of spills for the map part
- As much as possible keep intermediate reduce data in memory

Speculative execution:

- A task is detected as much slower than average
- Re-run it on a different node
- Kill all the other duplicates as soon as one completes

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Key points

- Explain the basic structure of Hadoop
- What are the main principles behind HDFS?
- How is YARN used by higher level applications?
- Describe the structure of a MapReduce job

Notes

3. Drill, Spark, and more

Notes

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Chapter organisation

```
graph TD; A((Drill, Spark, and more)) --- B((Spark)); A --- C((Drill)); A --- D((More tools));
```

Notes

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Drill job

Parties always involved in a Drill job:

- A client which initiates the job
- Zookeeper

Parties optionally involved in a Drill job:

- YARN
- HDFS
- Hive
- HBase

Notes



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Drill on YARN workflow

Running Drill as a YARN application:

- 1 Start Drill on the client machine
- 2 Upload resources to the FS and request resources for the application master
- 3 Ask a node manager to prepare and start a container for the application master
- 4 The application master contacts the resource manager to obtain more containers

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Drill on YARN workflow

Running Drill as a YARN application:

- 5 Request the start of Drill software on each assigned node
- 6 Start a "Drill process" called a *drillbit*
- 7 Each drillbit starts and registers with Zookeeper
- 8 The application master checks the health of each drillbit through Zookeeper
- 9 Use Zookeeper to retrieve information on the drillbits, run queries, etc.

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Drillbit

Foreman drillbit:

- Drillbit that receives the query
- It drives the entire query

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Logical plan

Initial SQL query:

