1. $(V,+,\cdot)$ vector space. $U, W \subset V$ (mention concept checking papers)

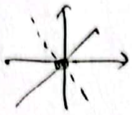
① $U \cap W \neq \phi$??

mention ~~element~~ ~~unit~~ neutral element AC

② $U \cap W$ ✓

$U \cup W$ ✗ (mention subset)

③ $U \setminus W$, $\boxed{\text{counter example}}$



~~unit~~ neutral element.

2. ask student: $\boxed{\text{norm}: \ x=0 \iff \|x\|=0}$

① $\|f\| = \|f\|_\infty$ ✓, of course

② $\|f+g\| = \sup |f'+g'| \leq$ ~~$\|f\|+\|g\|$~~ $\sup|f'| + \sup|g'|$

$f=-g$ $\quad$ $f=0 \iff \|f\|=0$

check one by one

③ $\|f+g\| = \sup |f+g| + \sup |f'+g'|$ ACD

$= \sup|f| + \sup|g| + \sup|f'| + \sup|g'|$

3. radius of convergence, $\boxed{p>1}$ how to caculate. AC

pay attention to boundary.

4. $f''(0)=0$ C

$f'(0) \geq 0$

$$f(x) = \begin{cases} \boxed{x^2 \sin^2\left(\frac{1}{x}\right)} & (x \neq 0) \\ 0 & (x=0) \end{cases}$$

$f(0+h) = f(0) + f'(0)\cdot h + o(h)$

$h^2 \sin^2\left(\frac{1}{h}\right) = o(h) + f'(0) h$

$\Rightarrow f'(0) = 0$.

$f'(x) = 2x \cdot \sin^2\left(\frac{1}{x}\right) + x^2 \cdot 2\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right) \cdot -\frac{1}{x^2}$

$= 2x \cdot \sin^2\left(\frac{1}{x}\right) - 2\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)$; $\sin\left(\frac{2}{x}\right)$

$f''(x) = 2\sin^2\left(\frac{1}{x}\right) + 2x \cdot 2\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) + \cos\left(\frac{2}{x}\right)\left(-\frac{2}{x^2}\right)$

$= 2\sin^2\left(\frac{1}{x}\right) + \dfrac{-2x\sin\left(\frac{2}{x}\right) - 2\cos\left(\frac{2}{x}\right)}{x^2}$.

5- U_1+U_2 definition

U_1-U_2 definition.

(A) what is $\underline{\underline{0}}$? $\{0\}$ neutral element

$(U_1+U_2) = (U_1-U_2) = (U_1-U_2)+U_2 \neq U_1$

6. (i) ratio test:

$$\frac{\frac{(2n+2)!\,(3n+3)!}{(n+1)!\,(4n+4)!}}{\frac{(2n)!\,(3n)!}{n!\,4n!}} = \frac{\cancel{(2n+2)}(2n+1)(3n+3)(3n+2)(3n+1)}{2(n+1)\cancel{(4n+4)}(4n+3)(4n+2)(4n+1)} \to \frac{108}{256} < 1$$

$= \cancel{\ldots}$

(ii) for $a \le 0$, $n \to \infty$, $\left(\frac{1}{n} - \sin\left(\frac{1}{n}\right)\right)^a \to \infty$, not converge

tell student: discuss simple case, ~~even~~ may worth some points

$$\sin\left(\tfrac{1}{n}\right) = \frac{1}{n} - \frac{1}{6n^3}(1+o(1)) \quad (n\to\infty).$$

$$\Rightarrow \frac{1}{n} - \sin\left(\tfrac{1}{n}\right) = \boxed{\frac{1}{6n^3}}(1+o(1))$$

find something larger / smaller $\boxed{\text{squeeze}}$ ~~but still~~

$$\frac{1}{12n^3} < \frac{1}{n} - \sin\left(\tfrac{1}{n}\right) < \frac{1}{3n^3}$$

$$\Rightarrow \frac{1}{12^a n^{3a}} < \left(\frac{1}{n} - \sin\left(\tfrac{1}{n}\right)\right)^a < \frac{1}{3^a n^{3a}} \quad (\text{for large } n)$$

$$\Rightarrow \left(a > \tfrac{1}{3}\right)$$

mention: Taylor series (if you know)

Definition of $\sin()$ & $\cos()$

7. mention local extrema / global extrema > value & point.

local extrema $\boxed{\text{point}}$

mention graph (rubric)

Idea: convergence is uniform → evaluate sup → ε

continuous + close → uniform → property

f_n perhaps not continuous → no continuous property

↳ you need "ε", only can ~~use~~ use "f_n"

ε-over-3-trick

8. Uniformly continuous on $[0,1] \Rightarrow |x-y| < 1/N \Rightarrow |f(x)-f(y)| < \varepsilon$

start here

$$\sup_{x\in[0,1]} |f_n(x)-f(x)| = \max_{1\le k\le N} \sup_{x\in[\xi_{k-1},\xi_k]} |f_n(x)-f(x)|$$

$$\le \max_{1\le k\le N} \sup_{x\in[\xi_{k-1},\xi_k]} |f_n(x)-f_n(\xi_{k-1})|_{①} \quad \|f_n-f\|_\infty \xrightarrow{n\to\infty} 0$$

$+ \max |f_n(\xi_{k-1}) - f(\xi_{k-1})|_{②} < \varepsilon$

$+ \max \sup_{x\in[\ldots]} |f(\xi_{k-1}) - f(x)|_{③} \to < \varepsilon$, for uniformly continuous

Ah-uh, f_n is not continuous, then divide it

f_n increasing, $① = \max_{1\le k\le N} |f_n(\xi_k) - f_n(\xi_{k-1})|$

$\le \max |f_n(\xi_k) - f(\xi_k)| + \max |f(\xi_k) - f(\xi_{k-1})| + \max |f(\xi_{k-1}) - f_n(\xi_{k-1})|$

"$<\varepsilon$" choose n large

"$<\varepsilon$" choose N large

"$<\varepsilon$" choose n large