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1 SELECT course, student_id, gpa
2 FROM clist.json l, hive.students s
3 WHERE l.student_id = s.student_id

```

Logical plan:

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101
Drill query execution

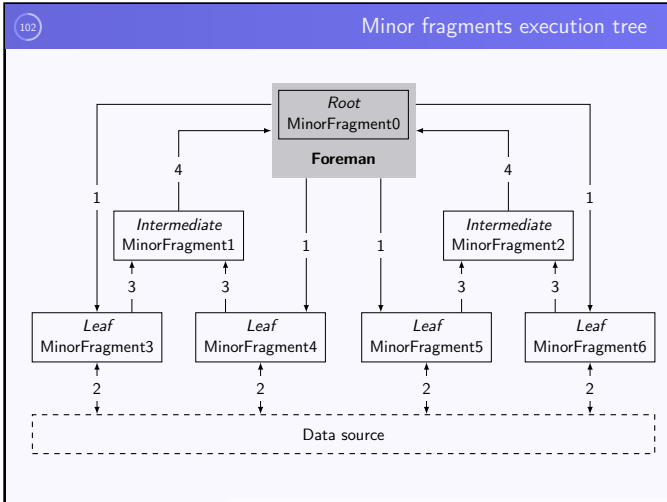
Major fragment:

- Concept representing a phase of the query execution
- Composed of minor fragments

Minor fragment:

- Logical unit of work running in a thread
- Contain one or more relational operators
- Usually as numerous as the number of available drillbits
- Scheduled based on data locality when possible and round-robin otherwise

Notes



Notes

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Drill's strength

Architecture:

- No central server, no master-slave concept
- Each drillbit contains all the services and capabilities of Drill
- Nodes can be added or removed at no cost

Columnar execution:

- Avoid access for columns not involved in the query
- Directly performs SQL processing on columns

Notes

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Drill's strength

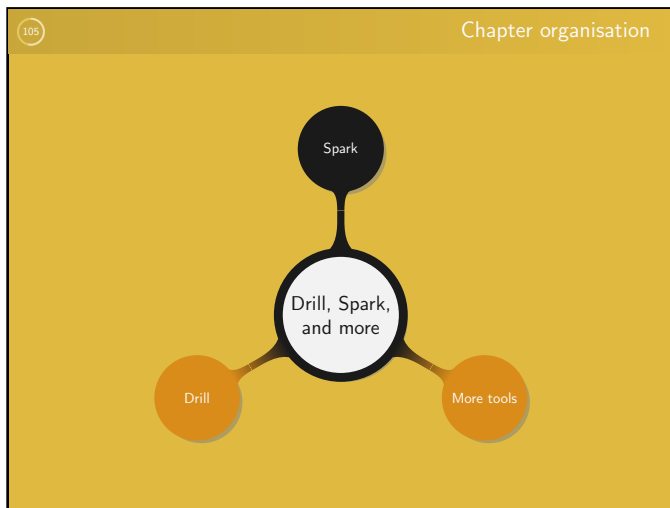
Optimistic query execution:

- Assume no failure will occur during query execution
- Rerun the query in case of failure
- Only write on disk when memory overflows

Vectorization: allow the CPU to operate on vectors

Runtime compilation: generate efficient code for each query

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106 Spark job

Spark organisation:

- Application: user program built on Spark and composed of:
  - A driver program: process running the main function
  - Executors: processes launched for an application on worker nodes
- The driver program connects to a cluster manager
  - Standalone: cluster manager provided with Spark
  - YARN
  - Mesos
  - Kubernetes

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107 Spark on YARN workflow

Two modes available:

- Client mode:
  - Driver runs in the client
  - Required in the case of interactive programs
  - Useful when building a Spark program
- Cluster mode:
  - The entire application runs in the cluster
  - Appropriate from production jobs
  - YARN application master failure strategy (slide 2.90) is applied

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108 YARN client mode

Spark job workflow:

- 1 Start the driver program on a client node
- 2 The driver requests a container to the resource manager
- 3 A container starts and runs an Executor Launcher application master
- 4 The Executor Launcher requests more resources to start Executor backends processes in new containers
- 5 Each Executor Backend registers with the driver

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YARN cluster mode

Workflow similar to client mode but:

- The driver program runs in a YARN application master process
- The client submit a job but does not run any user code
- The application master starts the driver program
- The driver program “replaces” the Executor Launcher

Remark. Data locality:

- Executors are launched before data locality information is available
- The driver can optionally specify preferred locations

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Resilient Distributed Dataset

Resilient Distributed Dataset (RDD):

- Core abstraction in Spark
- Collection of objects distributed across a cluster:
  - Read-only: do not alter a dataset, transform it into a new one
  - Resilient: no disk write, reconstruct the RDD in case of partition loss
- Loaded as input:
  - Created from an external dataset
  - From an existing RDD
  - Parallelising an existing collection

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RDD operations

Two types of operations on an RDD:

- Transformation:
  - Create a new dataset from an existing one
  - Only compute the result when an action is run
  - Do not return any result to the driver program
- Action:
  - Run a computation on a dataset
  - Return the value to the driver program

Benefits of this approach:

- Transformed RDD is in memory when performing an action
- No large dataset to send back to the driver program

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RDD persistence

Datasets are cached in memory across operations:

- An RDD is stored on the node where it was computed
- An old RDD is dropped following the LRU algorithm
- A lost RDD is automatically recomputed if needed

Caching levels:

- Memory only: no compression, lost partitions are recomputed
- Memory and disk: partitions that do not fit in the memory are spilled on disk
- Memory only serialized: compression enabled
- Replication: all the above but also replicate on another node

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113

Functions and variables

Serialization of data and functions:

- Used to share information among the executors
- Transparent to the user

Task closure:

- Cannot share variables among executors
- Determine what variables and methods an executor needs
- Serialize this closure and send it to the executor
- Each executor receives a copy of the original variable
- Variables are not updated on the driver

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Shared variables

Broadcast variables:

- Read-only variables broadcasted to each executor
- Data sent in an efficient way to minimize traffic
- Useful for data needed over several stages of the computation

Accumulators:

- Variables that can be added to, using associative and commutative operations
- The driver can retrieve their value
- They are only updated on action tasks
- Update only occurs once, even if an action is rerun

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Running of a Spark job

Job submission and execution:

- A job is submitted when an action is performed on an RDD
- The transformations on the RDD are organised into a logical execution plan
- Spark DAG scheduler transforms the logical plan into a execution physical plan
- The physical plan defines stages, split into tasks
- Spark task scheduler constructs a mapping of tasks to executors
- The executor runs the task
- Executors send status updates to the driver when a task is completed or has failed

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Higher level abstractions

Newer and higher level abstractions relying on RDD:

- Datasets:
  - Foundational type of the structured APIs
  - Provide type-safety and allow much flexibility
  - Only available in Java and Scala and comes at a performance cost
- DataFrames:
  - Similar to a spreadsheet split into partitions
  - Internally defined as DataSets of type Row
  - Most common structured API, supported in all languages
- SQL Tables:
  - Data structure similar to DataFrames but defined within a database
  - Unmanaged table: defined from a file on the disk
  - Managed table: imported in and managed by Spark

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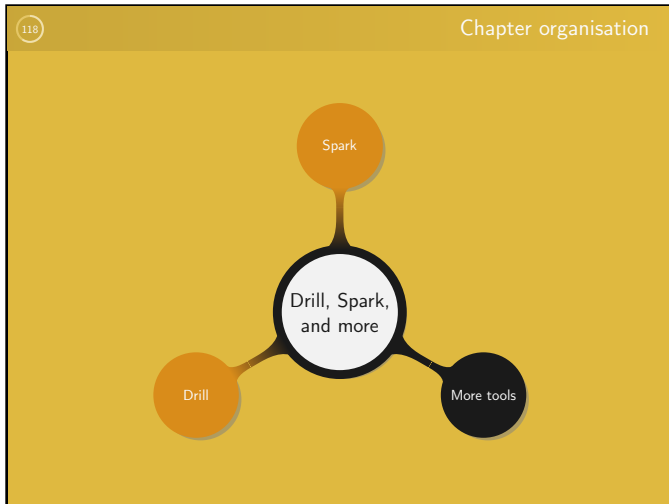
117
Spark's strength

Advantages of DAG:

- Any lost RDD can easily be recovered
- Offers more possibilities than a simple Map and Reduce approach
- Transformation on RDDs are not directly applied:
  - Allows better optimizations
  - Decreases disk writes and data transfer

Remark. Spark generalises the MapReduce approach, is much faster, and features many more high-level operators

Notes



Notes

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Scaling up software

Simple observations:

- Over 20 billions devices are connected to the internet
- The complexity of software keeps increasing
- New security challenges need to be addressed
- Package manager dependencies are complex to handle

Alternative package management systems:

- Flatpak, Snap, AppImage:
  - Distribution agnostic packages
  - Applications are sandboxed, i.e. isolated from each others and the host
- Nix: *all* packages are isolated from each others

Notes

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Linux containers

LinuX Containers (LXC):

- Operating-system-level virtualization method
- Relies on the kernel's *cgroup* and *namespace* isolation functionalities
- Concurrently run multiple isolated Linux OS on a machine
- Each container must be individually maintained
- Containers can be either privileged or unprivileged
- Containers access the bare machine and rely on the host kernel

*LXC requires the setup of a whole OS for each container*

Notes

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Docker

Docker uses a different approach than LXC:

- Initially based on LXC but *now* completely independent
- A daemon manages the docker containers
- A container is an encapsulated environment that runs applications
- An image containing an application and its dependencies is built based on a "configuration file"
- To update simply replace the old image with a new one

*Docker needs to be "manually" deployed, managed, and scaled*

Notes

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Kubernetes

Kubernetes is a container orchestration tool:

- Mostly used with, but not limited to Docker
- Initially developed as an internal Google project
- Kubernetes is a cluster application that handles a cluster:
  - Master*: controls all other machines in the cluster
  - Nodes*: the machines onto which applications are running
  - Pods*: single instance of an application or running process

Notes

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Kubernetes

Main tasks of Kubernetes:

- Monitor the health of the running applications
- Balances the load
- Manages hardware resources allocation
- Eases the deployment of preconfigured applications
- Allows access to storage in the same way as any other resources

During the lifespan of an application:

- Containers can live, die, be resurrected
- Kubernetes handles everything without any human interaction

*Kubernetes can be coupled to Hadoop or work independently*

Notes

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Key points

- When to use Drill?
- Explain the organisation of a drillbit
- When to use Spark?
- How does Spark interact with YARN?
- What are containerized solutions?

Notes

## 4. Dimensionality reduction

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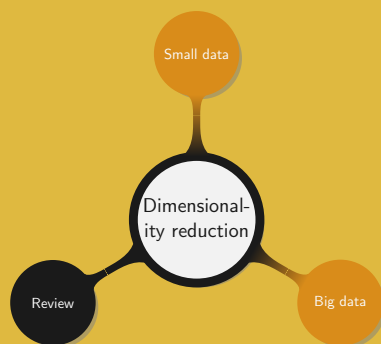
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### Chapter organisation



### Statistical strategy

Before the big data age:

- Extract a sample from a large population, e.g. phone survey
- Derive measurements on the population based on the sample
- Classify the population with respect to the measurements
- Take decisions based on some given goals

Limitations:

- Not all people in a same category behave similarly
- Goals must be general and cannot be personalised

### Statistical reminders

Given a dataset  $X = [X_1, \dots, X_n]$ , we define the

- *Median*: which corresponds to the "middle" value
- *Mean*:  $\bar{X} = \sum_{i=1}^n \frac{X_i}{n}$
- *Standard deviation* as the average distance between  $X_i$  and  $\bar{X}$ :
  - It represents the spread of the dataset
  - It is given by  $\sigma = \sqrt{\frac{(\sum_{i=1}^n X_i - \bar{X})^2}{n-1}}$
- *Variance* which also measures the spread and is given by  $\sigma^2$

*Those measures are one-dimensional*



129 Statistical reminders

Given a dataset  $[X_1, X_2] = [[X_{1,1}, \dots, X_{1,n}], [X_{2,1}, \dots, X_{2,n}]]$ , *covariance*:

- Is given by
 
$$\sigma_{X,Y} = \frac{\sum_{i=1}^n (X_{1,i} - \bar{X}_1)(X_{2,i} - \bar{X}_2)}{n-1}$$
- Verifies  $\sigma_{X_1, X_2} = \sigma_{X_2, X_1}$  and  $\sigma_{X, X} = \sigma^2$
- Evaluates how much two dimensions vary from the mean with respect to each other
- Provides information on whether both dimensions “increase together”

Given  $n$  variables  $X_1, \dots, X_n$ , their covariance matrix is given by

$$\begin{pmatrix} \sigma_{X_1, X_1} & \cdots & \sigma_{X_1, X_n} \\ \vdots & \ddots & \vdots \\ \sigma_{X_n, X_1} & \cdots & \sigma_{X_n, X_n} \end{pmatrix}$$

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130 Algebraic reminders

Let  $\mathcal{B}$  be a basis of a vector space  $V$ . For  $M \in \mathcal{M}_n(\mathbb{K})$ ,  $\lambda \in \mathbb{K}$  is an *eigenvalue* of  $M$  if and only there exists an *eigenvector*  $X \in \mathcal{M}_{n,1}(\mathbb{K})$  such that  $MX = \lambda X$ .

Important properties related to eigenvectors and eigenvalues:

- If  $M$  has rank  $n$  then it has  $n$  non-zero eigenvalues
- If  $M$  has  $n$  eigenvalues, distinct two by two, then it is diagonalizable
- $M$  is diagonalizable if and only if its eigenvectors form an orthogonal basis  $\mathcal{B}'$
- $M = PDP^{-1}$ , where  $D$  is a diagonal matrix featuring the eigenvalues of  $M$  on its diagonal, and  $P$  is the transition matrix from  $\mathcal{B}'$  into  $\mathcal{B}$
- For any  $k \in \mathbb{N}$ ,  $M^k = PD^kP^{-1}$

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131 Algebraic reminders

The *characteristic polynomial* of  $M$  is defined as

$$\begin{aligned} \chi_M : \mathbb{K} &\longrightarrow \mathbb{K} \\ \lambda &\longmapsto \det(M - \lambda I_n). \end{aligned}$$

Properties of  $\chi_M$ :

- If  $\lambda$  is an eigenvalue of  $M$ , then it is a root  $\chi_M$
- $M$  is diagonalizable if and only if  $\chi_M$  splits on  $\mathbb{K}$  and the dimension of the eigenspace associated to each eigenvalue  $\lambda$  is equal to the multiplicity of  $\lambda$

General remarks:

- Not all matrices are diagonalizable
- Not all square matrices are diagonalizable
- In general the determinant is “complicated” to compute
- A matrix that is not invertible is called *singular*

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132 Chapter organisation

```

graph TD
    SD((Small data)) --- DR((Dimensionality reduction))
    DR --- R((Review))
    DR --- BD((Big data))
  
```

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*I admire the elegance of your method of computation; it must be nice to ride through these fields upon the horse of true mathematics while the like of us have to make our way laboriously on foot.*

Albert Einstein

## Notes

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## Principal component analysis

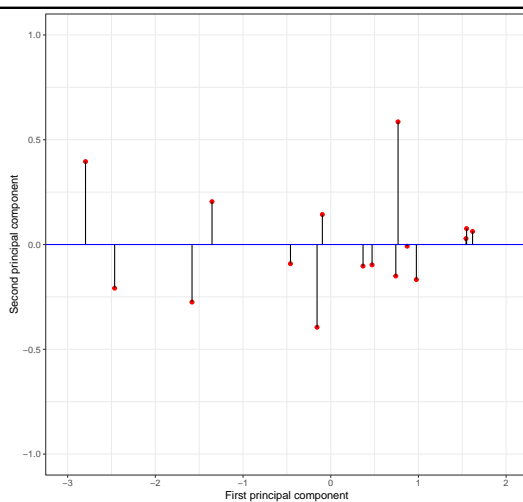
Basic idea behind Principal Component Analysis (PCA):

- Provides the best “perspective” that emphasises similarities and differences in the data
- This new perspective combines the original “characteristics” in order to best summarize the data

Example. Data that can be collected about teaching classrooms:

- The area
- The number of students who can seat in
- The number of blackboards
- The number of desktop computers

*Which are useful, useless, or redundant?*



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## Principal component analysis

Process:

① Standardize the range of the variables:

- Prevents large differences in the range of the variables
- Ensures all variables “contribute equally”
- Compute

$$Z_{i,j} = \frac{X_{i,j} - \bar{X}_j}{\sigma_{X_j}}$$

② Determine the “relationship” between the variables:

- Find correlations between the variables
- Construct the covariance matrix over all the variables

③ Identify the principal components:

- Compute the eigenvalues and eigenvectors of the covariance matrix
- Reorder the eigenvalues in non-increasing order

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Numerical stability

The stability of a method defines how it “reacts” to small perturbations

Since  $M = PDP^{-1}$  we have  $D = P^{-1}MP$ , and for a small perturbation  $\delta M$  we get

$$D + \delta D = P^{-1}(M + \delta M)P.$$

This yields  $\delta D = P^{-1}\delta MP$ , which in term of norms translates as

$$\|\delta D\| \leq \|P^{-1}\| \|P\| \|\delta M\|, \quad (4.1)$$

where the  $p$ -norm of an  $m \times n$  matrix  $A = (a_{i,j})_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}}$  is defined as

$$\|A\|_p = \left( \sum_{i=1}^m \sum_{j=1}^n |a_{i,j}|^p \right)^{\frac{1}{p}}.$$

Equation (4.1) means that a perturbation  $\|\delta M\|$  might be magnified by a factor as large as  $\|P^{-1}\| \|P\|$ .

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Linear transformations

For the sake of simplicity we consider the dimension 2 case

The diagram illustrates four linear transformations in 2D:

- Shear:** Matrix  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ . The coordinate system shows the x-axis shifted by the y-axis value.
- Dilution:** Matrix  $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ . The coordinate system shows the x-axis scaled by a factor of 2.
- Rotation:** Matrix  $\begin{pmatrix} \cos \frac{\pi}{4} & \sin \frac{\pi}{4} \\ -\sin \frac{\pi}{4} & \cos \frac{\pi}{4} \end{pmatrix}$ . The coordinate system shows a 45-degree counter-clockwise rotation.
- Reflection:** Matrix  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ . The coordinate system shows the x-axis reflected across the y-axis.

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Singular value decomposition

Let  $\{v_1, v_2\}$  be an orthonormal basis, and  $M$  be the matrix of a linear transformation. Then for two unit vectors  $u_1$  and  $u_2$  we obtain

$$Mv_1 = u_1\sigma_1 \quad \text{and} \quad Mv_2 = u_2\sigma_2,$$

with  $\sigma_1$  and  $\sigma_2$  in  $\mathbb{R}$ . For a vector  $x = \langle x, v_1 \rangle v_1 + \langle x, v_2 \rangle v_2$ , we have

$$\begin{aligned} Mx &= \langle x, v_1 \rangle Mv_1 + \langle x, v_2 \rangle Mv_2 \\ &= \langle x, v_1 \rangle u_1\sigma_1 + \langle x, v_2 \rangle u_2\sigma_2. \end{aligned}$$

Recalling that  $\langle x, v_1 \rangle = x^\top v_1 = v_1^\top x$ , we get  $Mx = u_1\sigma_1 v_1^\top x + u_2\sigma_2 v_2^\top x$ , which yields  $M = u_1\sigma_1 v_1^\top + u_2\sigma_2 v_2^\top$ . In term of matrices this translates into

$$M = \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} v_1^\top \\ v_2^\top \end{pmatrix}.$$

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Singular value decomposition

More generally, if  $M$  is an  $m \times n$  real matrix we can write  $M = U\Sigma V^\top$ , where both  $U = (u_1 \cdots u_m)$  and  $V = (v_1 \cdots v_n)$  are rotation matrices, and  $\Sigma$  is diagonal.

Size of the matrices:

- $M$ :  $m \times n$
- $U$ :  $m \times m$
- $\Sigma$ :  $m \times n$
- $V$ :  $n \times n$

The elements on the diagonal of  $\Sigma$  are called *singular values*, the columns of  $U$  *left singular vectors*, and the rows of  $V^\top$  *right singular vectors*.

If  $U\Sigma V^\top$  is an SVD for an  $m \times n$  matrix  $X$  of rank  $r$ :

- $\Sigma$  has exactly  $r$  strictly positive elements which are the square roots of the  $r$  eigenvalues of  $X^\top X$ , with corresponding multiplicities
- The columns of  $V$  are eigenvectors of  $X^\top X$
- The columns of  $U$  are eigenvectors of  $XX^\top$

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Singular value decomposition

General remarks on the SVD:

- SVD is not unique and exists for any matrix, whether invertible or not
- Singular values can be re-ordered as long as their corresponding singular vectors are re-arranged accordingly
- If  $U\Sigma V^T$  is an SVD for  $X^T$ , then  $V\Sigma^T U^T$  is an SVD for  $X$

As  $U$  and  $V$  are rotation matrices, they are orthogonal, i.e.  $U^T U = I_m$  and  $V^T V = I_n$ . In particular, referring to slide 4.137, we see that in term of stability a small perturbation gets increased by a factor:

- $\|P^{-1}\| \|P\|$ , for eigen decomposition
- $\|U^{-1}\| \|(V^T)^{-1}\| = \|U^T\| \|V\| = 1$ , for SVD

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QR decomposition

Any  $X \in \mathcal{M}_{m,n}(\mathbb{K})$ , with linearly independent columns, can be written  $X = QR$  where:

- $Q \in \mathcal{M}_{m,n}(\mathbb{K})$  is orthogonal
- $R \in \mathcal{M}_{n,n}(\mathbb{K})$  is upper triangular

Remark. If  $X$  is invertible, then so is  $R$ .

Various approaches can be applied to compute the QR decomposition of  $X$ . The most famous one, Gram-Schmidt is unfortunately unstable. The two other most common options are based on Givens rotations and on Householder reflections. The former is slower but simpler to parallelize.

Given a vector  $u$ , a *Householder reflection* is a linear transformation  $H_u = I - \frac{2uu^T}{u^T u}$ . For instance in an orthonormal basis  $\{u, v\}$  we can write  $w = c_1 u + c_2 v$ , with  $c_1 = \frac{\langle w, u \rangle}{\|u\|_2^2}$  and  $c_2 = \frac{\langle w, v \rangle}{\|v\|_2^2}$ .

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QR decomposition

Applying  $H_u$  to  $w$  we get

$$\begin{aligned} H_u w &= \left( I - \frac{2uu^T}{u^T u} \right) (c_1 u + c_2 v) = c_1 u + c_2 v - 2 \frac{uu^T}{u^T u} (c_1 u + c_2 v) \\ &= c_1 u + c_2 v - 2c_1 = -c_1 u + c_2 v - 2c_2 \frac{u}{u^T u} \langle u, v \rangle \\ &= -c_1 u + c_2 v. \end{aligned}$$

This example can be visually represented as a reflection about the plane orthogonal to  $u$  and  $v$ .

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QR decomposition

Householder reflections can be used to obtain the QR decomposition of a matrix  $X$ . More specifically, *Householder reflection theorem* states that for two vectors  $x$  and  $y$  with similar norm, there exists an orthogonal matrix  $Q$  such that  $y = Qx$ . This provides a method for iteratively constructing  $R$  and  $Q$ .

Idea on how to apply the above discussion:

- Determine an orthogonal matrix  $Q_1$  such that  $Q_1 c_1 = (\gamma_1, 0, \dots, 0)^T$ , where  $\gamma_1$  has similar norm as  $c_1$ . At this stage we have

$$R_1 = \left( \begin{array}{c|cccccc} \gamma_1 & * & \dots & \dots & * \\ 0 & & & & \\ \vdots & & & & \\ \vdots & & & & \\ 0 & & & & \end{array} \begin{array}{c} X_1 \\ \\ \\ \end{array} \right).$$

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149 Distributed systems

An  $m \times n$  matrix  $X$  can be:

- Dense
- Square
- Short and fat
- Sparse
- Tall and skinny

Ways to represent a matrix over many nodes:

- By row or column
- By elements
- By blocks

Remarks.

- The matrix does not fit anymore in memory
- The shape and representation of the matrix impacts the speed

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150 Tall and skinny

Setup:

- $X$  does not fit in memory
- $X$  has many more rows than columns ( $m \gg n$ )
- $n^2$  fits in memory on a single machine

When running PCA:

- Complexity of a full SVD:  $\mathcal{O}(mn^2)$
- We only need the  $k$  most significant singular values and vectors
- The work needed is  $\mathcal{O}(mk^2)$

*Can we take advantage of the shape of  $X$  to be faster?*

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151 Tall and skinny

From slide 4.146, we know that

$$C = X^T X = V \Sigma^2 V^T. \quad (4.2)$$

Also  $C$  can fit in memory since it has dimension  $n \times n$  and by assumption a single machine has  $n^2$  memory. Thus we *hope to compute*  $X^T X$ , without any dependence on  $m$ . Then using equation 4.2 we can find the eigenvalues of  $C$  and retrieve  $V$  and  $\Sigma$ . The main challenge is to efficiently compute  $X^T X$ , while ensuring the singular values of  $C$  are not significantly altered in the process.

Setup preparation:

- Store matrices row-by-row on disk
- Ensure all elements are in  $[-1, 1]$ , i.e. divided by the largest element

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152 Tall and skinny

Simple MapReduce approach to compute  $X^T X$ :

- Mapper: for all pairs  $(x_{ij}, x_{ik})$  on row  $i$ , return the *(key, value)* pair  $((c_j, c_k), x_{ij}x_{ik})$ , where  $c_j$  and  $c_k$  correspond to columns  $j$  and  $k$ , respectively.
- Reducer: consider all the *(key, value)* pairs  $((c_i, c_j), \langle v_1, \dots, v_R \rangle)$ , where each of the  $v_k$  corresponds to a product of elements in  $X$  and  $R$  is the number of non-zero products, and compute  $\sum_{i=1}^R v_i$ .

Considerations on the complexity:

- Communication: *shuffle size*  $\mathcal{O}(mL^2)$ , with  $L$  the maximum number of non-zero elements on a row
- Overload on a single machine: *reduce-key complexity*  $\mathcal{O}(m)$

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Measuring similarity

Measuring the distance between several documents:

- Count the number of occurrence for all the words
- Compare the scores
- If documents feature many common words, then they are similar

Limitation:

- Take three documents  $d_1, d_2$ , and  $d_3$ , where  $d_1, d_2$  are long and  $d_3$  is an expert of  $d_1$
- What is likely to happen?

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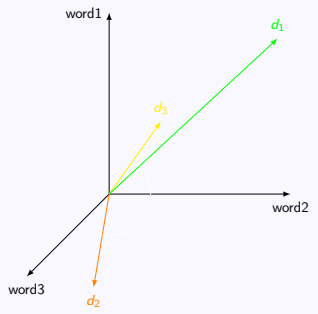
Cosine similarity

For two vectors  $d_i$  and  $d_j$  we define their *cosine similarity* as

$$\cos(d_i, d_j) = \frac{\langle d_i, d_j \rangle}{\|d_i\| \|d_j\|}.$$

Basic idea:

- The closer two documents, the smaller the angle
- The smaller the angle, the larger the cosine



More refined strategy:

- Classify words by meaning
- Consider the cosine similarity based on word semantic

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Tall and skinny with cosine similarity

We fix  $\gamma$ , a parameter that can be adjusted when running MapReduce.

MapReduce using cosine similarity to compute  $X^T X$ :

- Mapper: for all pairs  $(x_{ij}, x_{ik})$  on row  $i$ , return the *(key, value)* pair  $((c_j, c_k), x_{ij}x_{ik})$ , with probability  $\min\left(1, \frac{\gamma}{\|c_j\| \|c_k\|}\right)$ , where  $c_j$  and  $c_k$  correspond to columns  $j$  and  $k$ , respectively.
- Reducer: consider all the *(key, value)* pairs  $((c_j, c_k), \langle v_1, \dots, v_R \rangle)$ , where each of the  $v_k$  corresponds to a product of elements in  $X$  and  $R$  is the number of non-zero products and proceed as follows.
  - If  $\frac{\gamma}{\|c_j\| \|c_k\|} > 1$ , then return  $\frac{1}{\|c_j\| \|c_k\|} \sum_{i=1}^R v_i$
  - Otherwise, return  $\frac{1}{\gamma} \sum_{i=1}^R v_i$

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Tall and skinny with cosine similarity

What the reducer returns is in fact not  $X^T X$ , but a matrix  $X'$  which contains the cosine similarities between the columns of  $X$ . Defining  $\tilde{v}_i$  to be  $v_i$ , if a *(key, value)* pair is returned and 0 otherwise, this can be seen by looking at the expectation of an output

$$E\left(\frac{1}{\gamma} \sum_{i=1}^R \tilde{v}_i\right) = \frac{1}{\gamma} P(\tilde{v}_i = v_i) \sum_{i=1}^R v_i = \frac{1}{\|v_j\| \|v_k\|} \sum_{i=1}^R v_i.$$

To recover  $X^T X$  from the matrix  $X'$ , it suffices to define a diagonal matrix  $D$  whose elements  $d_{ii}$  are exactly  $\|c_i\|$ , and then compute  $X^T X \approx DX' D$ .

Using Latala's theorem it is possible to prove that when using cosine similarities to sample columns, singular values are preserved with "high enough" probability.

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157
Tall and skinny with cosine similarity

Remarks.

- Since  $\|c_i\|$  is used in the mapper it means the norm of all the columns must be pre-computed. This requires an all-to-all communication.
- The parameter  $\gamma$  can be adjusted depending on what is expected:
  - Preserve similar entries in  $X^T X$ :  $\gamma = \Omega\left(\frac{\log n}{s}\right)$ , where  $s$  is the lowest cosine similarity
  - Preserve singular values of  $X^T X$ :  $\gamma = \Omega\left(\frac{n}{\varepsilon^2}\right)$ , where  $\varepsilon$  is the relative error

Complexity, with  $h$  the smallest non-zero value after normalization:

- Shuffle size:  $\mathcal{O}(nL\gamma/h^2)$
- Reduce key complexity:  $\mathcal{O}(\gamma/h^2)$

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158
Distributed QR

As explained in slide 4.145 first decomposing  $X$  into  $X = QR$  can speed up the SVD of  $X$ . The basic idea related to Householder reflection theorem can in fact be extended into a “tiled QR decomposition” algorithm, which is applicable when the matrix  $X$  does not fit in memory.

In this algorithm, the tiles correspond to matrix blocks which can be stored and processed on different nodes.

Represent  $X$  in tall and skinny blocks, then start process:

- Obtain the QR decomposition of block  $X_{k,k}$
- Adjust all the blocks  $X_{k,j}$  on the right of  $X_{k,k}$
- Adjust all the blocks  $X_{i,k}$  below  $X_{k,k}$  and the blocks  $X_{i,j}$  on their right
- Repeat for all blocks on the diagonal

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159
Looking at the big picture

Brief summary:

- Data has many variables
- Apply PCA to find a better perspective
- Ignore low contributions, only keep the most “prominent” dimensions
- Apply SVD in order to compute PCA
- Run further tasks based on the PCA “approximation”

Comments on PCA:

- It works well on small data as well as on tall and skinny big data
- It searches for a “best” solution

*How much better is the best solution compared to a random solution?*

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160
Random projections

*Construct a map preserving the pairwise distance between points*

Setup and goals:

- Map  $m$  points  $(u_i)_{1 \leq i \leq n}$  in  $\mathbb{R}^n$  into  $m$  points in  $\mathbb{R}^d$ , with  $d \ll n$
- Ensure that for all  $i, j$ ,
$$\|v_i\|_2 \approx \|u_i\|_2 \text{ and } \|v_i - v_j\|_2 \approx \|u_i - u_j\|_2 \quad (4.3)$$

*Johnson-Lindenstrauss lemma* states that after a random projection, with high probability the distance between any two points is “distorted” by a factor at most  $1 \pm \varepsilon$ .

If  $X' = \frac{1}{\sqrt{d}}XR$ , where  $R \in \mathbb{R}^{n \times d}$  is random with independent identically distributed entries and zero mean, then  $X'$  is much smaller and preserves all pairwise distances in expectation.

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161
Random projections

Feeling behind the random projection idea:

- Project a random vector onto a fixed subspace:
  - Consider  $m$  points in  $\mathbb{R}^n$  and fix  $k$  coordinates uniformly at random
  - Two vectors differing by only few coordinates can see their distance totally changed after projection
- Project a fixed vector onto a random subspace:
  - Consider  $m$  points in  $\mathbb{R}^n$  and project them in a  $k$ -dimensional subspace
  - Two vectors differing by only few coordinates will "spread out" all coordinates and prevent missing out on "important" coordinates

Generation of the matrix  $R = (r_{i,j})$ :

- Different  $r_{i,j}$  can impact the variance and the error tail bounds
- The  $r_{i,j}$  are independent identically distributed with zero mean
- The  $r_{i,j}$  are often selected following a symmetric distribution about zero with unit variance

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Random projections

Remarks.

- A random projection is likely not a projection in the algebraic sense:
  - Most often:  $R^2 \neq R$
  - Eigenvalues of  $R$  are not necessarily in  $\{0, 1\}$
  - It is  $\varepsilon$ -close to a projection
- Requirement (4.3) does not extend to other norms
- The norm of a projected vector is  $\sqrt{\frac{d}{n}}$  in expectation, with an error exponentially small in  $d$
- Time complexity:  $\mathcal{O}(mnd)$
- The projection can be made very sparse with little loss of accuracy and a major speedup

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163
Dimensionality reduction for big data:

Preserving important structural properties of the data:

- Tall and skinny can be handle using SVD
- For other cases random projections are a good alternative
- PCA will ensure a good result, random projections might not
- Randomized PCA:
  - Randomly project  $X$  to obtain  $X'$
  - Perform a QR decomposition on  $X' = QR$
  - Compute  $B = Q^T X$
  - Run SVD on  $B = U_1 \Sigma V^T$
  - Approximate the SVD of  $X$  by  $QQ^T X = Q(U_1 \Sigma V^T) = U \Sigma V^T$
  - Complete PCA the usual way

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Key points

- How to perform dimensionality reduction?
- What is the relationship between SVD and PCA?
- What is cosine similarity?
- How to deal with tall and skinny big data?
- What are random projections?

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## 5. Optimization

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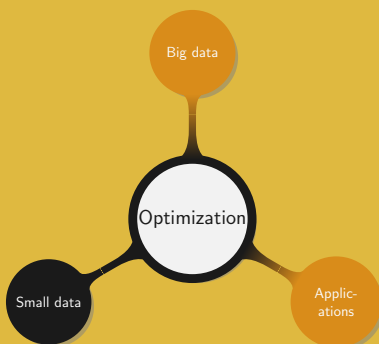
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### Chapter organisation



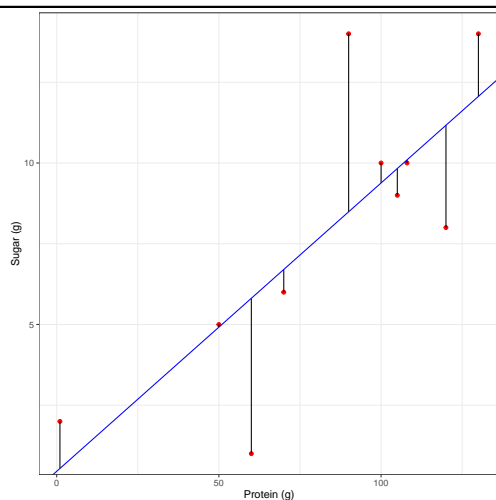
### Regression analysis

#### Basics on regression:

- Find the relationship between:
  - A *dependent variable*, i.e. the outcome or response variable
  - Independent variables*, i.e. predictors or explanatory variables
- Many types of regression exist, most common ones are linear, logistics, and polynomial
- The goal is to minimize the error of the prediction

#### Error evaluation:

- Error is often measured using the *the sum of squares*, i.e. summing up the square of the error at each point
- The minimum of the sum of squares is called *least squares*
- The smaller the error the better the model



169 Optimization problems

Reminders on optimization:

- The goal is to maximize or minimize a function  $f$  depending in its input  $x$
- The function  $f$  is called *objective function* or *criterion*
- During the minimization process  $f$  is often referred to as *cost*, *loss*, or *error function*

Remark on minimization with respect to least square:

- Whether positive or negative a large error is bad
- Squaring allows to penalize larger residuals more than smaller ones
- Error is often composed of *systematic* and *random* noises
- Minimizing the sum of squared errors is the same as minimizing the variance

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170 Notes on optimization problems

Common examples of optimization problems:

- Chip design: ensure no tracks cross on a computer chip
- Timetable: given a list of students in each course, minimize the number of collisions
- Traveling salesman: given a list of cities, minimize the distance necessary to visit all of them

No free lunch theorem:

- There is no best solution to all search problems
- Algorithms performing better on certain problems, do worse on others
- Work is often needed to find the most suitable algorithm

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171 Gradient descent

*Description is usually done in term of minimization*

Basic setup for a function  $f(X)$ , where  $X = (x_1, \dots, x_n)$ :

- Iteratively create a sequence of  $X_i$
- At each step, determine the gradient in each direction
- Select a direction and keep going down
- Repeat until the gradient is 0 in all directions

Remark. The function  $f$  should be convex, i.e. for any  $x, y \in \mathbb{R}^d$ , and  $\alpha \in [0, 1]$ ,

$$f(\alpha x + (1 - \alpha)y) \leq \alpha f(x) + (1 - \alpha)f(y),$$

to ensure it has a global minimum, and the algorithm does not return a local one.

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172 Gradient descent

Example. For  $f(X) = 0.5x_1^2 + 0.2x_2^2 + 0.6x_3^2$ , we obtain

$$\nabla f(x) = (x_1, 0.4x_2, 1.2x_3).$$

Using a step of length 1, and starting with  $X_0 = (-2, 2, -2)$ , the steepest downhill direction is  $(-2, 0.8, -2.4)$ , yielding  $X_1 = (0, 1.2, 0.4)$ .

After a few iterations we find  $X_6 = (0, 0.0569, 0.0000256)$ .

Remarks.

- Using a constant step could lead to either going very slowly or overstepping the minimum
- Computation can be speeded up by using Taylor expansion. This involves inverting the Hessian matrix of  $f$  which has a cost of  $\mathcal{O}(n^3)$ . However this method allows to keep a constant step of 1, simplifying other parts of the computation

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Comparison of gradient descent methods

Gradient descent:

- Uses the whole dataset
- Deterministic method
- Slow but fast to converge
- Yields an optimal solution
- Slow to escape local minima

Stochastic gradient descent:

- Randomly selects a sample
- Stochastic method
- Fast but slow to converge
- Yields a good enough solution
- Faster to escape local minima

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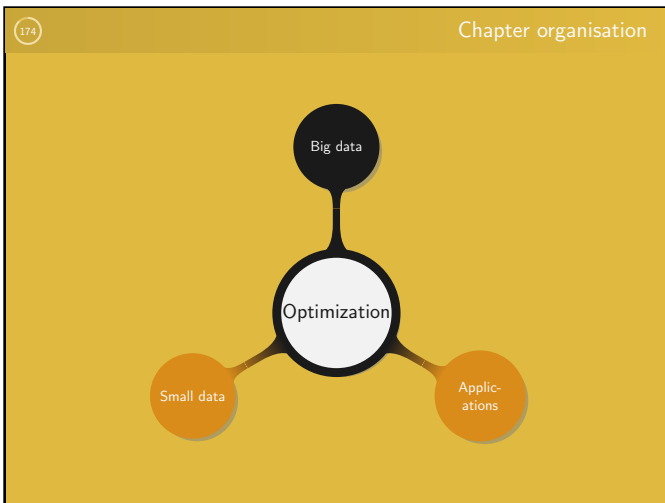
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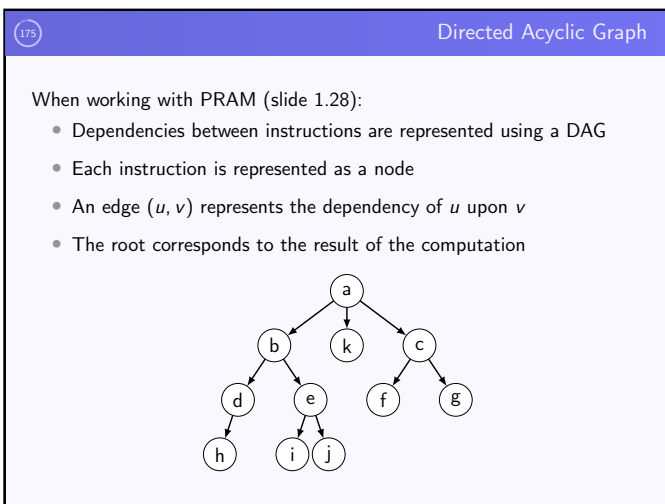
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176

Work-depth model

Computation finishes when the last processor completes its job:

- The amount of time when using one CPU is referred to as  $T_1$
- The amount of time when using  $p$  CPUs is referred to as  $T_p$
- The amount of time when using infinitely many CPUs is referred to as  $T_\infty$

Remarks.

- The *depth of an algorithm* is defined with respect to the last CPU to complete its tasks
- The *work of an algorithm* corresponds to the amount of time necessary to complete all tasks multiplied by the number of CPUs

*Does  $T_\infty$  tend to zero?*

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Bounds on  $T_p$

Brent's theorem:

$$\frac{T_1}{p} \leq T_p \leq \frac{T_1}{p} + T_\infty$$

Meaning of the result:

- If work is evenly shared among all  $p$  CPUs then we get  $\frac{T_1}{p}$
- $T_\infty$  helps define how far we are from the ideal case

Remarks.

- Together  $T_1$  and  $T_\infty$  provide information on how well an algorithm performs on  $p$  CPUs
- Increasing the number of CPUs will never impact performance

*The work-depth model helps designing better parallel algorithms*

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178

Back to gradient descent

From a general point of view gradient descent is an optimization problem

$$\min_w (F(w)) = \sum_{i=1}^m F_i(w, x_i, y_i),$$

where  $x_i \in \mathbb{R}^n$ ,  $y_i \in \mathbb{R}$  is a "label", and  $w$  is the parameter we expect to optimize over.

In essence gradient descent starts with a random initial  $w$  and has the goal of iteratively improving on it, by computing  $w_{k+1} = w_k - \alpha \nabla F(w_k)$ , for some small  $\alpha$ . In our case the objective function  $F$  corresponds to a loss function.

Remark. From a theoretical point of view, this works especially well when  $F$  is strongly convex, differentiable, and  $\nabla F$  is  $L$ -Lipschitz continuous. In such a case taking  $\alpha < \frac{1}{L}$  leads to an exponential convergence rate to a global minimum!

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179

Gradient descent – Sum of squares

For the sum of squares loss function, we want to minimize

$$F(w) = \sum_{i=1}^m F_i(w, x_i, y_i) = \sum_{i=1}^m \|x_i^\top w - y_i\|_2^2,$$

with  $x_i$ ,  $y_i$ , and  $w$  as above.

Notes on our setup:

- $F$  is strongly convex and Lipschitz continuous
- $m$  corresponds to the data parallelism
- $n$  corresponds to the model parallelism

*How well does gradient descent scale up?*

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Parallel summation

*How to sum up  $n$  elements in parallel?*

Algorithm. (*Basic summation*)

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**Input** :  $a$  an array with  $n$  elements  
**Out-** :  $s$  the sum over all the elements of  $a$   
**put** :

```

1  $s \leftarrow 0$  ;
2 for  $i \leftarrow 1, \dots, n$  do
3    $s \leftarrow s + a[i]$  ;
4 end for
5 return  $s$ 

```

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Basic summation:	How to achieve:
• Work: $\mathcal{O}(n)$	• Work: $\mathcal{O}(n)$
• Depth: $\mathcal{O}(n)$	• Depth: $\mathcal{O}(\log n)$

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181 Gradient descend – Work and depth

Complexity of computing

$$F(w) = \sum_{i=1}^m \|x_i^\top w - y_i\|_2^2$$

- Work:  $\mathcal{O}(mn)$
- Depth:  $\mathcal{O}(\log mn)$

Complexity of computing

$$\nabla F(w) = 2 \sum_{i=1}^m x_i^\top (x_i^\top w - y_i)$$

- Work:  $\mathcal{O}(mn)$
- Depth:  $\mathcal{O}(\log mn)$

Performing a complete gradient descend:

- Error  $\frac{1}{\epsilon}$  can be achieved after  $\mathcal{O}\left(\log \frac{1}{\epsilon}\right)$  iterations
- Total depth:  $\mathcal{O}\left(\log \frac{1}{\epsilon} \log mn\right)$

*How suitable is gradient descent for parallelization and big data?*

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182 Stochastic gradient descend

*Instead of computing a full gradient, apply it to a randomly selected point*

Stochastic gradient descent (using notations from slide 5.178):

- Call  $s_k$  the index uniformly sampled at iteration  $k$
- Compute the sequence

$$w_{k+1} = w_k - \alpha \nabla F_{s_k}(w_k)$$

Notes on stochastic gradient descent:

- Error  $\epsilon$  will be achieved after  $\mathcal{O}\left(\frac{1}{\epsilon}\right)$  iterations
- Work decreases linearly with the number of points considered
- The number of iterations does not increase linearly with the number of sample points

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183 Stochastic vs. batch gradient descent

<div>Gradient descent:</div> <ul style="list-style-type: none"> <li>• Per iteration: <ul style="list-style-type: none"> <li>• Work: <math>\mathcal{O}(mn)</math></li> <li>• Depth: <math>\mathcal{O}(\log mn)</math></li> </ul> </li> <li>• Total: <ul style="list-style-type: none"> <li>• Work: <math>\mathcal{O}\left(mn \log \frac{1}{\epsilon}\right)</math></li> <li>• Depth: <math>\mathcal{O}\left(\log \frac{1}{\epsilon} \log mn\right)</math></li> </ul> </li> </ul>	<div>Stochastic gradient descent:</div> <ul style="list-style-type: none"> <li>• Per iteration: <ul style="list-style-type: none"> <li>• Work: <math>\mathcal{O}(n)</math></li> <li>• Depth: <math>\mathcal{O}(\log n)</math></li> </ul> </li> <li>• Total: <ul style="list-style-type: none"> <li>• Work: <math>\mathcal{O}\left(\frac{n}{\epsilon}\right)</math></li> <li>• Depth: <math>\mathcal{O}\left(\frac{\log n}{\epsilon}\right)</math></li> </ul> </li> </ul>
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*Which is best, gradient descent or stochastic gradient descent?*

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184 Parallelizing Stochastic gradient descent

Setup in PRAM:

- Save  $w$  in a shared piece of the memory
- All CPUs have access to  $w$  and the whole dataset

Things which could go wrong:

- A model is read, transformed, and written in the memory but in the meantime the model has been updated by another CPU
- An updated model is overwritten by an older one

*How bad is this situation?*

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Parallelizing Stochastic gradient descent

Adding locks:

- Solves the race condition problem
- Only one CPU can access  $w$  at a time
- All the benefits from the parallelism get lost

*Are locks really needed?*

In stochastic gradient descent:

- Both  $x_i = (x_i^{(1)}, \dots, x_i^{(n)})$  and  $w = (w^{(1)}, \dots, w^{(n)})$  are  $n$ -dimensional vectors
- If  $x_i$  is sparse, then only a few  $w^{(k)}$  will be updated

*In a big data setup the probability of collision will likely be low*

Notes

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Going Hogwild!

Hogwild! strategy for  $p$  processors:

- Proceed in parallel over all available CPUs
- Until the expected error condition is met:
  - Select a random index  $j$  from  $\{1, \dots, m\}$
  - Concurrently compute  $F_j(w)$  and  $\nabla F_j(w)$  for the  $w$  currently in the shared memory
  - For all  $k$  such that  $x_i^{(k)} \neq 0$ , update  $w^{(k)}$  with  $w^{(k)} - \alpha [\nabla F_j(w)]^{(k)}$

Remarks.

- No locks are used, leading to speed up nearly linear in term of CPUs
- The locking overhead should be avoided when processing big data
- Hogwild! requires the cost function to be sparse

Notes

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Stochastic vs. batch gradient descent

Looking back at gradient descent in PRAM:

- Stochastic has lower depth but is hard to parallelize
- Batch is easier to parallelize but slower
- Hogwild! renders stochastic almost *embarrassingly parallel*

Stochastic and batch on a distributed system:

- Communication based on the number of iterations:
  - Batch:  $\mathcal{O}\left(\log \frac{1}{\epsilon}\right)$
  - Stochastic:  $\mathcal{O}\left(\frac{1}{\epsilon}\right)$
- Stochastic is often preferred on a single computer with many GPUs
- Batch is more common on distributed systems

Notes

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Chapter organisation

```

graph TD
    BigData((Big data)) --- Optimization((Optimization))
    SmallData((Small data)) --- Optimization
    Applications((Applications)) --- Optimization
  
```

Notes

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Implementing gradient descent

Questions to consider first:

- What tool to use?
- Is batch or stochastic gradient descent most appropriate?
- How large or sparse is the data, i.e. can  $n$  and  $m$  fit in memory?

In our setup we expect to:

- Use Spark
- Check how both gradient descents strategies behave
- Work with  $n$  small enough to fit in memory but no restriction on  $m$

*How to store the data on the cluster?*

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190
Batch gradient descent in Spark

High-level idea for a Spark implementation:

- 1 Organise the data by row and store it in an RDD
- 2 Use a `map` transformation to generate a closure for each point
- 3 Use the `cache` action to ensure Spark keeps the RDD in memory
- 4 For each point  $p$ , apply a `map` to transform  $p$  into  $\nabla F_p(w)$
- 5 Apply a `reduce` to sum up all the  $\nabla F_p(w)$
- 6 Update  $w$  and repeat from step 4 until the expected error is reached

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191
Batch gradient descent in Spark

Optimizing our approach:

- How many times is  $w$  sent?
- Is  $w$  modified by the mappers?
- With respect to bandwidth and memory usage, how large is  $w$ ?
- How should  $w$  be shared among all the machines?

Basic analysis:

- Where is the bottleneck in our approach?
- How good or bad is the communication cost?

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Stochastic gradient descent in Spark

Reminders on stochastic gradient descent:

- Total depth is much larger than for batch gradient descent
- Apply Hogwild! to speed up the process when data is sparse

Hogwild! on Spark:

- Broadcast needs to be completed to start the mappers
- All mappers and reducers must be done before broadcasting again
- Spark achieves fault-tolerance through synchronisation barriers

Minimizing an objective function in Spark:

- Try random updates and accept any one lowering the objective
- Use mini-batches:
  - At each iteration select “many” samples, instead of one
  - Apply batch gradient descent to them

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193
Gradient descent and PCA

Common strategy for gradient descent when data is too large:

- Extract the principal components of the big dataset
- Apply gradient descent on the resulting approximation

Relating gradient descent to PCA:

- Find the axes which maximize the variance
- Find the direction for which the expectation of  $XX^T$  is maximized
- Find the direction which minimizes the residuals

For mean centered data:

- Pythagorean theorem:  $V^2 = M^2 + m^2$
- PCA maximizes  $M$
- Gradient descent minimizes  $m$

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194
Linear programming

*Given a mathematical model where an objective is represented as a linear function and some constraints are expressed as equalities or inequalities, maximize the objective while respecting the constraints.*

A gradient descent reading of linear programming:

- Any maximization problem can be rephrased into a minimization one
- The simplex method solves linear programming problems:
  - A basic solution is used to start with
  - A "local search" finds a *pivot* to improve on the basic solution
  - Different choices of pivot lead to different convergence rates
  - Any pivot leads to an optimal solution
- Finding the best pivot in the simplex is equivalent to finding the best direction to improve on the objective

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Key points

- What are batch and stochastic gradient descents?
- How to batch and stochastic gradient descent compare?
- What are PRAM, work, and depth?
- Explain how Hogwild! works
- Discuss gradient descent implementation on Spark

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Thank you, enjoy the Summer break!

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References I

1.30 [https://upload.wikimedia.org/wikipedia/commons/0/0e/Hadoop\\_logo.svg](https://upload.wikimedia.org/wikipedia/commons/0/0e/Hadoop_logo.svg)

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